Mead

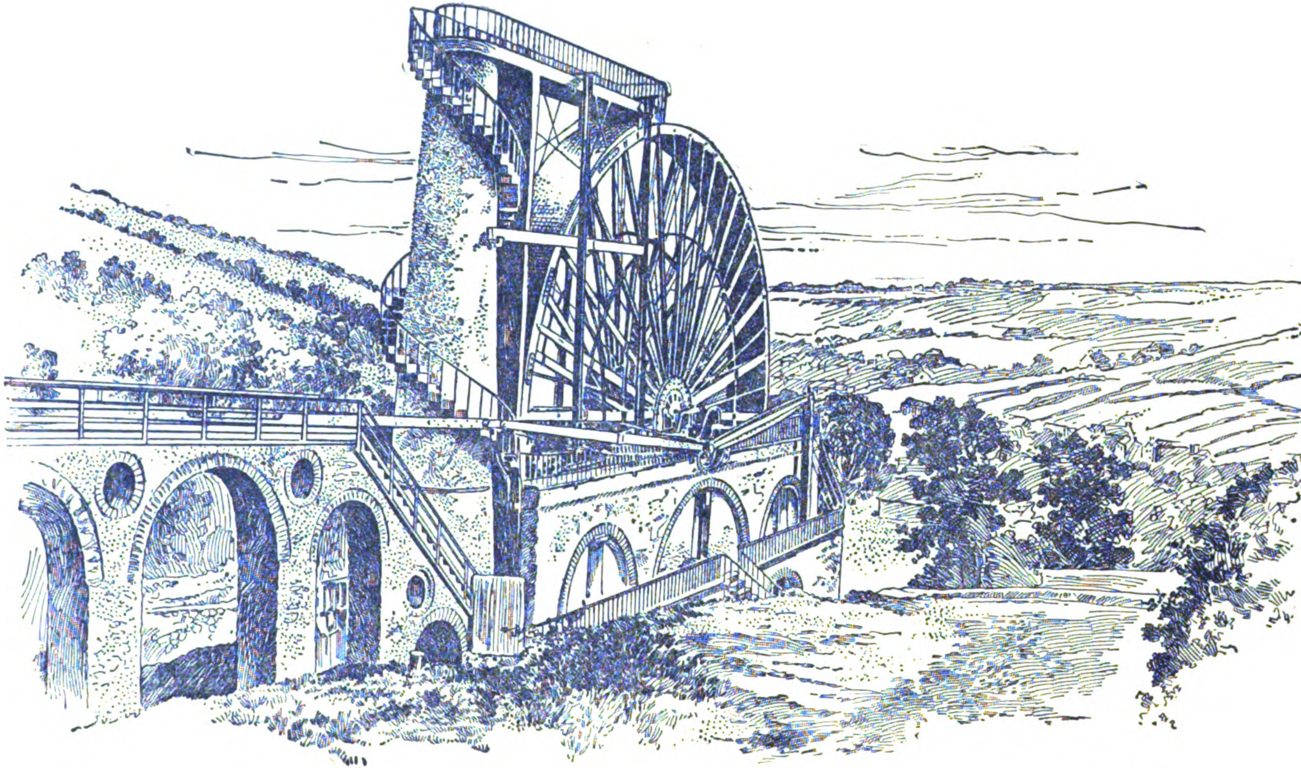

Fig. 1.—Laxy Overshot Water Wheel, Isle of Man.

WATER POWER ENGINEERING

THE THEORY, INVESTIGATION AND DEVELOPMENT
OF WATER POWERS.

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PREFACE

In the development of a water power project the engineer is freqently called upon to do more than design and construct the power plant. He may be required to report on the adequacy of the supply, the head and power available and the probable variations in the same, the plan for development, the cost of construction and operation, and the advisability of the investment. A study of the entire project, therefore, becomes essential, and each factor must be carefully considered in detail to assure ultimate success. Each of the features of the development is of equal importance to the commercial success of the project. The majority of the failures in water power development have occurred from causes other than structural defects, and a knowledge of these other important and controlling factors is therefore quite as essential as a knowledge of design and construction. It must be said, however, that in respect to some of these controlling factors current practice has not been what it should be. This has resulted in many over-developments and illy advised installations, from which the power generated has not been equal to that anticipated, and in many poor financial investments amounting frequently to practical failures. The engineer has given much attention to design and construction but too little attention to the other fundamental considerations mentioned above on which the success of the project depends to an equal extent.

In the preparation of this book the author has endeavored to consider, briefly at least, all fundamental principles and to point out the basis on which successful water power development depends. The method of study and investigation outlined herein was developed by the author during twenty-five years of professional practice and in his efforts to illustrate the principles underlying the subject in his lectures to the senior class in water power engineering at the University of Wisconsin. A somewhat extended acquaintance with the literature relating to water power engineering leads the author to believe that in a number of features the principles and methods described in this book are somewhat in advance of present practice.

In current practice, the hydraulic engineer, to determine the extent of a proposed hydraulic development, commonly depends on a study of the monthly averages of stream flow and of observed maximum and minimum flows. He usually assumes from his previous knowledge and study that the development should be based on a certain minimum or average stream discharge per square mile of drainage area. The value of this method depends on the breadth of the engineer's local knowledge of rainfall and run-off relations. With a sufficient knowledge of these conditions, this method may form a safe basis for water power development but it fails to give the complete information which is essential for a full comprehension of the subject. In other cases the development is predicted on a single, or on a very few, measurements of what is believed, or assumed to be, the low water flow of the stream. This method, even when accompanied by careful study of rainfall records, is a dangerous one to employ as many over-developed water power projects demonstrate. Neither of these methods compares favorably with the more exact method of studying flow by actual or comparative hydrographs as is described in Chaps. IV, V, VIII and IX.

In current practice the head available is usually determined for average conditions, or, perhaps, occasionally for low, average and high water conditions, and no detailed study of the daily effect on power is attempted. In Chaps. IV and V this subject is presented in detail and a method of the investigation of this important subject, under all conditions of flow and all conditions of use, is outlined.

On the basis of the knowledge gained from the study of flow and head, the study of the power that can be developed for each day in the year and during each year for which actual or comparative hydrographs are available, is outlined. An outline of a method for the consideration of possible variations in flow during periods for which no measurements are available based on the available rainfall records, is also given in Chaps. VI, VII and VIII. A study of the effect of pondage on power, a most important matter, though not always carefully considered or appreciated, is also discussed in considerable detail in Chaps. IV, V and XXVI.

In the selection of turbines for a water power project, the current practice has been for the engineer, while drawing certain conclusions from the tables of manufacturers' catalogues, to present to the manufacturer the conditions under which the power is to be developed and to rely largely or entirely on the manufacturer for advice as to machinery to be used. In such cases he is dependent for results on guarantees which are usually quite indefinite in character and seldom verified by actual tests, under working conditions, before the wheels are accepted and paid for. This has resulted in many cases in the installation of wheels which are entirely unsuited to the particular conditions under which they are installed.

Practical turbine analysis has not been treated except in the most general way in any publications except the various German treatises on the turbine in which the subject is discussed from the basis of turbine design. The author has developed the method of turbine analysis and selection, outlined in Chapters XIV and XVI, which applies to all wheels when tests of wheels of the series or class considered are available. These methods are based on the practical operating conditions of actual tests and are both theoretically and practically correct. The engineer should be able to intelligently select the turbines needed for the particular conditions of his installation and to determine, with a considerable degree of accuracy, the results on which he can depend during all conditions of head and flow.

It is believed that this treatment of the subject is sufficiently complete to place the selection of turbines on a better footing and that, when adopted, it will lead to the selection of better and more improved designs and assure more satisfactory results.

The subject of turbine governing has, for electrical reasons, become an important one. While a number of important papers have appeared on this subject, there is, so far as the author knows, no discussion in English which offers the engineer a basis for a complete consideration of this subject. Chap. XVIII, on the principles of turbine governing together with appendixes A, B and C, offer, it is believed, suggestions for the consideration of this subject which may prove of value to water power engineers.

The report on a water power project should involve a careful and complete investigation of the entire subject, and should be based on the broadest considerations of the project in all its relations. Many reports which have come to the author's attention have been too limited in scope and have included only general opinions which have not, to his mind, been sufficiently specific or based on sufficient information to warrant approval without extended investigations. In Chap. XXVIII the author has outlined his idea of the extent and scope of such investigation and report, which he believes is essential for an intelligent investigation and a reliable opinion on this subject.

ACKNOWLEDGMENTS.

There can be little which is strictly new or original in any technical work, and in offering this book to the profession, the author wishes to acknowledge his indebtedness to the large number of technical articles that have already appeared on various phases of the subject. Many references to such literature have been given at the end of the various chapters.

Many illustrations have been taken, with more or less change from Engineering News, Engineering Record, Cassier's Magazine and Electrical World and Engineer. Various manufacturers have furnished photographs and, in some cases, cuts of their wheels, governors and apparatus, in connection with which their names appear.

The author has been greatly aided by his assistants, both of his own private office and of the University staff. He wishes especially to acknowledge the assistance of Mr. L. F. Harza to whom Chap. XVIII on The Speed Regulation of Turbine Water Wheels and appendixes A, B and C are largely due. Mr. Harza has also been of much assistance in the editorial work of publication. Especial acknowledgment is also due to Professor G. J. Davis, Jr., for the preparation of the diagrams of friction of water in pipes and of Bazin's and Kutter's coefficients, etc. Mr. Robert Ewald assisted in the selection of material for illustrations, in the investigation of German literature, and the preparation of various graphical diagrams, including the first development of the characteristic curve.

The author also desires to acknowledge his indebtedness to his principal assistant, Mr. C. V. Seastone, for advice and assistance in the arrangement of many of the chapters in this work and assistance in the editorial work of publication.

The sources of various other tables, illustrations, etc., are acknowledged in their proper places. D. W. M.

Madison, Oct. 1, 1908.

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WATER POWER ENGINEERING.

CHAPTER I.

INTRODUCTION.

THE HISTORY OF WATER POWER DEVELOPMENT.

1. **Early Development of Water Power.**—Most methods of power generation can be traced to an origin at no very remote period. Their development has been within historic times. The first development of water power, however, antedates history. Its origin is lost in remote antiquity.

Air and water, both physical agents most essential to life, have ever been the most obvious sources of potential energy and have each been utilized for power purposes since the earliest times. Beside the Nile, the Euphrates, and the Yellow Rivers, thousands of years ago the primitive hydraulic engineer planned and constructed his simple forms of current wheels and utilized the energy of the river current to raise its waters and irrigate the otherwise arid wastes into fertility. Such primitive wheels were also utilized for the grinding of corn and other simple power purposes. From these simple forms and primitive applications have gradually been developed the modern water power installations of to-day.

2. **The Earliest Type of Water Wheel.**—The crude float wheel driven directly by the river current developed but a small portion of the energy of the passing stream. The Chinese Nora, built of bamboo with woven paddles, is still in use in the east (see Fig. 1), and was probably the early form of development of this type of wheel. The type is by no means obsolete for it is yet used for minor irrigation purposes in all countries. These wheels, while inefficient, served their purpose and were extensively developed and widely utilized. One of the greatest developments of which there is record was the float wheel installa-

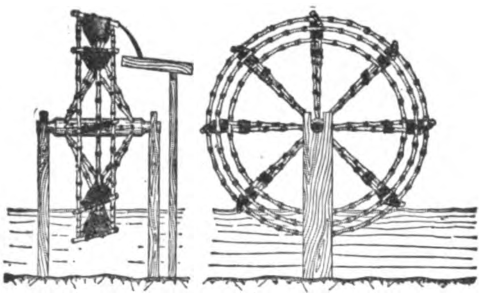


Fig. 1.—Chinese Nora or Float Wheel Used From Earliest Times to Present.

tion used to operate the pumps at London Bridge for the first water supply system of the city of London, and constructed about 1581 (see Fig. 2). In all such wheels the paddles dip into the unconfined current which, when impeded by the wheel, heads up and passes around the sides of the wheel and thus allows only a small part of the current energy to be utilized.

3. **The Undershot Wheel.**—The introduction of a channel confining the water and conducting it to a point where it could be applied directly to the undershot wheel, was an improvement that permitted the utilization of about thirty per cent. of the theo-

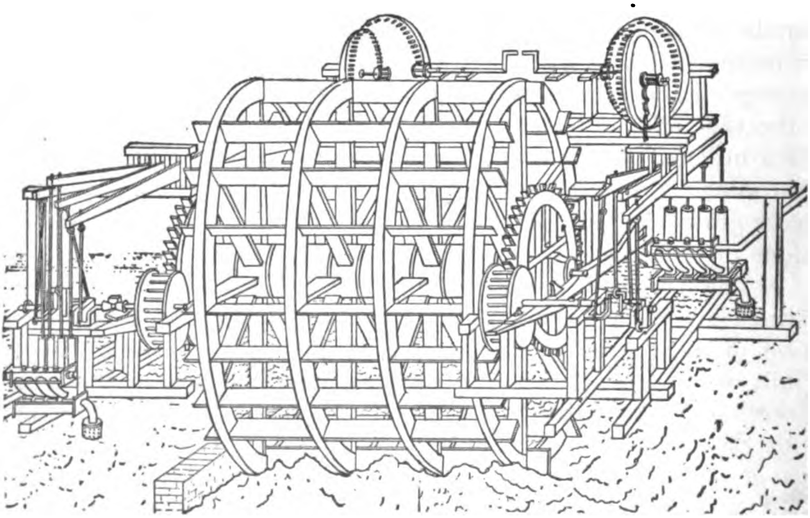


Fig. 2.—Float Wheel Operating Pumps for Water Supply of London 1581. (From Matthews' Hydraulia Lond. 1835.)

retical power of the water. This form of water wheel was most widely used for power development until the latter half of the eighteenth century.

In the float and undershot wheels the energy of water is exerted through the impact due to its velocity. The heading up of the water, caused by the interference of the wheel, results also in the exertion of pressure due to the weight of the water, but this action has only a minor effect. The conditions of the application of the energy of water through its momentum is not favorable to the high efficiency of this type of wheels and the determination of this fact by Smeaton's experiments undoubtedly was an important factor in the introduction and adoption of the overshot water wheel.

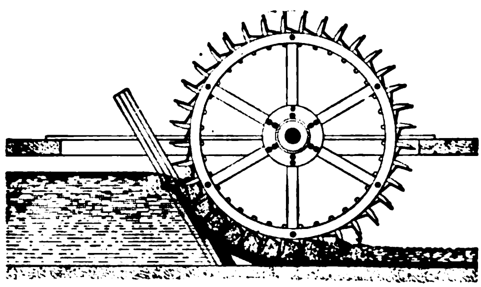


Fig. 3.—Breast Wheel Used From About 1780 to About 1870.

4. **The Overshot and Breast Water Wheel.**—In the overshot water wheel the energy of water is applied directly through its weight by the action of gravity, to which application the design of the wheel is readily adapted. Such wheels when well constructed have given efficiencies practically equal to the best modern turbine, but on account of their large size and the serious effects of back-water and ice conditions, they are unsatisfactory for modern power plants (see Fig. 11).

Following the work of Smeaton, the breast wheel (see Fig. 3) was developed in England largely through the work of Fairbairn and Rennie. The latter in 1784 erected a large wheel of this type to which he applied the sliding gate from which the water flowed upon the wheel instead of issuing through a sluice as formerly. About this time the fly-ball governor, which had been designed and adapted as a governor for steam engines by Watt, was applied to the governing of these wheels and by means of these governors the speed of the wheel under varying loads was

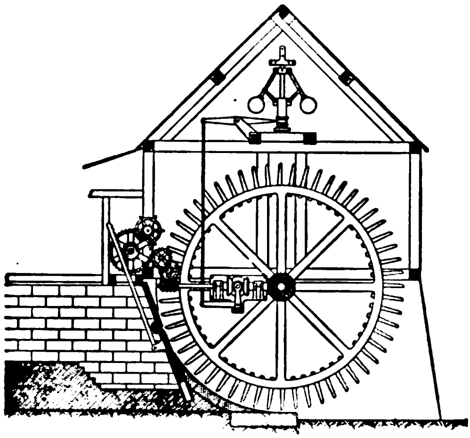


Fig. 4.—Breast Wheel About 1790 Showing Early Application of Governor. (After Glynn.)

kept sufficiently constant for the purpose to which they were then applied. (See Fig. 4.)

Another mode of applying water to wheels under low falls was introduced by M. Poncelet. (See Fig. 5.) Various changes and improvements in the form of buckets, in their ventilation so as to permit of complete filling and prompt emptying, and in their structure, took place from time to time, and until far into the middle of the nineteenth century these forms of wheels were widely used for water power purposes.

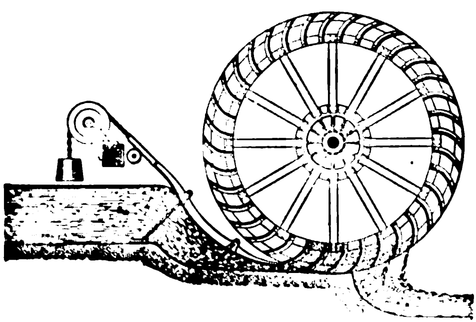


Fig. 5.—Poncelet's Wheel.

5. The Development of the Turbine.—The invention of any important machine or device is rarely the work of a single mind. In general such inventions are the result of years of experience of many men which may be simply correlated by some designer, to whom often undue credit is given. To the man who has gathered together past experiences and embodied them in a new and useful invention and perhaps through whose energy practical applications are made of such inventions, the credit is frequently assigned for ideas which have been lying dormant, perhaps through centuries of time. Every inventor or promotor of valuable improvements in old methods and old construction is entitled to due credit, but the fact should nevertheless be recalled that even in the greatest inventions very few radical changes are embodied, but old ideas are utilized and rearranged and a new and frequently much more satisfactory combination results. Improvements in old ideas are the improvements which are the most substantial. Inventions which are radically new and strictly original are apt to be faulty and of little practical value.

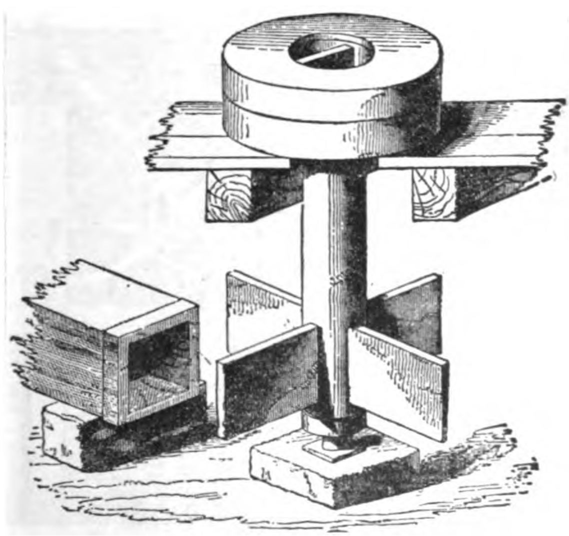


Fig. 6.—Ancient Indian Water Wheel. (After Glynn.) Containing Fundamental Suggestion of Both Turbine and Impulse Wheels.

6. **Fundamental Ideas of the Turbine.**—The embryo turbine may be distinguished in the ancient Indian water mill (see Fig. 6). A similar early type of vertical wheel used in Europe in the sixteenth century, the illustration of which was taken from an ancient print (see Sci. Am. Sup. Feb. 17, '06) is shown in Fig. 7. Barker's mill in its original form or in the form improved by M. Mathon de Cour, embodied the principal idea of the pressure

turbine, and was used to a considerable extent for mill purposes. In 1845 James Whitlaw suggested an improved form which was used in both England and Germany early in the nineteenth century. (See Fig. 8.) Many elements of the modern turbine were conceived by Benjamin Tyler, who received letters patent for what he termed the "Wry Fly" wheel in 1804. The description of this wheel as contained in the patent specifications is as follows:

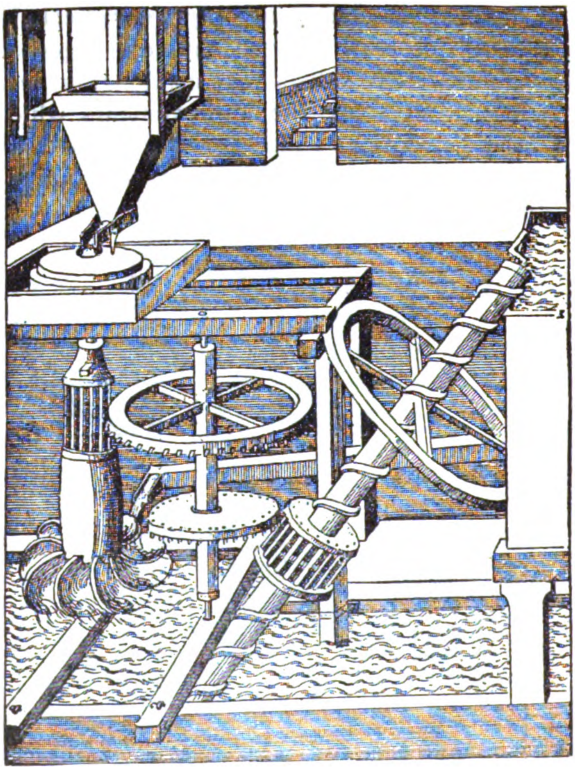
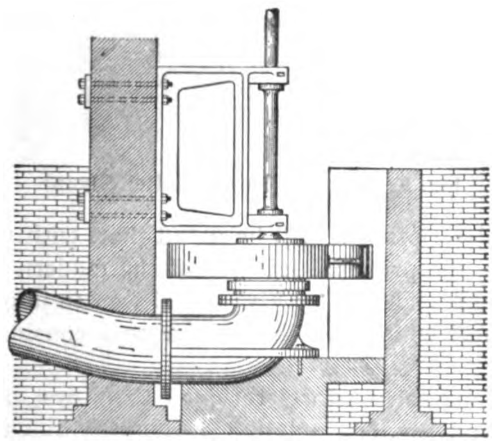
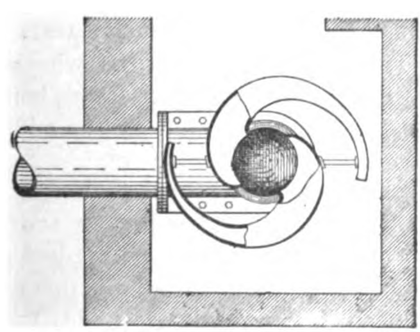


Fig. 7.—Early Vertical Wheel. Containing fundamental suggestion of the Turbine.

'The Wry Fly is a wheel which, built upon the lower end of a perpendicular shaft in a circular form, resembles that of a tub. It is made fast by the insertion of two or more short cones, which, passing through the shaft, extend to the outer side of the wheel. The outside of the wheel is made of plank, jointed and fitted to each other, doweled at top and bottom, and hooped by three bands of iron, so as to make it water-tight; the top must be about one-fifth part larger than the bottom in order to drive the hoops, but this proportion may be varied, or even reversed, according to the situation of place, proportion of the wheel, and quantity of water. The buckets are made of winding timber, and placed inside of the wheel, made fast by strong wooden pins drove in an oblique direction; they are fitted to the inside of the tub or wheel, in such a manner as to form an acute angle from the wheel, the inner edge of the bucket inclining towards the water, which is poured upon the top, or upper end of it about twelve and a half degrees; instead of their standing perpendicular with the shaft of the wheel they are placed in the form of a screw, the lower ends inclining towards the water, and against the course of the stream, after the rate of forty-five degrees; this, however, may be likewise varied, according to the circumstances of the place, quantity of water, and size of the wheel."



Elevation.



Plan and Partial Section.

Fig. 8.—Early Vertical Wheel. Containing Fundamental Suggestion of the Turbine. (After Glynn.)

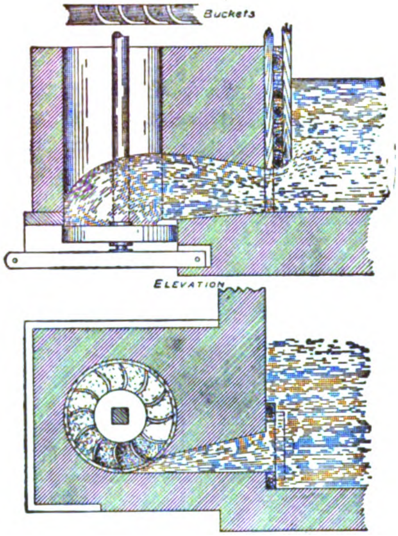


Fig. 9.—Roue A' Curves (After Glynn).

From the description it will be noted that, with the exception of the chutes, the principal features of the modern turbine were here anticipated. The "Wry Fly" wheel was an improvement on the "tub" wheel which was then in use to a considerable extent in the country.

These various early efforts received their first practical consummation and modern solution through various French inventors early in the nineteenth century. The "Roue à Cuves" (Fig. 9) and the "Roue Volant" (Fig. 10) had long been used in France, and were the subject of extensive tests by MM. Piobert and Tardy at Toulouse. Those various wheels received the water tangentially through an opening or spout, being practically an improvement on the old Indian mill by the addition of a rim and the modification of the form of buckets.

7. **The Modern Turbine.**—The next improvement in the United States consisted in the addition of a spiral or scroll case to the wheel, by means of which the water was applied equally to all parts of the circumference passing inward and downward through the wheel. To the French inventors, Koechlin, Fourneyron and Jonval, is largely due the design of the turbine in a more modern and practical form. By the middle of the nineteenth century these wheels had met with wide application in France and been

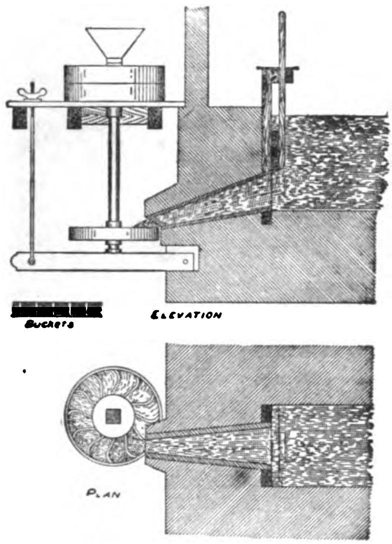


Fig. 10.—Roue Volant (After Glynn).

adopted and considerably improved by American and German engineers, but were scarcely known in England. (See "Power of Water," by Jos. Glynn, 1852.) The turbine was introduced into the United States about 1843 by Elwood Morris, of Pennsylvania, but was developed and brought to public attention more largely through the inventions of Uriah A. Boyden, who in 1844 designed a seventy-five horse-power turbine for use at Lowell, Mass. (See Fig. 132, page 251.) The great advantage of the turbine over the old style water wheel may be summarized as follows: (See Figs. 11 and 12).

First: Turbines occupy a much smaller space.

Second: On account of their comparatively high speed they can frequently be used for power purposes without gearing and with a consequent saving in power.

Third: They will work submerged.

Fourth: They may be utilized under any head or fall of water. (Turbines are in use under heads as low as sixteen inches and as high as several hundred feet.)

Fifth: Their efficiency, when the wheel is properly constructed, is comparatively high.

Sixth: They permit a greater variation in velocity without material change in efficiency.

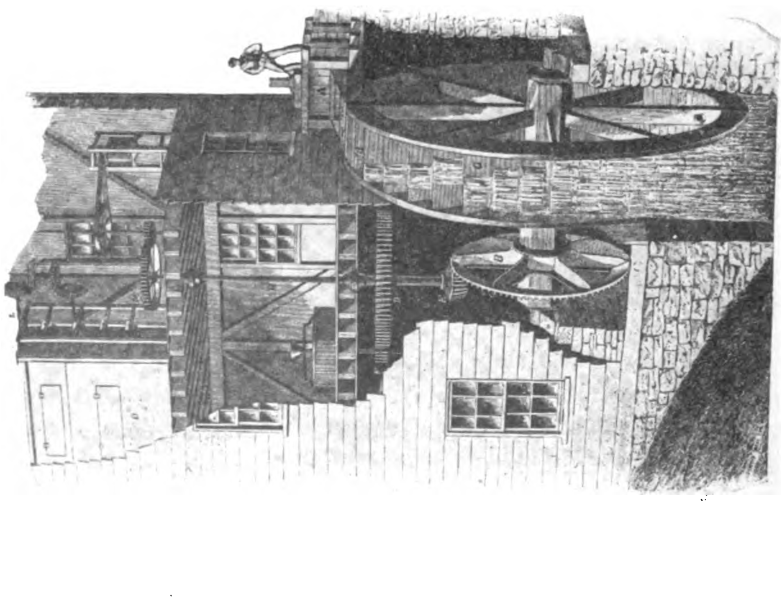


Fig. 11.—Installation of Overshot Wheel.

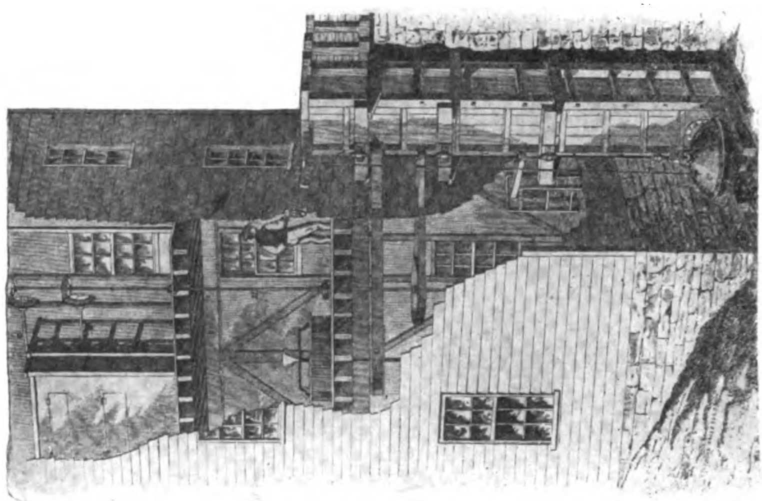


Fig. 12.—Installation of Turbine.

Comparative Installation of Water Wheel and Turbine (After Grimshaw).

Seventh: They are more readily protected from ice interference.

8. The American or Francis Turbine.—Through the efforts of Uriah A. Boyden and James B. Francis (1849), the Fourneyron turbine became the leading wheel in New England for many years.

In 1838 Samuel B. Howd of Geneva, New York, patented the "inward flow" wheel, in which the action of the Fourneyron turbine was reversed. This seems to have been the origin of the American type of turbine, and the Howd wheel was followed by a large number of variations of the same general design on which American practice has been based for many years. About 1849, James B. Francis designed an inward flow turbine of the same general type as the Howd wheel. Two of these wheels

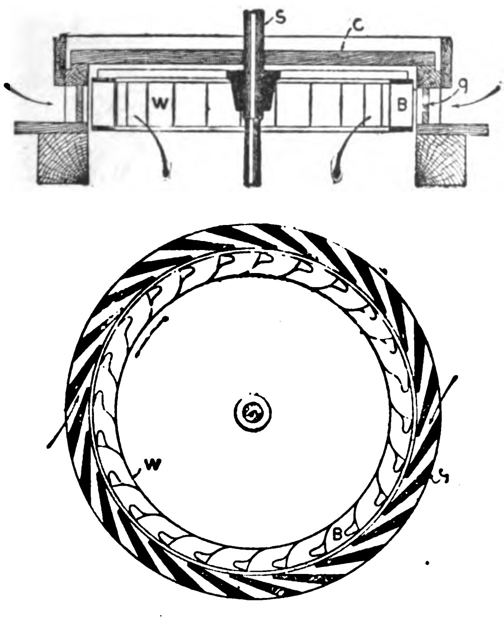


Fig. 13.—Inward Flow Wheel by S. B. Howd (After Francis).

were constructed by the Lowell Machine Shop for the Boott Cotton Mills. In the Lowell hydraulic experiments (page 61) Mr. Francis refers to the previous patent of Howd and says: "Under this patent a large number of wheels have been constructed and a great many of them are now running in different parts of the country. They are known in some places as the Howd wheel, in others as the United States wheel. They have uniformly been constructed in a very simple and cheap manner in order to meet the demands of the numerous classes of millers and manufacturers who must have cheap wheels if they have any."

Fig. 13 shows a plan and vertical section of the Howd wheels as constructed by the owners of the patent rights for a portion of the New England states. In this cut g indicates the wooden

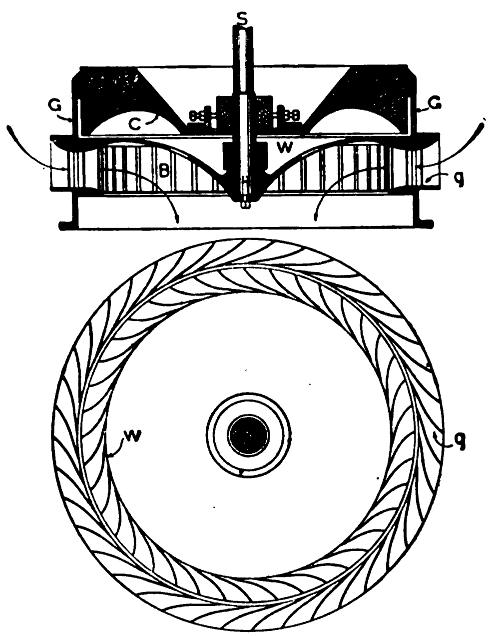


Fig. 14.—Original Francis Turbine.

guides by which the water is directed on to the buckets; W indicates the wheel which is composed of buckets of cast iron fastened to the upper and lower crowns of the wheel by bolts. The upright crown is connected with the vertical shaft S by arms. The regulating gate is placed outside of the guides and is made of wood. The upright shaft S runs on a step at the bottom (not shown in the cut). The projections on one side of the buckets, it was claimed, increased the efficiency of the wheel by diminishing the waste of the water.

The wheel designed by Francis was on more scientific lines, of better mechanical construction (see Fig. 14) and is regarded by many as the origin of the American turbine. The credit of this design is freely awarded to Francis by German engineers, this type of wheel being known in Germany as the Francis Turbine. The Francis wheel was followed by other inward flow wheels of a more or less similar type. The Swain wheel was designed by A. M. Swain in 1855. The American turbine of Stout, Mills and Temple (1859), the Leffel wheel, designed by James Leffel in 1860, and the Hercules wheel, designed by John B. McCormick in 1876, are among the best known and earliest of the wheels of this class.

9. Modern Changes in Turbine Practice.—A radical change has taken place in later years in the design of turbines by the adoption of deeper, wider and fewer buckets which has resulted in a great increase of power as shown by the following table from a paper by Samuel Webber (Transactions of Am. Soc. M. E. Vol. XVII):

TABLE I.—*Showing Size, Capacity and Power of Various Turbines Under a 26-foot Head.*

	Inches Diameter.	Cubic Feet Water per Second.	Horse Power.
Boyden-Fourneyron	36	22.95	55
Risdon	36	35.45	89
Risdon "L. C."	36	48.27	121
Risdon "L. D."	36	80.	199
Leffel, Standard	36	40.45	96
Leffel, Special	35	60.	148
Tyler	36	40.7	95.8
Swain	36	58.2	140
Hunt, "Swain bucket"	36	48.8	121
Hunt, New Style	36	98.	239.74
Leffel, "Samson"	35	109.1	264
"Hercules"	36	107.6	253.5
"Victor"	25	108.8	266
New Swain	36	89.5	215

By 1870 the turbine had largely superseded the water wheel for manufacturing purposes at the principal water power plants in this country. The old time water wheel has since become of comparatively small importance, but it is still used in many isolated places where it is constructed by local talent, and adapted to local conditions and necessities.

The current wheel is still widely used for irrigation purposes and in many instances is a useful and valuable machine.

10. Historical Notes on Water Power Development.—Water mills were introduced at Rome about seventy years B. C. (see Strabo Lib. XII), and were first erected on the Tiber. Vitruvius describes their construction as similar in principle to the Egyptian Tympanum. To their circumference were fixed floats or paddles which when acted upon by the current of the stream drove the wheel around. Attached to this axis was another vertical wheel provided with cogs or teeth. A large horizontal wheel toothed to correspond with it worked on an axis, the upper head of which was attached to the mill stone. The use of such water wheels became very common in Italy and in other countries subject to Roman rule.

Some of the early applications of water power are of interest. In 1581 a pump operated by a float wheel was established at London Bridge to supply the city of London with water. In 1675 an elaborate pumping plant driven by water wheels was established on the Seine river near Saint Germain. For this plant a dam was constructed across the river and chutes were arranged to conduct the water to the undershot water wheels. These were twelve or more in number, each operating a pump that raised the waters of the Seine into certain reservoirs and aqueducts for distribution.

The pumping of water for agricultural irrigation and drainage, domestic supplies and mine drainage, was undoubtedly the first application of water power, and still constitutes an important application of water. Fig. 15, from an article by W. F. Dupfee, published in Cassier's Magazine of March, 1899, illustrates a primitive application of the water wheel to the pumping of water from mines. The frontispiece also shows the great Laxy overshot water wheel in the Isle of Man which is still used for mine drainage. The wheel is about seventy feet in diameter and the water is brought from the hills a considerable distance for power purposes.

11. Development of Water Power in the United States.—In this country one of the first applications of water power was the old tidal mill on Mill Creek near Boston, constructed in 1631, which was followed by the extensive developments of small powers wherever settlements were made and water power was available. Often availability of water power determined the location of the early settlement.

About 1725 the first power plant was established along the Niagara River. This was a water-driven saw-mill constructed

Chronological Development of Water Power of the United States to 1898.

	Year.	Fall Ft.	Minimum Horse Power.	Drainage Area Sq. Miles.
Lowell, Mass	1822	35	11,845	4,083
Nashua, N. H	1823	36	1,200	516
Cohoes, N. Y	1826	104	9,450	3,490
Norwich, Conn	1828	16	700	1,240
Augusta, Me	1834	17	3,500	5,907
Manchester, N. H	1835	52	12,000	2,839
Hooksett, N. H	1841	14	1,800	2,791
Lawrence, Mass	1845	30	11,000	4,625
Augusta, Ga	1847	50	8,500	8,830
Holyoke, Mass	1848	50	14,000	8,000
Lewiston, Me	1849	50	11,900	3,200
Columbus, Ga	1850	25	10,000	14,900
Rochester, N. Y	1856	236	8,000	2,474
St. Anthony Falls, Minn	1857	50	15,500	19,736
Niagara, N. Y. (Hy. canal)	1861	90	15,000	271,000
Turner's Falls, Conn	1866	35	10,000	6,000
Fox River, Wis	1866	185	....	6,449
Birmingham, Conn	1870	22	1,000	2,000
Bangor, Me	1876	9	1,767	7,200
Augusta, Ga	1876	50	8,500	6,830
Palmer's Falls, N. Y	1882	30	1,125	2,650
Mechanicsville, N. Y	1882	20	3,636	4,476
St. Cloud, Minn	1885	14	4,500	13,250
Little Falls, Minn	1887	14	4,000	11,084
Spokane, Wash	1888	70	18,000	4,180
Howland, Me	1888	22	6,000	....
Great Falls, Mont	1890	42	16,000	22,000
Austin, Texas	1891	60	10,000	40,000
Sault Ste. Marie, Ont	1891	18	10,000	51,600
Folsom, Cal	1891	55	6,200	....
Concord, N. H	1894	13	5,000	2,350
Niagara, N. Y. (tunnel)	1894	170	50,000	271,000
Ogden, Utah	1896	446	2,940	360
Helena, Mont	1897	32	10,000	14,900
Minneapolis, Minn	1897	18	6,000	19,737
Mechanicsville, N. Y.	1898	18	3,270	4,478

by the French to furnish lumber for Fort Niagara. Mr. J. T. Fanning gives the following list of the dates of establishing some of the principal water powers of the United States:

The last few years have witnessed a still more rapid development. The increase in manufacturing industries and other demands for power and energy, the increased cost of coal, and the improvement in electrical methods of generation and transmission have all united to accelerate the development of water power plants. Water powers once valueless on account of their distance from centers of manufacturing and population are now accessible and such powers are rapidly being developed and their energy brought into the market.

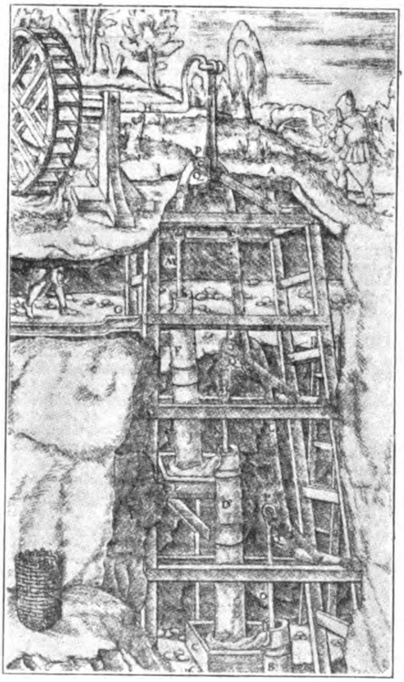


Fig. 15.—Early Application of Undershot Water Wheel to Mine Drainage, Date Unknown (from *Cassiers Mag.* March, 1899).

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CHAPTER II.

POWER.

12. **The Development of Potential Energy.**—The development of natural sources of potential energy, the transformation of such energy into forms which can be utilized for power, and its transmission to points where it can be utilized for commercial purposes, constitutes a large portion of the work of the engineer. The water power engineer primarily deals with energy in the form of flowing or falling water, but his knowledge must extend much further for he encounters other forms of energy at every turn. Much of the energy available from the potential source will be lost by friction in bringing the water to and taking it from the wheel. Much is lost in hydraulic and mechanical friction in the wheel; additional losses are sustained in every transformation, and, if electric or other forms of transmission are used or auxiliary power is necessary for maintaining continuous operation, the engineer will be brought in contact with energy in many other forms.

13. **Definition of Energy.**—Energy is the active principle of nature. It is the basis of all life, all action, and all physical phenomena. It is the ability to exert force, to overcome resistance, to do work. All physical and chemical phenomena are but manifestations of energy transformations, and all nature would be rendered inactive and inanimate without these changes.

14. **Solar Energy the Ultimate Source.**—A brief consideration of the various sources of potential energy makes the fact manifest that solar energy is the ultimate source from which all other forms are directly or indirectly derived. The variations in solar heat on the earth's surface produces atmospheric currents often of tremendous power. This form of energy may be utilized, in its more moderate form, to drive the sailing vessel and the windmill, and in other ways to be of service to man. The energy of fuel is directly traceable to solar action. Through present and past ages it has been the active cause of chemical and organic

change and growth. From this has resulted fuel supplies available in the original form of wood, or in the altered forms, from ancient vegetation to the forms of coal, oil and gas, and from which a large portion of the energy utilized commercially is derived.

A brief study of meteorological conditions shows that through the agency of solar heat, and the resulting atmospheric movement, a constant circulation of water is produced on and near the earth's surface. Hundreds of tons of water are daily evaporated from the seas, lakes, rivers and moist land surface, rise as vapor into the atmosphere, circulate with the winds, and, under favorable conditions, are dropped again upon the earth's surface in the rainfall. Those portions of the rain that fall upon the land tend to flow toward the lower places in the earth's crust, where lie the seas and oceans, and such portions of these waters as are not absorbed by the strata, evaporated from the surface or utilized in plant growth, ultimately find their way to these bodies of water to again pass through this cycle of changes which is constantly in progress. Thus we find water always in motion. and always an active agent in nature's processes. Due to its peculiar physical properties and chemical relations, it is one of the essential requisites of life, and is also of great importance in nature's processes through the energy of which it is the vehicle.

15. No Waste of Energy in Nature.—Active continuous energy transformation is a most important natural phenomenon. Changes from one form to another are constantly in progress. In nature's transformations energy is always fully utilized. As the running stream plunges over the fall, the potential energy, due to its superior elevation, is transformed into the kinetic energy of matter in motion, and through the shock or impact the kinetic energy is transformed into thermal energy due to a higher temperature, which again may be partially changed in form by radiation or vaporization. Thus the quantity of energy is continually maintained, while its quality or conditions constantly vary. There is, and can be, no waste or loss of energy as far as nature itself is concerned. Wasted or lost energy are terms that apply only to energy as utilized in the service of man. Nature itself never seems to utilize the entire quantity of energy from one source for the development of energy of a single form, but always differentiates from one form into a number of other forms. When the engineer therefore attempts to utilize any source of potential energy for a single purpose, he at once encounters this natural law of differentiation and finds it impossible to utilize more than a portion of the energy used in the manner in which he desires to utilize it. Much of this loss may be due to the form of energy available, much to the medium of transformation and transmission, and much to physical difficulties which it is impossible to overcome.

16. **Laws of Energy Conservation.**—Primarily it should be fully understood and clearly appreciated that matter and energy can neither be created nor destroyed. Both may be changed in form or they may be dissipated or lost so far as their utilization for commercial needs is concerned. But in one form or another they exist, and their total amount in universal existence is always the same. In any development for the utilization, transformation or transmission of energy, the following fundamental axioms must be thoroughly understood and appreciated:

First: That the amount of energy which can be actually utilized in any machine or system can never be greater than the amount available from the potential source.

Second: That the amount of energy which can be utilized in any such system can never be greater than the difference between the amount entering the system and the amount passing from the system as waste in the working medium.

17. **Efficiency.**—Efficiency is the ratio or percentage of energy utilized to energy applied in any system, part of a system, machine or in any combination of machines.

The efficiency of a given machine or mechanism, or the percentage of available energy which can be obtained from a given system of generation and transmission therefore can never be greater than represented by the equation:

$$\text{Efficiency or amount of available energy} = \frac{E - E'}{E} \text{ in which}$$

E equals the energy in the working medium entering the machine
E′ equals the energy in the working medium passing from the machine.

18. **Natural limit to efficiency.**—The total energy in a working medium such as water, steam, air, etc., is the energy measured from the basis of the absolute zero for the medium which is being considered. For example, the average surface of Lake Michigan is 580 feet above sea level; each pound of water, therefore, at lake level contains 580 foot pounds of potential energy. This amount of energy must therefore be expended in some man-

ner by each pound of water passing from the lake level to the ocean level, which may be regarded as the absolute zero reference plane for water power. This energy cannot be utilized at Chicago for there no fall is available. A small portion of this energy is now utilized in the power plants at the falls of Niagara. Some energy will be ultimately utilized on the Chicago Drainage Canal, where a fall of some thirty-four feet is available from the controlling works to Joliet. Perhaps ultimately in its entire course one hundred and seventy feet of fall may be utilized by the waters of the drainage canal, in which case the absolute available energy of each pound of water cannot be greater than shown by the following equation:

$$\text{Available energy} = \frac{580 - 410}{580} = \frac{170}{580} = .2931, \text{ or } 29.31 \text{ per cent.}$$

With any other form of energy the same conditions also prevail. Consider a pound of air at 760 degrees absolute temperature Fahr., and at 75 pounds absolute pressure. The number of heat units contained will be given by the equation:

$$\text{Heat units} = \text{temperature} \times \text{weight} \times \text{specific heat.}$$
$$\text{B. T. U.} = 760 \text{ degrees} \times 1 \times .169 = 128.$$

To utilize all of the energy in this air, it would be necessary to expand it down to a temperature of absolute zero and exhaust it against zero pressure. In any machine for utilizing compressed air, it will be necessary to exhaust it against atmospheric pressure. This will expand the air 3.10 times, and if expanded adiabatically it will have a final temperature of 474 degrees. The heat units in the exhaust will therefore be as follows:

$$\text{B. T. U.} = 474 \text{ degrees} \times 1 \times .169 = 80,$$

and the available energy will be as follows:

$$\text{Available energy} = \frac{128 - 80}{128} = \frac{48}{128} = .375, \text{ or } 37.5 \text{ per cent.}$$

In this case also the temperatures vary directly as the heat units, and are therefore a measure of available energy:

$$\text{Available energy} = \frac{760 - 474}{760} = .375 \text{ or } 37.5 \text{ per cent.}$$

In the ideally perfect furnace the efficiency is somewhat higher. The fuel may be consumed at a temperature of about 4,000 Fahr. absolute, and the gas may be cooled before escaping to about 600 Fahr. In this case the possible efficiency or available energy is:

$$\text{Available energy} = \frac{4000 - 660}{4000} = .832 \text{ or } 83.2 \text{ per cent.}$$

The above examples show, therefore, the limits which nature itself places on the proportion of energy which it is theoretically possible to utilize. For such losses the engineer is not accountable except for the selection of the best methods for utilizing such energy. The problem for his solution is, what amount of this available energy can be utilized by efficient machines and scientific methods.

19. **Practical Limits to Efficiency.**—The preceding equations are the equations of ideally perfect machines. Of this available energy only a portion can be made actually available. In practice we are met with losses at every turn. Some energy will be lost in friction, as radiated heat, some in the slip by pistons, or as leakage from defective joints. In many other ways the energy applied may be dissipated and lost. From this it follows:

The amount of energy which can be utilized can never be greater than the difference between the amount supplied to any given machine or mechanism, and the amount lost or consumed in such machines by friction, radiation or in other ways. Hence it follows that the efficiency of a given machine, or the percentage of energy available, or which can be obtained from the machine, can never be greater than the following:

$$\text{Efficiency} = \frac{E - (E' + E'' + E''' + E'' \text{etc.})}{E} \text{ in which}$$

E = total energy available

E′ E″ E‴ etc. = the energy lost in friction and in various other ways, in the machine or system, and rejected in the exhaust from the same.

Every transmission or transformation of energy entails a loss, hence, starting with a given quantity of energy, it gradually disappears by the various losses involved in the mechanism or machines used. Other things being equal, the simpler the transmission or transformation, the greater the quantity of the original amount of energy that can be utilized.

The term efficiency as here applied represents always the ratio between the energy obtainable from the mechanism or machine and the actual energy applied to it.

Therefore the efficiency of a pumping engine is the ratio between the energy of the water leaving the pump and the energy of the steam applied to the engine.

The efficiency of a hydro-electric plant is the ratio between the energy in the electric current delivered at the switch board and the energy in the water entering the water wheel.

The efficiency of the dynamo in the same plant is the ratio between the energy furnished by the dynamo and the energy applied to it.

If a shaft receives from an engine 100 horse power and delivers 90, ten horse power being lost in friction, etc., the efficiency of the shaft transmission is 90 per cent.

If a steam engine receives 1,000,000 heat units from the steam it uses, and is able to deliver only the equivalent of 10,000 heat units; i. e., 7,780,000 foot pounds of work, the efficiency of the engine is only one per cent.

20. Efficiency of a Combined Plant.—In any plant or connected arrangement of mechanisms and machines for the transformation or transmission of energy the efficiency of the plant is the product of the efficiency of each of its parts.

Hence, to estimate total efficiencies, the efficiency of each part may be estimated, and the combined efficiency then obtained. From the same calculation, the necessary relations between the input and the output of energy can be obtained. Thus, if a boiler has an efficiency of 50 per cent., and an engine has an efficiency of 10 per cent., the combined efficiency will be $.50 \times .10 = .05$ or five per cent.

In the following examples the loss and efficiency of the unit and the combined efficiency of the various units in the system are shown.

FIRST EXAMPLE.

Example of Energy Loss in Well-Designed Steam Power Plant.

	Per Cent Lost.	Per Cent Efficiency	Net Efficiency from Potential Source.
Furnace	20	80	80
Boiler	15	85	68
Steam Pipe	5	95	64.5
Engine	94	6	3.87
Belt	5	95	3.67
Shafting, Belts and Counter Shafts	40	60	2.2
Lathes or other Machine Tools	50	50	1.1
Percentage of original energy utilized in useful work	....	....	1 1

SECOND EXAMPLE.

Example of Energy Loss in Hydraulic Plant for Electric Lighting.

	Per Cent Lost.	Per Cent Efficiency	Net Efficiency from Potential Source.
Head and Tail Races	5	95	95
Turbine	20	80	76
Gearing	15	85	64.6
Shaft	5	95	60.37
Belt	5	95	57.35
Generator	8	92	52.76
Line Loss	10	90	47.48
Transformer	·20	80	37.98
Lamp	80	20	7.60
Percentage of original energy utilized in useful work	....	....	7.60

THIRD EXAMPLE.

Example of Energy Lost in Steam and Electric Pumping Plant.

	Per Cent Lost.	Per Cent Efficiency	Net Efficiency from Potential Source.
Boiler and Furnace	30	70	70
Steam Pipe	5	95	66.6
Engine	90	10	6.65
Belt	5	95	6.32
Generator	20	80	5.05
Line	10	90	4.55
Motor	10	90	4.09
Pump	25	75	3.06
Suction and Discharge Pipe	20	80	2.45
Percentage of original energy utilized in useful work	....	....	2.45

21. **Capacity of Each Part of a System Not Identical.**—In each of the transmission systems outlined above a much larger amount of energy enters the first unit of the system than is delivered by the last. Each unit in the system receives a decreasing amount of energy.

In consequence, the first units in the system must be of greater proportional capacity, and in practice each unit must be selected of a size or capacity suited for its position in the system. Thus in the first example, for each 100 units of energy received by the furnace, the engine receives but 64.5, and the shafting but 4.

22. The Analysis of Losses.—In estimating power losses the loss in each step from the generation to the utilization of the power should be carefully examined. Four steps may ordinarily be considered in any system:

1. Generation of power from potential source.
2. Conversion of power into form for transmission.
3. Transmission of power.
4. Utilization of power.

An analysis of the first three items is shown in Table II. In Table III is shown the ordinary maximum and minimum efficiencies obtained from various motors and machines in practical work. Higher efficiencies are sometimes obtained under test conditions where great attention is given to secure favorable conditions, and, in many places where careless work is permitted, neglect and unsatisfactory conditions will result in much lower efficiencies than the minimum shown.

23. The Losses in a Hydro-electric Plant.—To emphasize and point out in greater detail the various losses encountered in the generation and transmission of energy, especially as applied to hydro-electric plants, attention is called to Fig. 16. In this diagram is traced the losses from the potential energy of the water in the head race of the power plant to the power available at the point where it is used. In each case considered it is assumed that 1,000 horse-power of energy is applied to the particular work considered.

First, consider the transmission of power for traction purposes. If a certain head is available when no water is flowing in the raceways, that head becomes reduced at once when the wheels begin to operate. A certain amount of head is also lost in order to overcome the friction of flow through raceways, racks and gateways. In the problem here considered it is assumed that the above losses are five per cent. of the total energy available in the head-race, and that this loss occurs before the water reaches the turbines: hence, 95 per cent. of the potential energy is available at the turbine. The turbine loss is here assumed to be about 20 per cent. First-class turbines under three-quarter to full load conditions, will commonly give 80 per cent. efficiency, or a little better.

Professor Unwin, in his "Development and Transmission of Power," page 104, gives the following percentage of loss in turbines:

Shafting, friction and leakage	3 to 5 per cent.
Unutilized energy	3 to 7 per cent.
Friction in shaft, guides and passages	10 to 15 per cent.
Total loss of energy	16 to 27 per cent.

TABLE II.

POWER LOSSES IN GENERATION AND TRANSMISSION OF ENERGY.

		Method of Generation.	Losses.
Generation from Potential Source.	Fuel	Internal Combustion Engine Gas—Oil	Engine losses.
		Steam: Direct (Vacuum Pump); Indirect	Furnace. Boiler. Piping.
	Water Power	Direct (Ram)	Ram losses.
		Indirect (Wheels)	Velocity losses. Wheel losses.
	Minor Sources.	Electric (Primary Batteries); Wind (Mills); Waves (Motors); Sun Heat (Solar Engines)	Various mechanical and other losses due to method used.
Conversion into Form for Transmission.	Internal Combustion Engine		Included in engine losses.
	Steam		Engine and connection losses.
	Electrical		Dynamos and wire losses.
	Hydraulic		Pump losses.
	Pneumatic		Compressor losses.
Method of Transmission.	Mechanical	Direct connected,—Shaft; Cables, Ropes, Chains; Electric; Combination	Various losses due to method used.
	Hydraulic		Entrance head. Pipe friction. Motor losses. Connections.
	Electrical		Tranformer losses. Wire losses. Motor losses. Connections.
	Pneumatic		Pipe friction. Air cooling. Motor losses. Connections.

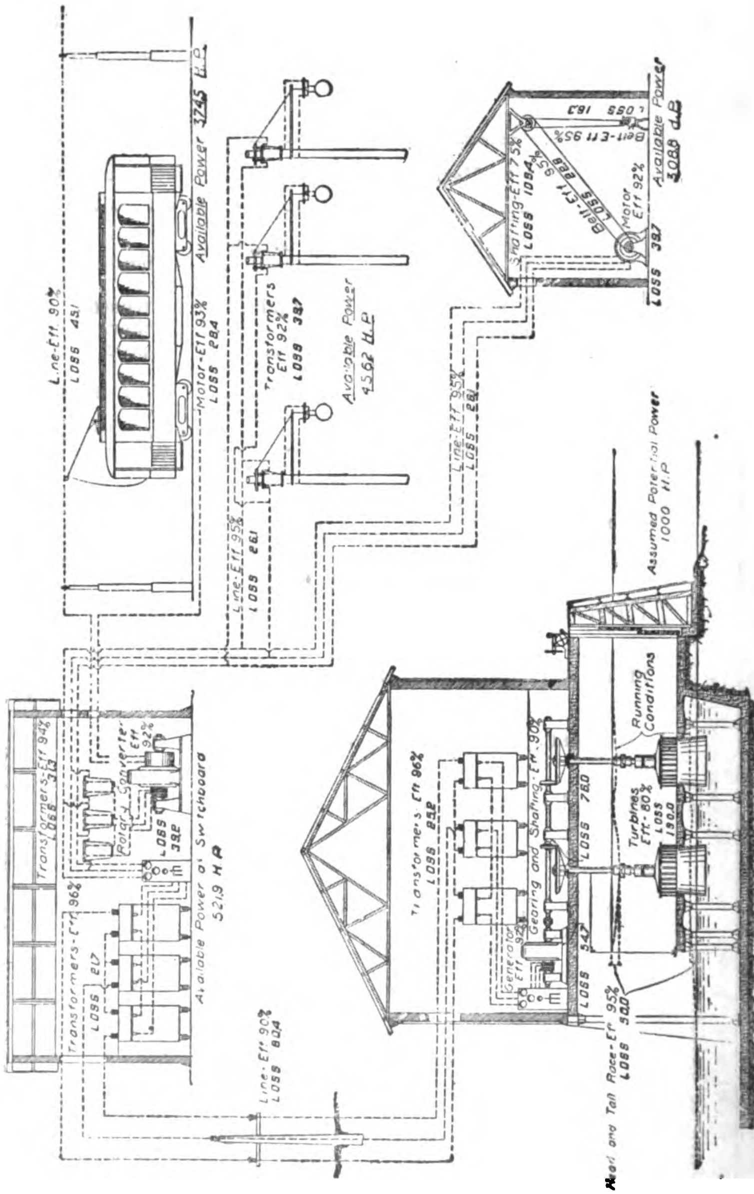

Line-Eff. 90%
LOSS 45.1
Motor-Eff 93%
LOSS 28.4
Available Power 374.5 H.P.
Transformers
Eff 92%
LOSS 39.7
Available Power
456.2 H.P
Line-Eff 95%
LOSS 26.1
Line-Eff 95%
LOSS 28.1
Shafting-Eff 75%
LOSS 108.4
Belt-Eff 95%
LOSS 16.3
Motor
Eff 92%
LOSS 39.7
Available Power
308.8 H.P
Assumed Potential Power
1000 H.P
Running Conditions
Turbines
Eff. 80%
LOSS
190.0
Head and Tail Race-Eff 95%
LOSS 50.0
Generator
Eff 92%
LOSS 54.7
Gearing and Shafting-Eff. 90%
LOSS 76.0
Transformers-Eff 96%
LOSS 25.2
Transformers-Eff 94%
LOSS 31.3
Rotary Converter
Eff 92%
LOSS 39.2
Available Power at Switchboard
521.9 H.P
Transformers-Eff 96%
LOSS 21.7
Line-Eff 90%
LOSS 60.4

Fig. 16.

The next loss shown on the diagram is the loss in transmitting the energy through the bevel gear and the shafting to the generator. The loss in gearing, shafting, etc., is shown as 10 per cent., which is probably much less than actually takes place in most plants of this kind, but may be considered as representing the results of good practice.

The loss in the transformation of power in the generator is given as 8 per cent. The generator is an alternator, and the current generated would be at about 2,300 volts. This current must be raised to a higher voltage, by means of transformers, for long distance transmission. These transformers would give an efficiency of about 96 per cent. The line loss is dependent on the size of the copper used, but would probably not exceed 10 per cent. At the distributing point, where the energy is to be used, the high voltage current must be transformed again into suitable voltage for distribution. The same energy loss is estimated for these transformers. If the current is to be used for traction purposes, it will be necessary to convert it into direct current by means of a rotary converter, the efficiency of which is estimated at 92 per cent. The voltage from the general distribution system would probably be too high for direct use in the rotary converter, and would have to be transformed to a lower voltage before passing into the converter. A loss of about 6 per cent., therefore, should be allowed for this transformation.

The current from the rotary converter is subject to a line loss which may be again assumed at 10 per cent. The loss in the car motor may be estimated at 7 per cent. The percentage of loss and the percentage of efficiency for each unit in this generation and transmission system is based, of course, on the actual energy supplied by the unit next previous to it in the system, so that the percentages mentioned are not based on the total potential power available in the head-race but on the power actually reaching the machine.

In the solution of any actual problems of this character it is necessary to determine the efficiencies of the various units of the plant under the condition of actual service. The efficiency will be found to vary under various conditions of load. It may therefore be desirable to determine the probable losses under various working conditions.

In the selection of the various machines which are to form a part of such a system of transmission, the choice should be based on an effort to establish a plant which will give the maximum economy when all conditions of loading are considered. The losses in the transmission of power for traction purposes, as shown on the diagram, may be traced through in tabular form as follows:

	Total Energy Available.		1,000 Horse Power.
	Per Cent Loss.	Per Cent Efficiency.	Loss in horse power
Head race	5	95	50
Turbine	20	80	190
Shaft and gearing	10	90	76
Generator	8	92	54.7
Transformers	4	96	25.2
Transmission line	10	90	60.4
Step-down Transformers	4	96	21.7
Secondary Transformers	6	94	31.3
Rotary Converters	8	92	39.3
Line	10	90	45.1
Traction Motor	7	93	28.4

Power utilized for operating the cars, or 37½ per cent of the original energy .... 374.5 Horse Power.

In the generation and transmission of power for lighting purposes, the losses will be similar to those above mentioned, up to and including the step-down transformers at the point of distribution. In this case, however, no secondary transformers or rotary converters would be necessary. The only loss between the step-down transformers and the light will be the line loss assumed at 5 per cent. The loss in the individual transformer for the light will be about 8 per cent., leaving the available energy for actual use in the lamp at about 456.2 horse power, or a little less than 46 per cent. of the total energy in the head-race.

In the case of the utilization of this energy for manufacturing purposes, the loss would be the same up to and including the step-down transformers at the point of distribution. The line loss in the distribution from the transformer house to the manufacturing establishment may be assumed at 5 per cent. The motor, if properly selected, may be run at the line voltage, and no transformer losses need be considered. The motor efficiency is here shown at 92 per cent., although in most cases the percentage of efficiency would be considerably less.

The belt loss in transmitting the power from the motor to the line shafting is estimated at 5 per cent.

TABLE III.—*Ordinary Efficiency of Generators and Motors.*

	Class of Machinery.	Efficiency Per Cent at Full Load. Maximum.	Minimum.
Water Wheels	Overshot Wheels	75	65
	Breast Wheels	65	60
	Undershot Wheels	40	25
	Turbines	85	60
	Impulse Wheels	85	75
Steam Generators	Boilers	75	50
	Steam Pipe	95	75
Condensing Steam Engines	Triple Expansion Corliss	18	15
	Compound Corliss	15	12
	Simple Corliss	12	10
	Compound High Speed	12	10
Non-Condensing Steam Engines	Compound Corliss	12	10
	Simple Corliss	9	7
	Compound High Speed	9	7
	Simple High Speed	7	6
	Simple Slide Valve	7	5
Heat Engines	Gas or Oil Engines	20	16
	Diesel Motor	30	25
Steam Air Compression	Compound Con. Corliss	12	10
	Simple Con. Corliss	9	7
	Simple Corliss	6	5
	High Pressure	4.5	3
	Small Straight Line	3	2
Air Motor	Air, cold	50	30
	Air, reheated	70	60
Electrical Machinery	Dynamos	92	80
	Motor, large	90	80
	Motor, small	85	75
	Transformer	95	50
Transmitting Mechanisms	Belt	95	85
	Rope	97	90
	Cable	95	75
	Direct connection	99	95
	Shafting	95	70
	Gearing	85	50
	Bevel Gearing	75	50
Transmission Methods	Pneumatic, per mile	97	92
	Hydraulic, per mile	98	90
	Electric, usual	95	85

The shafting necessary for the general distribution of power through the factory is estimated at 75 per cent. efficiency.

The belt loss from the shaft to the individual machine is estimated at an additional 5 per cent., leaving the total energy available for use in the machine at 308.8 horse power, or about 31 per cent. of the original energy in the head-race.

It should be noted that in each of the three transmission systems mentioned above, the actual power utilized at the point of application is less than half of the energy available in the head-race. It is the function of the engineer to see that these losses are reduced to the greatest practicable extent. These losses must be limited in both directions. They must not be too great, nor too small. They must be adjusted at the point where true economy would dictate. This limit is the point where the capitalized value of the annual power lost is equal to the capitalized cost of effecting further saving. In other words, true economy means the construction of a plant that will save all the power or energy which it is financially desirable to save, and will permit such waste of energy as true economy directs.

24. Units of Energy.—Energy is known by many names and exists in many forms which seem more or less independent. The principal forms of energy are measured by various units. Those most commonly considered in power development and transmission are as follows:

Work is energy applied to particular purposes. In general it is energy overcoming resistance, mechanically it is the exertion of force through space.

Power is the rate of work, or the relative amount of work done in a given space of time.

The unit of work is the foot pound, or the amount of work required to raise one pound one foot. One pound raised one foot, one-tenth pound raised ten feet, ten pounds raised one-tenth of a foot, or any other sub-division of pounds and feet whose product will equal one requires one foot-pound of work to perform it.

The unit of power is based on the unit of work, and is called "horse power." It is work performed at the rate of 550 foot pounds per second, or 33,000 foot pounds per minute.

Units of Heat. The unit of heat is the amount of heat which will raise one pound of water from 39 degrees Fahr. to 40 degrees Fahr. at atmospheric pressure. It is called the British Thermal Unit, and is indicated by the initials B. T. U.

Electric Unit. The unit of quantity of electricity is the coulomb. One coulomb per second is called an ampere, and one ampere under a volt pressure is equal to a watt, the unit of electric power.

Water Power. Water power is the power obtained from a weight of water moving through a certain space. In water power the unit of quantity may be the gallon or the cubic foot; the unit of head may be the foot; and the unit of time may be the second or minute. The weight of water, unless highly mineralized, at ordinary temperature, varies from 62.3 to 62.5 pounds per cubic foot. As these weights vary from each other less than one-third of one per cent., the difference is insignificant in practical problems where the errors and uncertainties are often large. In the further discussion of this subject, therefore, the weight of 62.5 pounds is used as the most convenient in calculation.

Steam Power. The unit of steam power in ordinary use is the pound of steam, its pressure, and rate of use. It is, however, based on the heat unit, and must be so considered for detailed examination.

Definite quantities of work are also designated by the "horse power hour," equivalent to 1,980,000 foot pounds, and the "kilowatt hour," equivalent to 2,654,150 foot pounds.

The pound of steam may be considered as containing an average of 1,000 British thermal units, which may be utilized for power. This is equivalent to 778,000 foot pounds.

25. Conversion of Energy Units.—The various forms of energy as expressed by the units named are convertible one into another in certain definite ratios which have been determined by the most careful laboratory methods. In considering these ratios, however, it must be remembered that, as shown in the preceding examples, in the transformation from one form of energy into another the ratios given cannot be attained in practice on account of losses which can not be practically obviated. Such losses must be, in good practice, reduced to a minimum, and the ratios given are, therefore, the end or aim toward which good practice strives to attain as nearly as practicable when all conditions and facts are duly considered.

Energy must be considered in two conditions as well as in the above named forms, viz.: passive and active or potential and kinetic.

Potential energy is energy stored and does not necessarily involve the idea of work. Kinetic energy is energy in action and involves the idea of work done or power exerted and for its measurement must be considered in relation to time.

The most common units of potential energy and their equivalents are as follows:

The footpound (one pound raised one foot).
=1/62.5 or .016 foot cubic foot (of water).
=1/8.34 or .12 foot gallon (of water).
=1/2655.4 or .0003766 volt coulombs.
=1/778 or .001285 British thermal units.

The foot cubic foot (one cubic foot of water raised one foot).
=62.5 foot pounds.
=7.48 foot gallons.
=.08 British thermal units.
=.02353 volt coulombs.

The foot gallon (one gallon of water raised one foot)
=8.34 foot pounds.
=.01072 British thermal units
=.00314 volt coulombs.
=.1334 foot cubic feet.

The volt coulomb
=2655.4 foot pounds.
=42.486 foot cubic feet.
=318.39 foot gallons.
=3.414 British thermal units.

The British thermal unit
=778 foot pounds.
=12.448 foot cubic feet.
=93.28 foot gallons.
=.2929 volt coulombs.

Quantities of energy available, used or to be used, and either potential or kinetic may be measured in the above units.

When the rate of expenditure is also stated these units express units of power. Some of the equivalent values of power are as follows, those most commonly used being printed in black-face type:

The horse power
=1980000 foot pounds per hour.
=33000 foot pounds per minute.
=550 foot pounds per second.
=31680 foot cubic feet per hour.
=528 foot cubic feet per minute.

=8.8 foot cubic feet per second.
=237600 foot gallons per hour.
=3960 foot gallons per minute.
=66 foot gallons per second.
=746 watts.
=2545 British thermal units per hour.
=42.41 British thermal units per minute.
=.707 British thermal units per second.

The foot pound per minute
=1/33000 or .0000303 horse power.
=1/778 or .00129 British thermal units per minute.
=.0226 watts.
=1/8.34=.12 foot gallons per minute.
=1/62.5=.016 foot cubic feet per second.

The foot cubic foot per minute
=62.5 foot lbs. per minute.
=1/528=.00189 horse power.
=1.412 watts.
=7.48 foot gallons per minute.
=.0803 British thermal units per minute.

The foot cubic foot per second
=3750 foot lbs. per minute.
=62.5 foot lbs. per second.
=1/8.8=.1136 horse power.
=448.8 foot gallons per minute.
=7.48 foot gallons per second.
=4.820 British thermal units per minute.
=.0803 British thermal units per second.

The watt
=44.24 ft. lbs. per minute.
=.00134 horse power.
=.0568 British thermal units per minute.
=5.308 gallons feet per minute.
=.7089 ft. cu. ft. per minute.

The British thermal units per minute
=778 ft. lbs. per minute.
=.02357 horse power.
=17.58 watts.
=93.28 ft. gal. per minute.
=12.48 ft. cu. ft. per minute.

26. Motion in General.—In moving a body against a given force or resistance the work done in foot pounds is the product of the space passed through (in feet) and the resistance (in pounds). Thus in raising a ten-pound weight 100 feet high, 1,000 foot-pounds of work is performed. But this is not the only work performed. To produce motion in a body or to bring a body to a state of rest necessitates a transfer of energy. For all moving bodies are endowed with kinetic energy—the energy of motion—and this energy must be given to them to produce motion, and must be taken from them to produce a state of rest.

Hence, Newton's laws of motion:

1. "Every body continues in a state of rest, or of uniform motion in a straight line except in so far as it may be compelled by impressed forces to change that state."
2. "Change of motion is proportional to the impressed force and takes place in the direction of the straight line in which the force acts."
3. "To every action there is always an equal and contrary reaction."

The acceleration of gravity is the acceleration due to the weight of a body acting on its mass.

The weight of a body W (on account of centrifugal effect of the earth's revolution) varies, being least at the equator and greatest at the poles. From Newton's second law it follows that the acceleration in motion designated by g and caused by the weight of any body acting on its mass will be proportional to its weight, i. e., $g=$ constant $\times$ W, and hence the weight of a body divided by the acceleration will always be constant. This constant quotent designated by the letter M is termed the mass of the body.

(1) $M=\frac{W}{g}$

Let W=The weight of a body.

M=Mass.

g=Acceleration due to gravity=velocity of a falling body at end of first second, and is ordinarily taken as 32.2 ft. per sec. per sec.

A=Acceleration of moving body=velocity of body at end of first second.

W'=Weight acting.

W''=Weight acted on.

V=Velocity at end of time t.
V_a=Average velocity.
t=Time force has acted.
S=Space passed through.
h=Height passed through by falling body.
V'=Initial velocity.
S'=Initial space passed through.

27. **Uniform Motion.**—In uniform motion the moving body passes through equal spaces in any equal divisions of time.

Hence by definition:

The space passed through (S) equals the product of the velocity (V) and the time (t).

(2) $S = V\,t.$

(3) $V = \frac{S}{t}$

28. **Uniformly Varied Motion.**—If the velocity of a body is increased or diminished uniformly, the motion is termed uniformly varied motion and is termed uniformly accelerated motion in the first case and uniformly retarded motion in the latter case.

In all such cases the following relations hold:

(4) $A = \frac{W'}{W''}\,g.$

(5) $V = At = \frac{W'}{W''}\,g\,t.$

(6) $V_a = \frac{At}{2}$

(7) $S = V_a\,t = \frac{At^2}{2} = \frac{V^2}{2A}$

(8) $V = \sqrt{2\,A\,S}.$

With falling bodies:

$S = h.$

$A = g.$

From which equation (8) becomes

(9) $V = \sqrt{2\,gh}$, the well known basis of hydraulic calculations.

(10) $\text{Work} = W\,h = W\,V^2/2g = M\,V^2/2.$

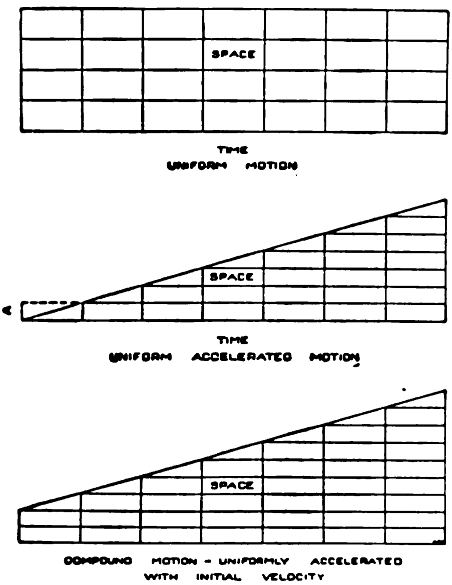
29. **Compound Motion.**—When bodies are already in motion and additional force is applied, the following relations hold:

(11) $V = V' + A\,t.$

(12) $S = S' + V'\,t + \frac{At^2}{2}$

30. Graphical Representation of the Laws of Motion.—In each case—

The vertical ordinates represent velocity
Abscissas represent time.
Areas represent space passed through.



$$V = \text{constant}$$
$$S = Vt$$
$$V = \frac{S}{t}$$

$$V = At = \frac{W'}{W}\,gt$$
$$V_a = \frac{At}{2}$$
$$S = V_a t = \frac{At^2}{2} = \frac{V^2}{2A}$$
$$V = \sqrt{2AS}$$

$$V = V' + At$$
$$S = S' + V'\,t + \frac{At^2}{2}$$

Fig. 17.—Graphical Representation of the Laws of Motive.

31. Transformation.—The transformation of potential to kinetic energy is well illustrated by water acting upon a water wheel. The energy in a body is always constant whatever its form, except as said energy be given up to other bodies or lost and wasted in various ways. Consequently the sum of the potential and kinetic energies in any body is a constant quantity unless the difference be accounted for by energy loss or transfer as above noted.

Water that has fallen to sea level has lost all the energy it may have once possessed, its energy having been expended in performing some kind of work.

If, in a hydraulic plant, we have an available fall of 8.8 ft. every cubic foot of water falling each second should produce 550 ft. lbs. of work per second or one horse power. After the water has passed through a well designed turbine it flows sluggishly away, having used up nearly all its energy in the turbine to which it has transferred its energy. If, however, on account of bad design the water flows away at a rapid rate, say at 10 feet per second, the head lost, $h=v^2/2g$ i. e. $h=10^2/64.4=1.55$ ft. of vertical fall. Under these conditions the energy due to this fall still remains in the water, after it has left the wheel, and is lost, the loss being 17.8 per cent. of the original energy.

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CHAPTER III.

HYDRAULICS.

32. **Basis of Hydraulics.**—The science of hydraulics is an empirical, not an exact science, but is based on the exact sciences of hydrostatics and dynamics. Its principal laws are therefore founded on theory, but on account of the multitude of modifying influences and of our necessarily imperfect theoretical knowledge of their varying characters and extent, the formulas used must be derived from or at least modified by observation and experience and cannot be founded solely on theoretical considerations. The conditions under which hydraulic laws must be applied are so varied in both number and kind that the application of the laws must be modified to suit those various conditions and for this reason their successful application depends largely on the practical experience of the engineer.

In the following discussion the letters used will have the significance shown below:

E=Energy (abstract).
P=Horse power.
W=Total weight of water.
h=The total available head in feet.
h_1=The velocity head.
h_2=The entrance head or influx head.
h_3=The friction head.
q=The quantity of water (in cubic feet per second).
w=The weight of each unit of water (cu. ft.=62.5 lbs.).
a=Area (in square inches) against which pressure is exerted.
s=The space (in lineal feet) through which the area moves under pressure.
v=The velocity of flow (in feet per second).
g=Acceleration due to gravity (32.2 feet per second per second.)
t=The time in seconds.

33. **Mathematical Expression for Energy.**—Mechanically, energy is the exertion of force through space. The amount of available

energy of water that may be theoretically utilized is measured by its weight (the force available) multiplied by the available head (the space through which the force is to be exerted), i. e., (1) $E = Wh$. From this it will be noted that the energy of water is in direct proportion to both the head and quantity. This energy may be exerted in three ways which may be regarded as more or less distinct but which are usually exercised, to some extent at least, in common. The exertion of this energy in the three ways mentioned, expressed in terms of horse power, are as follows:

First: By its weight which is exerted when a definite quantity of water passes from a higher to a lower position essentially without velocity. This method of utilization is represented by the equation

$$P = \frac{qwh}{550} \tag{2}$$

Second: By the pressure of the water column on a given area exerted through a definite space. This method of utilization is represented by the equation

$$P = \frac{.434h\ as}{550t} \tag{3}$$

Third: By the momentum of the water exerted under the full velocity due to the head. The energy of a moving body is represented by the formula:

$$E = \frac{Wv^2}{2g} \tag{4}$$

The equation for the horse power of water under motion is therefore represented by the equation:

$$P = \frac{qwv^2}{550 \times 2g} \tag{5}$$

An analysis of these formulas will show that under any given conditions the theoretical power exerted will be the same in each case.

34. **Velocity Head (h_1).**—It has already been pointed out (chapter II) that energy must be expended in order to produce motion in any body and that the head (h_1) necessary to produce a velocity (v) is

$$h_1 = \frac{v^2}{2g} \tag{6}$$

This proportion (h_1/h) of the available head h has to be expended to produce and keep in motion the flow of water. This head (h_1) is not necessarily lost (it has simply been converted into kinetic energy, and it may be re-converted into potential energy by correct design or it may be utilized in some other way, as, for example, by pressure or impact in hydraulic motors).

Whatever head (h_1) is necessary to maintain the velocity (v), with which the water leaves the plant, will be lost to the plant. It is, therefore, desirable to keep v at this point as low as may be found practicable when other conditions are considered.

Sudden enlargements or contractions in pipes or passages may wholly or partially destroy the velocity and cause the permanent loss of the corresponding head (h_1).

In this case an additional amount of the available head (h_1) must be used to again generate the velocity (v) required to convey the water through the remainder of its course. Gradual change in the cross-section of all channel conduits or passages is, therefore, desirable in order that the transformation from kinetic to potential energy, and the reverse, shall be made without material loss.

Not only the head (h_1) but still other portions of the total available head (h) may be lost in the channels and passages of a machine or plant by improper design.

35. **Entrance Head.**—The loss of head (h_2) which occurs at entrance into a raceway, pipe or passage may be called the "influx head." The amount of this loss differs considerably with the shape and arrangement of the end of the pipe or passage. In general, the influx head may be determined by the formula:

(7) $$h_2 = \left(\frac{1}{c^2} - 1\right)\frac{v^2}{2g}$$ (Merriman's Hydraulics, Art. 56)

In this formula the coefficient can be obtained from table IV, in which the variations of the constant under various conditions, with reference to a pipe inlet, are shown, and from which it will be noted that its magnitude depends on the shape and arrangement of the inlet.

TABLE IV.

Arrangements of a pipe inlet with corresponding coefficients.

Arrangement of Pipe.	c	$\frac{1}{c^2} - 1$
A. Projecting into reservoir	.715	.956
B. Mouth flush with side of reservoir	.825	.469
C. Bell shaped mouth, from	.950	.108
C. Bell shaped mouth, to	.990	.020

To find the value of h_2, the value of $\frac{1}{c^2} - 1$ corresponding to the given conditions, is to be selected from Table IV and substituted in formula (7). The ordinary arrangement of suction pipes is for a square ended pipe to project directly into the suction pit. In reservoirs the pipe may be flush with the masonry or project as in the case of suction pipes. With condition (A) formula (7) becomes

$$(8) \qquad h_2 = .956 \frac{v^2}{2g}$$

The value of h_2 can be readily obtained from equation (8), as it will be 95.6 per cent. of the velocity head.

With the mouth of the pipe flush with the side of the reservoir the loss would be 46.9 per cent. of the velocity head, and with a bell mouth pipe the loss would be decreased to from two per cent. to 10.8 per cent. according to the design of the bell mouth entrance.

The arrangements of inlet pipes as referred to in Table IV are shown in Fig. 18.

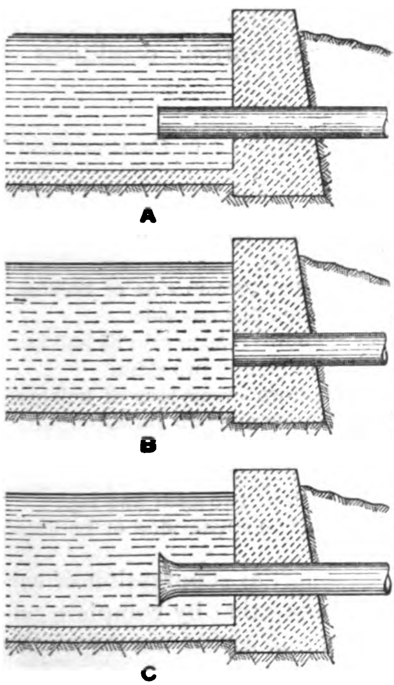


Fig. 18.

36. **Submerged Orifices.**—A similar loss is sustained in the flow through gates or submerged openings or in the flow past any form of obstruction which may be encountered by the water in its flow through channels, pipes or other forms of passages. Openings or obstructions with square edges may cause a serious loss of head which may, however, be reduced.

First: By increasing the opening, thus causing a reduction in velocity and consequently a saving in head, or

Second: By rounding the corners of the opening or obstruction, thus causing a gradual change in velocity and a partial recovery of any head necessarily used for creating greater velocity through such passage or past such obstruction.

But few experiments have been made on submerged orifices and tubes. These indicate a coefficient of about .62 for complete contraction which increases to .98 or even .99 with the contraction completely suppressed. Certain experiments have recently been made at the hydraulic laboratory of the University of Wisconsin, on the discharge through orifices and tubes four feet square and of various thicknesses or lengths and with various conditions of contraction. The values of the coefficients as determined in these experiments with various losses of head and various conditions of entrance, are shown in Table V.*

The Forms of Entrance and Outlet Used for the Tubes in the experiment were as follows:'

A. Entrance; all corners 90°.
Outlet; tube projecting into water on down stream side of bulkhead.

a Entrance; contraction suppressed on bottom.
Outlet; tube projecting into water on down stream side of bulkhead.

b Entrance; contraction suppressed on botton and one side.
Outlet; tube projecting into water on down stream side of bulkhead.

c Entrance; contraction suppressed on bottom and two sides.
Outlet; tube projecting into water on down stream side of bulkhead.

c' Entrance; contraction suppressed on bottom and two sides.
Outlet: square corners with bulkhead to sides of channel preventing the return current along the sides of the tube.

d Entrance; contraction suppressed on bottom, two sides and top.
Outlet; tube projecting into water on down stream side of bulkhead.

From this table it will be noted that a partial suppression of contraction does not always improve results, and that by complete suppression, the coefficient is greatly increased with a corresponding decrease in head lost.

37. **Friction Head (h_3)**—In raceways and short pipes the velocity head (h_1) and the influx head (h_2) are frequently the sources of the greatest losses of head. In canals and pipes of considerable length the friction of flow may become the most serious sources of energy loss.

The principles of flow in such channels may be considered as follows:

First Principle: In any frictionless pipe, conduit, channel or passage of any length the flow may be expressed by the formula:

$$h_3 = \frac{v^2}{2g} \text{ or } v = \sqrt{2gh} \qquad (9)$$

In practice, however, we find friction is always present and a friction factor must be introduced in the above formula in order to

*From experiments by Mr. C. B. Stewart at the Hydraulic Laboratory of the University of Wisconsin.

represent the actual conditions of practice. (9) therefore becomes:

$$h_2 = c' \frac{v^2}{2g} \text{ or } v = c\sqrt{2gh} \tag{10}$$

TABLE V.

Value of the Coefficient of Discharge for flow through horizontal submerged tube, ¼ feet square, for various lengths, losses of head and forms of entrance and outlet.

Loss of head, h_2 in feet.	Forms of Entrance and Outlet	Length of tube, in feet.						
		0.31	0.62	1.25	2.50	5.00	10.0	14.0
		Value of the coefficient, c.						
.05.........	A	.631	.650	.672	.769	.807	.824	.838
	a	.762			.742	.810		.848
	b	.740			.769	.832		.862
	c	.834			.769	.875		.890
	c'							.875
	d	.948			.943	.940	.927	.931
.10.........	A	.611	.631	.647	.718	.763	.780	.795
	a	.636			.698	.771		.801
	b	.685			.718	.791		.813
	c	.772			.718	.828		.841
	c'							.830
	d	.932			.911	.899	.892	.893
.15.........	A	.609	.628	.644	.708	.758	.779	.794
	a	.630			.689	.767		.803
	b	.677			.708	.787		.814
	c	.765			.708	.828		.839
	c'							.829
	d	.936			.910	.899	.893	.894
.20.........	A	.609	.630	.647	.711	.768	.794	.809
	a	.632			.694	.777		.819
	b	.678			.711	.796		.833
	c	.771			.711	.838		.856
	c'							.846
	d	.948			.923	.911	.906	.905
.25.........	A	.610	.634	.652	.720	.782	.812	.828
	a	.634			.705	.790		
	b	.683			.720	.809		
	c	.779			.720	.854		
	c'							
	d	.965			.938	.928		
.30.........	A	.614	.639	.660	.731	.796	.832	.850
	a	.639						
	b	.689						
	c	.788						
	c'							
	d	.984						

The formulas (9) and (10) represent one of the important fundamental principles from which many hydraulic formulas are developed.

Second Principle: In any pipe, conduit, channel or passage we may fairly assume:

First: From axiomatic considerations the resistance to the flow of water may be regarded as directly proportional to the area of the surface in contact with the water.

Second: From observed conditions the resistance is found to be directly proportional to the square of the velocity of flow.

Third: Experience leads to the conclusion that the resistance to flow is inversely proportional to the cross-section of the stream.

These conclusions may be expressed by the following equation:

$$\text{Resistance} = \frac{(\text{Velocity})^2 \times \text{area of contact}}{\text{area of section}}$$

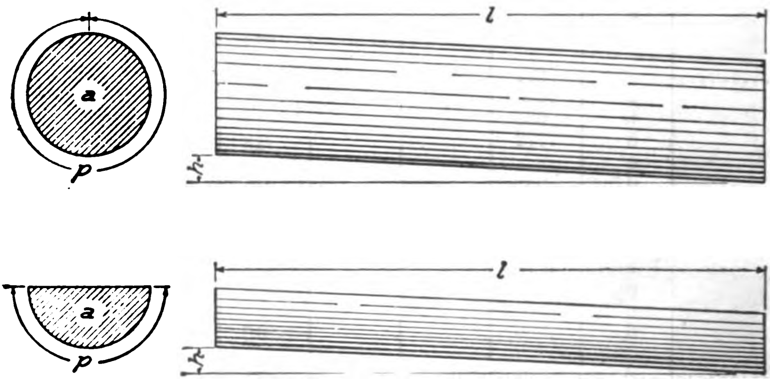


Fig. 19.

The area of the surface of a channel is the product of the wetted section or wetted perimeter (p) times the length of the section, or, to p x l. (See Fig. 19.) The velocity is represented by v and the cross-section by a. Hence, from the above considerations, we may write for the friction head:

$$\text{(11)} \qquad h_s = \frac{v^2 pl}{a} \text{ and by transposition } v^2 = \frac{ah_s}{pl}$$

That is to say, the square of the velocity is in direct proportion to the area of the section and to the friction head and inversely proportional to the wetted perimeter and to the length of the section.

In practice it is found that there are numerous factors which affect the theoretical conditions, as above set forth, which must therefore be modified in accordance with the conditions which obtain. In formula (11) therefore it is necessary to apply a coefficient (c′) which represents the summation of such other influences. The form in which this last equation is ordinarily written is

$$(12) \qquad h_s = c' \frac{v^2 pl}{a} \text{ or } v = c\sqrt{\frac{ah_s}{pl}}$$

Ordinarily this form is somewhat abbreviated by substituting for a/p the hydraulic radius which represents this ratio. That is to say,

$$\frac{\text{area of cross section}}{\text{wetted perimeter}} = \frac{a}{p} = r$$

The "hydraulic radius" is also sometimes termed the "mean depth" or the "mean radius." For the ratio of the resistance head to the length of section the equivalent slope s is substituted. That is to say:

$$\frac{\text{Resistance head}}{\text{Length of section}} = \frac{h_s}{l} = s$$

With these substitutions the formula (12) assumes the final form of:

$$(13) \qquad v = c\sqrt{rs}$$

In the use of this formula three factors must be determined by measurement or estimate in order to derive the fourth. v, r and s can be determined experimentally or measured directly. The factor c is the most difficult to ascertain as it depends upon a very great variety of conditions which can only be known and appreciated by a thorough knowledge of the conditions under consideration, and by comparison of such conditions with similar observed conditions. Various attempts have been made to derive a formula which would give the value of c in accordance with the varying conditions. The principal formulas for the values of c are those of Ganguillet and Kutter and of Bazin. Ganguillet and Kutter's formula for the value of c is as follows:

38. Kutter's Formula.—

$$(14) \qquad c = \frac{41.6 + \frac{1.811}{n} + \frac{0.00281}{s}}{1 + \left(41.6 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}}$$

From this formula it will be seen that Ganguillet and Kutter assume c to vary with the slope, with the square root of the hydraulic radius and with a new factor "n" which is termed the coefficient

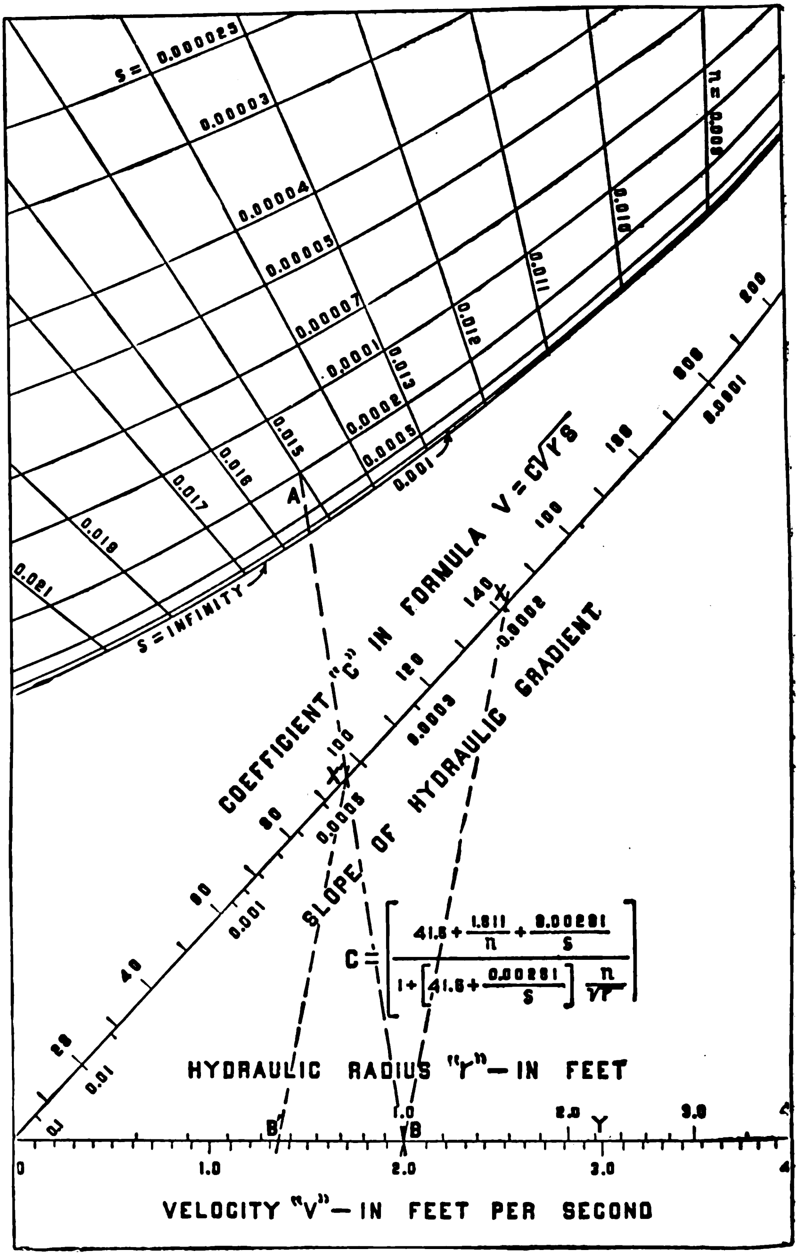

S = 0.000025
0.00003
0.00004
0.00005
0.00007
0.0001
0.0002
0.0005
0.001
S = INFINITY
n = 0.009
0.010
0.011
0.012
0.013
0.015
0.016
0.017
0.019
0.021
A
COEFFICIENT "C" IN FORMULA V=C√rs
SLOPE OF HYDRAULIC GRADIENT
C = [(41.6 + 1.811/n + 0.00281/s) / (1 + [41.6 + 0.00281/s] n/√r)]
HYDRAULIC RADIUS "r"— IN FEET
VELOCITY "V"— IN FEET PER SECOND
B
B'
Y
1.0
2.0
3.0
4
0

Fig. 20.

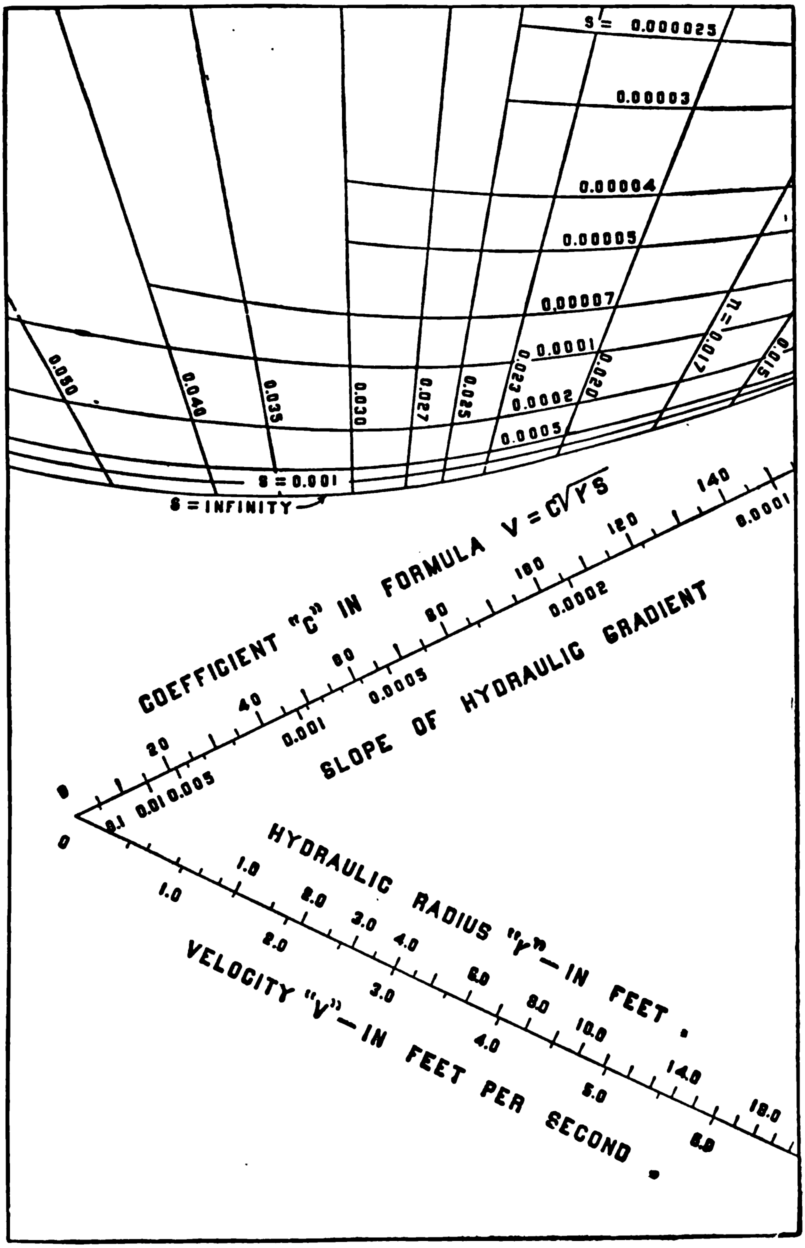

S = 0.000025
0.00003
0.00004
0.00005
0.00007
0.0001
0.0002
0.0005
S = 0.001
S = INFINITY
n = 0.017
0.015
0.050
0.040
0.035
0.030
0.027
0.025
0.023
0.020
COEFFICIENT "C" IN FORMULA V = C√rS
0
20
40
60
80
100
120
140
SLOPE OF HYDRAULIC GRADIENT
0.1
0.01
0.005
0.001
0.0005
0.0002
0.0001
HYDRAULIC RADIUS "r"— IN FEET.
1.0
2.0
3.0
4.0
6.0
8.0
10.0
14.0
18.0
VELOCITY "V"— IN FEET PER SECOND.
1.0
2.0
3.0
4.0
5.0
6.0

Fig. 21.

4

of roughness. The value of this coefficient as determined by these experiments is as follows:

For large pipe with the following characteristics:

Exceptionally smooth cast iron pipe	n =	.011
Ordinary new cast iron or wooden pipe		.0125
New riveted pipes and pipes in use		.014
Pipes in long use		.019

For open channels of uniform sections:

For planed timber sides and bottom	n=	.009
For neat cement or glazed pipe		.01
For unplaned timber		.012
For brick work		.013
For rubble masonry		.017
For irregular channels of fine gravel		.02
For canals and rivers of good section.........		.025
For canals and rivers with stones and weeds ...		.030
For canals and rivers in bad order		.035

The relation of the above factors may be determined by the diagrams, Figs. 20 and 21. If with a known slope and a known value of n (for example, let n=0.15 and s=.0002, as at A, Fig. 20), a straight line be drawn on this diagram to the scales of the hydraulic radius (at B) it will show at the intersection with the scale for the coefficient (c) the relative value of this coefficient for these conditions, or with a known **c** and the known hydraulic radius and the given slope the value of **n** of a channel may be likewise determined. After a line has once been drawn connecting these four known values the velocity can be determined by drawing a line from the hydraulic radius scale (B) to the proper point on the scale of slope or hydraulic gradient at x, and then from the point of intersection of the line A B with the coefficient scale at x′ drawing a line parallel with xB which will intersect the velocity scale at the point B′, giving the velocity (in this case equal to 1.34 ft. per second). These formulas only apply with accuracy where the channels or passages are uniform and if applied to channels or passages which are not uniform the sections selected must be fairly representative. If the sections selected are not fairly representative the value of c or n determined from observations and experiments may vary considerably from the values which would otherwise be anticipated. That is to say, the calculations based on c and n will take into account irregularities in channels and other unknown or unrecognized conditions, including curves, bends and obstructions which may not

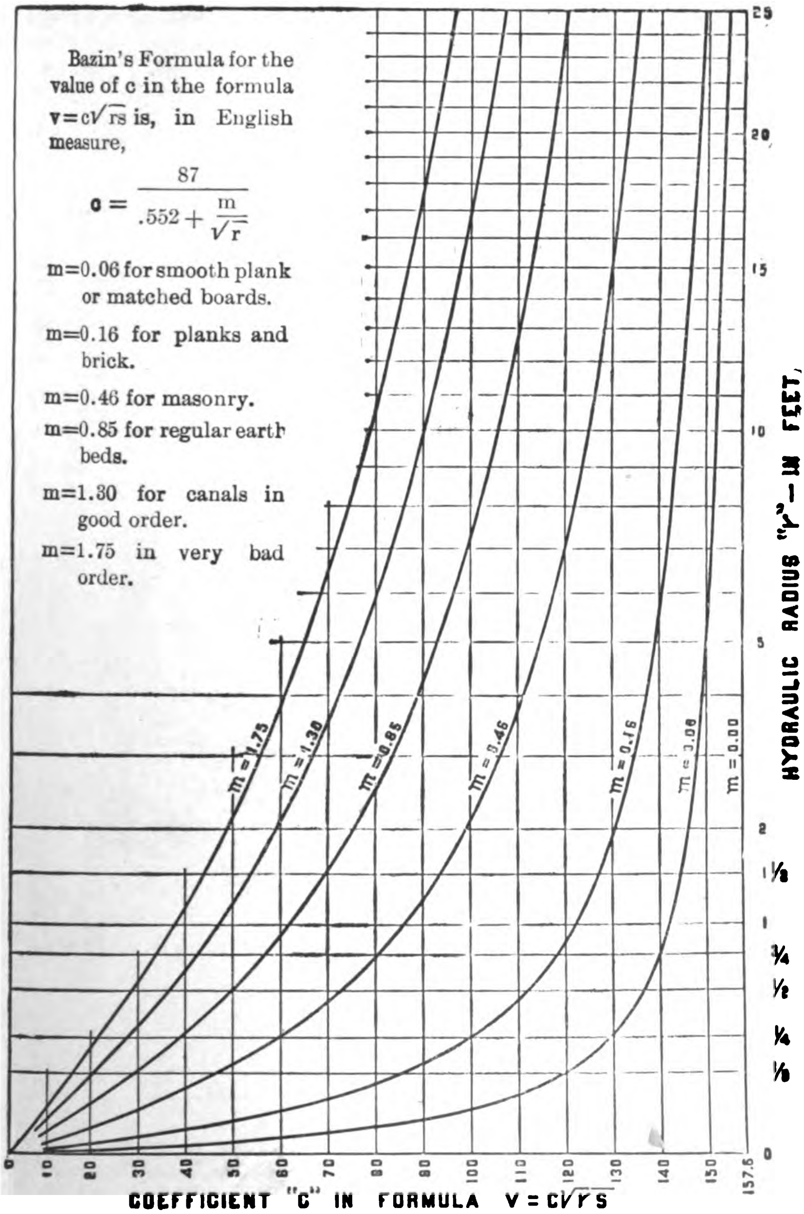


Fig. 22.—Diagram For Solution of Bazin's Formula.

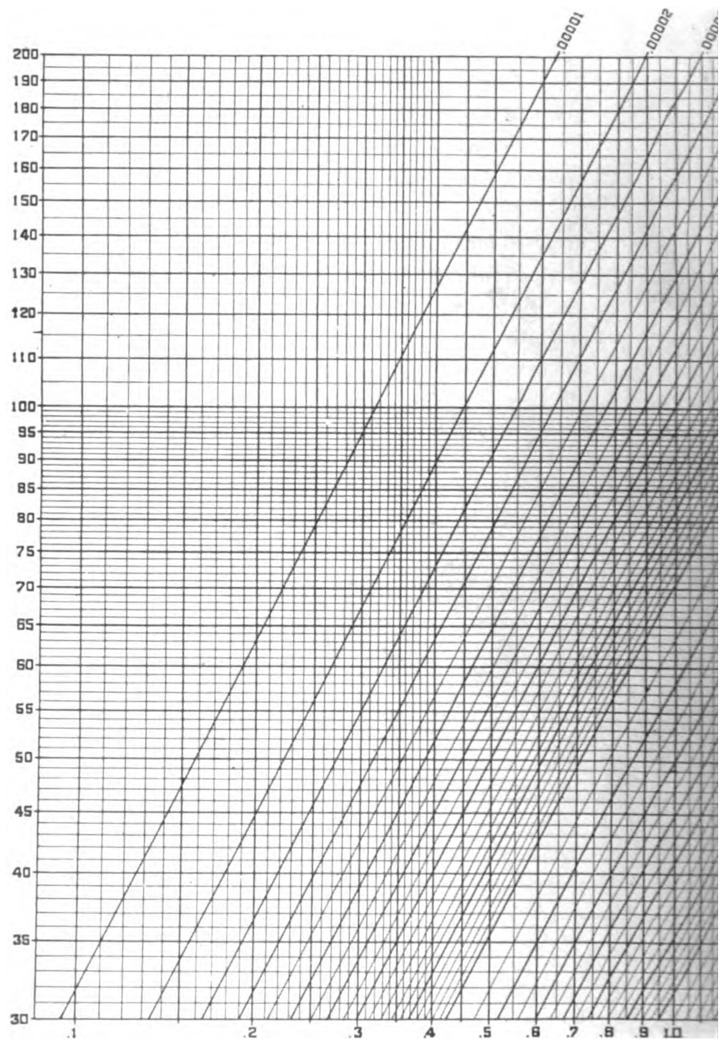
GRAPHICAL SOLUTI

$$V = C\sqrt{}$$

V = VELOCITY IN FEET PER SECOND.
C = COEFFICIENT.
R = HYDRAULIC RADIUS IN FEET = $\frac{a}{p}$.
S = SINE OF SLOPE = $\frac{h}{l}$.

a = A
p = W
h = F.
l = L

VALUE



C = COEFFICIENT

V = VELOCITIES

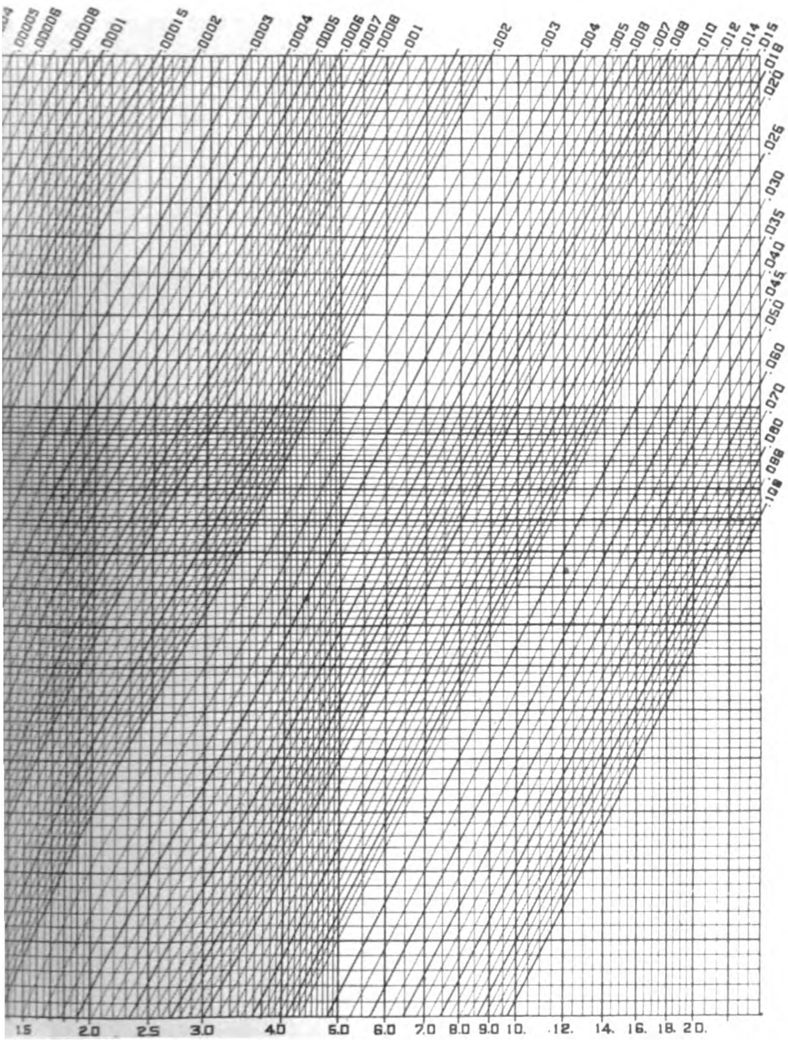
1

I OF CHEZY'S FORMULA

S = $C\sqrt{\frac{a}{p}\frac{h}{l}}$

,IN SQ. FEET OF CHANNEL SECTION.
TED PERIMETER OF CHANNEL SECTION IN LINEAL FEET.
. IN FEET BETWEEN POINTS CONSIDERED.
TH OR DISTANCE, BETWEEN POINTS CONSIDERED, IN LINEAL FEET.

OF R . S.



I FEET PER SECOND

re 23

have been considered at the time the original observations were made.

39. **Bazin's Formula.**—It has been questioned by many observers whether the slope of the channel has any material influence on the value of the coefficient c. Bazin has derived a formula based on his examination of this subject in which he assumes that c does not vary with the slope. His formula, which is intended for the calculation of flow in open channels is shown, together with a graphical table based thereon, in Fig. 22. This figure illustrates the law of variation of c and is applicable in principle in a general way to all channels and passages.

The graphical diagram, Fig. 23, which was prepared by the writer in connection with Mr. J. W. Alvord, affords a ready method of solving Chezy's formula (13).

40. **Efficiency of Section.**—From equations (12) and (13)

$$v = c\sqrt{rs} = c\sqrt{\frac{ah_3}{pl}}$$

$$q = \text{velocity} \times \text{area} = va \tag{15}$$

$$\text{or } q = ca\sqrt{rs} = ca\sqrt{\frac{ah_3}{pl}}$$

With c and s constant q varies as $a\sqrt{r}$ or as $\sqrt{\frac{a^3}{p}}$

From this the conclusion may be drawn that other things being equal the maximum quantity of water will pass through any section of any river or other channel in which the hydraulic radius is a maximum or the wetted perimeter a minimum. Where a choice exists as to the class of material with which the channel is to be lined c becomes a variable and q will vary as

$$ca\sqrt{r} \text{ or as } c\sqrt{\frac{a^3}{p}}$$

That is to say, under circumstances where different characters of lining may be used the maximum quantity will pass a given section with c and r maximum or with c a maximum and p a minimum for given a.

41. **Determination of Canal Cross-section.**—The velocity of the water in any artificial channel must be limited by the class of material used in its construction and the head which it is found practicable to use. As noted above the efficiency of a section is greatest with the value of p minimum. Therefore, the semi-circular section is the most advantageous cross-section that can be used in a channel where resistance alone is considered and when the canal

is to be lined with material which can be readily shaped into this form. If the canal is to be lined with stone masonry it is frequently more advantageous to make the face perpendicular and to place the batter of the wall at the back. Where the canal is cut from stone or shales which will not readily disintegrate in contact with the water, a slope of 90° to 40° may be sometimes used. Quite steep slopes can also be used with dry masonry walls. In material which can be handled with pick and shovel, slopes may be used from 1 to 1.25 to 1 to 1.50. With artificial banks of dirt and gravel a less slope angle is necessary and the slope must frequently be made as low as one to two.

Table VI, which is taken partially from "Uber Wasserkraft und Wasser Versorgungsanlagen," by Ferdinand Schlotthauer, is of considerable value in determining the most advantageous cross-section in various sections which may be adopted in the construction of a canal. As seen in the discussion above, the most advantageous cross-section, other things being equal, is that in which the

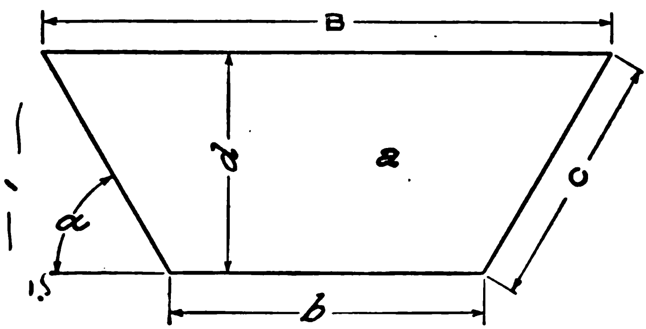


Fig. 24.

wetted perimeter is a minimum or the hydraulic radius is a maximum. The following general discussion of the relations is based on Fig. 24. From this figure it will be seen that

(16) $$a = bd + d^2 \cot\alpha$$

(17) $$p = b + 2d \operatorname{cosec} \alpha$$

The transposition of (17) gives

(18) $$b = p - 2d \operatorname{cosec} \alpha$$

Substituting (18) in (16)

(19) $$a = dp - 2d^2 \operatorname{cosec}\alpha + d^2 \cot\alpha$$

The above equation now contains the area, depth, wetted perimeter and functions of the slope angle, in this case a constant. The conditions of maximum efficiency of a canal section require that the wetted perimeter be a minimum or what amounts to the same thing with a given wetted perimeter the area a must become a maximum. The value of d which makes a the maximum is determined by putting $\frac{d(a)}{d(d)} = 0$

(20) $$\frac{d(a)}{d(d)} = p - 4d \operatorname{cosec} \alpha + 2d \cot\alpha$$

(21) $$0 = p - 4d \operatorname{cosec} \alpha + 2d \cot\alpha$$

(22) $$d = \frac{p}{4 \operatorname{cosec}\alpha - 2 \cot\alpha}$$

Substituting for p its value in (17)

(23) $$d = \frac{b + 2d \operatorname{cosec} \alpha}{4 \operatorname{cosec} \alpha - 2 \cot\alpha}$$

Equation (16) transposed reads

(24) $$b = \frac{a - d^2\cot\alpha}{d}$$

Substituting this value in (23) we have

(25) $$d = \frac{\frac{a}{d} - d \cot\alpha + 2d \operatorname{cosec} \alpha}{4 \operatorname{cosec} \alpha - 2 \cot\alpha}$$

Clearing:

(26) $$4d^2\operatorname{cosec} \alpha - 2d^2\cot\alpha = a - d^2\cot\alpha + 2d^2\operatorname{cosec}\alpha$$

Transposing:

(27) $$d^2 = \frac{a}{2\operatorname{cosec}\alpha - \cot\alpha}$$

Transforming trigonometric functions

(28) $$d^2 = \frac{a}{\frac{2}{\sin \alpha} - \cos \alpha \operatorname{cosec} \alpha}$$

(29) $$= \frac{a}{\frac{2 - \sin \alpha \cos \alpha \operatorname{cosec} \alpha}{\sin \alpha}}$$

(30) $$= \frac{a \sin \alpha}{2 - \cos \alpha}$$

Finally:

(31) $$d = \sqrt{\frac{a \sin \alpha}{2 - \cos \alpha}}$$

Equation (24) may be written

(32) $$b = \frac{a}{d} - d \cot \alpha$$

Table VI is calculated from the formulas:

(31) $$d = \sqrt{\frac{a \sin \alpha}{2 - \cos \alpha}}$$

TABLE VI.

Economic Dimensions of Canal Cross-Sections with Various Slopes. Dimensions Expressed as Functions of Area of Cross-Section.

Side Slope.	Slope Angle. α	Relative Slope. $\cot \alpha$	Depth. d	Bottom Width. b	Width of Water Surface. B	Length of Slope Line. C	Wetted Perimeter. p	Hydraulic Radius. r	Character of Material to which Side Slope is Adapted.
Vertical	90°00′	0.0000	$.707\sqrt{a}$	$1.414\sqrt{a}$	$1.414\sqrt{a}$	$.707\sqrt{a}$	$2.828\sqrt{a}$	$.353\sqrt{a}$	Concrete and Stone Masonry.
1:1¾	60°15′	0.5714	$.760\sqrt{a}$	$.882\sqrt{a}$	$1.750\sqrt{a}$	$.875\sqrt{a}$	$2.633\sqrt{a}$	$.380\sqrt{a}$	Dry Masonry Walls
1:1	45°00′	1.0000	$.740\sqrt{a}$	$.613\sqrt{a}$	$2.093\sqrt{a}$	$1.046\sqrt{a}$	$2.705\sqrt{a}$	$.369\sqrt{a}$	Clayey Gravel or Hardpan.
1¼:1	38°40′	1.2500	$.716\sqrt{a}$	$.503\sqrt{a}$	$2.293\sqrt{a}$	$1.146\sqrt{a}$	$2.795\sqrt{a}$	$.358\sqrt{a}$	Well Compacted Clay.
1½:1	33°41′	1.5000	$.689\sqrt{a}$	$.418\sqrt{a}$	$2.485\sqrt{a}$	$1.243\sqrt{a}$	$2.904\sqrt{a}$	$.344\sqrt{a}$	Coarse Gravel, Clayey Loam.
2:1	26°34′	2.0000	$.636\sqrt{a}$	$.300\sqrt{a}$	$2.844\sqrt{a}$	$1.422\sqrt{a}$	$3.144\sqrt{a}$	$.318\sqrt{a}$	Loose Earth, Coarse Sand.
Semi-Circular	....	....	$.798\sqrt{a}$	....	$1.596\sqrt{a}$	....	$1.778\sqrt{a}$	$.563\sqrt{a}$	Brick Concrete, Metal.

(32) $$b = \frac{a}{d} - d \cot \alpha$$

(33) $$B = b + 2d\cot \alpha$$

(34) $$p = b + \frac{2d}{\sin \alpha}$$

In the above, a=cross-section area; d=depth of water in channel; b=bottom width; B=width at water level; p=wetted perimeter; c=the length of slope which is equal to $\frac{d}{\sin \alpha}$

In Table VI the relation of these functions, for the slopes ordinarily used in practice have been calculated as well as for the semicircular section. The use of the table may be illustrated as follows: The quantity of water which it is desired to deliver is determined by the conditions of the problem or by measurement. The velocity to be maintained in the channel is determined by the existing slope, the nature of material encountered, or the friction head which it is found desirable to maintain. The area of the cross-section required to carry the quantity q with velocity v is $a=\frac{q}{v}$ After the slope angle has been selected, for the material in which the channel is to be constructed, the corresponding values may be taken out of the table from their respective columns and multiplied by the square root of a. The result thus obtained gives the desired dimensions. If, for example, we desire to carry 100 cu. ft. of water per second in a canal at a velocity of 2 1/2 ft. per second at which velocity small pebbles are unaffected, and with a side slope of 1.5 to 1, which is suitable for loose earth, has been decided upon, the required area of cross-section will be 100/2.5 =40 sq. ft. The square root of 40 is 6.33. The required dimensions of canal as taken from the table are

Depth d=.689 x 6.33=4.36 ft.
Bottom width b=.418 x 6.33=2.65 ft.
Top width B=2.485 x 6.33=15.73 ft. and
The wetted perimeter p=2.904 x 6.33=18.38 ft.

Computation of the area from the above dimensions gives 40 sq. ft. Hence the work has been checked.

42. The Back Water Curve.—One of the problems which becomes very important in many water power installations is the effect on the elevations of the stream produced by the erection of a dam or other obstruction therein. The back water curve can best be determined by the use of the simple formula of flow, equation (13).

$$v = c\sqrt{rs} \tag{13}$$

From this, as shown in equation (15)

$$q^2 = v^2a^2 = \frac{c^2a^2h_s}{pl} \tag{15}$$

From this equation can be derived

$$h_s = \frac{q^2\,pl}{c^2a^3} = \frac{q^2l}{c^2} \times \frac{p}{a^3} \tag{35}$$

With $\frac{q^2l}{c^2}$ constant, $h_s : h'_s :: \frac{p}{a^3} : \frac{p'}{a'^3}$, therefore

$$h'_s = \frac{h_s\,p'a^3}{a'^3p} = \frac{h_s\,a^2r}{a'^2r'} \tag{36}$$

That is to say, with the quantity of water and length of section constant, if the coefficient remains constant the head due to any obstruction will vary in accordance with equation (36).

Where the water is greatly deepened in proportion to its original depth the value of c will not remain constant but will vary. Where such is the case and where q^2l is constant, under which condition

$$h'_s = \frac{h_s\,p'a^3}{a'^3p} \times \frac{c^2}{c'^2} = \frac{h_s\,a^2\,rc^2}{a'^2\,r'\,c'^2} \tag{37}$$

The difficulties in the determination of the value of c are, of course, obvious, but it is believed that the back water curve can be closely calculated by this simple formula in which the new value of c is the only factor to be estimated, and where the other elements of the problem can be determined by actual measurements. In using this formula the original value of c under existing condition of flow can be determined by calculation based on actual observation of flow under different conditions of water and the conditions of the channel under the new regimen can be closely estimated. New values of c can be very closely estimated on the basis of the values known to exist under other similar circumstances. This method will permit of a more practical solution of the problem than by the use of formulas based on entirely theoretical consideration of conditions which can never be approximated in practice.

43. **Flow of Water in Pipes.**—Mathematical expressions for the flow of water in pipes may be derived from either of the fundamental hydraulic formulas

$$v = c\sqrt{rs} \quad \text{or} \quad v = c\sqrt{2gh_s}$$

Starting with the former equation, in the case of a pipe flowing full the hydraulic radius $r = \frac{d}{4}$ where d is the diameter of the pipe and for s we may substitute $\frac{h_s}{l}$ We then have

$$v = c\sqrt{\frac{dh_s}{4l}} \tag{38}$$

In a pipe of unit length and unit diameter without friction the flow would be expressed by the formula

$$v = \sqrt{2gh} \quad \text{or} \quad h = \frac{v^2}{2g}$$

To modify this for friction a friction factor f is introduced and the equation then reads:

$$h_s = f\frac{v^2}{2g}$$

The friction varies directly as the length and is assumed to vary inversely as the diameter. Hence, for any pipe of length l and diameter d the complete equation is:

$$h_s = f\frac{l}{d}\,\frac{v^2}{2g} \quad \text{or} \quad v = \sqrt{\frac{h_s d\, 2g}{fl}} \tag{39}$$

Placing (38) and (39) equal it will be found that

$$c = \frac{16.04}{\sqrt{f}}$$

so that the equations can be made equivalent by the proper modifications of friction factors. An extensively used formula for the determination of c in equation (38) is that of Darcy. It reads:

$$c = \frac{1}{\sqrt{\alpha + \frac{\beta}{\alpha}}} \tag{40}$$

For new pipe $\alpha = .00007726$ and $\beta = .00009647$.
For old pipe $\alpha = .0001543$ and $\beta = .00001294$.

These coefficients were determined from experiments on small pipes and therefore in the case of large pipes with high velocities the velocities computed by this formula are too small.

Various modifications of the Chezy formula, having the general form

$$v = cr^n s^m \tag{41}$$

have been proposed or derived from experiments. Lampés and Flamant's are the best known of this type. Lampés reads

$$v = 77.68\, d^{0.694}\, s^{0.555} \tag{42}$$

and Flamant's

$$v = cd^{\frac{5}{7}}\, s^{\frac{4}{7}} \tag{43}$$

in which c=76.28 for old cast iron pipe and 86.3 for new pipe.

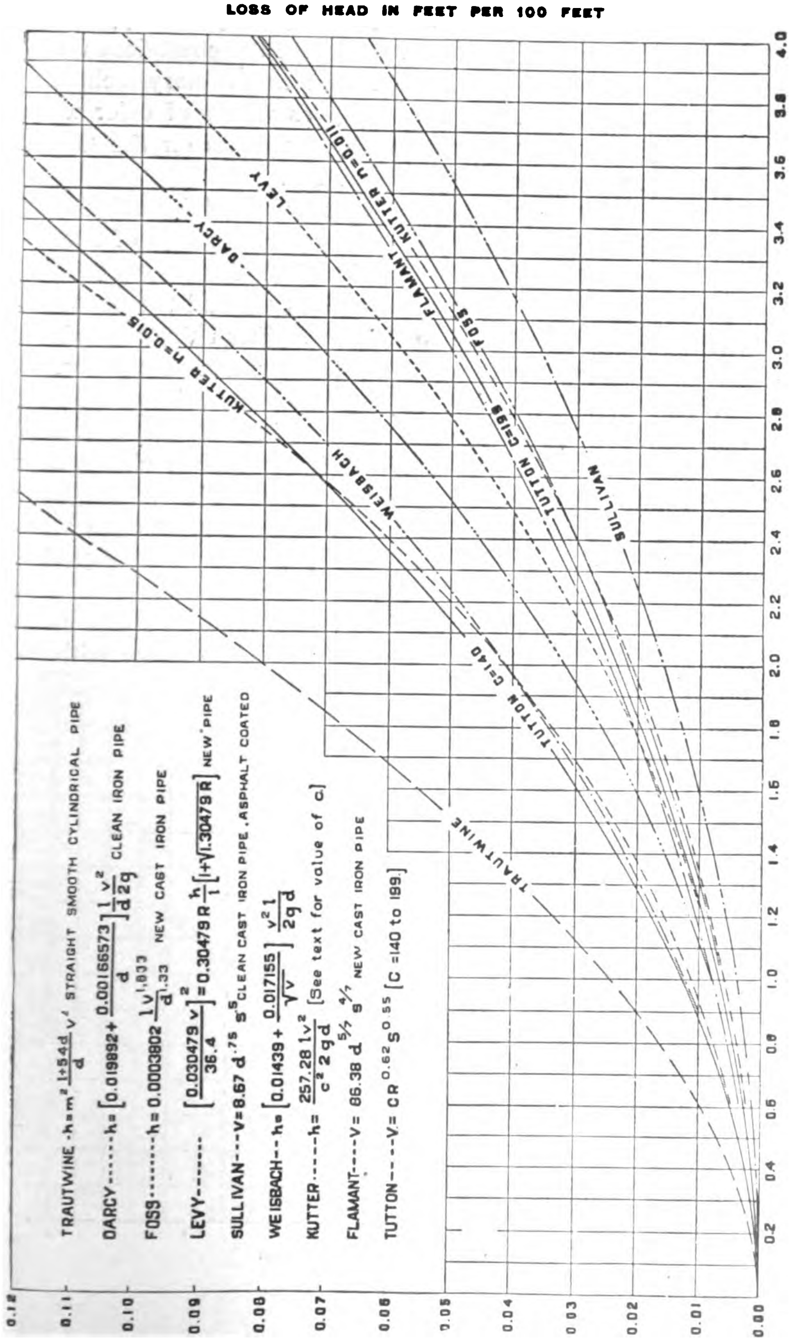


Fig 25.—Comparison of Formulas for the Flow of Water through a four foot cast iron pipe.

The value of c in the formula $v=c\sqrt{rs}$ may vary from 75 to 150 for large cast iron pipe. For riveted steel pipe the coefficient varies but little with velocity and diameter and at ordinary velocities ranges from 100 to 115. A. L. Adams gives values of c for wood stave pipe ranging from 100 to 170. Experiments on the Ogden pipe line showed average values of about 120.

An examination of the various formulas proposed for calculating the flow of water in pipes will show a very wide range of results. For example, for calculating the head lost in a four-foot new cast iron pipe, some of the principal formulas offered and the graphical solution of the same are shown by Fig. 25. From these results it will be seen that the data from which the formulas were derived are evidently obtained under widely varying conditions and that in the relation of such formulas for use on important work, they must be chosen after a careful consideration of all the elements of the problem, and that it is usually much better, when possible, to utilize the original data and observation along similar lines when such can be obtained, and derive the formula to be used instead of accepting one whose basis may be obscure or unknown.

In construction where pipes are short and comparatively unimportant, a formula may be selected which seems to agree with the

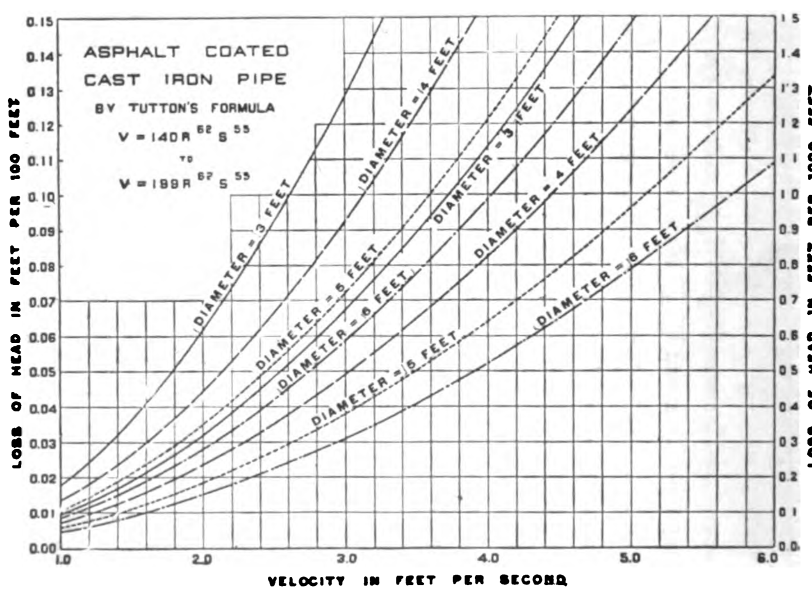


Fig. 26.

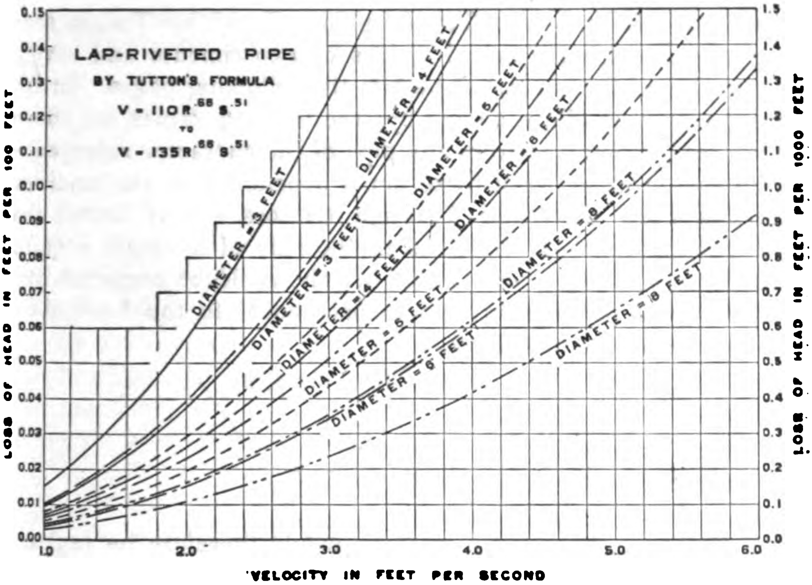

LAP-RIVETED PIPE
BY TUTTON'S FORMULA
V = 110R.66 S.51
TO
V = 135R.66 S.51
DIAMETER = 3 FEET
DIAMETER = 4 FEET
DIAMETER = 5 FEET
DIAMETER = 6 FEET
DIAMETER = 8 FEET
LOSS OF HEAD IN FEET PER 100 FEET
LOSS OF HEAD IN FEET PER 1000 FEET
VELOCITY IN FEET PER SECOND

Fig. 27.

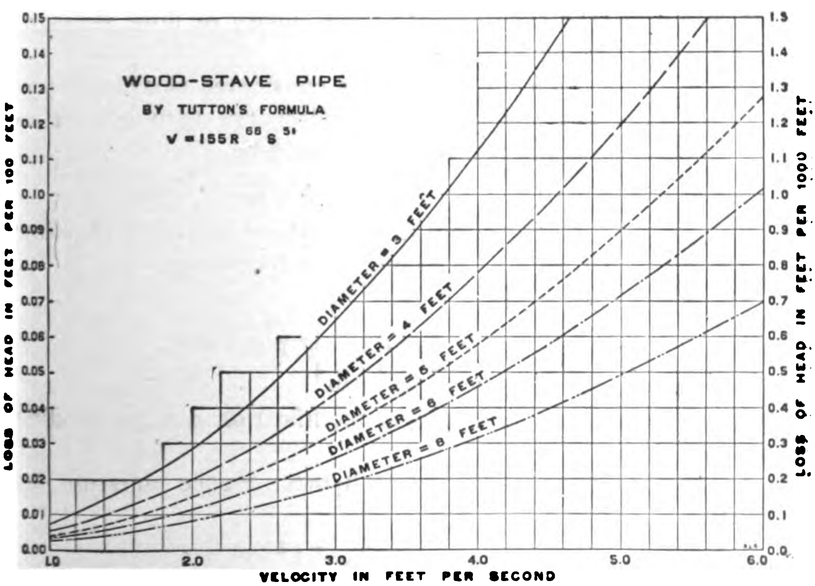

WOOD-STAVE PIPE
BY TUTTON'S FORMULA
V = 155R.66 S.51
DIAMETER = 3 FEET
DIAMETER = 4 FEET
DIAMETER = 5 FEET
DIAMETER = 6 FEET
DIAMETER = 8 FEET
LOSS OF HEAD IN FEET PER 100 FEET
LOSS OF HEAD IN FEET PER 1000 FEET
VELOCITY IN FEET PER SECOND

Fig. 28.

elements of the problem. The formulas offered by Tutton seem to agree well with the actual results of experiments and several diagrams based thereon are shown in the following pages. In two of these diagrams (Figs. 26 and 27) the limiting values are shown and the results obtained from any pipe of the character represented therein should lie between these limits depending on its condition.

44. The Flow of Water Through Orifices.—It is found that water flowing through an orifice in the side of a vessel acquires a velocity practically equal to that which would be acquired by a falling body in passing through a space equal to the head above the center of the opening, i. e.,

$$v = \sqrt{2gh} = 8.025\sqrt{h} \qquad (44)$$

in which

v=velocity of spouting jet.
g=acceleration of gravity=32.2.
h=head on opening.

The discharge through the opening would therefore be (45) $q = va = a\sqrt{2gh}$ or practically (46) $q = ca\sqrt{2gh}$ where c is a coefficient varying with the size and shape of the orifice and with various other factors.

A more accurate determination of the theory of flow through a given orifice is derived as follows:

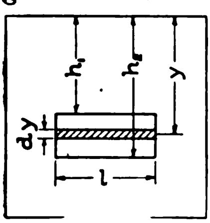


Fig. 29.

If a thin opening is considered at a depth y below the surface the discharge through the elementary section $l\,dy$ would be

$$dq = l\,dy\sqrt{2gy} \qquad (47)$$

Integrating this equation between the limits h_2 and h_1 we obtain the following:

$$q = \tfrac{2}{3}l(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})\sqrt{2g} \quad \text{or practically} \qquad (48)$$

$$q = m\tfrac{2}{3}\,l\sqrt{2g}\,(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}) \qquad (49)$$

m being the coefficient of practical modification due to condition of the orifice.

45. Flow Over Weirs.—In a weir h_1=o. Hence equation (49) becomes

$$q = m\left(\tfrac{2}{3}\right) l \sqrt{2g}\, h^{\frac{3}{2}} \qquad (50)$$

in which h is the head on the crest of the weir. That is, the vertical distance from the water level above to the crest of the weir.

For practical use the coefficient m together with the constants $\frac{2}{3}$ and 2g are combined as follows:

$$c = m \tfrac{2}{3}\sqrt{2g} = M\sqrt{2g} \text{ and equation (50) becomes}$$

$$(51) \qquad q = c\, l h^{\frac{3}{2}}$$

The value of m and consequently of c varies with the shape of the weir and with other factors and must be determined experimentally. This has been done with weirs of many forms, both by Bazin in France and by Rafter and Williams at the Cornell hydraulic laboratory. The results of these experimental determinations are given by Figs. 30 to 34, inclusive. These figures are reduced directly from the diagrams of Mr. Rafter in the Report of the Board of Engineers of Deep Waterways, 1900.

In practice many weir formulas are in use, based on various experiments and observations. The formula of Francis', equation (52), is probably the best known in this country. It is best adapted to long, sharp crested weirs without end contractions.

$$(52) \qquad q = 3.33\, l h^{\frac{3}{2}}$$

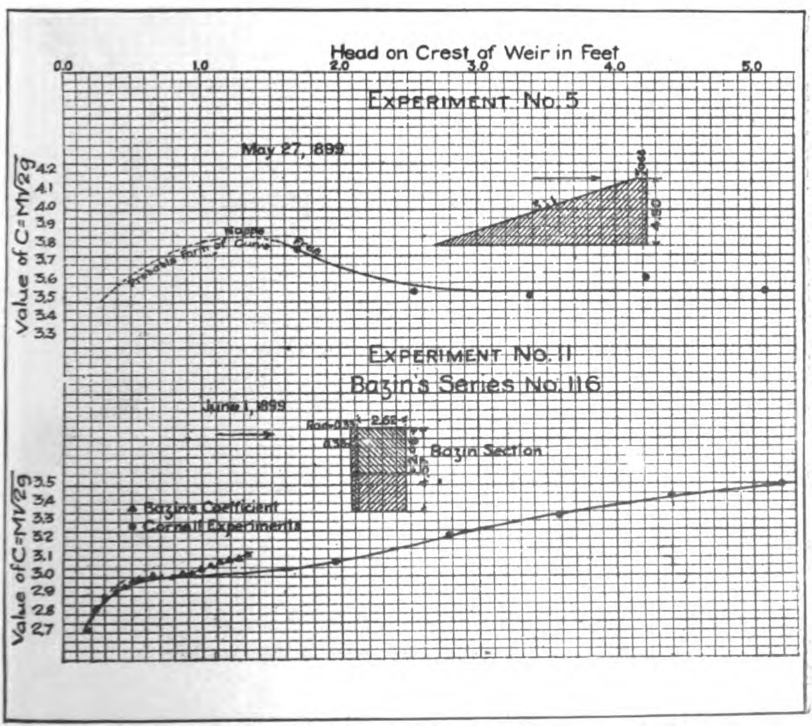


Fig. 30.—Weir Coefficients for Weirs of Various Shapes.

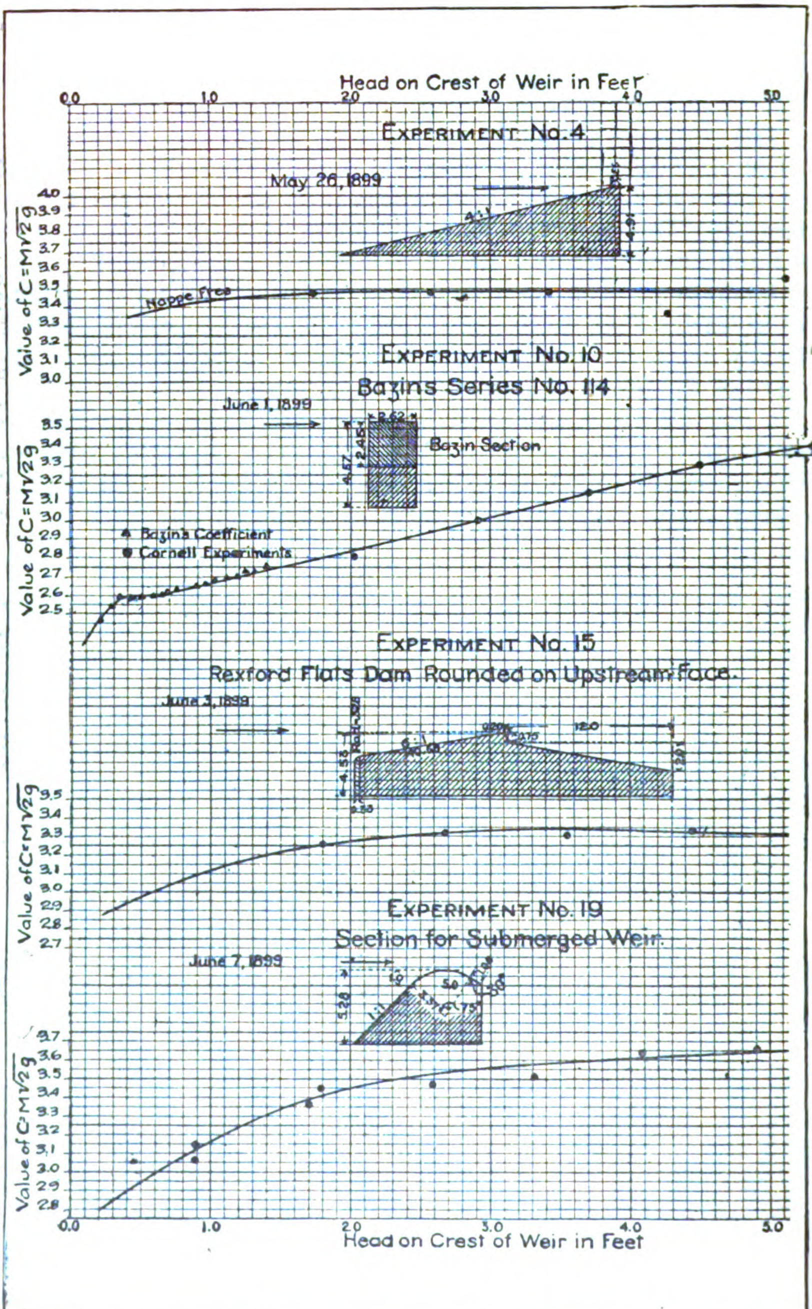


Fig. 31.—Weir Coefficients for Weirs of Various shapes.

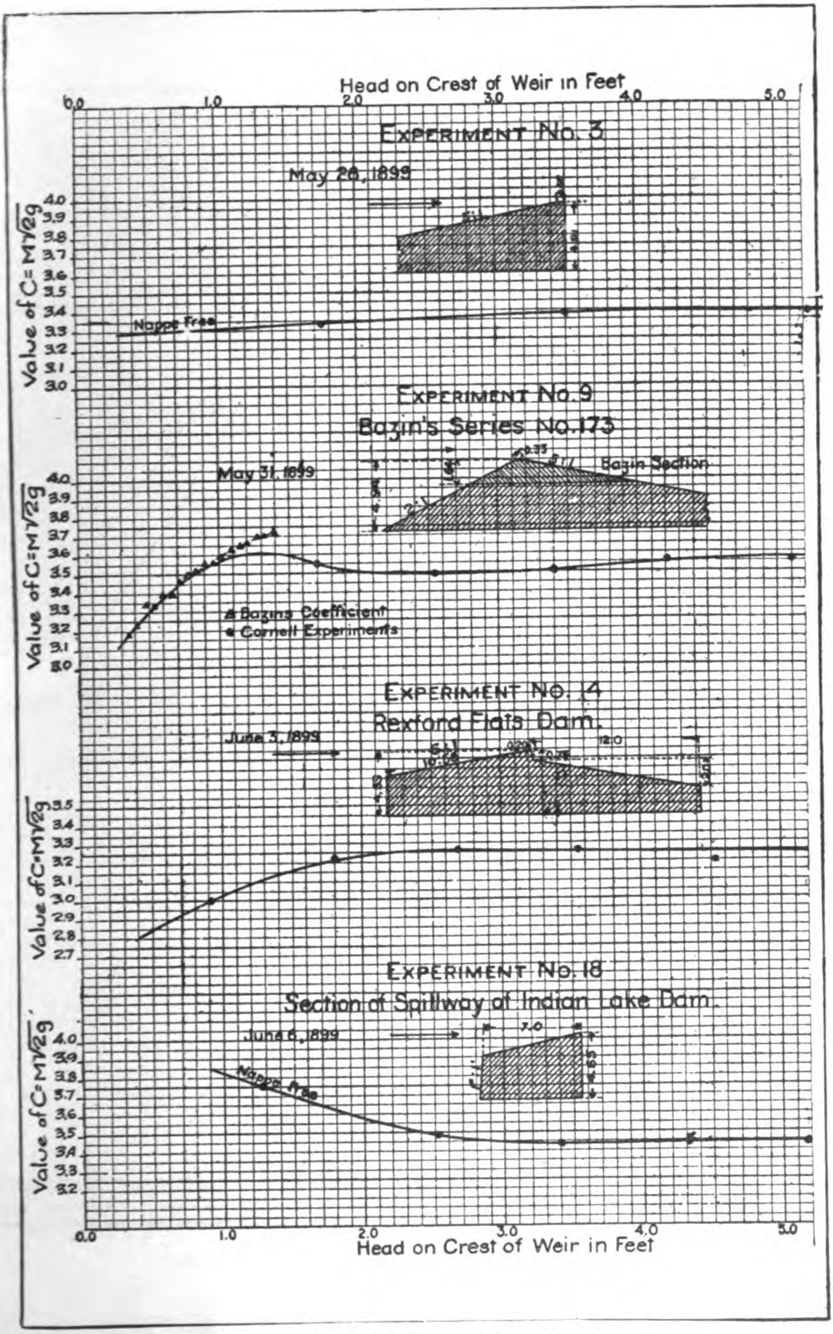
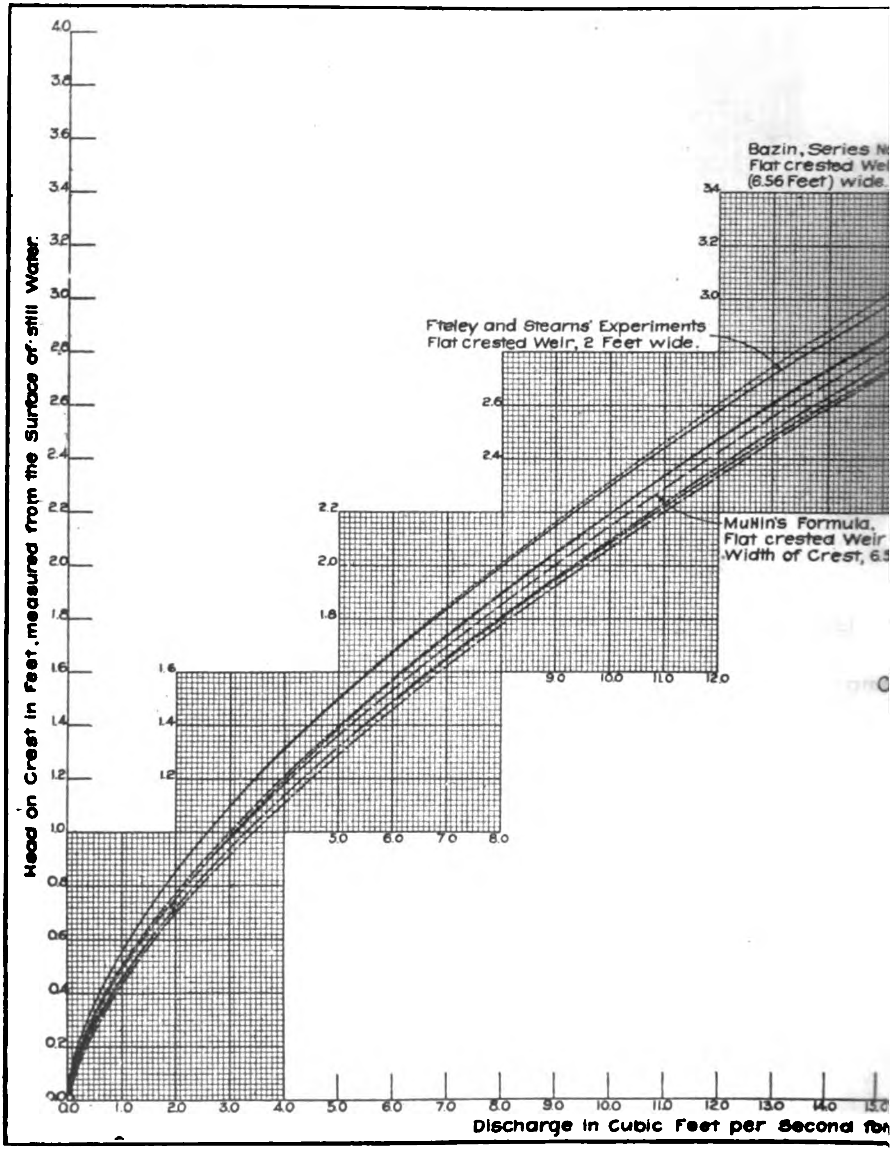
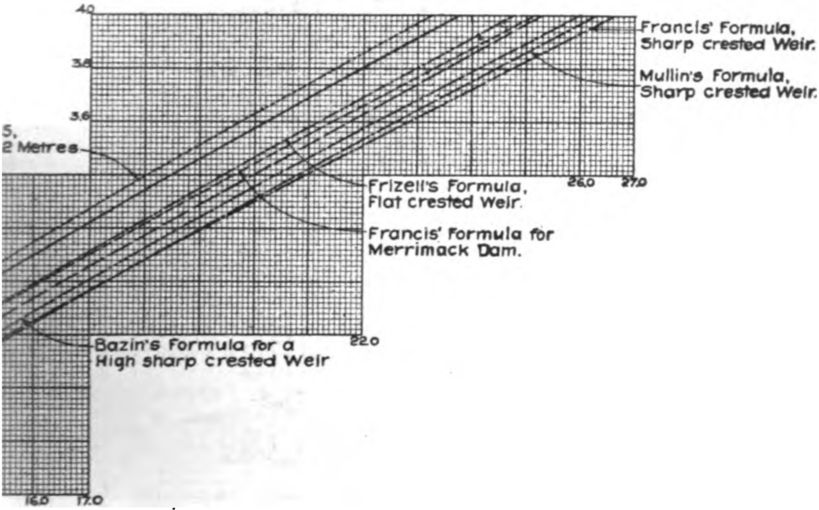


Fig. 32.—Weir Coefficients for Weirs of Various Shapes.



Head on Crest in Feet, measured from the Surface of still Water.
Bazin, Series N
Flat crested We
(6.56 Feet) wide.
Fteley and Stearns' Experiments
Flat crested Weir, 2 Feet wide.
Mullin's Formula.
Flat crested Weir
Width of Crest, 6.5
Discharge in Cubic Feet per Second for

Figure



'eet

U.S. BOARD OF ENGINEERS ON DEEP WATERWAYS
WATER SUPPLY DIVISION.

mparative Discharge over Weirs, by different Formulae,
For a Single Foot of Crest.

Bazin's Formula, $Q = mLH\sqrt{2gH}$.

Q = Discharge in Cubic Feet per Second
L = Length of Crest in Linear Feet.
H = Head on Crest measured from the Surface of still Water, in Feet.
m = Coefficient of Discharge, derived by Experiment for each Form of Weir.

Francis Formula for a Sharp Crested Weir.
$Q = 3.33\,LH^{3/2}$.

Francis Formula for Dam on the Merrimac River, at Lawrence, Mass.
$Q = 3.01208\,LH^{1.53}$

Frizell's Formula for a Flat Crested Weir
$Q = 3.09\,LH^{3/2}$

Mullin's Formula, used by East Indian Engineers
$Q = 5.35\,LCH^{3/2}$

For a Sharp Crested Weir
$C = 1 - \left(\frac{0.04(34.6 + H)}{2}\right)$
$= 0.654 - 0.01H$

For a Flat Crested Weir.
$C' = C - \left(\frac{0.025C(10 + H)}{H + 1}\right)$

To accompany Report on Special Water Supply Investigation
Geo. W. Rafter.

16.0 17.0 18.0 19.0 20.0 21.0 22.0 23.0 24.0 25.0 26.0 27.0
one linear Foot of Crest.

H.D. Alexander

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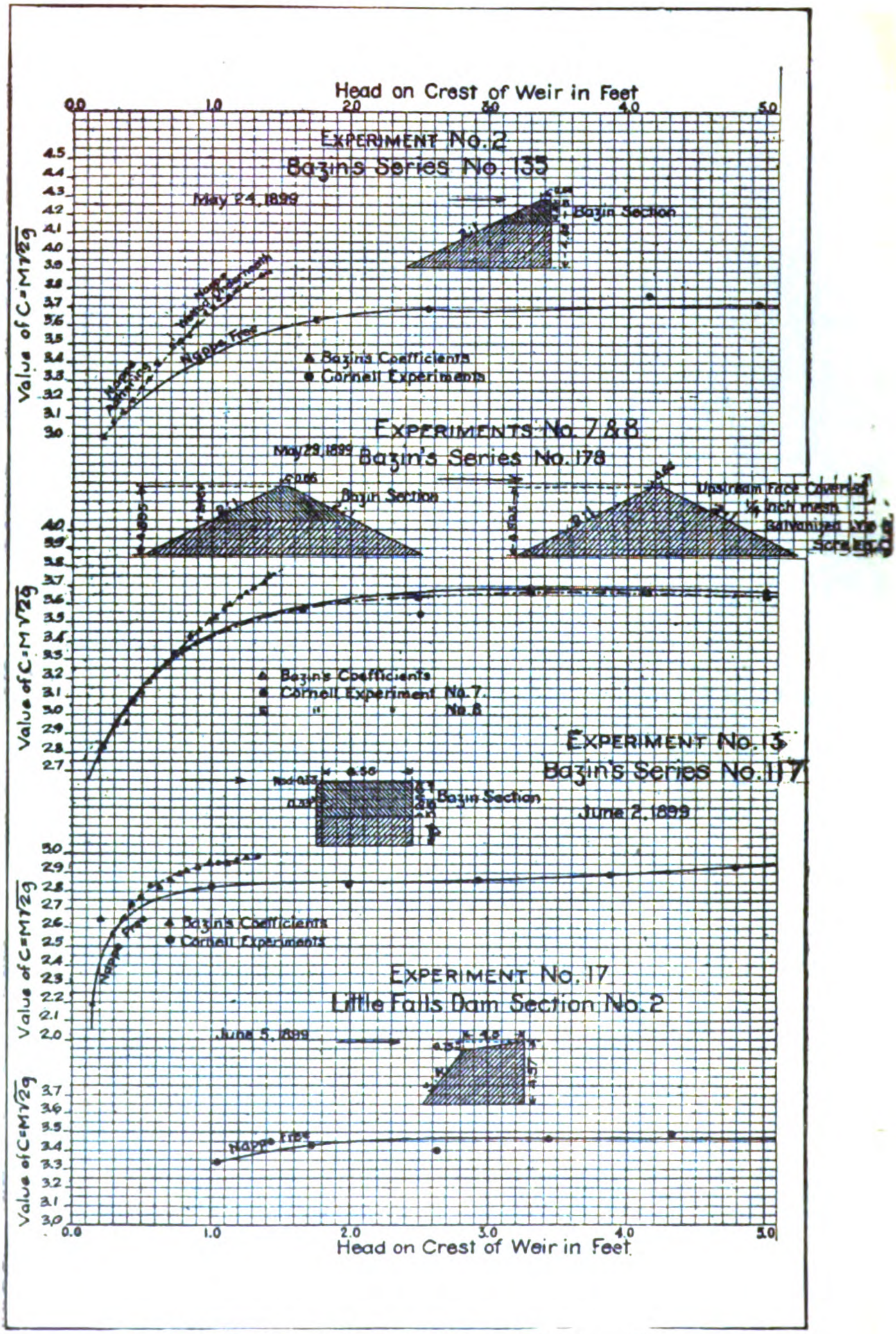


Fig. 33.—Weir Coefficients for Weirs of Various Shapes.

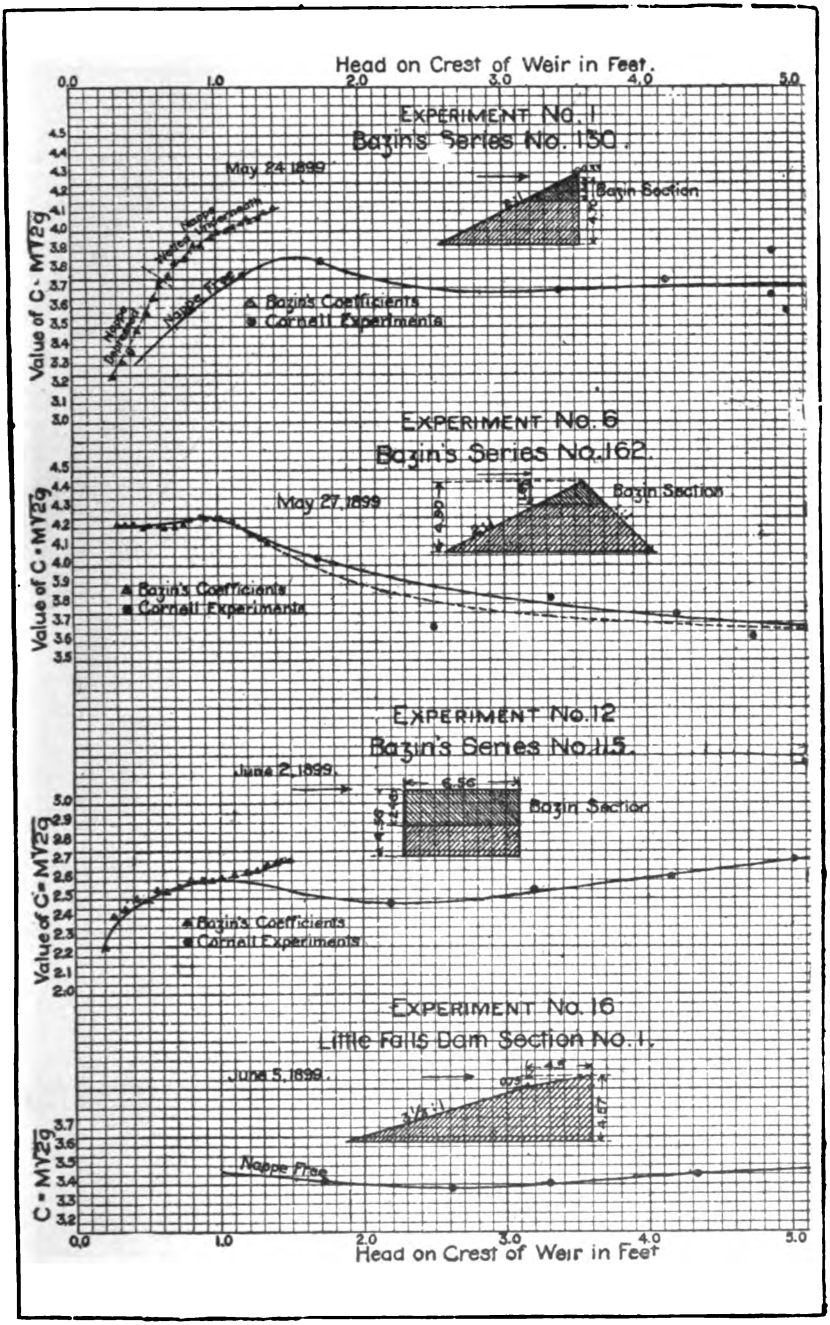
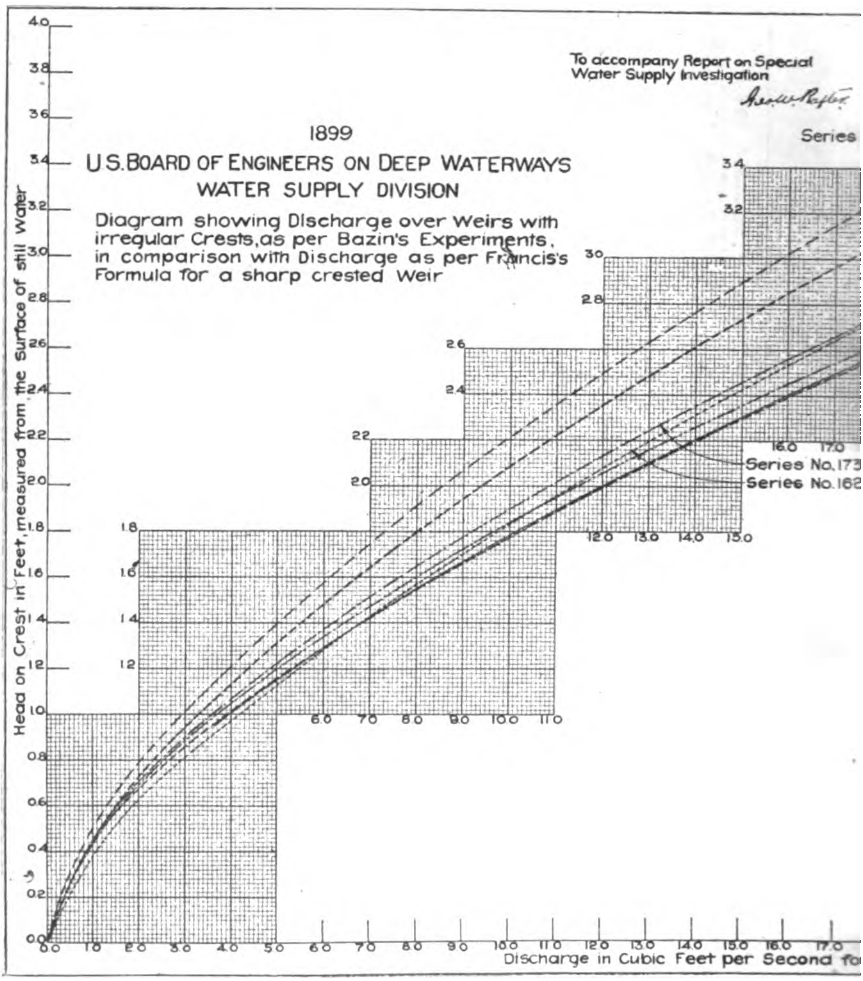
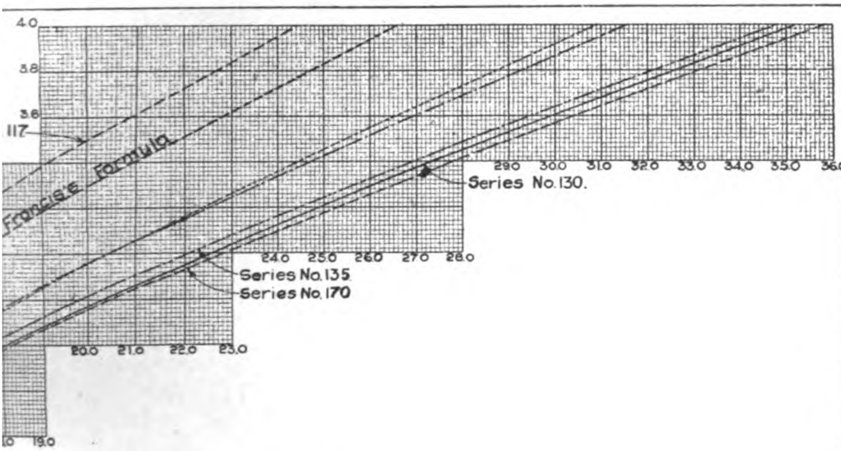


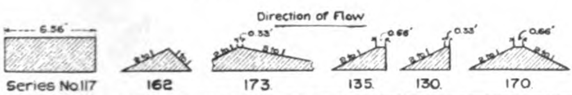
Fig. 34.—Weir Coefficients for Weirs of Various Shapes.


To accompany Report on Special
Water Supply Investigation
1899
U.S. BOARD OF ENGINEERS ON DEEP WATERWAYS
WATER SUPPLY DIVISION
Diagram showing Discharge over Weirs with irregular Crests, as per Bazin's Experiments, in comparison with Discharge as per Francis's Formula for a sharp crested Weir
Head on Crest in Feet, measured from the Surface of still Water
Discharge in Cubic Feet per Second
Series
Series No. 173
Series No. 162

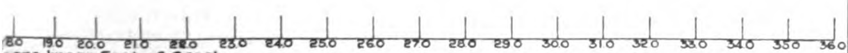
Figu



Sections of experimental Weirs.



Head on Crest of Weir in Feet	Discharge over Weir, for a length of one Foot, in Cubic Feet per Second.(Approximate)						
	Series No.117	162	173	135	130.	170	Francis's Formula Sharp Crested Weir
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.98	1.42	1.16	1.20	1.22	1.20	1.18
1.0	2.95	4.20	3.58	3.70	3.96	3.90	3.33
1.5	5.55	7.50	6.86	7.20	7.60	7.62	6.12
2.0	8.56	11.35	10.85	11.45	12.00	12.05	9.42
2.5	12.05	15.68	15.40	16.50	17.00	17.16	13.16
3.0	15.82	20.40	20.38	22.04	22.56	22.85	17.30
3.5	20.00	25.43	25.80	28.15	28.60	29.10	21.90
4.0	24.45	30.90	31.55	34.75	35.15	35.88	26.64



one linear Foot of Crest.

H. D. Alexander del.

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A number of different formulas for the flow over weirs are given on Fig. 35 and the flow as calculated by these formulas is shown on the diagram. L in these formulas represents the length of the weir crest which in the dimension above is represented by l.

Figure 36 shows graphically the results of the application of the value of c as given on Figs. 30 to 34 as compared with Francis formula.

In small weirs the effect of end contraction and of the velocity of approach becomes important and corrections to the formulas must be applied in order to allow for those influences.

If n = the number of end contractions and the effect of each is to reduce the effective length of the weir by one-tenth the head on the weir, equation (51) will become

$$q = c\left(l - n\frac{h}{10}\right) h^{\frac{3}{2}} \tag{53}$$

The effect of the velocity of approach, for a given quantity, is to reduce the head on the weir by the velocity head. This reduction is given by the formula:

$$h' = \frac{v'^2}{2g} \tag{54}$$

in which v′ = velocity of approach and h′ = velocity head.

TABLE VII.

Coefficient of discharge C for use with Hamilton Smith, Jr.'s formula (56) for flow of water over sharp crested weirs having full contraction.

l = length of weir.

Effective head = h	.66	1(?)	2	2.6	3	4	5	7	10	15	19
.1	.632	.639	.646	.650	.652	.653	.653	.654	.655	.655	.656
.15	.619	.625	.634	.637	.638	.639	.640	.640	.641	.642	.642
.2	.611	.618	.626	.629	.630	.631	.631	.632	.633	.634	.634
.25	.605	.612	.621	.623	.624	.625	.626	.627	.628	.628	.629
.3	.601	.608	.616	.618	.619	.621	.621	.623	.624	.624	.625
.4	.595	.601	.609	.612	.613	.614	.615	.617	.618	.619	.620
.5	.590	.596	.605	.607	.608	.610	.611	.613	.615	.616	.617
.6	.587	.593	.601	.604	.605	.607	.608	.611	.613	.614	.615
.7	.585	.590	.598	.601	.603	.604	.606	.609	.612	.613	.614
.8	......	......	.595	.598	.600	.602	.604	.607	.611	.612	.613
.9	......	......	.592	.596	.598	.600	.603	.606	.609	.611	.612
1.0	......	......	.590	.593	.595	.598	.601	.604	.608	.610	.611
1.1	......	......	.587	.591	.593	.596	.599	.603	.606	.609	.610
1.2	......	......	.585	.589	.591	.594	.597	.601	.605	.608	.610
1.3	......	......	.582	.586	.589	.592	.596	.599	.604	.607	.609
1.4	......	......	.580	.584	.587	.590	.594	.598	.602	.606	.609
1.5	......	......	......	.582	.585	.589	.592	.596	.601	.605	.608
1 6	......	......	......	.580	.582	.587	.591	.595	.600	.604	.607
1.7	......	......	......	......	......	......	......	.594	.599	.603	.607
2.0	......	......	......	......	......	......	......	......	......	....	....

To allow for the influence of velocity of approach h′ must be added to h and equation (53) becomes

$$q = c\left(1 - n\frac{h}{10}\right)(h + h^1)^{\frac{3}{2}} \tag{55}$$

Experimental results at the hydraulic laboratory of the University of Wisconsin show that for small sharp crested weirs, with end contraction, the formula (56) of Hamilton Smith, Jr., is practically correct:

$$q = c\,\tfrac{2}{3}\sqrt{2g}\,lh^{\frac{3}{2}} \tag{56}$$

In this formula

c=coefficient of discharge (to be taken from Table VII).

h=observed head on crest (H) plus correction due to velocity of approach.

Variations in the forms of the crest of weirs and in the arrangement of sides and bottom of the channel of approach cause considerable variation in their discharging capacity. It is therefore apparent that unless the conditions closely agree with those on which experimental data is available that the error of calculation may be considerable.

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CHAPTER IV.

WATER POWER.

THE STUDY OF THE POWER OF A STREAM AS AFFECTED BY FLOW.

46. **Source of Water Power.**—Water power depends primarily on the flow of the stream that is being considered for power purposes, and on the head that can be developed and utilized at the site proposed for the power plant. Both head and flow are essential for the development of water power, but both are variable quantities which are seldom constant for two consecutive days at any point in any stream. The variations in head and flow radically affect the power that can be generated by a plant installed for power purposes. These variations also greatly affect the power that can be economically developed from a stream at any locality. The accurate determination of both head and flow therefore becomes very important in considering water power installations and hence should receive the careful consideration of the engineer. The neglect of a proper consideration of either or both of these factors has frequently been fatal to the most complete success of water power projects.

47. **Factors of Stream Flow.**—The quantity of water flowing in a stream at any time, which is more briefly termed "stream flow" or "run-off," depends primarily upon the rainfall. It is, however, influenced by many other elements and conditions. It depends not only upon the total quantity of the yearly rainfall on the drainage area, but also on the intensity and distribution of the rainfall throughout the year. In addition to these factors the geological structure of the drainage area, the topographical features, the surface area of the catchment basin, the temperature, the barometric condition, all influence and modify the run-off. Sufficient data is not available for a full understanding of this subject, but enough is available so that the general principles involved can be intelligently discussed and the problems considered in such a way as to give a fairly satisfactory basis for practical work. A knowledge of the importance of the factors above mentioned and the extent to which they modify, influence or control stream flow, is essential

to a broad knowledge of water power engineering. These factors are discussed in more detail in chapters VI, VII and VIII.

48. **Broad Knowledge of Stream Flow Necessary.**—The flow of a stream is constantly changing and any single measurement of that flow will not furnish sufficient data on which to base an intelligent estimate of the extent of its possible or even probable economical power development. A knowledge of the economical possibilities of such development must be based upon a much broader knowledge of the variations that take place in the flow of the stream. In order to fully appreciate the power value of a stream, the character and extent of its daily fluctuations must be known or estimated. Averages for the year, monthly averages, and estimates of average power have been ordinarily taken as a basis for water power estimates, but they are more or less misleading, unsatisfactory and uncertain for the reason that such averages include extremes, the maximum of which are often unavailable for water power purposes without more extensive pondage than is usually practicable. These maximum and minimum flows which affect the power of a stream not only through the quantity flowing but also through the head as well, as will be hereafter discussed, are of the utmost importance for a broad consideration of water power. So also is a knowledge of the various stages of flow and the length of time that each will prevail. Such knowledge demands daily observations or estimates of daily flow which can be represented in graphical form by the hydrograph.

49. **The Hydrograph.**—The hydrograph, constructed for the study of stream flow and its influence on water power, may be drawn by representing the daily flow in cubic feet per second at the point of observation by the ordinates of the diagram and the element of time by the abscissas. (See Fig. 37.) The result is a graphical diagram which shows the character and extent of the daily fluctuations in the flow of a stream at the point of observation during the period for which the hydrograph has been prepared.

A single observation of the flow of a stream represents a totally inadequate and unsatisfactory criterion for water power consideration. By reference to Fig. 37 it will be seen that, if the discharge of the Wisconsin River at Necedah had been measured only on August 5, 1904, the conclusion would have been reached that the discharge of the river was about 2,100 cubic feet per second. If the measurement had been taken only on August 15, 1904, the flow would have been determined at about 5,850 cubic feet per second, or almost three times as great as on the first date. The

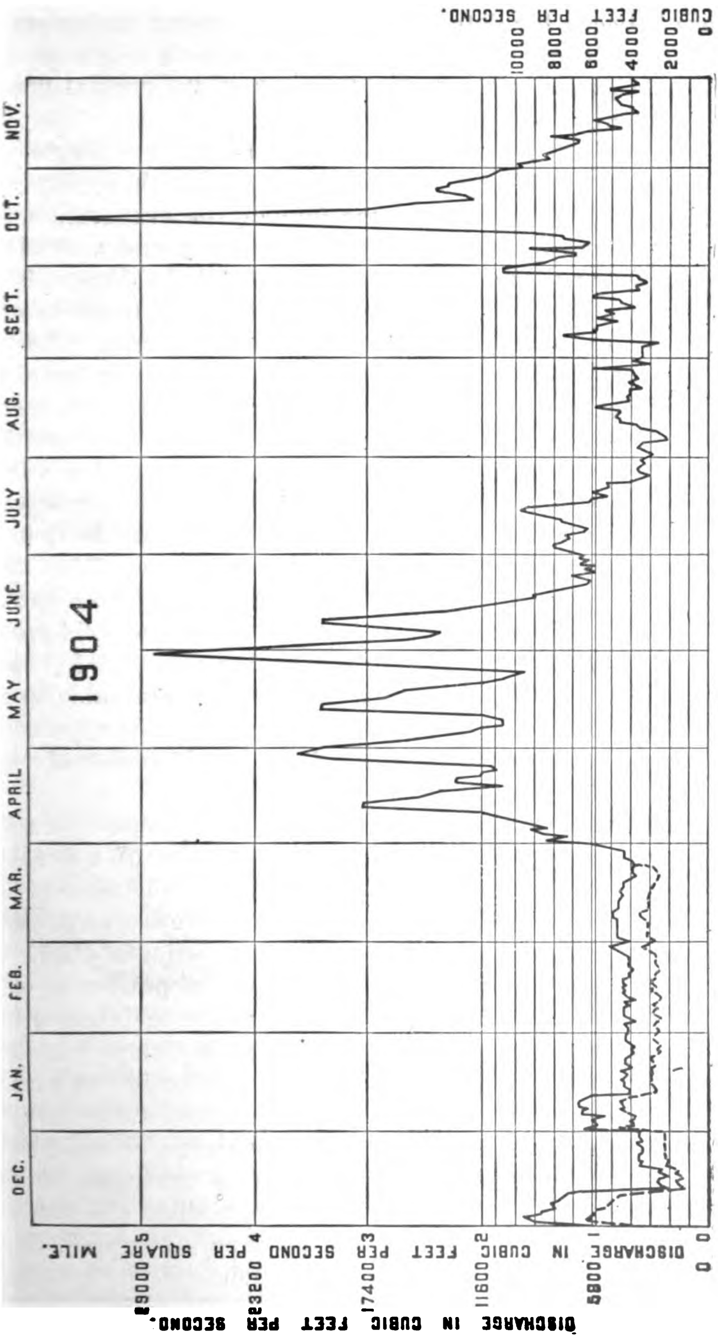


Fig. 37.—Hydrograph of the Wisconsin River at Necedah, Wis.

difference between the dates might be even greater, and no single measurement nor any series of measurements for a single week or month would give a fair criterion from which the normal flow of the river could be judged.

The hydrograph of the daily flow of a river for a single year gives a knowledge of the variation in flow for that year only, under the peculiar conditions of the rainfall, the evaporation, and the other physical factors that modify the same and that obtain for that particular year. Such information, while important, is not altogether sufficient for the purpose of a thorough understanding of the availability of the stream flow for power purposes. Observations show that stream flow varies greatly from year to year, and while, with a careful study of the influences of the various factors on stream flow, together with a knowledge of the past variations in such factors, the hydrograph for a single year may give a fairly clear knowledge of the variations to be expected in other years where conditions differ considerably, still it is desirable that the observations be extended for as long a period as possible. Such long time observations may remove the estimates of flow entirely from the domains of speculation and place them on the solid ground of observed facts. Hydrographs of a river that cover the full range of conditions of rainfall, temperature, etc., which are liable to prevail on its drainage area, give a very complete knowledge of the flow of the stream for the purpose of the consideration of water power.

It is rare, however, that observations of stream flow for a long term of years are available at the immediate site of a proposed power plant. Such observations are ordinarily made only at locations where power has been developed and where water power or similar interests have been centered for a long period of time. Occasionally, however, the future value of potential powers is recognized and appreciated, and local observations are maintained for a series of years by interested parties, having a sufficient knowledge of the subject to recognize the value and importance of such information. The variation of flow for some considerable time previous to construction is thus available upon which to base the design.

In considering new installations, one of four conditions obtains:

First: Hydrographs are available at the immediate site proposed

Second: Hydrographs are available at some other point on the river above or below the proposed installation.

Third: Hydrographs are not available on the river in question but are available on other rivers where essentially similar conditions of rainfall and stream flow prevail.

Fourth: No hydrographs, either on the river in question or on other rivers of a similar character and in the immediate vicinity, are available.

50. The Use of Local Hydrographs.—When hydrographs, constructed from observations taken at the immediate site of the proposed water power installation, are obtainable, for a considerable number of years, the most satisfactory character of information is available for the consideration of a water power project. Under such conditions the engineer is not obliged to consider the relation of rainfall to run-off or to speculate as to the relative value of the stream in question compared with other adjacent streams, or as to the effects of the physical conditions of drainage area, evaporation, temperature and other factors on stream flow. The actual daily flow of the stream from day to day, perhaps through all ranges of rainfall, temperature, evaporation and other physical conditions, is known and the principal points which must be considered are: First, the head available; Second, the effects of the variations of flow on the variations in head; and Third, the extent to which the flow can be economically developed or utilized. Generally, however, even where local hydrographs are available, they are not sufficiently extended to cover all the variations in river flow which must be anticipated, and it is ordinarily desirable to compare the available data with the flow at other points on the stream in question or with other streams in the immediate vicinity.

51. Use of Comparative Hydrographs.—Hydrographs taken at other points on the same river, or on other adjacent rivers where conditions are reasonably similar, are of great value in considering the local stream flow,—provided all modifying conditions are understood and carefully considered. Hydrographs are ordinarily prepared to show the cubic feet per second of actual flow at the point at which observations are made. If the observations (and the hydrographs based thereon) made at some other point on a stream, or on some other streams, are to be used for the consideration of the flow at a point where a water power plant is to be installed or considered, the relation of the flows at the several points must be determined.

As a basis for such comparison of stream flow, it may be assumed that the flow per unit of area at different points on the same

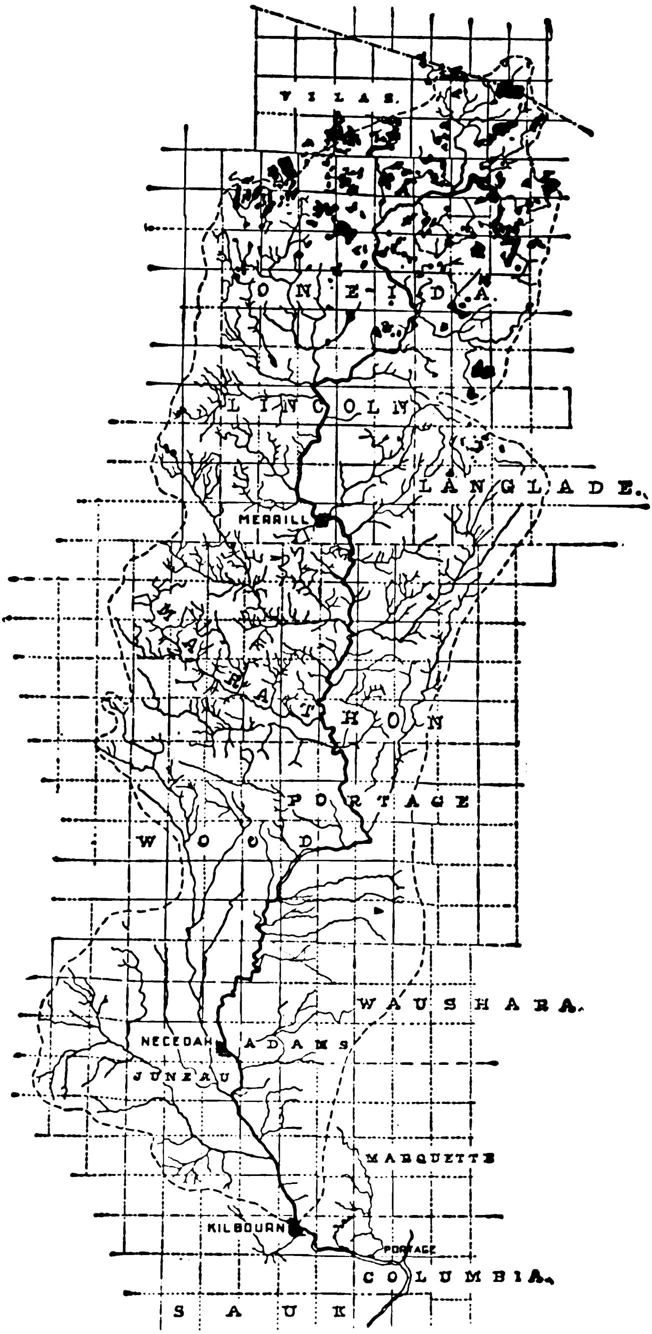


Fig. 38.—Drainage Area of Wisconsin River Above Kilbourn, Wis.

stream, or at points on different streams under similar circumstances, is essentially the same. This is not strictly true, or perhaps it may be more truly said that the apparent similarity of conditions is only approximate and hence differences in results must necessarily follow. For a satisfactory consideration of the subject of comparative hydrographs, the variations from this assumption, as discussed in another chapter, must be understood and appreciated. For practical purposes, however, the assumption is often essentially correct and forms a basis for an intelligent consideration of stream flow where local hydrographs are not available. Fig. 37 is a hydrograph constructed from observations made on the Wisconsin River at Necedah, Wisconsin, by the U. S. Geological Survey for the water year, 1904, and shows the daily rate of discharge of the Wisconsin River at that point for the year named. The area of the Wisconsin River (see Fig. 38) above Necedah is 5,800 square miles. If, therefore, we draw a horizontal line from the point representing 5,800 cubic feet per second on the discharge scale (see Fig. 37), the line so drawn will represent a discharge at Necedah of one cubic foot per second per square mile of drainage area, and a similar line drawn from the 11,600 cubic foot point on the vertical scale will represent a discharge of two cubic feet per second per square mile, and so on. These lines may be fairly regarded not only as indicating the flow per unit of area of the river at Necedah, but also the relative flow per unit of area of the Wisconsin River at points not greatly distant therefrom. At Kilbourn, (see Fig. 38) located on the same river about forty miles below Necedah, the flow may be assumed to be similar and proportionate to the flow at Necedah. Above Kilbourn the drainage area is 7,900 square miles, and with similar flow the discharge would be proportionately greater. The fact must be recognized, and acknowledged, that the hydrograph is strictly applicable only to the point at which it is taken, and that certain errors will arise in considering its application to other points, yet observations and comparisons show that, while such errors exist, they are not nearly so important as the errors which arise from the consideration of averages, either annually or monthly.

Consider, therefore, on this basis the Necedah hydrograph as shown in Fig. 37. On this diagram a flow of one cubic foot per second per square mile at Necedah, representing an actual flow of 5,800 cubic feet per second at that point, would, by proportion, represent a flow of 7,900 cubic feet per second at Kilbourn and,

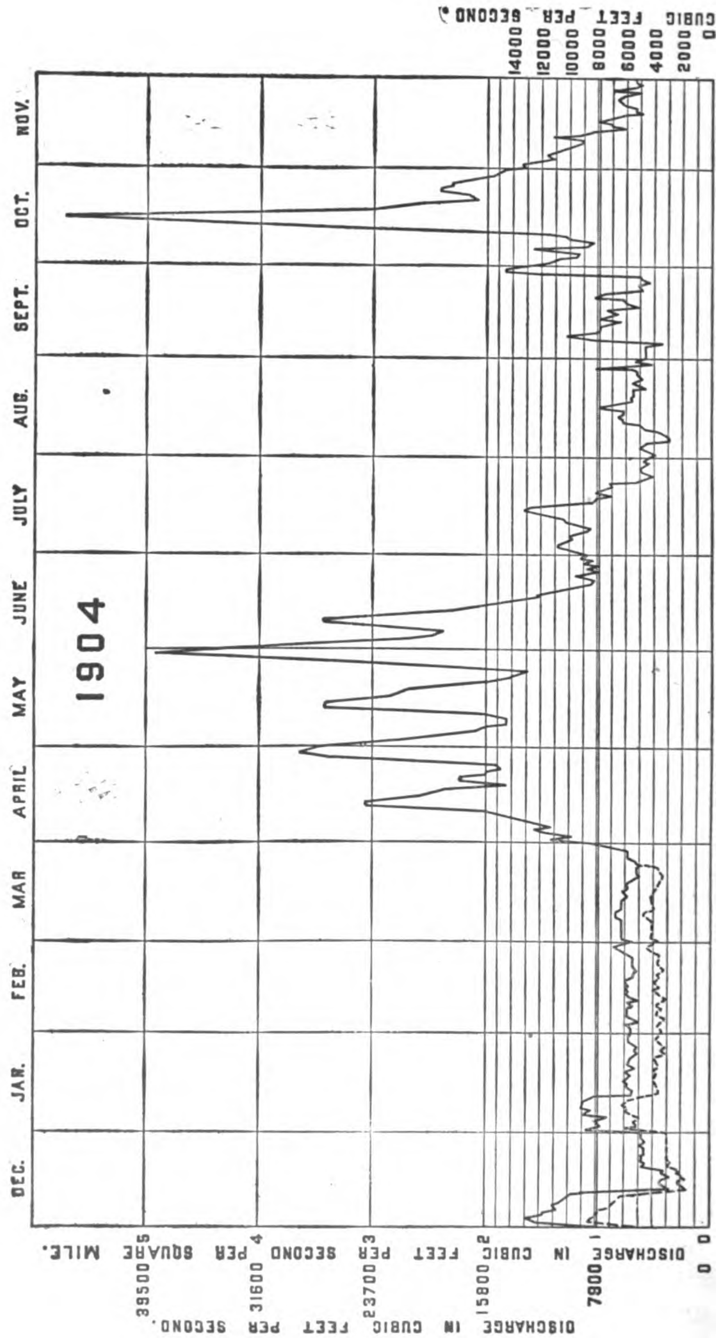


Fig. 39.—Hydrograph of Wisconsin River Based on Observations at Necedah, Wis., Adapted by Change of Scale to Conditions at Kilbourn, Wis.

with a suitable change in scale, the diagram may be redrawn to represent the flow at Kilbourn as shown in Fig. 39. This same method can be applied to any point on the same river or to comparative points on different rivers.

52. **Reliability of Comparative Hydrographs.**—It must be clearly understood that comparisons as above described hold good only as the conditions are essentially similar at the various points compared.

Stream flow at the best is very irregular and varies greatly from year to year. The actual departure from the truth can best be understood and appreciated from an actual comparison of flows on adjacent drainage areas where observations have actually been made for a term of years. From such an investigation, which can be made as extended as desirable, the true weight to be given to the comparative hydrograph can best be judged. It is not believed that the actual variations from the truth, as shown by carefully selected comparative hydrographs, will be any greater than the flow variations which actually take place from a drainage area from year to year under the varying conditions of rainfall and climate. This method, therefore, is believed to be a scientific and systematic one for the consideration and discussion of probable variations in stream flow at any given point, if its limitations and the modifying influences known to exist on different drainage areas and under different geographical, geological and meteorological conditions are known and appreciated.

53. **When no Hydrographs are Available.**—In a new country where no observations are available either on the drainage area under consideration or on other areas adjacent thereto, the study of comparative hydrographs is impossible and a different method of consideration must be used. If no data are available, time must be taken to acquire a reasonable amount of local information which should include not less than one year's observation. In addition to such observation a study as thorough as practicable should be made of the geology, topography, and other physical conditions that prevail on the water shed. Rainfall data is commonly available for a much greater range of time than the observations of stream flow. The relations of rainfall to run-off are hereafter discussed and approximate fixed relations are shown to exist between them. From such relations, and from a single year's observations, conclusions may be drawn as to the probable variations from the observed flow which will occur during the years where the rainfall

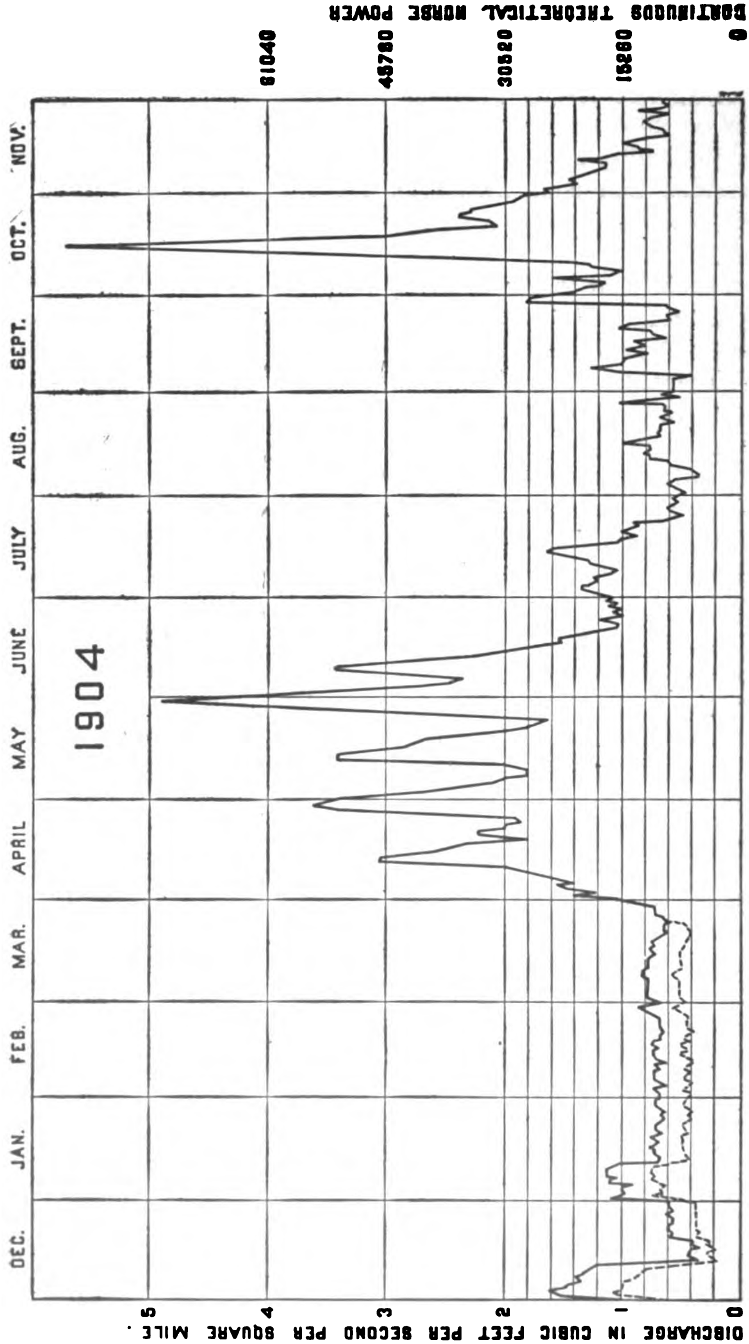


Fig 40.—Hydrograph Showing Continuous (24 Hour) Theoretical Horse Power at Kilbourn, Wis., With 17 Foot Head.

varies greatly from that of the year during which observations are available. Such conclusions are necessarily unsatisfactory, or at least much less satisfactory than conclusions based on actual stream flow. The consideration of the best information available on any project is the basis on which the engineer should always rest his conclusions, and all relations which will throw light on the actual conditions should be given careful attention. If a water power plant must be immediately constructed upon a stream concerning which little or no information is available, then the risk is proportionately greater, and safety is obtained only by building in such a conservative manner that success will be assured for the plant installed and on plans that will permit of future extensions should the conditions that afterward develop warrant an extension of the same.

54. **The Hydrograph as a Power Curve.**—The hydrograph, by a simple change in the vertical scale similar to that already considered, may also be made to show graphically the variations in the power of the stream. If, for example, at Kilbourn, a constant fall of seventeen feet be assumed, then a flow of one cubic foot per second per square mile represents a total flow of 7,900 cubic feet per second, and this flow, under 17 foot head, will give a theoretical hydraulic horse power as follows:

$$\text{H.P.} = \frac{7900 \times 17}{8.8} = 15261$$

Now if a hydrograph be constructed on such a scale that the line of flow of one cubic foot per second per square mile will also represent 15,261 horse power, the result will be a power hydrograph (see Fig. 40), which represents the continuous (24 hours per day) theoretical power of the river under the conditions named.

On account of losses in the development of power the full theoretical power of a stream cannot be developed, and hence the actual power that can be realized is always less than the theoretical power of the stream. If it is desired to consider the actual power of the stream on the basis of developing the same with turbines of 80 per cent efficiency, the line representing the flow of one cubic foot per second per square mile will represent the actual horse power to an amount determined as follows:

$$\text{A.H.P.} = \frac{7900 \times 17 \times .80}{8.8} = \frac{7900 \times 17}{11} = 12209$$

A hydrograph platted so that the line of one cubic foot per square mile will represent this amount, will represent the actual

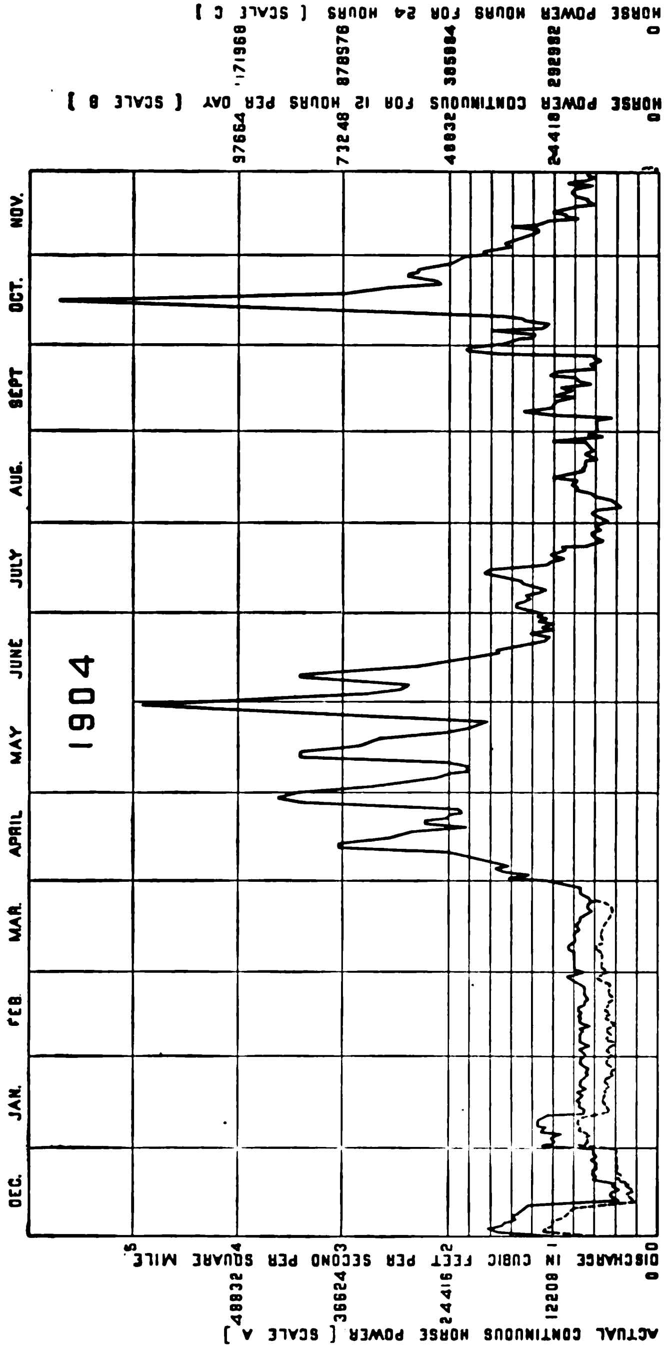


Fig. 41.—Power Hydrograph at Kilbourn, Wis. (17 Foot Constant Head).

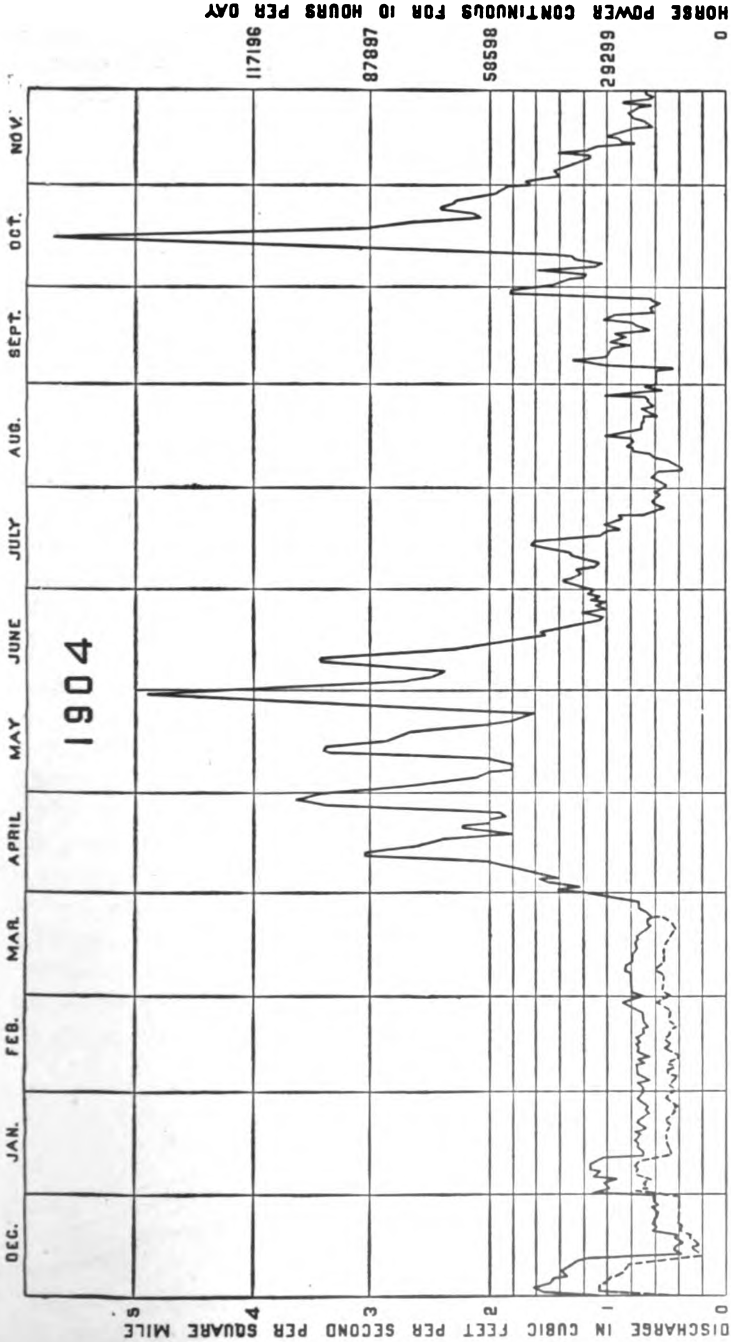


Fig. 42.—Power Hydrograph at Kilbourn, Wis. (Ten hour power with perfect pondage and seventeen foot constant head).

horse power of the river at Kilbourn with the wheels working with the efficiency and under the head named. Such a hydrograph is shown by Fig. 41, referred to by the left-hand scale (A). Power, however, is not always used continuously for twenty-four hours. If pondage is available the night flow may be stored and utilized during the day. If the flow of twelve hours at night is impounded and used during the day under the seventeen foot head, the power will be double that shown on scale A, and can be represented by another change in scale as shown by Fig. 41, referred to scale B. If the flow for the fourteen hours of night is stored and utilized in the ten hours of day, then the hydrograph can be made by another change in scale to represent the ten hours power as shown by Fig. 42.

The total horse power hours which are available from a stream for each day may be represented (either theoretically or actually) by multiplying the scale of continuous power by 24. The actual horse power available at Kilbourn under the conditions named is represented by scale C in Fig. 41. It will be noted that by pointing off one place in the figures of scale C, Fig. 41, the hydrograph will represent the same condition as shown in Fig. 42.

CHAPTER V.

WATER POWER (Continued.)

THE STUDY OF THE POWER OF A STREAM AS AFFECTED BY HEAD.

55. **Variations in Head.**—In the previous chapter the graphical representation of stream flow has been considered. A method for the expression of the power resulting from the fluctuations of stream flow and under a constant head has also been shown. Experience shows, however, that such a condition seldom if ever occurs. In some cases where the available head is a very large element of the possible power, the fluctuations may be so small as to be of little or no importance. In many other cases where the available heads are considerable, the importance of the fluctuation in head is comparatively small, under which condition the diagrams already discussed are essentially correct and are satisfactory for the consideration of the varying power of the stream. In power developments under the low heads available in many rivers, the fluctuation in head is almost or quite as influential on the continuous power that may be economically developed from a stream as the minimum flow of the stream itself.

The hydraulic gradient of a stream varies with the quantity of water flowing. At times of low water the fall available in almost every portion of its course is greater than is necessary to assure the flow between given points and frequent rapids result (see R. R. Fig. 43) which are commonly the basis for water power develop-

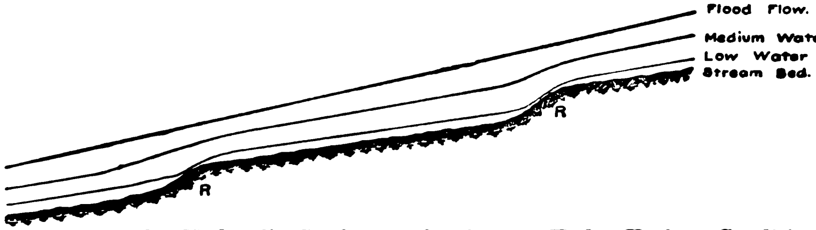


Fig. 43.—Hydraulic Gradients of a Stream Under Various Conditions of Flow.

ments. As the flow increases, however, a higher gradient and greater stream section is necessary in order to pass the greater quantity of water, and the rapids and small falls gradually become obscured (as shown by the medium water lines, Fig. 43) or disappear entirely under the larger flows (as shown by the higher water line, Fig. 43). Water power dams concentrate the fall of the

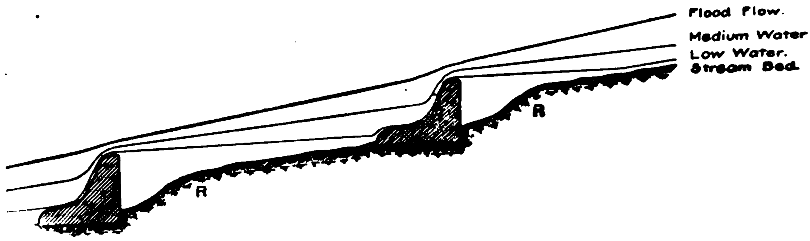


Fig. 44.—Hydraulic Gradients of the Same Stream After the Construction of Dam and Under Various Conditions of Flow.

river that is unnecessary to produce flow during conditions of low and moderate water (as shown in Fig. 44), and when the gradient of the water surface and the cross section of the stream are increased to accommodate the larger flow, the fall at such dams is frequently greatly reduced (as shown by the medium water line in Fig. 44) or, during high water, the fall is largely or completely destroyed (as shown by the high water lines in the Figure), or at least is so reduced as to be of little or no avail under practical water power conditions.

The cross section of the river bed, its physical character and longitudinal slope, are the factors which determine the hydraulic gradient of a stream under different flows. They are so variable in character and their detail condition is so difficult of determination that sufficient knowledge is seldom available, except possibly in the case of some artificial channels, to determine, with reasonable accuracy, the change of the surface gradient and cross section of the water under various conditions of flow. Where a power plant is to be installed, it is important to ascertain the relation of flow to head in order that the available power may be accurately determined. Where a river is in such condition as to make the determination of a discharge rating curve possible, either by direct river measurement at the point in question or by a comparison with the flow over weirs at some other point, such determination should be carefully made, as such knowledge is of the utmost importance in considering the problem of continuous power.

56. The Rating or Discharge Curve.—The rating curve, which will be discussed in some detail in a later chapter, is a hydrograph that represents the relation of the elevation of the water surface in a channel to the quantity of water passing a given cross section. The form of this curve varies with the various conditions of the cross section both at the immediate point and for a considerable distance above and below the location considered and can usually be determined only by detail observations. The rating curve is a uniform curve only for channels in which no radical change in form of cross section occurs with the increase of flow. (See A Fig. 45.) If, on account of overflow conditions, or sudden enlargements of the cross section, that cross section varies radically in form at a given height, then at this elevation a radical change in the slope of the rating curve is likely to occur. (See B and C Fig. 45.)

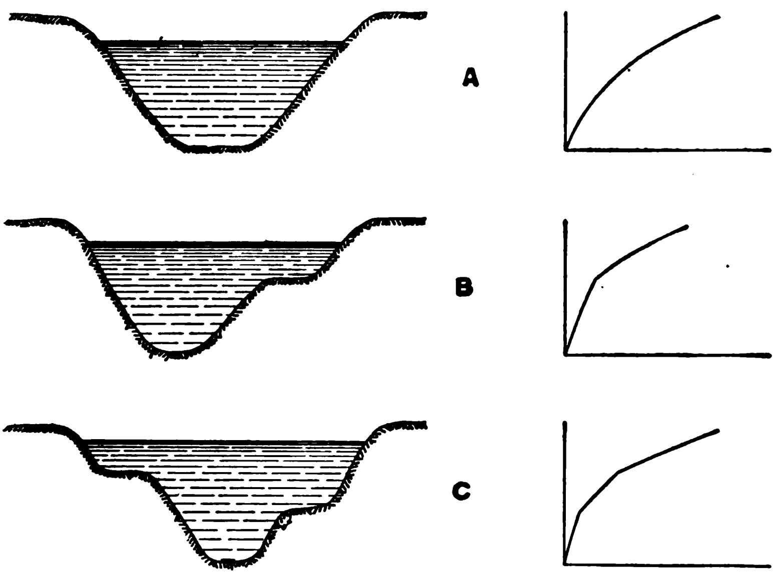


Fig. 45.—The Influence of the Stream Cross Section on the Rating Curve.

Any change in the bed of the stream may, and frequently does, modify to a considerable extent the rating curve, which must be expected to vary under such conditions to an extent that depends on the variations that take place in the cross section and elevation of the stream bed. Such variations, however, are not, as a rule, of great magnitude and consequently will not usually affect the head materially at a given point.

In Fig. 46, which shows the rating curve of the Wisconsin River at Necedah, Wis., as determined at different times during the years 1903 and 1904, an extreme change of head of about six inches will be noted for ordinary flows. When the change in head is of sufficient importance to warrant the expense, the river channel may be so dredged out as to restore the original head when the reduction in head is occasioned by the filling of the section.

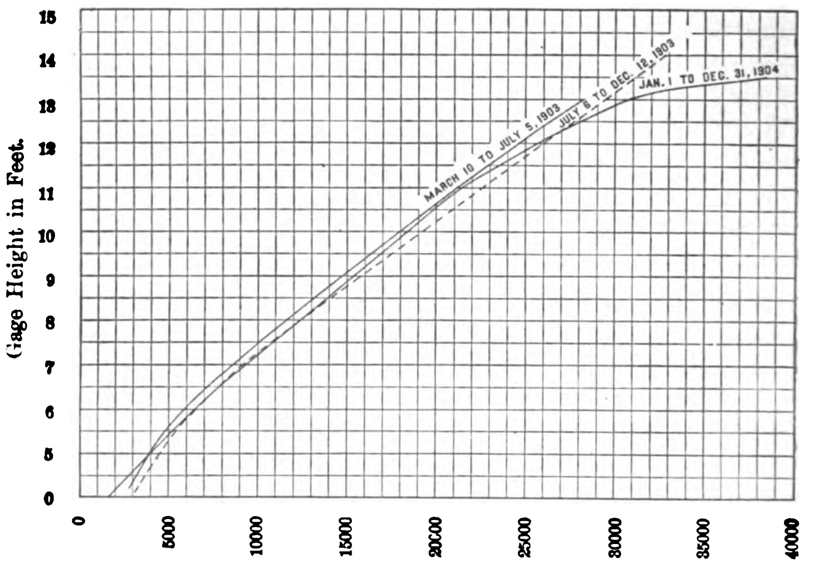


Fig. 46.—Rating Curves, Wisconsin River at Necedah, Wis., Showing Changes in Head Due to Changes in Cross Section.

57. The Tail Water Curve.—It will be readily seen that while the rating curve shows the relation between stream flow and river height prior to the construction of a dam, it will still represent the condition of flow below the dam after construction is completed. The water flowing over the dam will create a disturbed condition immediately below. If the velocity of the flow is partially checked or entirely destroyed, a heading-up of the water may result below the dam sufficient to give the velocity required to produce the flow in the river below, but it will soon reach a normal condition similar to that which existed previous to the construction of the dam.

58. The Head Water Curve.—In Chapter III is shown (see Figs. 35 and 36) the discharge curves over weirs of various forms and the formulas representing them are also quite fully discussed. From these formulas or diagrams a discharge curve can be readily calculated, with reasonable exactness, for a dam with a certain form and length of crest. Such a curve will show the height of the head waters above the dam and under any assumed conditions of flow. From the rating curve of the river at the point considered, and the discharge curve of the weir proposed, the relative positions of head and tail waters under varying conditions of discharge can be readily and accurately determined, and if a weir is to be built to a certain fixed height, it will be seen that the head under any given conditions of flow may be thus determined.

59. Graphic Representation of Head.—Fig. 47 shows the rating curve of the Wisconsin River (see lower curve marked "Tail Water

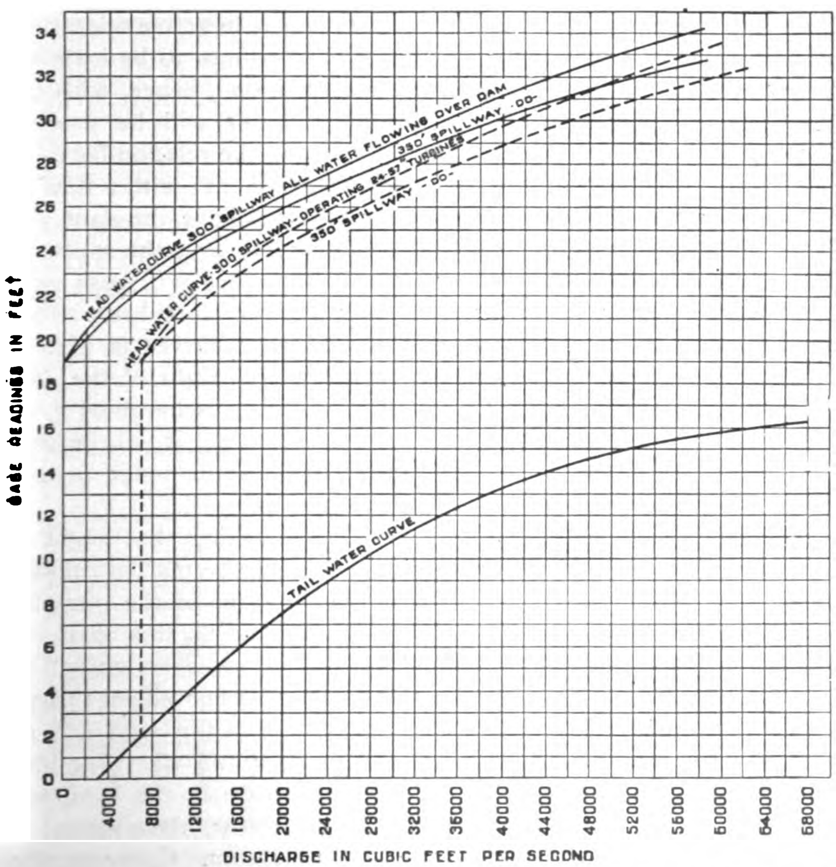


Fig. 47.—Showing Head at the Kilbourn Dam Under Various Conditions of Flow.

Curve") at Kilbourn. On this diagram has also been platted several discharge curves, two being for a weir of 300 feet in length and two for a weir of 350 feet in length. Both weir curves in the upper set are based on the assumption that the entire flow of water is passing over the weir. The crest of the dam is shown as raised to gauge 19, and the distance between the rating curve, which now represents the height of the tail water, and the weir discharge curves, which represent the height of the head water (with two different lengths of weir) under different conditions of flow, will show the heads that obtain at all times under these assumptions.

The entire discharge of the stream, however, will not pass over the dam except when the plant is entirely shut down, which would seldom be the case. The essential information which is desired therefore is the available head when the plant is in active operation. At the Kilbourn plant the discharge of the turbines to be installed under full head will be 7,000 cubic feet per second, hence, with the plant in full operation, this quantity of water will be passing through the wheels. Therefore in determining the relation between head water and tail water it must be considered that with a flow of 7,000 cubic feet per second, the water surface above the dam will be at the elevation of its crest, no flow occurring over the spillway, and that only the flows greater than this amount will pass over the dam. Another curve for each weir has therefore been added to the diagram in which the zero of the weir curves is platted from the point where the line representing the height of the dam (elevation 19) intersects the line representing a discharge of 7,000 cubic feet per second. From this diagram (Fig. 47) it will be seen that other heads, shown in Table VIII, will obtain under various conditions of flow.

It will readily be seen that the line representing the height of the dam is not essential and that the curves may be platted relative to each other, leaving the height of the dam out of the question entirely and indeterminate. A curve constructed on this basis but otherwise drawn in the same manner as in Fig. 47, is shown in Fig. 48. In Fig. 48, wherever the weir or head water curves pass above the tail water curve, it shows that an increase in the head will result under the corresponding condition of flow and wherever they pass below such curve, it shows that a decrease in the head will result under the corresponding condition of flow, the amount of which is clearly shown by the scale of the diagram. Consequently, having given the height of the dam above tail water at the point of no discharge, the head available under any other condition can be immediately determined from the diagram.

From this diagram the changes in head (as shown in table IX) can be determined and these, with a 17 foot dam, will give the total

TABLE VIII.

Gauge heights and heads available at Kilbourn Dam under various conditions of flow, with a length of spillway of 300 and 350 feet.

Flow in cubic feet per second.	Head Water.		Tail Water.	Head with	
	300 ft. dam.	350 ft. dam.		300 ft. dam.	350 ft. dam.
7000....................	19	19	2	17	17
14000....................	22.9	22.3	5.1	17.8	17.2
21000....................	25.2	24.6	8	17.2	16.6
28000....................	27	26.2	10.3	16.7	15.9
35000....................	28.5	27.7	12.2	16.5	15.5
42000....................	30.2	29.3	13.6	16.6	15.7
49000....................	31.5	30.4	14.7	16.8	15.7
56000....................	32.7	31.6	15.6	17.1	15.8

heads available under various conditions of flow as shown in the last two columns. These heads will be seen to correspond with the heads given in table VIII.

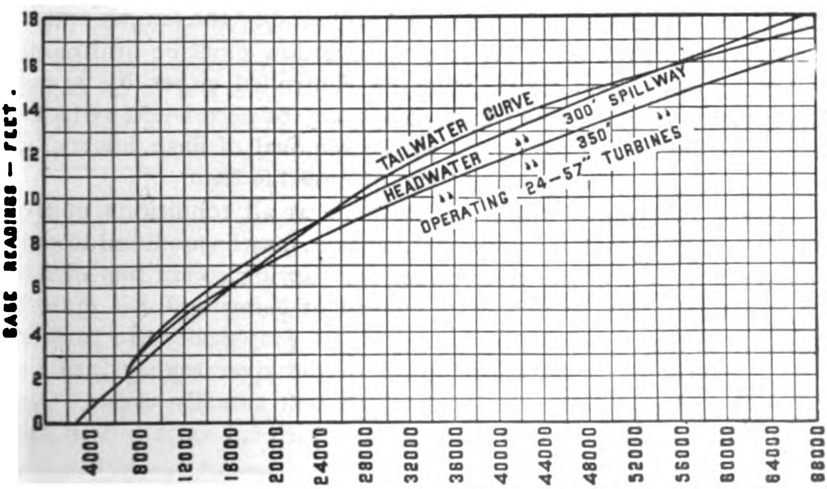


Fig. 48.—Showing Change in Head at Kilbourn Dam Under Various Conditions of Flow.

TABLE IX.

Changes in head at Kilbourn Dam with lengths of crest of 300 and 350 feet and under various conditions of flow with resulting total available head with 17 ft. dam.

Flow in cubic feet per second.	Changes in Head with		Total Head with	
	300 ft. dam.	350 ft. dam.	300 ft. dam.	350 ft. dam.
7000	0	0	17	17
14000	+ .8	+ .2	17.8	17.2
21000	+ .2	— .4	17.2	16.6
28000	— .3	—1.1	16.7	15.9
35000	— .5	—1.5	16.5	15.5
42000	— .4	—1.3	16.6	15.7
49000	— .2	—1.3	16.8	15.7
56000	+ .1	—1.2	17.1	15.8

60. Effects of Design of Dam on Head.—It should be noted in both of the last diagrams that the height of the water above the dam is readily controlled by a change in the form and length of the weir; that a contraction in the weir length produces a corresponding rise in the head waters as the flow increases, while the lengthening of the weir will reduce the height of the head water under all conditions of flow. The physical conditions relative to overflow above the dam will control the point to which the head waters may be permitted to rise and will modify the length and the construction of the dam. Where the overflow must be limited, the waters, during flood times, must be controlled either by a sufficient length of spillway or by a temporary or permanent reduction in the height of the dam such as the removal of flash boards, the opening of gates, or by some form of movable dam.

Having determined the head available at all conditions of river flow, the hydrograph, as previously shown, may be modified to show the actual power of the river under the varying conditions of flow. The vertical scale, in this case, instead of being uniform must be variable as the head varies. Fig. 49 shows graphically the variation in the continuous theoretical power of the river taking into consideration the variation in head which will actually occur. Compare this hydrograph with Fig. 40 in which no variation in head is considered.

61. Effect of Head on the Power of the Plant.—It is important at this point to take into consideration the effect of head and flow on the actual power of the plant. In most rivers, under flood condi-

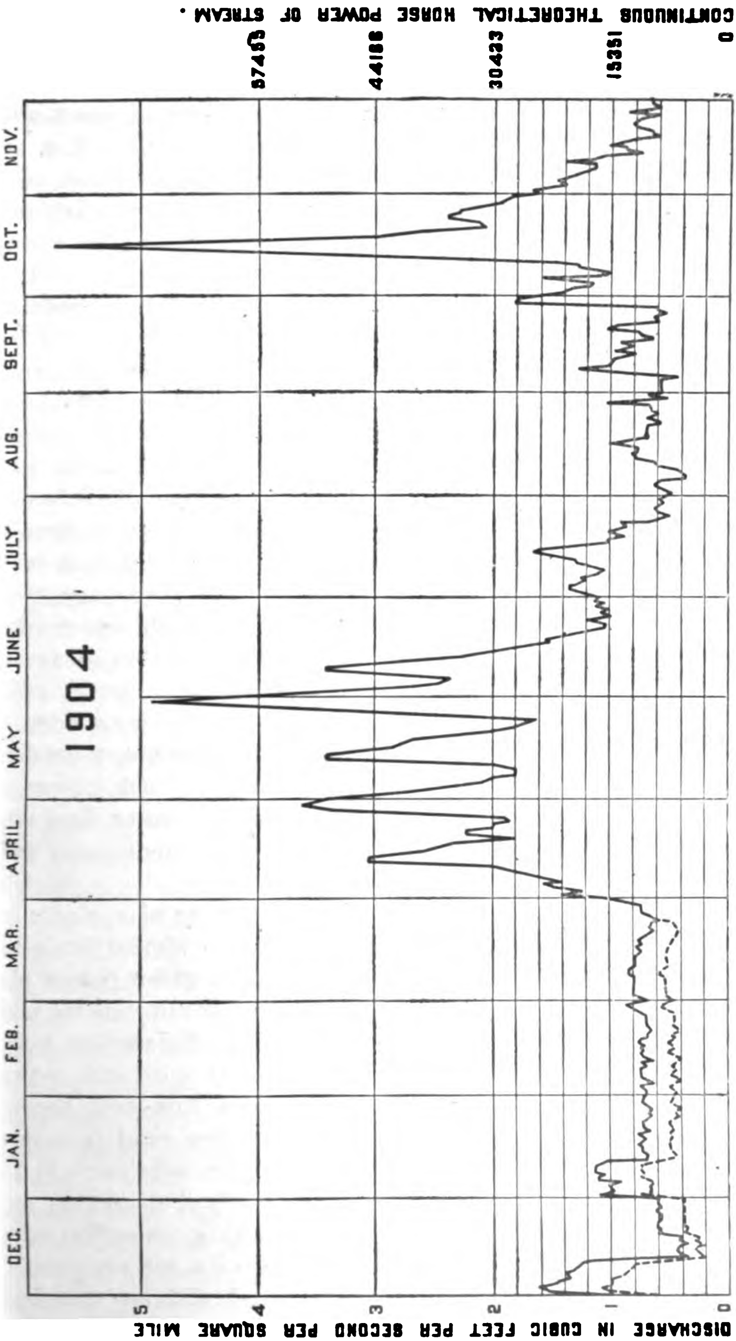


Fig. 49.—Hydrograph Showing Continuous (24 Hour) Theoretical Horse Power at Kilbourn, Wis., With Actual Head.

tions, the power theoretically available is largely increased, for, while the head may diminish, the flow becomes so much greater that the effect of head on the theoretical power is more than offset thereby. Practically, however, the conditions of head under which a given water wheel will operate satisfactorily (i. e. at a fixed speed) are limited, and, while the theoretical power of the river may radically increase, the power of the plant installed under such conditions will often seriously decrease, and under extreme conditions may cease entirely. The discharging capacity of any opening is directly proportional to the square root of the head, and the water wheel, or water wheels, simply offers a particular form of opening, or openings, and operates essentially under this general law. With a fixed efficiency, therefore, the power which may be developed by a water wheel is in direct proportion to its discharging capacity and to the available head. Hence, the power of the wheel decreases as the product of these two factors, and therefore the power available under conditions of high flow and small head are much less than where the head is large and the total flow of the river is less. The only way, therefore, to take advantage of the large increase in theoretical power during the high water conditions is to install a surplus of power for the condition of average water. This may sometimes be done to advantage, but its extent soon reaches a practical limitation on account of the expense. It often becomes desirable to take care of such extraordinary condition by the use of supplemental or auxiliary power. Such power can usually also be applied during conditions of low water flow when the power is limited by the other extreme of insufficient water under maximum head.

In considering the effect of head on the power of a plant, it is necessary to understand that water wheels are almost invariably selected to run at a certain definite speed for a given power plant and cannot be used satisfactorily unless this speed can be maintained. Also that any wheel will give its best efficiency at a fixed speed only under limited changes in head. If the head changes radically, the efficiency changes as well and this fact becomes more serious under a reduction in head. As the head is reduced, the discharging capacity of the wheel and its efficiency is also rapidly reduced so that the power of the wheel decreases more rapidly than the reduction in the discharging capacity would indicate. When the reduction of head reaches a certain point the wheel is able to simply maintain its speed without developing power, and when the head falls below that point, the speed can no longer be maintained. It is therefore plain that when the head of a stream varies greatly, it becomes an important and difficult matter to select wheels which will operate satisfactorily under such variations, and, when the variations become too great, it may be practically or financially impossible to do so. This subject is discussed at length in a later chapter, but is called to the attention of the engineer as an important matter in connection with the study of head.

62. Graphical Investigation of the Relations of Power, Head and Flow.—The relation of head and flow to the horse power of any stream on which a dam has been constructed, may be graphically investigated and determined by a diagram similar to Fig. 50. On this diagram are platted hyperbolic lines marked "horse power curves" which show the relation of horse power to head and flow within the probable limits of the conditions at Three Rivers, Mich. These lines are drawn to represent the actual horse power of a stream under limited variations in head and flow and on the basis of a plant efficiency of 75 per cent. These heads, which actually obtain at the Three Rivers dam, were observed under three conditions of flow, and these observations were platted on the diagram at e e e and a curve was drawn through them. From the intersection of this curve with the horse power curves, the actual power of the river available under the actual variations of head and flow, is determined. These measurements were taken with all of the water passing over the dam.

Let us assume that it is desired to investigate the effect of an installation of wheels, using 600 cubic feet per second, under a nine foot head. Under these conditions part of the water will pass through the turbines instead of over the crest of the dam, the available head will therefore be somewhat reduced, and the power curve of the river, under these new conditions, is shown on the diagram by the curve f f f. This curve was platted from the curve e e e by computing the amount the head on the crest of the dam would be lowered at different stages of the river by diverting through the wheels the quantity of water which they will pass under the reduced head. The actual power of the river at different heads and under these conditions is shown by the intersection of the line f f f with the horse power curve, and the actual power of the proposed plant under various conditions of flow is obtained by pro-

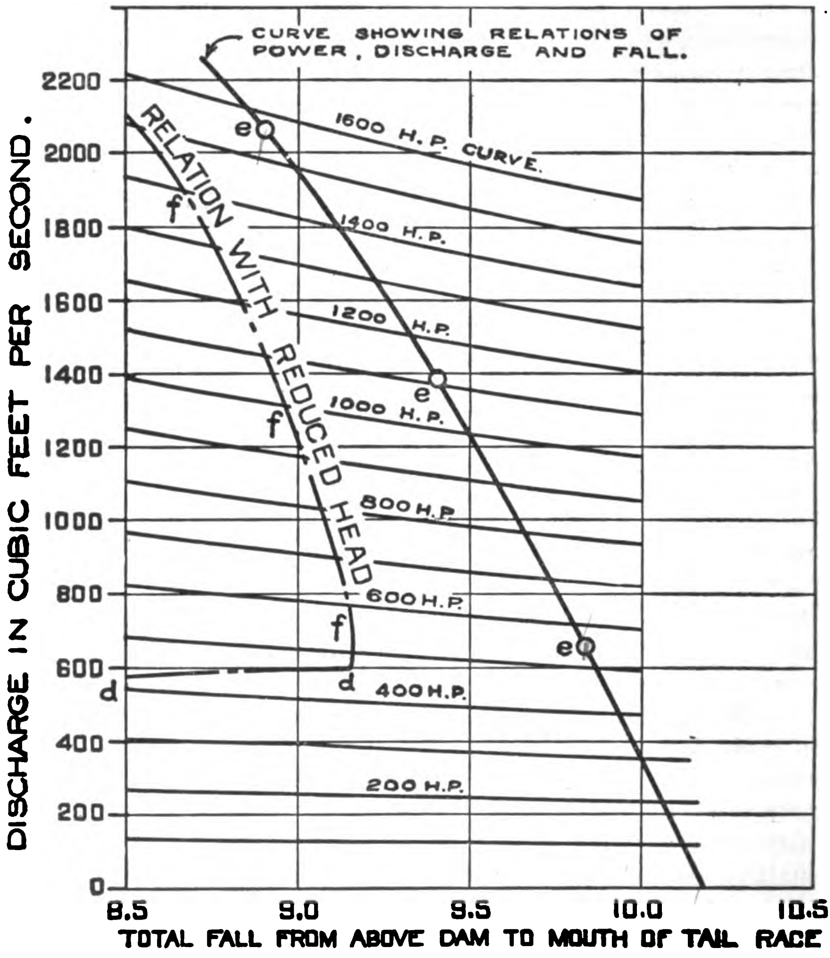


Fig. 50.—Graphical Study of Head.

jecting the point of intersection of the discharge line with the line f f f on the turbine discharge line d d.

Thus, with a flow of 600 cubic feet per second, the power of the plant would be about 470 horse power, while, with a flow of 2,100 feet per second, the power of the plant would decrease to about 420 horse power. At discharges below 600 cubic feet per second, the head would drop rapidly unless a portion of the installation was shut down.

63. **Graphical Study of Power at Kilbourn.**—A more detailed

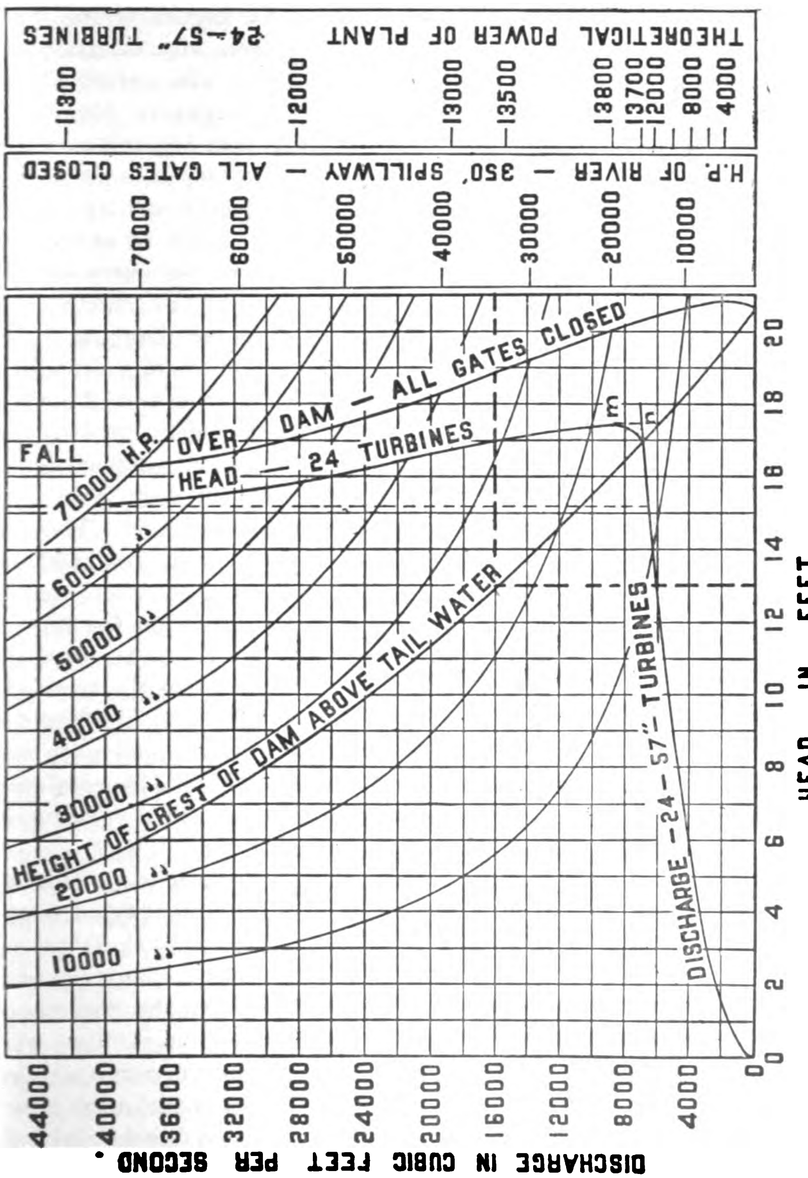


Fig. 51.—Graphical Analysis of Relation of Power, Head and Flow at Kilbourn Plant of Southern Wisconsin Power Co

study of head in connection with the conditions at Kilbourn, Wisconsin, is illustrated by Figures 51 and 52. In Figure 51 the theoretical horse power of any stream resulting from any variation between the head and flow is shown by the hyperbolic curves drawn from the upper to the right hand side of the diagram. Figure 47, already considered, shows the relation of the head and tail water at Kilbourn, where a dam with a crest 350 feet in length is projected.

The curve on Figure 51 marked "Height of crest of dam above tail water" was obtained by subtracting the height of tail water at the various river stages, as given by the rating curve of the river, from the height to which the dam is to be constructed and platting the same in their correct position on the diagram. The dam here considered is 17 feet in height above average water or with its crest at elevation 19 on the gauge. The curve on the right marked "Fall over dam,—all gates closed", is constructed in the same manner by laying off as abscissas the actual head as determined from Fig. 47 under various conditions of flow when the whole discharge of the river is passing over the dam. The abscissas, therefore, between these two curves show the head on the crest of the dam when the whole discharge of the river is passing over the dam. For any given river discharge (as for instance 16,000 cubic feet per second) the total fall can be obtained (in this case 18.8) and the theoretical horse power of the river (in this case 34,000 horse power) can be determined by finding the intersection of the line for 16,000 cubic feet per second with the curve marked "Fall over dam,—all gates closed", and determining the relation of this point to the power curves. This relation is more clearly indicated by the first scale to the right.

64. Power of the Kilbourn Wheels Under Variations in Flow.— When the gates to the turbines are open a less quantity of water will flow over the dam and the head on the crest will therefore be diminished. The amount of water which will pass through the proposed installation under various heads, is shown by the curve marked "Discharge 24-57″ turbines." The intersection of this curve, with the discharge lines, at all points to the left of the curve marked "Height of crest of dam above tail water" indicates that such flows will pass through the wheels at the head indicated by the point of intersection. The practical limit of the turbine capacity is the discharge indicated by the point of intersection of the turbine discharge curve with the "Height of crest of dam above tail water". It will be noted that this intersection shows a maximum discharge of 7,000 cubic feet per second under a head of 17 feet. A further increase in the discharge of the river up to 8,700 cubic feet per second, causes an increase in the head, which is found by following upward the curve marked "Head 24 turbines" to the point m where a maximum head is indicated. The discharge from the turbines under this condition increases but slightly and is indicated by the vertical projection of the point of greatest head (m) on the turbine discharge line (at n) which is so slightly above the 7,000 cubic feet line as to be hardly distinguishable on the diagram.

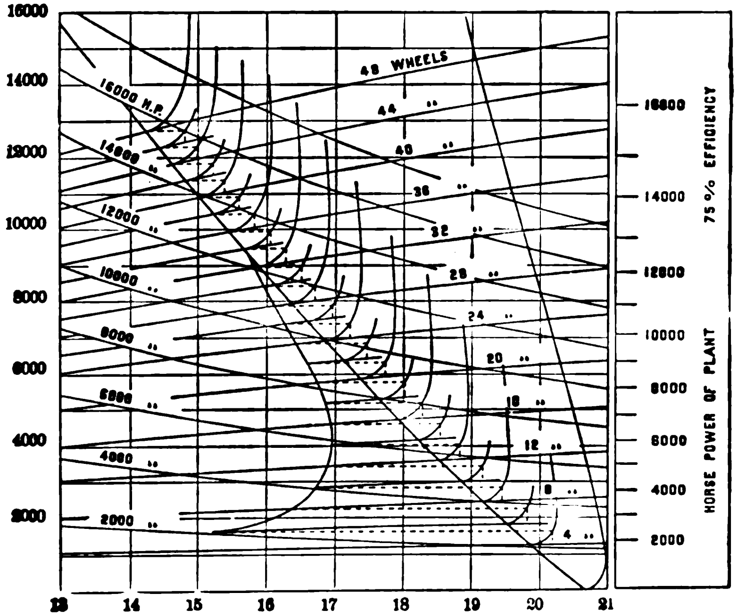
The power of the plant depends upon the head and the discharge through the wheels, hence the theoretical power which might be developed by the 24 turbines with a flow of 8,700 cubic feet per second would be about 13,800 horse power, which can be determined by calculation or is shown by the relation of the point n to the power curves. The actual value of these various points is more clearly shown on the second scale to the right, marked "Theoretical power of plant 24–57" turbines". A further increase in the discharge decreases the head until for the 24 turbines a minimum is reached at a discharge of 42,500 cubic feet per second. Under this condition of head the discharge through the wheels has also been somewhat reduced, and the corresponding horse power is reduced to 11,300 as shown by the intersection of the discharge curve and the line indicating the head existing under these conditions.

65. Effects of Low Water Flow.—In the case of low water when the flow is not sufficient to maintain the flow over the dam, if the turbines are run at full capacity, the water level behind the dam will drop until a point of equilibrium is attained where the head is just sufficient to force the entire discharge through the turbines. As the water level is lowered below the crest, the power of the plant rapidly diminishes owing to the great decrease in the head for a small decrease in the flow. When the head decreases beyond a certain point the power of the plant may be increased by closing some of the gates of the turbines until the discharge through the turbines is less than the discharge of the river, when the head will increase by the backing up of the water behind the dam.

Thus it will be seen by the diagram that, with only 6,000 cubic feet per second flowing in the river, if all of the turbines are operated the head will drop to about 12.7 feet, and the power of the plant under this head and flow would be about 8,660 horse power. If, under these conditions, one unit of six turbines, amounting to one-fourth of the plant, is shut down, the water will rise until the head is increased to about 18 feet. Under these conditions about 800 cubic feet per second of this water will waste over the dam, and the power developed by the remaining portion of the plant will be 10,630 horse power, or, about 2,000 horse power more with one unit shut down and with the resulting head than with all units in operation and the consequent lower head. The above discussion simply illustrates the point that it is rarely desirable to draw down the head of an operating plant, at least to any great extent, for the sake of operating a greater number of wheels, unless this is done for the purpose of impounding the night flow for use during the day or at times of maximum load. Even in this case too great a reduction in the head is undesirable and uneconomical.

66. Effects of Number of Wheels on Head and Power.—Fig. 52 is an enlarged section of that part of Fig. 51 shown by the dotted lines. This diagram shows how the head on the wheels may be maintained by shutting off some of the wheels in case the flow becomes so small as to entirely pass the wheels and thus reduce the head, as described above. It will be noted that with a total installation of 48 wheels, by closing the gates of two wheels at a time, the variation in the head would be only a fraction of a foot until as many as 24 wheels ahe closed. Hence it will be seen that when the power has been decreased by a rduction of head, the wheels should be closed off until the same power can be secured by the less number of wheels operating with the highest head that is available with the given discharge of the river. As the lower flows of the river are reached great fluctuation in the head will occur with the operation of the turbine gates. This diagram shows the actual delivered power of the plant and is based on a plant efficiency of 75 per cent. The power obtained for a given discharge is therefore less than shown by Fig. 51.

In order to secure more accurate results a small correction for the variations in efficiency under the variations in head may sometimes be desirable. In the problem under consideration this is unnecessary on account of the small variation which takes place. However, when the variations in head are considerable, this correction is essential if a close estimate of power at different heads is desired. Figure 53 is a power hydrograph similar to those already discussed but with such changes in the scale as to show the continuous power that could have been developed by these four groups of turbines at Kilbourn, Wisconsin, during the year 1904, under



Head in Feet

Note—H. P. Curves are based on 75% efficiency

Fig. 52.—Relation of Number of Wheels to Power and Head.

the variations in head which would actually have occurred with the dam it is proposed to construct.

From the previous discussion of the conditions at Kilbourn it is seen that with a dam with fixed crest the variations in head, due to variations in flow, are comparatively small. Consequently the power of the wheel to be installed will not decrease with an increase in flow to as great an extent as usually occurs in water power plants. If a system of flash boards or an adjustable crest is found desirable in order to prevent overflow at times of flood, the power of the plant when these are lowered will be still further reduced.

The hydrograph may be utilized for more detailed analysis of water power questions and will be further discussed in a future chapter.

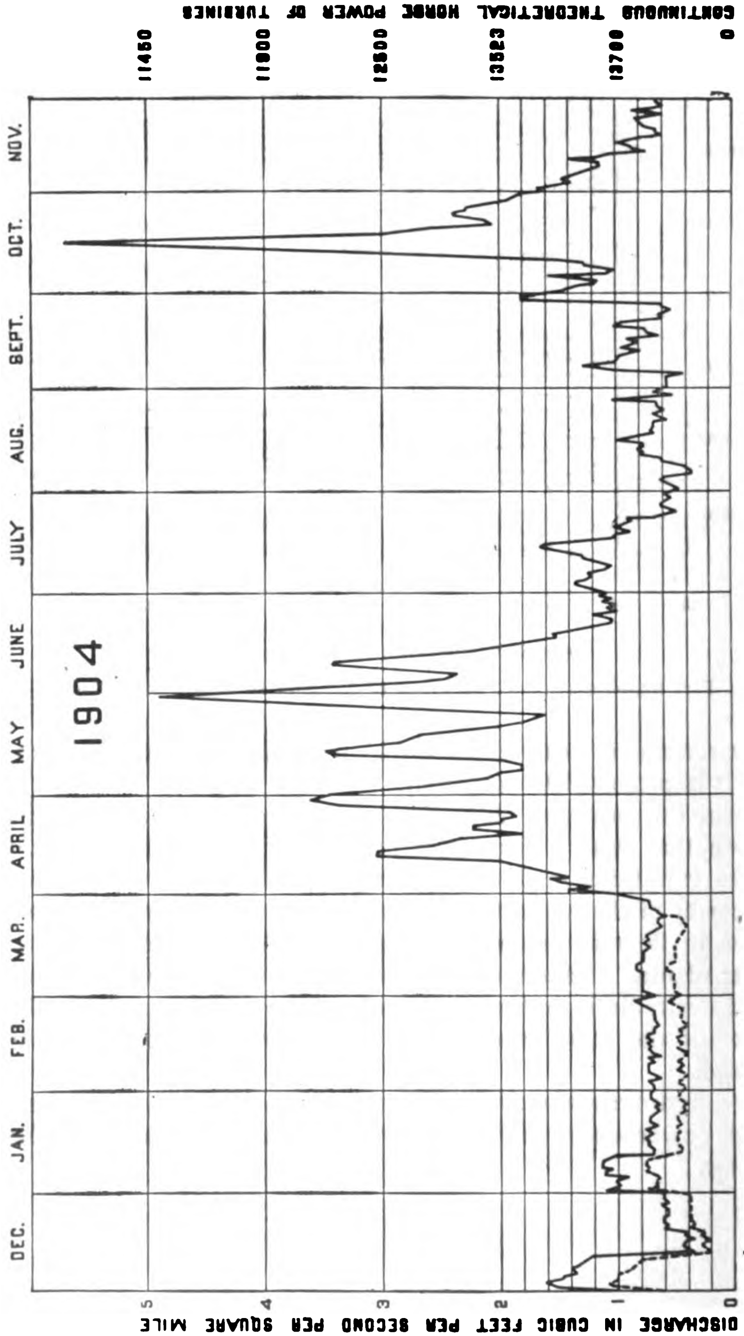


Fig. 53.—Hydrograph Showing Power of Plant as Influenced by Variable Head.

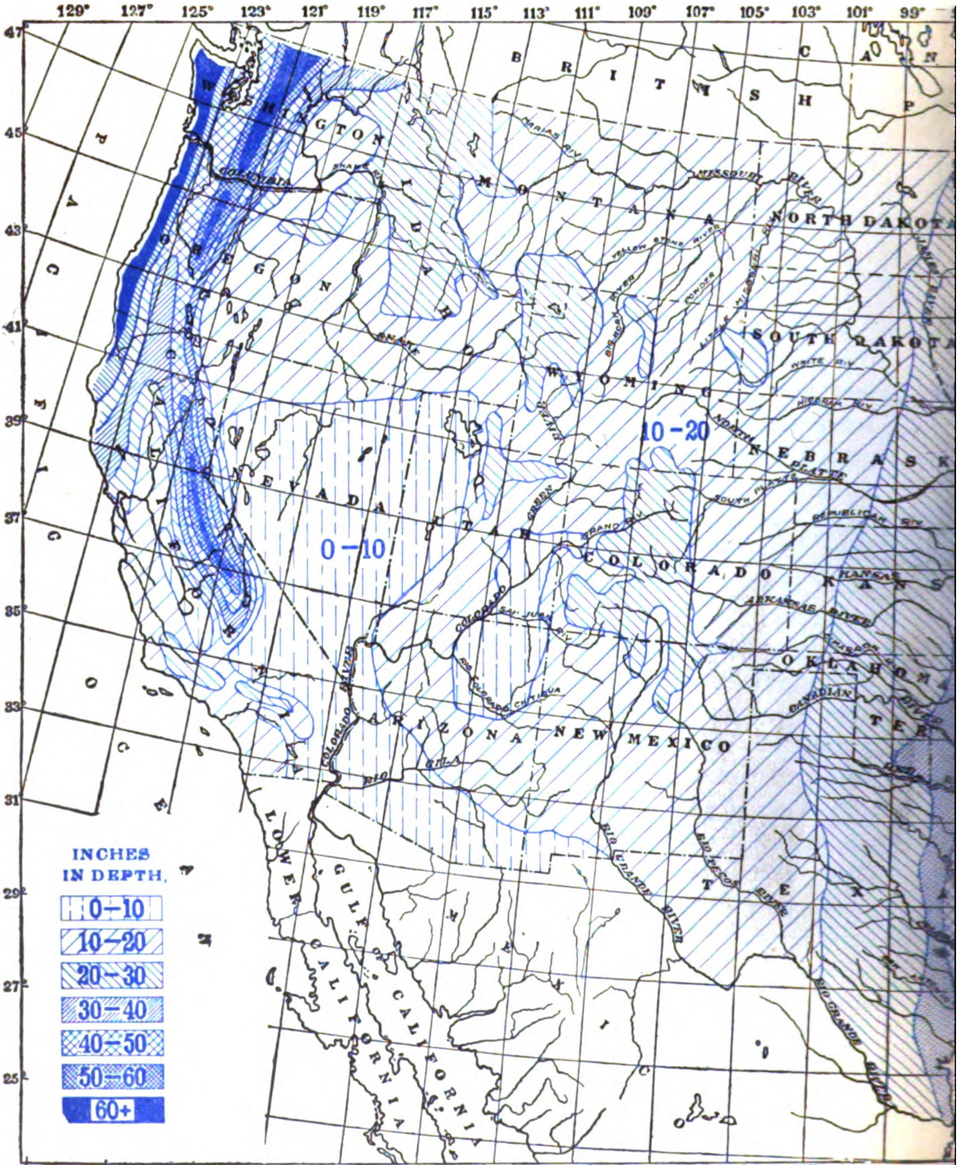
CHAPTER VI.

RAINFALL.

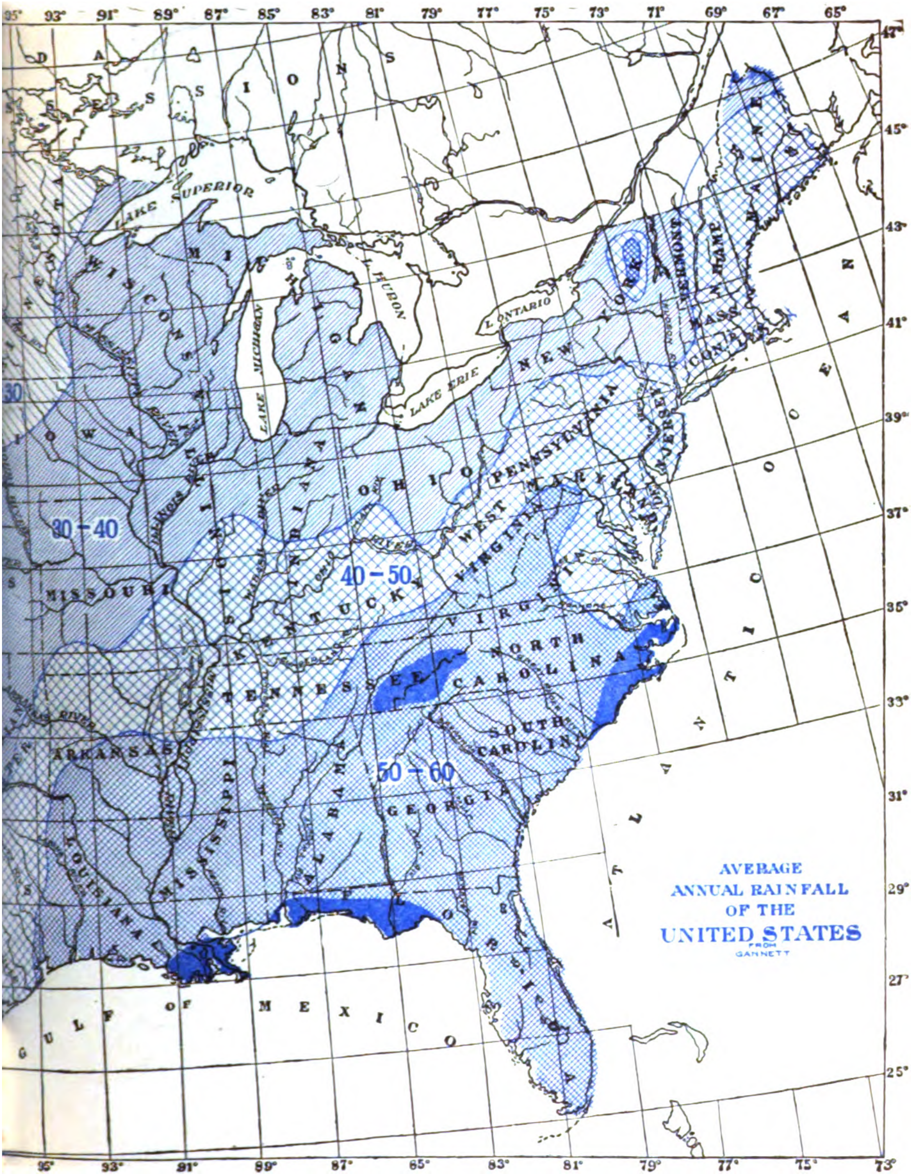
67. Importance of Rainfall Study.—The influence of rainfall on the flow of streams is so direct that those unfamiliar with the subject are apt to assume that the relation may be represented by some simple expression and that, therefore, if the rainfall for a period of years be known, the corresponding stream flow may be directly and readily calculated therefrom. With only a brief familiarity with the subject it is evident that no such simple relation exists. The relationship is, in fact, complicated by a multiplicity of other physical conditions which have an important if not an equal influence.

Observations of stream flow are quite limited both in time and geographical extent while the observations of rainfall have extended over a long period of time and the points of observation are geographically widely distributed. If, therefore, it is possible to trace such relationship between the flow of streams and the rainfall and other physical conditions on the drainage areas as will enable the engineer to calculate the flow even approximately, such relationships become of great value to the water-power engineer, on account of the lack of other more definite information. It is therefore important that the engineer inform himself as fully as possible on the relations that exist between rainfall and stream flow and the modifications of those relations by other physical factors. By such means the information regarding rainfall, already recorded for long terms of years, may be applied to the problem of stream flow in which the engineer is more directly concerned. For this reason the subject of rainfall is here discussed in as much detail as the space will permit.

68. Distribution of Rainfall.—A continuous circulation of water is in progress on the earth's surface. The evaporation from the water and moist earth surfaces rises into the atmosphere in the


INCHES IN DEPTH.
0-10
10-20
20-30
30-40
40-50
50-60
60+
0-10
10-20
BRITISH
WASHINGTON
OREGON
CALIFORNIA
NEVADA
IDAHO
MONTANA
WYOMING
UTAH
COLORADO
ARIZONA
NEW MEXICO
NORTH DAKOTA
SOUTH DAKOTA
NEBRASKA
KANSAS
OKLAHOMA
TEXAS
LOWER CALIFORNIA
GULF OF CALIFORNIA
MEXICO
PACIFIC OCEAN
COLUMBIA
SNAKE
MISSOURI RIVER
YELLOW STONE RIVER
ARKANSAS RIVER
RIO GRANDE RIVER
COLORADO RIVER
RIO GILA

Fig


AVERAGE
ANNUAL RAINFALL
OF THE
UNITED STATES
FROM
GANNETT
30 – 40
40 – 50
50 – 60
LAKE SUPERIOR
LAKE MICHIGAN
L. HURON
LAKE ERIE
L. ONTARIO
MISSOURI
ARKANSAS
KENTUCKY
TENNESSEE
OHIO
PENNSYLVANIA
WEST VIRGINIA
VIRGINIA
NORTH CAROLINA
SOUTH CAROLINA
GEORGIA
NEW YORK
GULF OF MEXICO
ATLANTIC OCEAN

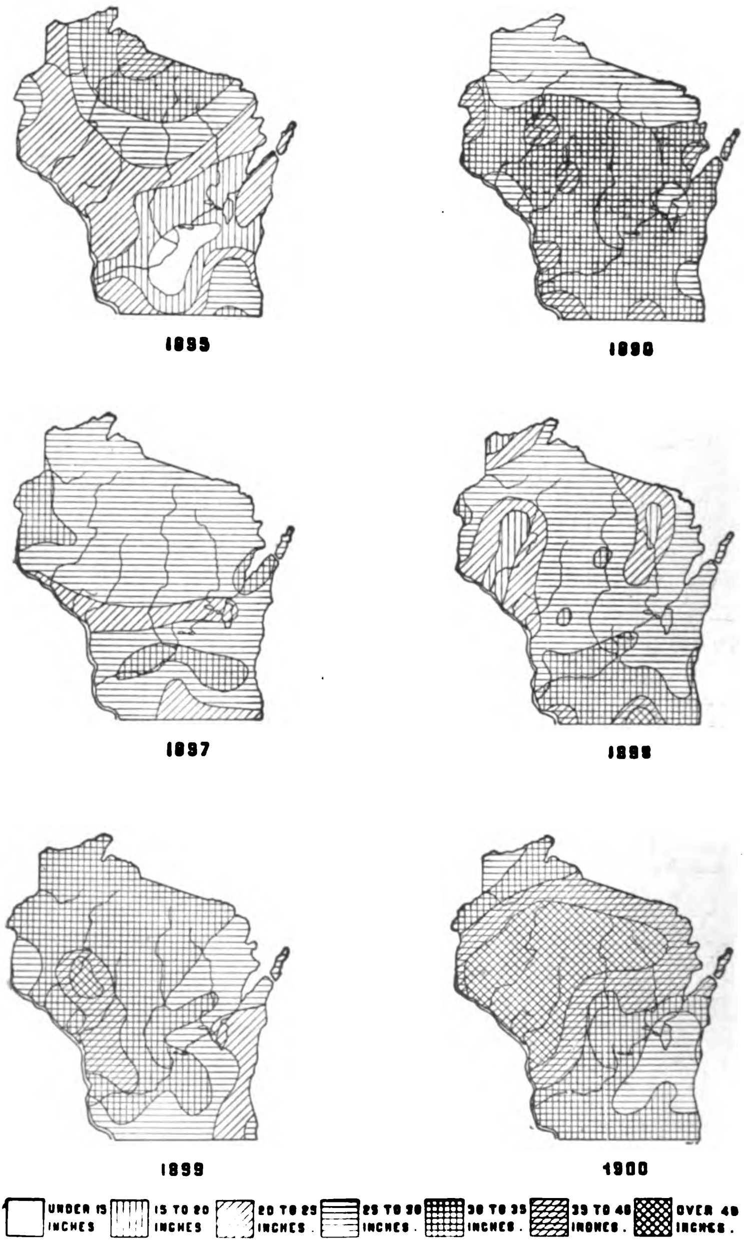


Fig. 55.—Distribution of Total Annual Rainfall in Wisconsin.

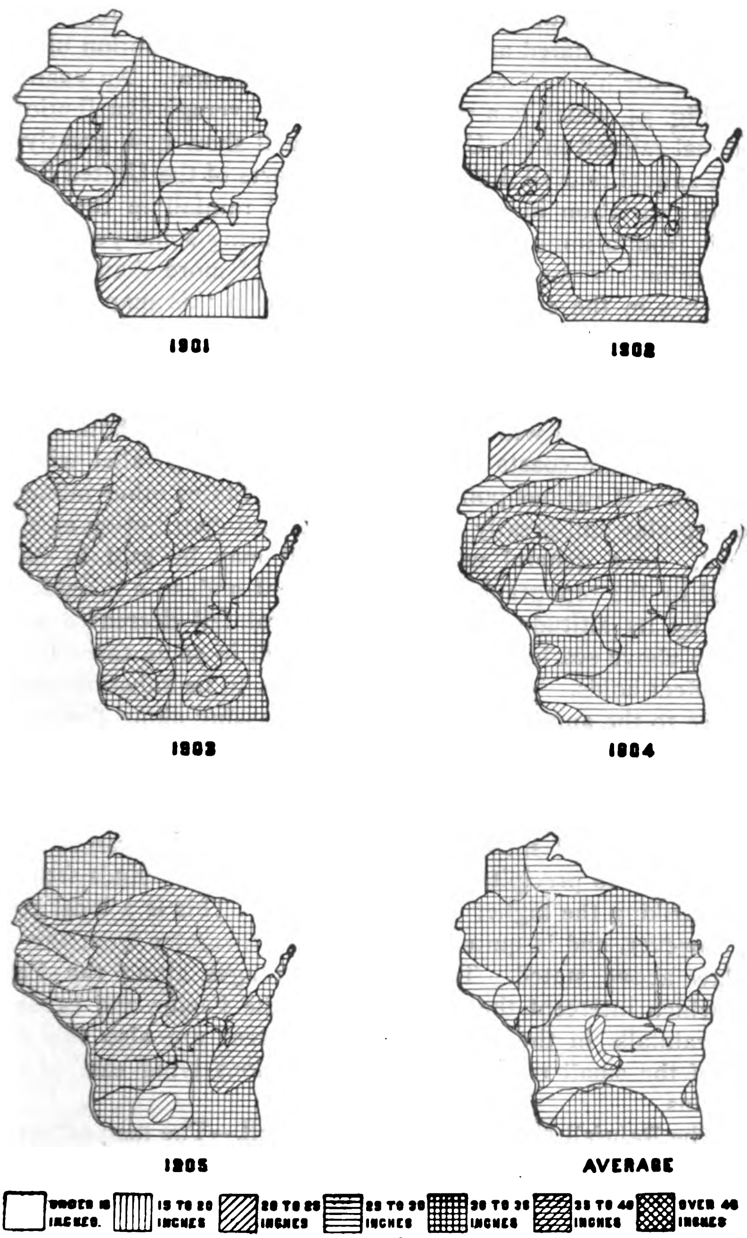


Fig. 56.—Distribution of Total Annual Rainfall in Wisconsin.

form of vapor, partially visible as clouds, mist and fog, and is afterwards precipitated as rain and dew. The distribution of rainfall on the earth's surface is by no means uniform. An examination of Fig. 54, which is a map showing the average distribution of the annual rainfall in the United States, will show how greatly the average annual rainfall differs in various parts of the United States. The local variation in the average annual rainfall in the United States is from a minimum of no rainfall, during some years in the desert regions, to an occasional maximum of more than one hundred inches in the extreme northwest. From this map it will be noted that from the Mississippi westward the lines of equal rainfall are approximately north and south and parallel with the mountain ranges. In the Southern states, east of Texas, they are approximately parallel with the Gulf of Mexico, and on both the Atlantic and Pacific coasts they are approximately parallel with the coast lines. At various points in the United States other influences come into play and greatly modify the general distribution as above outlined. In a general way the rainfall may be said to be influenced by the topography of the continent and, to a considerable extent, by its altitude. In general, the rainfall decreases as the elevation above sea level increases, although in some cases the opposite effect holds. This general law seems to be substantiated by reference to the annual rainfall map. In passing along the parallel of 40° as we ascend from the Mississippi River to the western mountains the annual rainfall decreases from about 35 inches annually to 10 inches or less. On the other hand, a reference to our Western coast will show that some of the heaviest rainfalls that occur are due to precipitation caused by the moist winds from the Pacific striking the higher mountain ranges. This is a local condition, however, and is quite different in its character from the general law above stated. The mountain ranges along the Pacific coast which intercept the moisture from the Pacific and cause the heavy rainfalls in the higher mountain areas are also the direct cause of the small rainfall in the arid regions lying east of these mountains.

69. **The Rainfall Must be Studied in Detail.**—The map of average annual rainfall is of value only for a general view of the subject. For special purposes a detail study of the local variations from the average conditions is necessary. Great variations take place in the annual rainfall of every locality. Sometimes the annual rainfall will be for a series of years considerably below the average, and

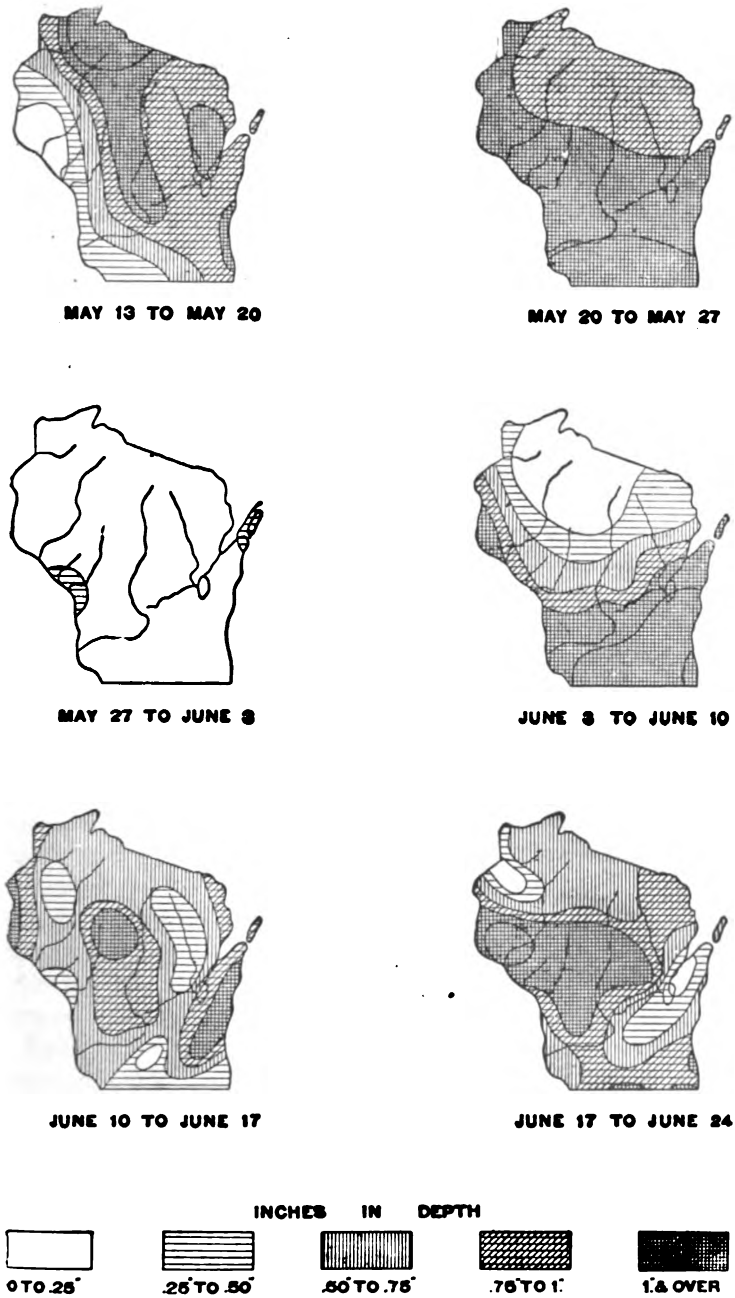


Fig. 57.—Distribution of Weekly Rainfall in Wisconsin.

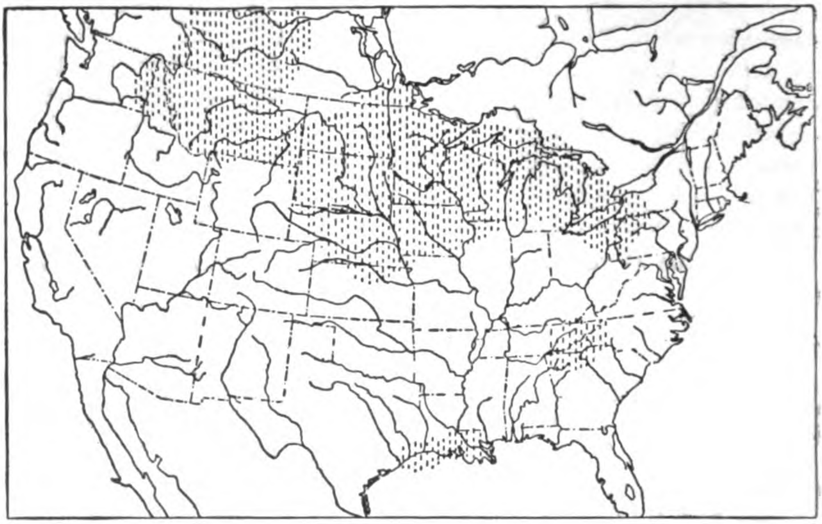


Fig. 58.—Rainfall Conditions in the United States at 8 A. M., July 16th, 1907.

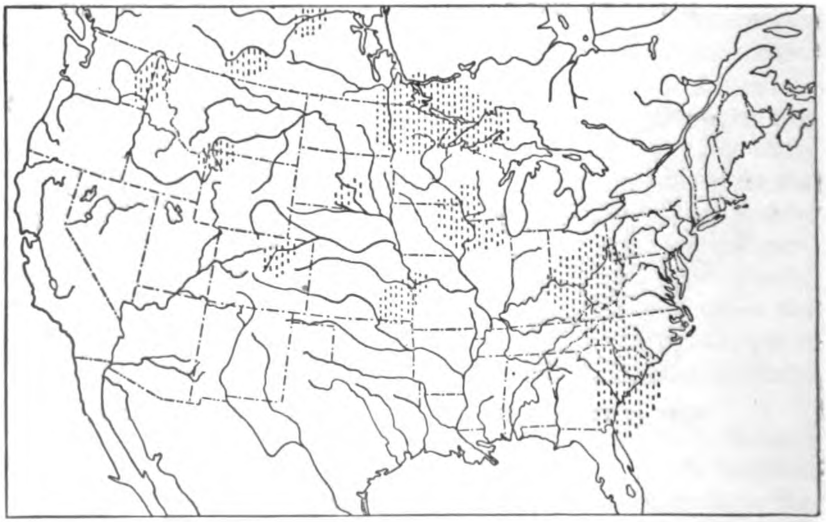


Fig. 59.—Rainfall Conditions in the United States at 8 A. M., July 17th, 1907.

then for a number of years the average may be considerably exceeded. No general law seems to hold, however, in regard to this distribution and the variation seems to occur either without law or by reason of laws so complicated as to defy determination. The variations in the distribution of the annual rainfall in the State of Wisconsin for eleven years are shown by Figs. 55 and 56. From these maps can be clearly seen how greatly the distribution of rainfall throughout the state differs in different years from the average annual rainfall as shown on the last map of the series. It should also be noted that in the same manner these annual rainfall maps are the results of the summation of an irregular distribution of numerous rainstorms, the irregularities of which can perhaps be more clearly shown by the maps on Fig. 57 which show the weekly distribution of rainfall in Wisconsin for six consecutive weeks in May and June, 1907. All such maps are but the result or summation of individual rainstorms which occur during the period considered. Individual rainstorms never occur twice over exactly the same geographical extent of territory nor with equal intensity at any points within the territory covered. They are not only irregular in their distribution but progressive in both their distribution and intensity, changing from hour to hour during their occurrence. The extent of a somewhat general rainstorm in progress at 8:00 A. M. (Washington time) over the Northwest on July 16th, 1907, is shown by Fig. 58. On the area over which this storm extended, the actual precipitation varied widely and the extent of the storm rapidly changed from hour to hour. At 8:00 A. M. on July 17th the general rainfall had ceased and the storm had become localized as shown by Fig. 59. The varying character and extent of the rainfall as illustrated by those two maps show the extremes of one storm which affected the Northwest, and illustrates, in a general way, the irregularity and lack of uniformity in rainfall occurrence and distribution.

70. Local Variation in Annual Rainfall.—By reference to Fig. 60, the variations which have occurred in the annual rainfall at various localities in the United States will be seen, and from this data the lack of uniformity in the annual rainfall will be more fully appreciated. By an examination of the records of a sufficient number of years the limiting conditions may be determined and an approximate determination of the relation between the extremely dry and extremely wet periods made.

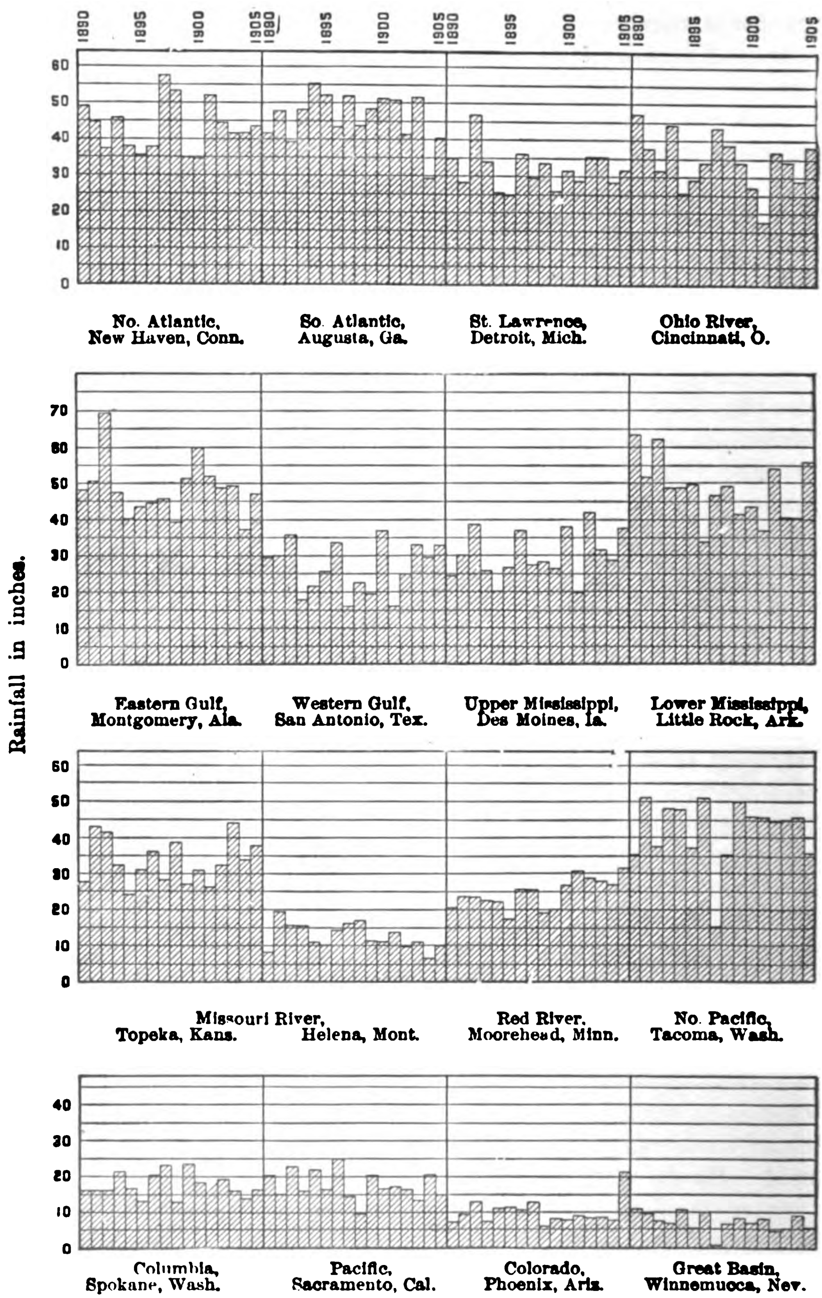


Fig. 60.—Variation in Annual Rainfall at Various Points in the United States.

Figure 61 i. a diagram showing the fluctuations that have occurred in the annual rainfall at Madison, Wisconsin, from 1869 to 1905. The variation at Madison has been from a maximum of about 52 inches in 1881 to a minimum of about 13 inches in 1895 which represents a greater range (4 to 1) than ordinarily obtains. As a general rule the maximum may be stated to be about double the minimum annual rainfall.

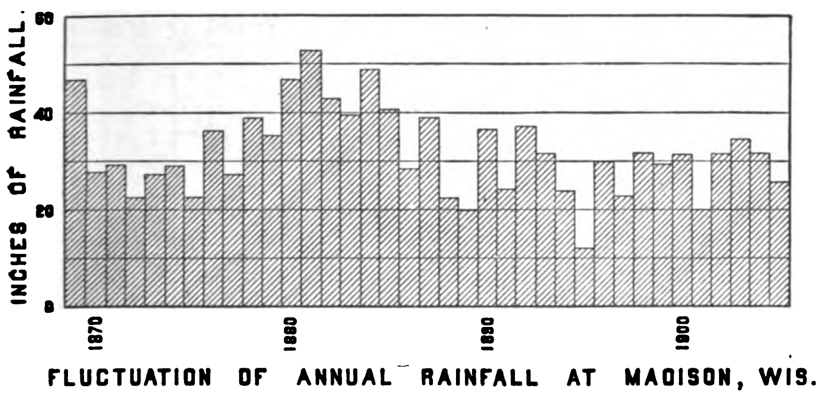
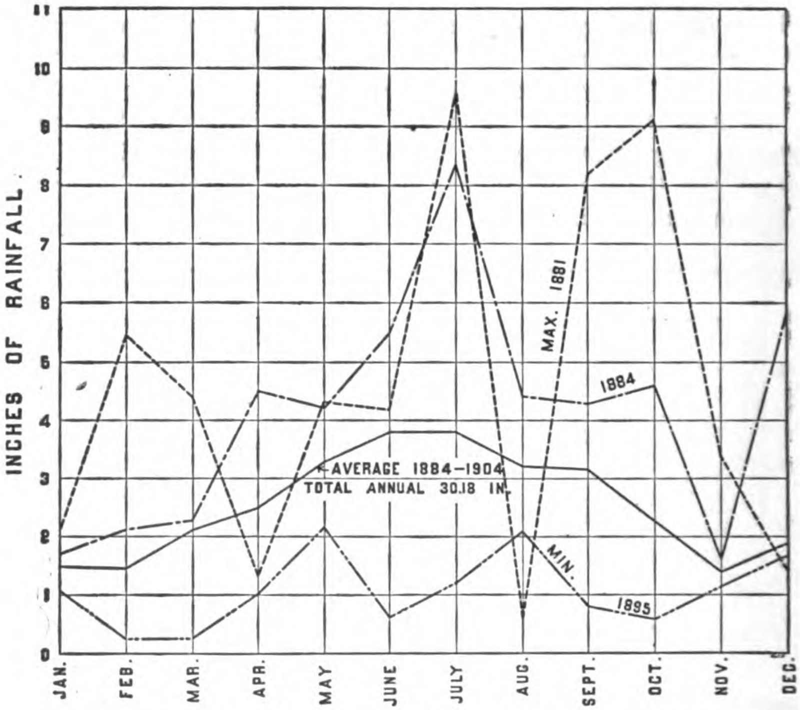


Fig. 61

71. Local Variations in Periodical Distribution of Annual Rainfall.—The amount of the annual rainfall is only one of the elements that influence the run-off. The time of occurrence or the periodical distribution of the rainfall is even of greater importance. The general character of the periodic distribution of the annual rainfall is similar each year in each locality, for the maximum and minimum monthly rainfalls occur in each locality at fairly definite periods. As the cycle of the seasons changes, conditions favorable or unfavorable to precipitation obtain, and, while these differ very largely from year to year and are subject to such wide variations as to render the character somewhat obscure, unless a number of seasons are considered, yet the same general character ordinarily prevails.

Figure 62 shows the extreme and the average variation of the monthly rainfall at Madison. The monthly rainfall in the various months differs widely in amount and is by no means proportional to the total annual rainfall for the year. It is especially observable that during the year of maximum rainfall, viz: for 1881, the rainfall for April was almost as low as for the April of the year 1895 when the total annual rainfall was at a minimum. It is also observ-



FLUCTUATION OF MONTHLY RAINFALL AT MADISON, WIS.

Fig. 62

able that the rainfall for August, 1881, was less than the rainfall for August of 1895. Figure 63 shows the typical average monthly distribution of precipitation at various points within the United States, and the general law to which even the variations mentioned partially conform. The character of the monthly distribution varies widely at different locations, but will be seen to have a similar character wherever similar conditions prevail. Thus the New England States present a similarity in the distribution of the monthly rainfall. A similarity in the monthly distribution is also found throughout the lake region and the Ohio Valley. The monthly distribution throughout the Great Plains is also similar, and a marked similarity exists at points along the Pacific coast.

72. **Accuracy of Rainfall Maps and Records.**—It must be understood that the rainfall maps, showing lines or belts of equal rainfall, are only approximately correct, and that it would be impossible to show by such lines small differences in annual rainfall of less

Types of Monthly Distribution of Precipitation in the United States.

Rainfall Distribution in the U. S. (Percentage of fall in each month represented by heavy lines.)

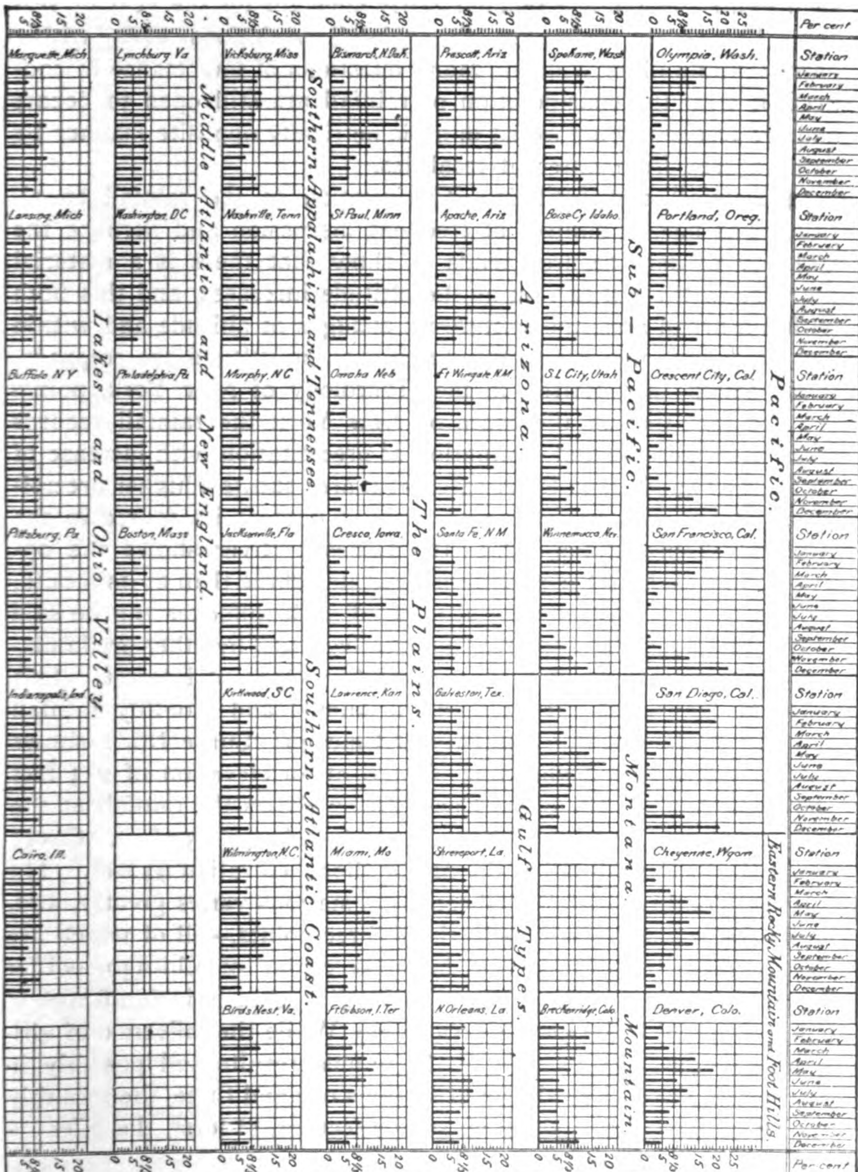


Fig. 63

than two or three inches. As a matter of fact, the rainfall actually differs considerably within comparatively small limits, but within such limits the average remains fairly constant for the year or season. Frequently, however, the rainfall variations even within narrow limits differ widely. Many questions of importance in connection with the consideration of rainfall are still open to debate and are frequently answered in a diametrically opposite manner by data secured from different localities.

73. **Rainfall and Altitude.**—The relation of the rainfall to altitude has been a subject of frequent discussion and perhaps the majority of data secured tends to show that there is a material decrease in the fall of rain as the altitude increases, and this both within a broad area and with great changes of altitude and within a limited area and where the differences in altitude are comparatively small. Mr. Rafter, in the Hydrology of New York, points out the fact that in the State of New York the rainfall records show both increase and diminution of precipitation with increase of altitude. The Hudson River catchment area shows a higher precipitation at the mouth of the river than it does at its source in the Adirondack mountains, while the Genesee River shows the opposite: that is, a higher precipitation at its source than at its mouth. In this case the influence of altitude, if such influence can be said to obtain on such limited areas, is overshadowed by other predominating influences. In this connection Fig. 64 is of interest. This diagram shows the variation in the annual and monthly rainfall at three stations within the City of Chicago. Curve No. 1 shows the rainfall at the Auditorium Tower, at an elevation of 233 feet above the level of the city. Curve No. 2 shows the rainfall at the Chicago Opera House Building, at an elevation of 132 feet. Curve No. 3 shows the rainfall at the Major Block, elevation 93 feet. The relative monthly rainfall at these three stations varies greatly, and, while the annual variations at these three points,—all of which are within a square mile in the business center of Chicago,—differ considerably from each other, still the difference is insignificant in comparison with the monthly variation. While the influence of altitude may possibly be seen in the annual results and possibly in the monthly results as shown at stations one and three, the monthly results at station two show no such effect, or, at least, the effect is greatly obscured by other influences.

74. **Value of Extended Rainfall Records.**—One of the points that becomes important in the consideration of rainfall records is the

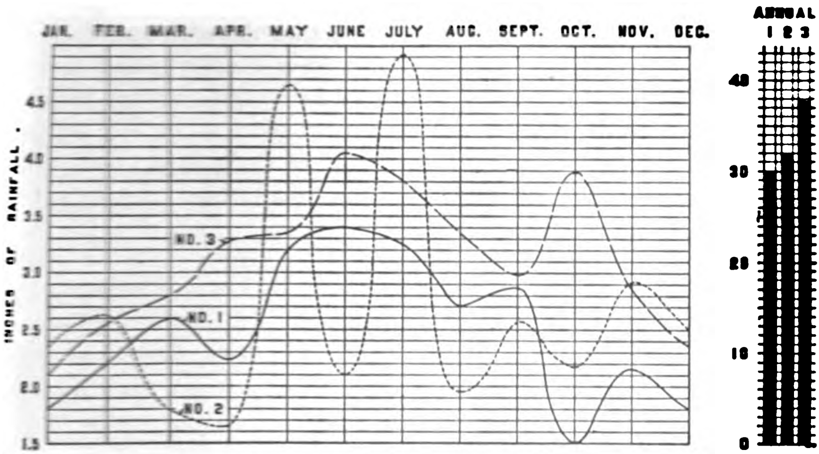


Fig. 64—Monthly and Annual Precipitation of Three Exposures in Chicago, Ill. 1. Auditorium Tower, Elevation 238 feet. 2. Chicago Opera House Building, Elevation 132 feet. 3. Major Block, Elevation 93 feet.*

length of time required to make such records safe as a basis for future estimates. This subject is well considered in a paper by Alexander A. Binnie, member of The Institute of Civil Engineers, published in the Proceedings, Vol. 109, pages 89 to 172. Mr. Binnie's conclusions are that:

"Dependence can be placed on any good record of 25 years' duration to give a mean rainfall correctly within 2 per cent of the truth."

Mr. Rafter, after reviewing this paper, concluded, that:

"For records from 20 to 35 years in length the error may be expected to vary from 3.25 per cent down to 2 per cent, and that for shorter periods of 5 to 10 and 10 to 15 years the probable extreme deviation from the mean would be 15 per cent to 4.75 per cent respectively."

Mr. Henry from his examination of this question in reference to various localities has drawn the following conclusion:

For a ten year period the following variations from normal have occurred:

New Bedford....................	+ 16 per cent.	— 11 per cent.
Cincinnati	+ 20 "	— 17 "
St. Louis......................	+ 17 "	— 13 "
Fort Leavenworth...............	+ 16 "	— 18 "
San Francisco..................	+ 9 "	— 10 "

* Reproduced from original slide published by Geographical Society of Chicago.

For a 25-year period Mr. Henry found that the extreme variation was 10 per cent both at St. Louis and New Bedford, and reached the conclusion that at least 35 to 40 years' variations are required to obtain a result that will not depart more than + or —5 per cent from true normal. The average variation of the 35-year period Mr. Henry found to be + or —5 per cent and for a total 40-year period + or —3 per cent.

75. **Accuracy in Rainfall Observation.**—It must also be understood that on account of the marked variations which actually occur in rainfall within limited areas and by reason of limited difference of elevation, the observation of actual rainfall is not without its difficulties. In order to secure great accuracy great care must be exercised in the placing of rain gauges so that they may receive and record the rain received in an accurate manner. Subject, as they are, to considerable variations, it would seem unwise to use great refinement in the calculations of rainfall, and in recording rainfall one decimal place is probably all that is warranted and two places is the ultimate limit of possible accuracy.

76. **District Rainfall.**—In determining the average rainfall on a drainage area an extended series of observations over the entire district considered become essential and conclusions drawn from more limited observations are subject to considerable inaccuracies. Rainfall stations, distributed as uniformly as possible over the drainage area, should be selected, and the average result of the observations of these stations should be used as the basis of calculation. Possibly a still more accurate method of considering this subject would be the selection of rainfall observations on each particular branch of the stream considered. The value to be given to each set of observations used should be in proportion to the territory drained by the tributaries.

77. **Study of Rainfall as Affecting Run-off.**—In considering the rainfall on a district in relation to the run-off of streams, it is desirable to study the rainfall records on the basis of what is termed "water year". The water year for most of the area of the United States, instead of coinciding with the calendar year may be best divided into periods beginning, approximately, with December and ending, approximately, with the following November. The first six months of this period, December to May inclusive, is termed the "storage" period. June, July and August constitute the "growing" period; September, October and November, the "replenishing" period. For the purpose of discussing rainfall in its

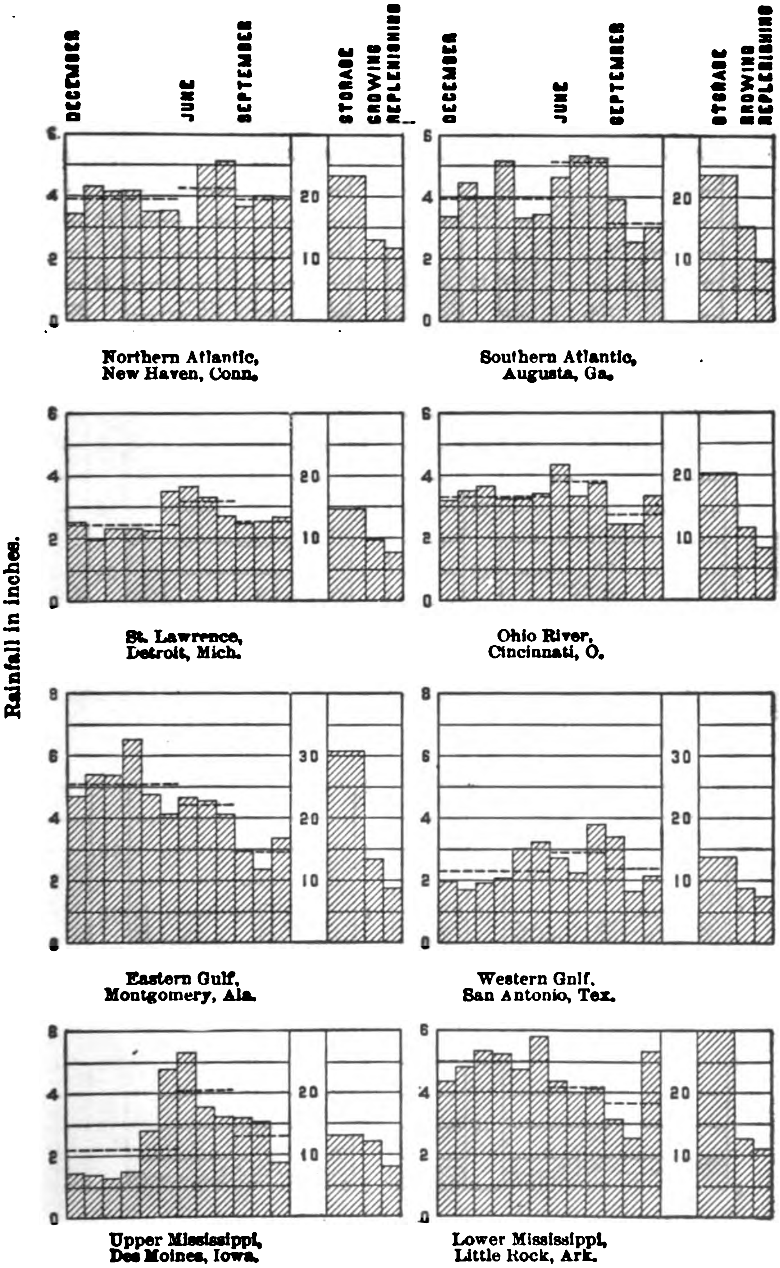


Fig. 65.—Mean Monthly Rainfall at Various Points in the United States.

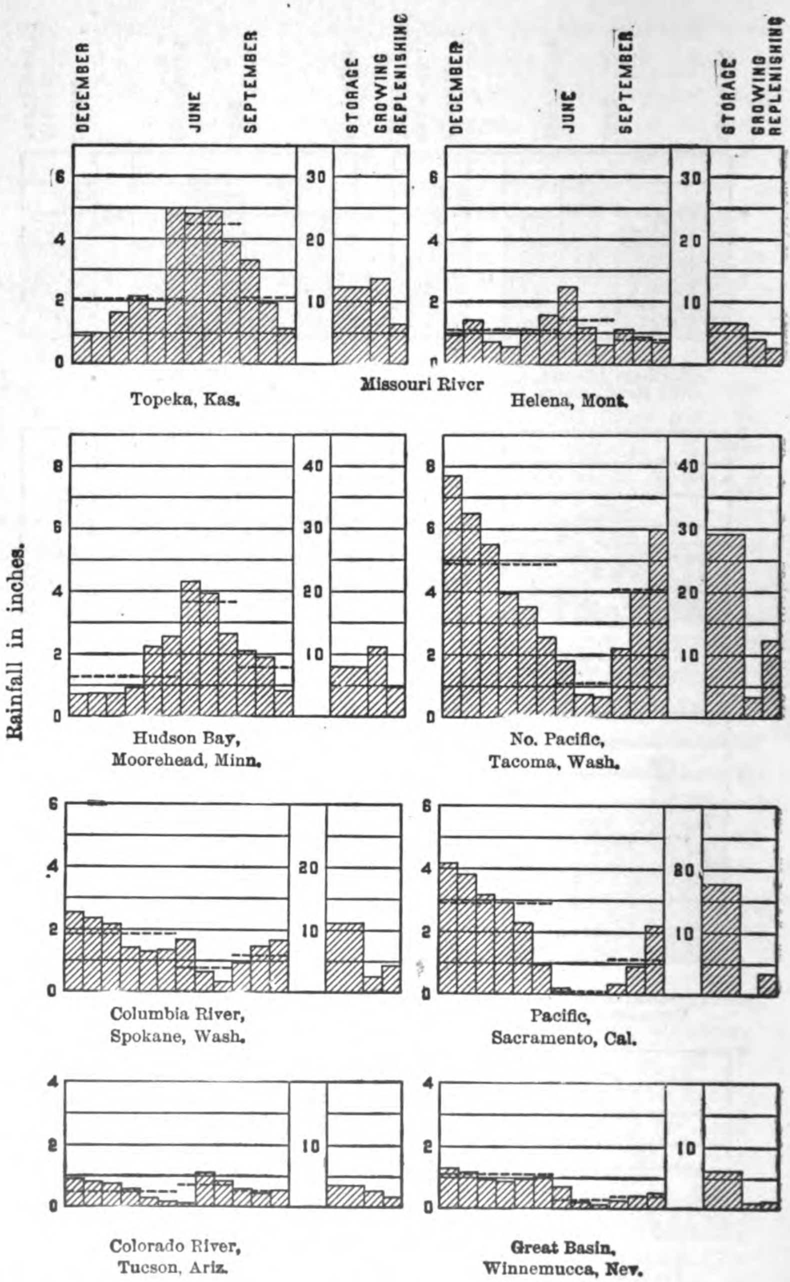


Fig. 66.—Mean Monthly Rainfall at Various Points in the United States.

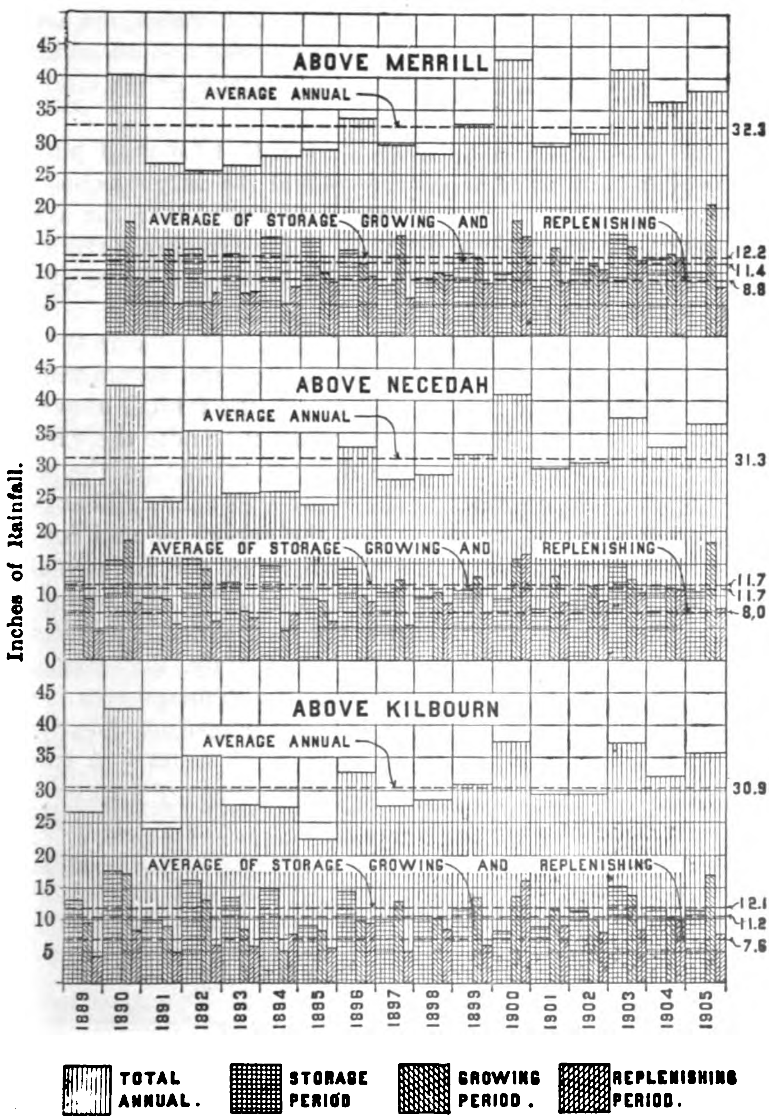


Fig. 67.—Rainfall on the Drainage Area of the Wisconsin River.

relation to run-off it is desirable to divide the annual rainfall in accordance with these periods. Figures 65 and 66 show the average monthly rainfall at various points in the United States, the average rainfall for each of the periods above mentioned and an additional diagram for each location showing the summation of the total rainfall for each period of the water year.

Here again attention is called to the fact that for most purposes of the engineer the extreme conditions and the varying conditions from year to year are of much greater importance than the average conditions as shown on these diagrams. Figure 67 shows the annual and periodic rainfall on the valley of the Wisconsin River at three different points, the relative location of which will be seen by reference to the map on page 84. The upper diagram shows the rainfall on the drainage area above Merrill, the center diagram the rainfall above Necedah, and the lower diagram, the rainfall above Kilbourn. In these three diagrams it is important to note the variation in the rainfall condition above the different points on the watershed. For example, considering the entire area above Kilbourn and above Necedah, it will be noted that the annual rainfall for 1895 was the lowest within the period shown, while for the area above Merrill the rainfall for 1892 was the lowest for the period discussed. This diagram will illustrate the fact, which is manifest on the investigation of most large streams, namely, that frequently the intensity of the rainfall upon part of the drainage area is radically different from that on other parts, and that, consequently, the various quantities of rain falling on a large watershed tend to balance each other and keep the total more constant than observation at any one point would seem to indicate, so that the minimum rainfall at any one point on the area is not necessarily coincident with the minimum rainfall that may occur at any other point or on the stream as a whole. From this it is evident that in an area of any magnitude it is necessary to consider the rainfall at a large number of stations well distributed over the area.

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CHAPTER VII.

THE DISPOSAL OF THE RAINFALL.

78. **Factors of Disposal.**—The portion of the rainfall in which the water power engineer is most directly interested is that which runs off in the surface flow or flow of streams. In order to form some idea of the amount of this run-off and the factors that control it, it is necessary, however, to investigate and consider the various ways in which the rainfall is distributed, for the ways in which the distribution occurs are mutually inter-dependent and of necessity modify and control each other. The rainfall disposal depends on a large number of factors or conditions among the most important of which may be named:

(1) The amount of the rainfall.
(2) The rate of rainfall.
(3) The condition of the surface on which the rainfall takes place.
(4) The condition of the underlying geological strata.
(5) The atmospheric temperature.
(6) The direction and velocity of the wind.
(7) The nature and extent of vegetation.
(8) The surface topography.
(9) The evaporation.

It will be noted that some of the factors mentioned above trespass more or less on others and are not clearly separable.

79. **The Rate or Intensity of Rainfall.**—It will readily be recognized that with very heavy or intense rainfall a larger percentage of the water will run directly into the streams and a smaller percentage will be taken up by the strata than would be the case were the rainfall very light. In very light rainfalls there is no run-off, the water being either taken directly into the strata or re-evaporated from the surface.

80. Condition of Receiving Surfaces and Geological Strata.—Next in importance in modifying the disposal of rainfall is the condition of the surface on which the rain falls and of the underlying geological strata. If the geological strata are porus in nature and comparatively free from water they will readily receive and transmit the rainfall if the surface is in proper condition to receive it. The condition of the surface itself modifies the reception of the rainfall in a very marked manner. With high surface slopes the rainfall may be large, even with somewhat porous strata, and yet very little water will be taken up by the strata. With low slopes and porus strata a large amount of water will be received directly by the surface and passed into the ground water and deep waters of underlying geological strata.

The temperature has an important influence on the condition of the strata, and consequently the disposal of the rainfall. Strata otherwise porous but with saturated and frozen surface will receive and retain practically no water and the consequence is that under these conditions even a low rainfall may produce a considerable run-off that under other temperature conditions would not occur.

81. Effects of Wind.—The wind has a marked effect on evaporation and consequently on the quantity of rainfall that passes away in the atmosphere. The average velocity of the wind will vary in different parts of the United States from three to seventeen miles per hour and, other things being equal, will increase evaporation as such average velocity increases.

82. Effects of Vegetation.—The nature and extent of the vegetation on a surface has a marked effect on the disposal of the rainfall. Experiments at the Wisconsin Agricultural Experimental Station show that barley, oats and corn require 15.2, 19.6 and 26.4 inches of rainfall, respectively, to produce a crop. This includes the transpiration and evaporation from the cultivated surface as well as the actual quantity used by vegetation. The amount actually retained as a part of the vegetable growth is, of course, very small. The water simply serves to convey the soluble foods of the soil to the various fibres of the plant. The actual amount of water used in irrigation is not a fair criterion of the amount needed for the development of plant life as in most cases crops are over-irrigated. The actual depth and the rainfall and irrigation water used on crops vary from as low as 12 inches to sometimes as high as 16 feet, frequently running into quantities much in excess of any ordinary rainfall in moist climates where irrigation is found to be unnecessary.

In the Report of the Kansas State Board of Agriculture for December 31, 1889, Mr. W. Tweeddale, C. E., gives the following table containing the results of investigations by M. E. Risler, a Swiss observer, upon the daily consumption of water by different kinds of crops:

TABLE. X

Daily Consumption of Water by Crops.

Crops.	Inches of Water.	
	Minimum.	Maximum.
Lucern grass	0.134	0.267
Meadow grass	0.122	0.287
Oats	0.140	0.193
Indian Corn	0.110	1.570
Clover	0.140	
Vineyard	0.035	0.031
Wheat	0.106	0.110
Rye	0.091	
Potatoes	0.038	0.055
Oak trees	0.030	0.038
Fir Trees	0.020	0.043

Mr. Tweeddale finds that this table agrees with careful experiments made in France and elsewhere, and calculates from it that from seed time to harvest cereals will take up 15 inches of water and grass may absorb as much as 37 inches.

This table shows also one of the important reasons why a decrease of stream flow follows the destruction of forests and their replacement by meadows and cultivated fields. It is quite evident also that if the watersheds were covered by grasses or cereals there would be comparatively little water left for the flow of streams. From this it will be seen that the character of the vegetation on a watershed exerts a considerable influence on the ultimate distribution of the rainfall.

The presence or absence of forests has also, as shown by a series of observations in Germany, a marked effect on evaporation. Prof. M. W. Harrington (see Bulletin No. 7, U. S. Dept. of Agriculture, p. 97) has compiled the accompanying diagram (Fig. 68), which illustrates clearly the effect of forests upon the monthly evaporation. The upper curve represents the evaporation from water surfaces in the open country, while the lower curve shows the evaporation from water surfaces in the woods. The shaded area thus illustrates the saving due to the cover and protection of forests.

83. Percolation.—On pervious and unsaturated strata a portion of the rainfall sinks below the surface until it reaches a saturated

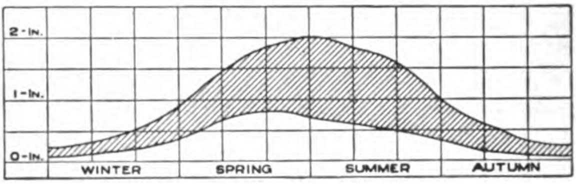


Fig. 68.—Reduction in Evaporation Due to the Presence of Forests.

or a relatively impervious stratum. The water then follows the dip of the stratum until it reaches an outlet along some stream or appears in the form of springs, frequently in an entirely different drainage area or possibly below the level of the sea itself. It is this ground water that gives rise to the dry weather flow of streams, and frequently is the only source from which stream flow is maintained during the dry seasons of the year. The same sources frequently maintain the winter flow at times when the rainfall is stored on the watershed in the form of snow and ice.

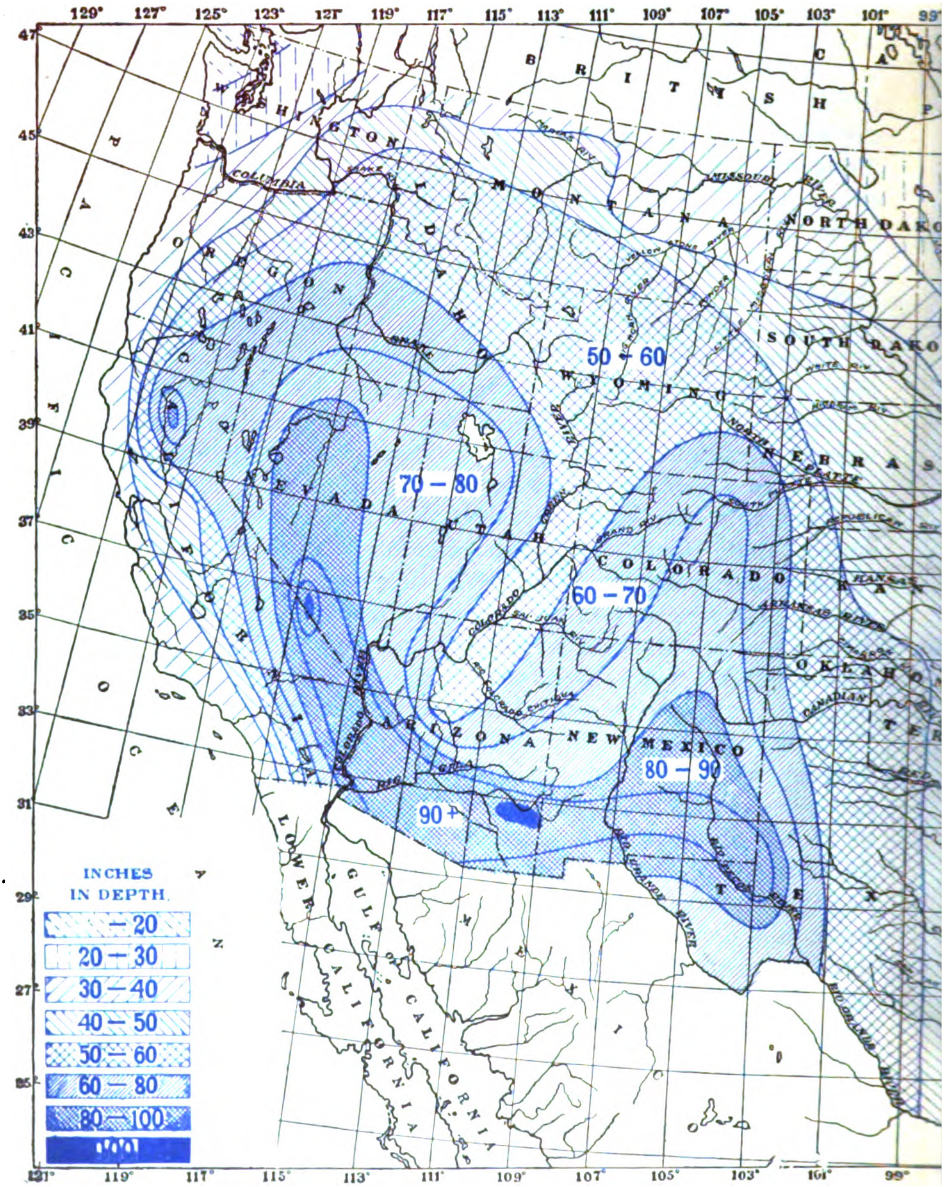
Percolation is an important factor in the storage of water and in the construction of raceways and canals and needs most careful attention when such works are under contemplation.

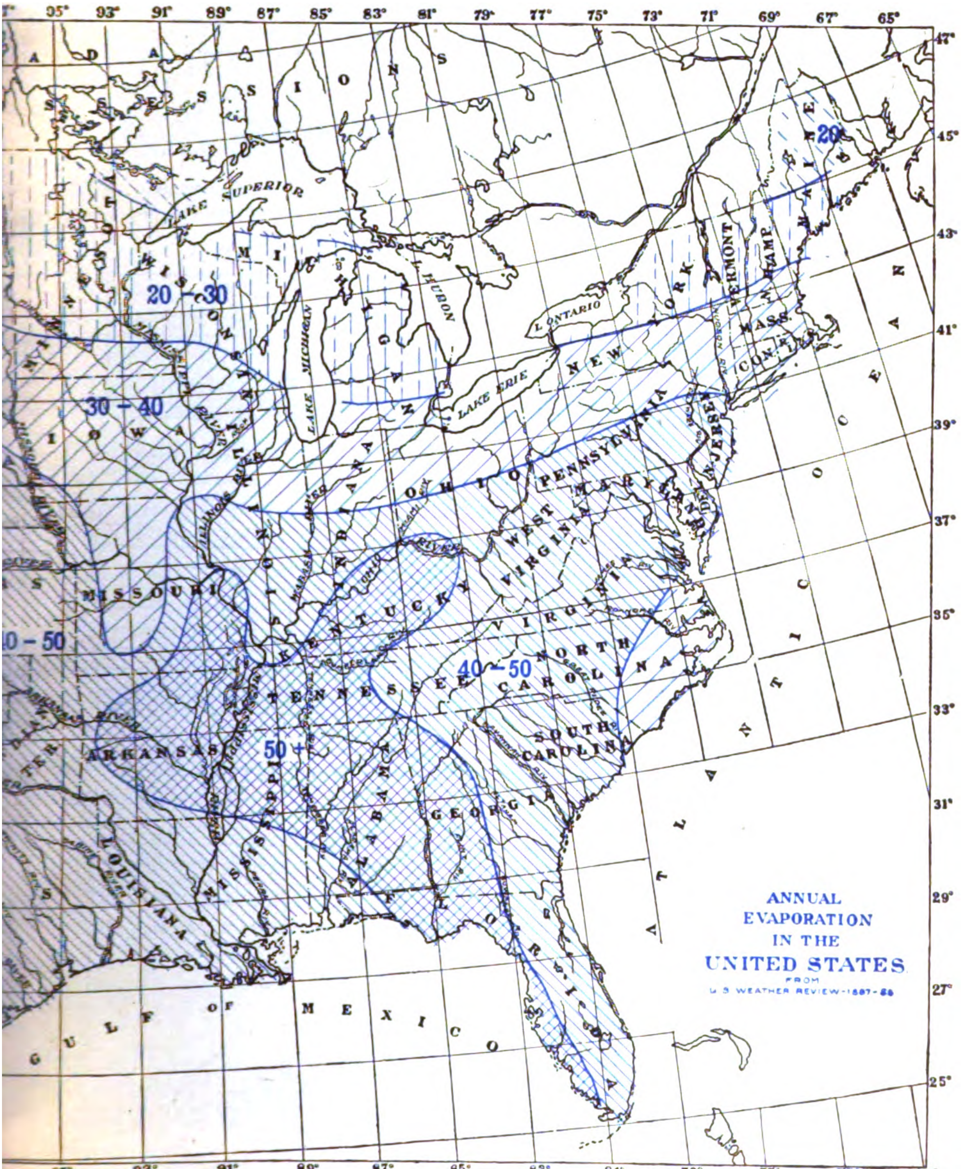
A large amount of valuable data concerning the losses due both to evaporation and seepage has been collected by Mr. E. Kuichling in connection with the study of the water supply for the New York Barge Canal and is reproduced in the Appendix.

A small portion of the ground water is taken up by the roots of plants and frequently feeds vegetation during dry periods. Water drawn from the soil for such purposes, after fulfilling its functions in vegetation, is transpired from the vegetable surfaces into the atmosphere. Streams fed from areas where large deposits of fine grained but porous material are developed, are usually more constant in flow and less subject to fluctuations either from flood or drought. The flows of the deeper strata usually pass far from the watershed on which the rainfall occurs and modify to a limited extent the stream flow in other valleys frequently far from the original rainfall source.

84. Evaporation.—Evaporation takes place from moist surfaces and from the water surfaces of swamps, lakes, streams and the oceans, whenever such surfaces are in contact with unsaturated atmosphere. The absorption of the rainfall by the strata effectively limits the amount of evaporation from a given area by reducing the area of contact of wet surface with the atmosphere, thus confining the evaporation largely to free water surfaces. Fig. 69 shows a map of the approximate annual evaporation which takes place from water surfaces at various points within the United States. It will be noted that this map shows, in the greater portion of the United States, evaporations equal to or greater than the annual rainfall at such localities. The total annual evaporation, as shown in the map, is based, however, on free water surfaces only, and evaporation from ground surfaces only takes place from occasional moist surfaces which occur after rains and when the humidity is high. The total amount of water evaporated, therefore, is very much less than that which the map would seem to indicate. This map and the table of monthly evaporation in the appendix are taken from data given in the Monthly Weather Review of September, 1888. The Weather Review observations are not based on absolute evaporation tests but are deduced from readings of dry and wet bulb thermometers as observed at various Signal Service Stations in 1887 and 1888. These deductions are supplemented by observations at several stations by means of the Piche evaprometer. While evaporation, like rainfall, varies from year to year in accordance with the variation in the controlling factors, yet in lieu of more extended observations this map and table indicate relative conditions at the various stations and approximately the evaporation from free water surfaces. The comparative monthly evaporation at sixteen stations distributed throughout the United States is shown graphically by Fig. 70. At a number of Eastern points, namely, Boston, Rochester and New York, evaporation observations have been made for a number of years and from the data thus collected a knowledge of the local variations that occur in evaporation at these points can be obtained

Evaporation is greatly promoted by atmospheric currents which have perhaps the most marked effect of any single influence. The temperature of the water and the humidity of the atmosphere also have a marked effect. Mr. Desmond Fitzgerald in a paper on evaporation (see Trans. Am. Soc. C. E., Vol. XV, page 581) offers the following formula for evaporation:


INCHES IN DEPTH
-20
20-30
30-40
40-50
50-60
60-80
80-100
100+
50-60
70-80
60-70
80-90
90+
BRITISH
WASHINGTON
MONTANA
NORTH DAKOTA
SOUTH DAKOTA
OREGON
IDAHO
WYOMING
NEBRASKA
NEVADA
UTAH
COLORADO
KANSAS
CALIFORNIA
ARIZONA
NEW MEXICO
OKLAHOMA
TEXAS
MEXICO
LOWER CALIFORNIA
GULF OF CALIFORNIA
PACIFIC OCEAN
COLUMBIA
MISSOURI RIVER
SNAKE
COLORADO RIVER
RIO GRANDE RIVER


ANNUAL
EVAPORATION
IN THE
UNITED STATES
FROM
U.S. WEATHER REVIEW-1887-88
20
20–30
30–40
40–50
50+
LAKE SUPERIOR
LAKE MICHIGAN
HURON
LAKE ERIE
L. ONTARIO
GULF OF MEXICO
ATLANTIC OCEAN
MISSOURI
IOWA
ARKANSAS
LOUISIANA
MISSISSIPPI
ALABAMA
GEORGIA
FLORIDA
SOUTH CAROLINA
NORTH CAROLINA
VIRGINIA
WEST VIRGINIA
KENTUCKY
TENNESSEE
OHIO
INDIANA
ILLINOIS
MICHIGAN
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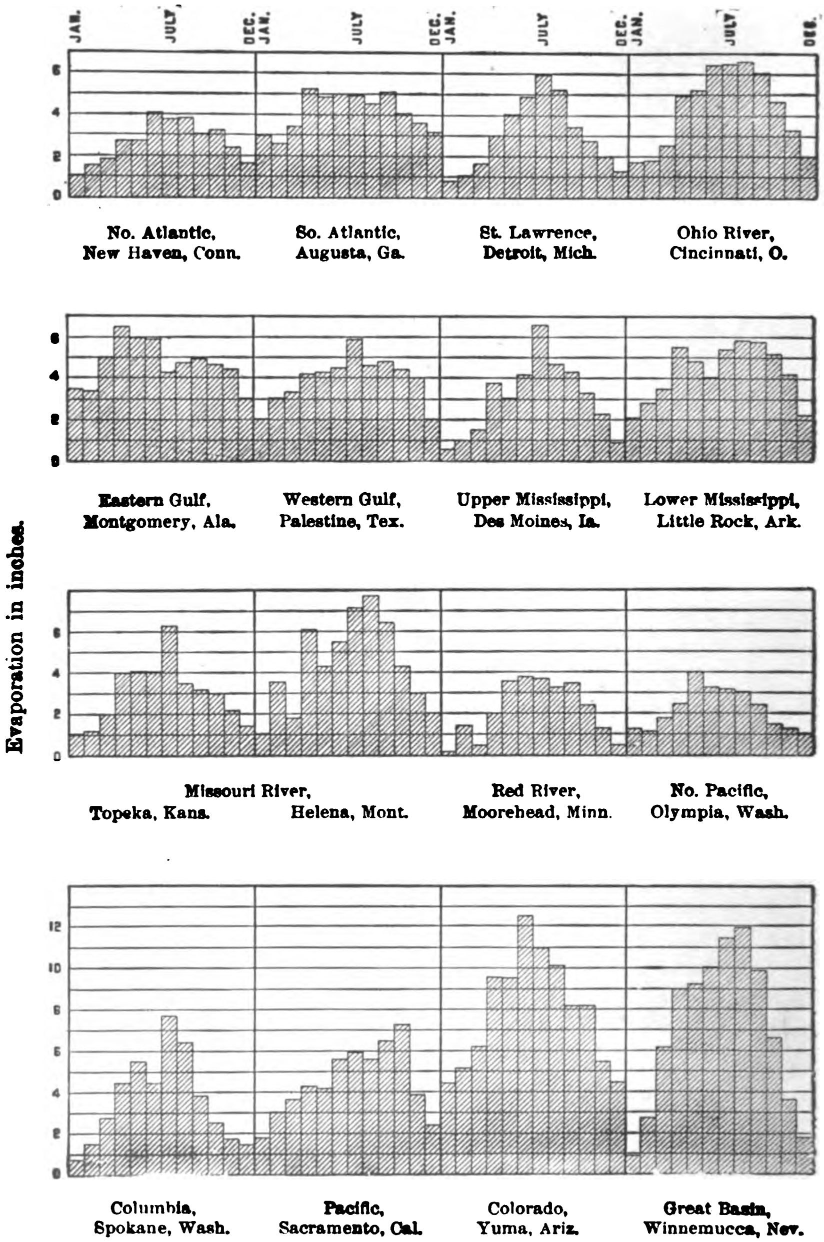

Fig. 70.—Monthly Evaporation From Free Water Surfaces at Various Points in the United States.

$$E = \frac{(V - v)\left(1 + \frac{W}{2}\right)}{60}$$

In this formula V equals the maximum force of vapor in inches of mercury corresponding to the temperature of the water; v, the force of the vapor present in the air; W, velocity of the wind in miles per hour; and E the evaporation in inches of depth per hour. The value of v depends on certain relations between the temperature of the air and the water. From a careful examination of the formula it will be seen that evaporation as represented thereby does not depend largely on temperature.

Table XI is taken from a paper on "Rainfall, Flow of Stream, and Storage" by Mr. Desmond Fitzgerald (Trans. Am. Soc. C. E., Vol. XXVII, No. 3), and shows the monthly evaporation from water surface at Boston, Massachusetts, for sixteen years. The table is partially made up from a diagram of mean monthly evaporation but only when the observation practically agreed with the same.

85. **Evaporation Relations.**—Professor Cleveland Abbe gives the following relations of evaporation, as established by Professor Thomas Tate:

(a) Other things being the same, the rate of evaporation is nearly proportional to the difference of the temperature indicated by the wet-bulb and dry-bulb thermometers.

(b) Other things being the same, the augmentation of evaporation due to air in motion is nearly proportional to the velocity of the wind.

(c) Other things being the same, the evaporation is nearly inversely proportional to the pressure of the atmosphere.

(d) The rate of evaporation of moisture from damp, porous substances of the same material is proportional to the extent of the surface presented to the air, without regard to the relative thickness of the substances.

(e) The rate of evaporation from different substances mainly depends upon the roughness of, or inequalities on, their surfaces, the evaporation going on most rapidly from the roughest or most uneven surfaces; in fact, the best radiators are the best evaporizers of moisture.

(f) The evaporation from equal surfaces composed of the same material is the same, or very nearly the same, in a quiescent atmosphere, whatever may be the inclination of the surfaces; thus a

TABLE XI.

Evaporation from Water Surface in Inches — Sixteen Years.

	1876	1877	1878	1879	1880	1875–1890 and 1881–1884	1885	1886	1887	1888	1889	1875–90 Total	1875–90 Mean
January	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	*0.96	15.36	0.96
February	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	*1.05	16.80	1.05
March	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	*1.70	27.20	1.70
April	*2.98	*2.98	*2.98	*2.98	*2.98	*2.98	*2.98	3.12	3.07	2.78	2.84	47.57	2.97
May	*4.45	4.05	4.14	5.89	5.22	*4.45	3.77	4.45	4.83	3.35	4.57	71.42	4.46
June	5.44	5.68	5.26	5.32	6.46	*5.55	7.01	5.25	5.05	5.98	3.94	88.69	5.54
July	7.50	4.82	6.04	6.41	5.82	*5.98	7.09	5.59	5.96	5.57	5.04	95.72	5.98
August	6.21	4.40	4.33	5.23	5.34	*5.50	7.41	5.80	6.20	5.81	4.25	87.98	5.50
September	3.48	4.08	4.04	3.80	4.04	*4.20	5.13	4.55	4.57	3.91	3.08	65.88	4.12
October	3.12	2.51	3.52	2.99	2.79	*3.11	2.79	4.13	3.61	3.27	3.13	50.52	3.16
November	0.66	*2.23	*2.23	*2.23	2.60	*2.23	*2.23	2.69	3.00	2.71	1.98	35.94	2.25
December	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	*1.51	24.16	1.51
Total	39.06	35.97	37.76	40.07	40.47	39.22	43.63	40.80	41.51	38.60	34.05	627.24	39.20

*From curve of mean evaporation, so adjusted as to vary very slightly from the means of observation. Unstarred numbers are from observations of Chestnut Hill Reservoir.

horizontal plate with its damp face upward evaporates as much as one with its damp face downward.

(g) The rate of evaporation from a damp surface (namely, a horizontal surface facing upward) is very much affected by the elevation at which the surface is placed above the ground.

(h) The rate of evaporation is affected by the radiation of surrounding bodies.

(i) The diffusion of vapor from a damp surface through a variable column of air varies (approximately) in the inverse ratio of the depth of the column, the temperature being constant.

(j) The amount of vapor diffused varies directly as the tension of the vapor at a given temperature, and inversely as the depth of the column of air through which the vapor has to pass.

(k) The time in which a given volume of dry air becomes saturated with vapor, or saturated within a given percentage, is nearly independent of the temperature if the source of vapor is constant.

(l) The times in which different volumes of dry air become saturated with watery vapor, or saturated within a given per cent, are nearly proportional to the volumes.

(m) The vapor already formed diffuses itself in the atmosphere much more rapidly than it is formed from the surface of the water. (This assumes, of course, that there are no convection currents of air to affect the evaporation or the diffusion.)

86. Practical Consideration of Losses.—From the previous discussion it will be readily realized that it would be impossible to differentiate all of the methods of the disposal of rainfall upon a drainage area. Evaporation differs widely from different classes of vegetation and from different classes of land surfaces; also on account of the slope and exposure. No two square miles upon a drainage area offer the same conditions as affecting evaporation which differs very widely with such conditions. Evaporation and seepage from any surface varies with the temperature, with the moisture in the air, and with the velocity of the wind. It is therefore impossible to compute, with any degree of accuracy, evaporation over an extended surface of a watershed or drainage area, or to ascertain with any degree of accuracy the probable losses that will take place in the same area.

For water power purposes, the rainfall can, therefore. be divided into two quantities in which the water power engineer is interested: First: The run-off on which the power developed directly depends, and, Second: The losses, by whatever means they occur, which are not available for such purposes. Evaporation is usually but not always the source of greatest loss on a drainage area and commonly other sources of loss are insignificant when compared with it. It is therefore a common practice to deduct the run-off from the rainfall on a given drainage area and to classify the difference as evaporation, including under this term all losses of this same general character, whether through seepage, evaporation or otherwise.

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CHAPTER VIII.

RUN-OFF.

87. **Run-Off.**—That portion of the rainfall that is not absorbed by the strata, utilized by vegetation or lost by evaporation, finds its way into streams as surface flow or run-off. The demands of the first named factors are always first supplied and the run-off is therefore the overflow or excess not needed to supply the other demands on the rainfall. The run-off, therefore, while a direct function of the rainfall, is not found to increase in direct proportion thereto, except perhaps in seasons such as early spring when from seasonal conditions the demands of vegetation, percolation and evaporation are not active and all or most all of the rainfall flows away on the surface. The remainder of the year the run-off may be said to increase with the rainfall but usually at a much less rapid rate and in many cases the rainfall is entirely absorbed by the strata or vegetation, and does not influence or affect the run-off. In this case the run-off is supplied from the ground water, stored from previous rainfalls, and is entirely or largely independent of the immediate rainfall conditions.

An examination of the observed run-off of streams, and the rainfall on their respective drainage areas, for annual, monthly and seasonal periods, will show that there is a relation more or less direct between the rainfall and run-off (see Fig. 71, et seq.). The relations are shown by various diagrams and mean curves from which many departures will be noted. The departure of individual observations from the mean curve expressing these relations shows the relative importance and influence of other factors in affecting such relations. The relations of the numerous factors which are known to influence the results are quite complex and are not well established and much more meteorological information in much greater detail and a careful consideration and study of the same will be necessary before such relations can be even approximately established.

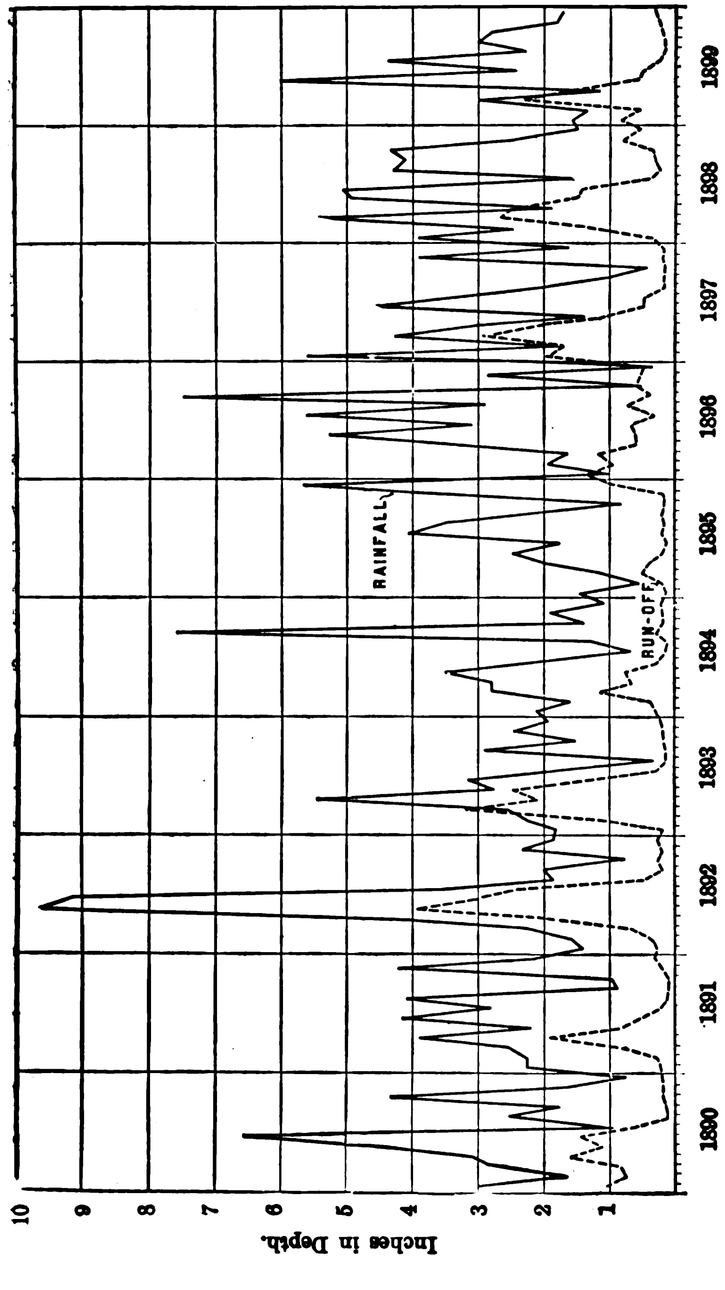


Fig. 71.—Comparison of Mean Monthly Rainfall and Run-off of Illinois River Basin.

88. Influence of Various Factors.—The influence of various factors of disposal was discussed in the last chapter. Evaporation is known to vary with temperature, the direction and velocity of the winds, barometric pressure, and various other meteorological influences, and yet no clearly defined relation has yet been shown to exist between these factors, by means of which their actual influence on the run-off can be approximately calculated. Mr. C. C. Vermeule (see Vol. III, Geol. Survey of New Jersey) considers that annual evaporation depends largely on the mean annual temperature and offers a formula for the calculation of the same, which, in many cases, gives results which seem to agree closely with the facts and data collected from a number of Eastern drainage areas. Mr. Vermeule's formula for the relation between annual evaporation and precipitation on the Passaic River, and some other Eastern drainage areas where conditions are similar, is:

$$E=15.50+0.16\ R$$

in which

E=The annual evaporation (including all losses on drainage area except from run-off)

and R=the annual rainfall.

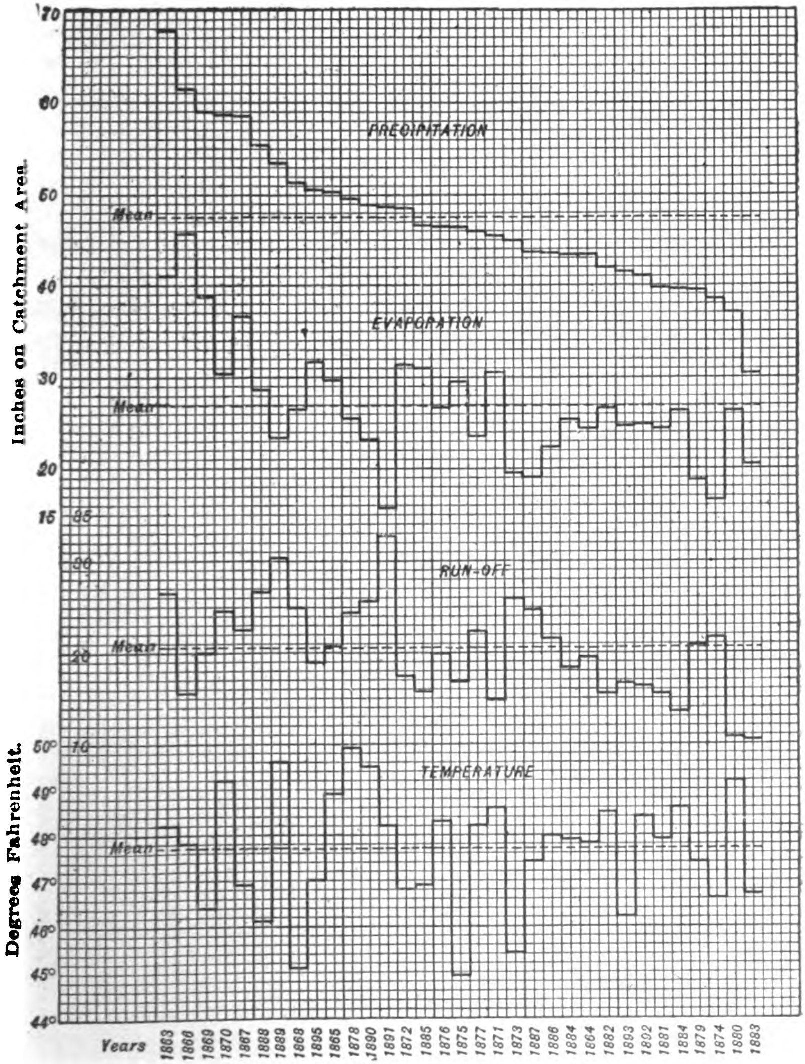
For general application to all streams he suggests the formula

$$E=(15.50+0.16\ R)\ (0.05\ T-1.48)$$

in which T= mean annual temperature.

Mr. Vermeule also offers a formula for the evaporation for each month and discusses at length the influence of ground storage on the flow of streams. Mr. Geo. W. Rafter (see Water Supply and Irrigation Paper No. 80) has made a careful analysis of available data which indicates that no such intimate relation can be found to exist. In general, the information available does not seem to show that other factors have a sufficiently definite relation to run-off or to each other to make such relation clearly manifest and yet such factors are known to have an unmistakable and constant influence. This fact is quite clearly demonstrated by a number of diagrams prepared by Mr. Rafter, which are here reproduced.

Figure 72 shows graphically the relation between precipitation, evaporation, run-off and temperature on the Lake Cochituate basin for thirty-three years. In this diagram the years are arranged in accordance with the precipitation. In a general way the evaporation and run-off for these years may be said to vary with the pre-



Years arranged in order of dryness.

Fig. 72.—Relation Between Precipitation, Evaporation, Run-off and Temperature on Lake Cochituate Basin.

9

cipitation. Evaporation, which, it must be remembered, here includes all losses except that due to run-off, increases in general as the rainfall on the area increases and decreases with the rainfall. For limited periods, however, this general law does not hold. Other factors affect the relations and cause material departures from the general law. This is particularly marked in the years 1891 and 1872. For these two years the rainfall was almost identical in amount. The evaporation for the same years, however, differed materially, being about 16 inches less in 1891 than in 1872. As a consequence the run-off for the year 1891 was about 15½ inches greater than in 1872.

In order to demonstrate the mutual relation between evaporation and temperature the data illustrated in the previous figure has been

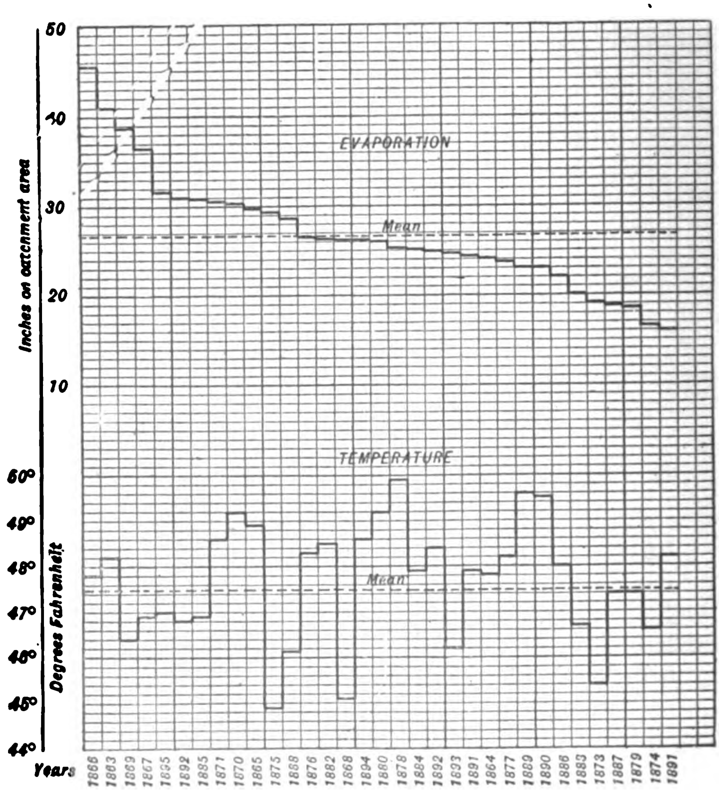
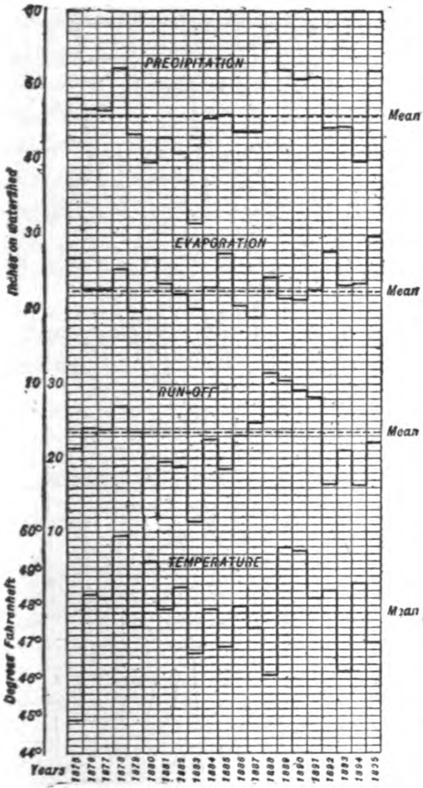
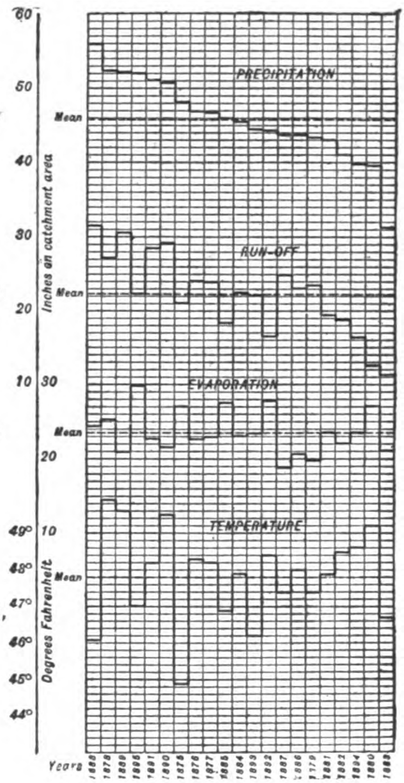


Fig. 73.—Relation Between Evaporation and Temperature on Lake Cochituate Basin.

Years being arranged according to amount of evaporation.





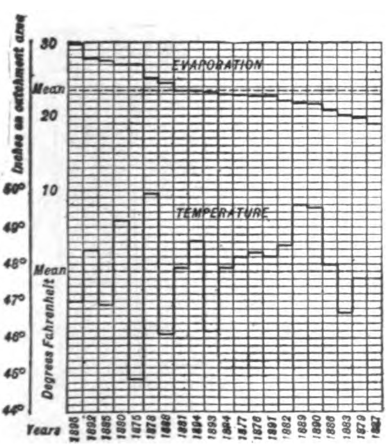
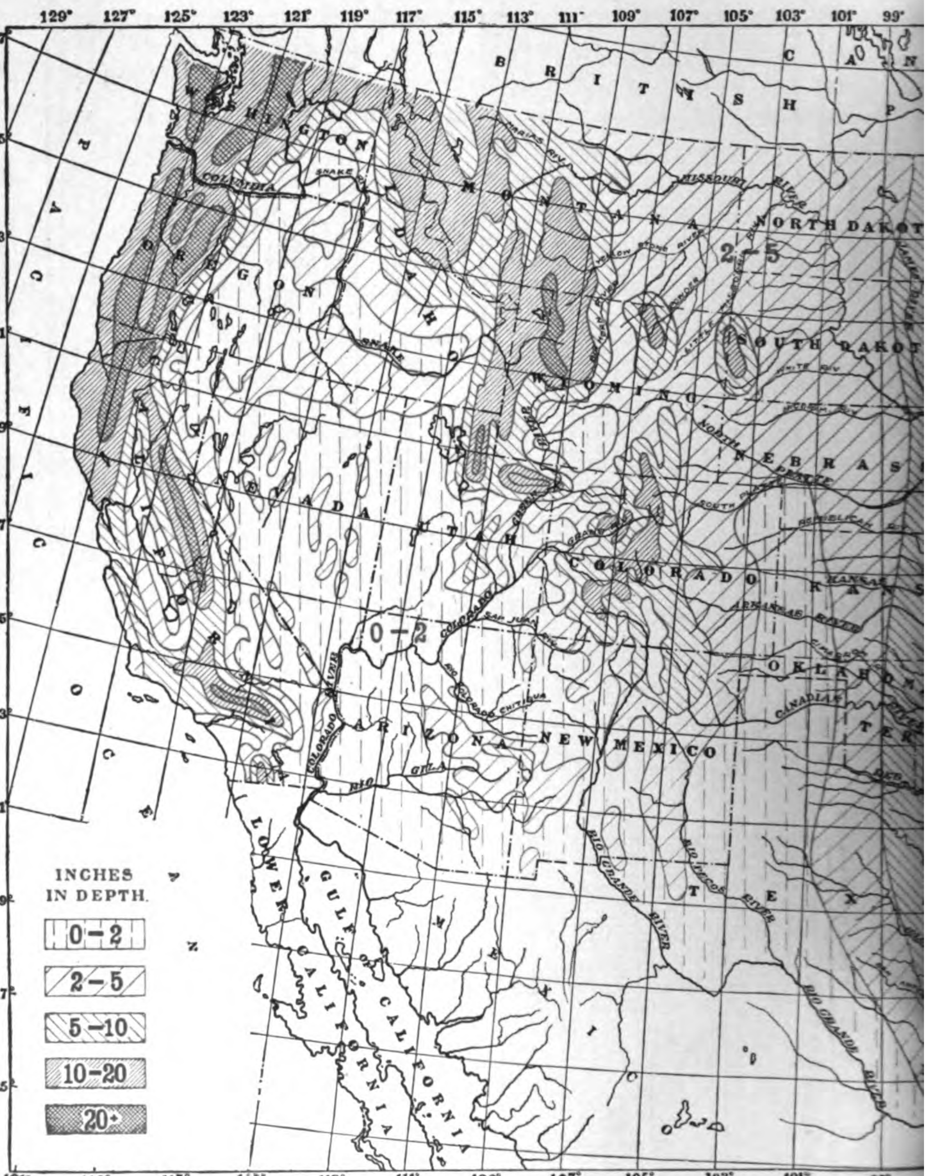


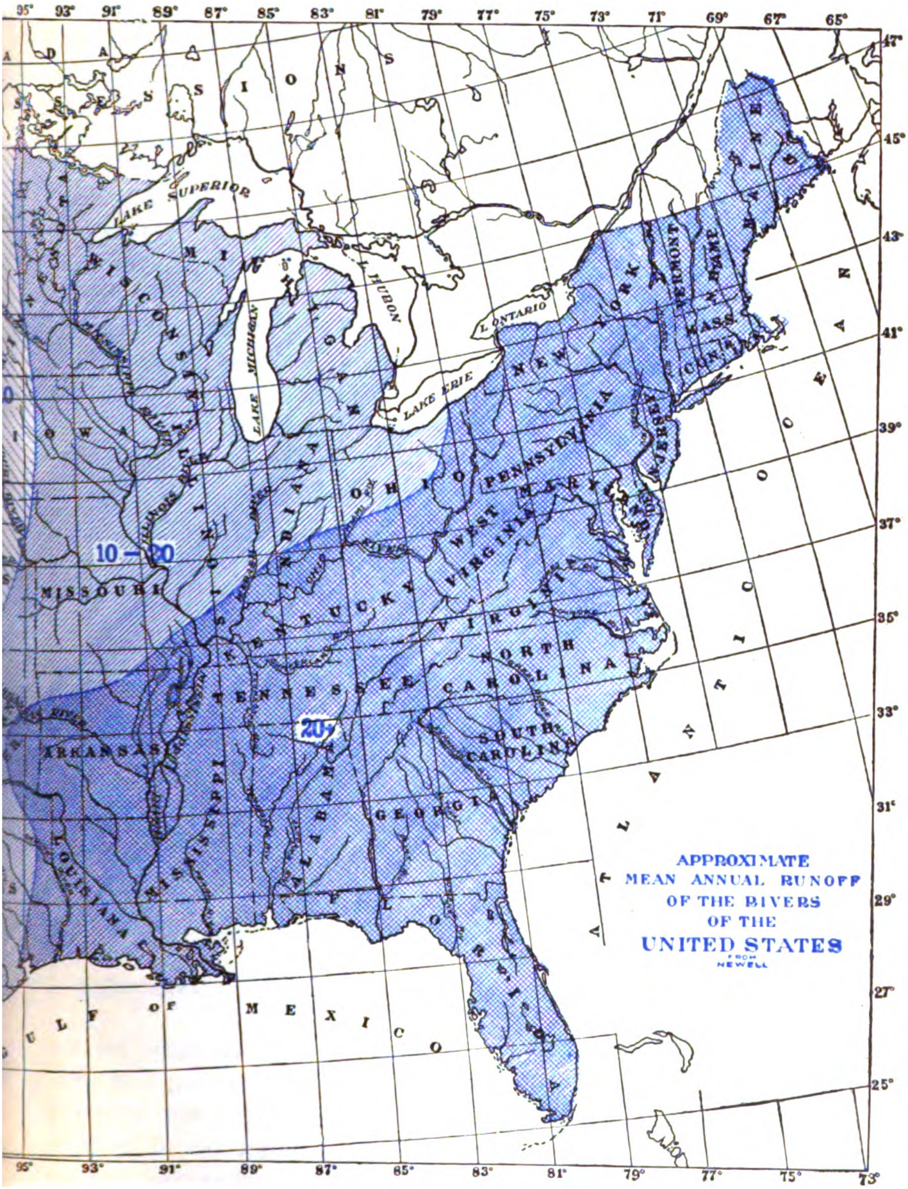
Fig. 74.—Relations between Precipitation, Run-Off, Evaporation and Temperature on Sudbury River Basin.

Years arranged according to regular order, dryness and decreasing evaporation respectively.



129° 127° 125° 123° 121° 119° 117° 115° 113° 111° 109° 107° 105° 103° 101° 99°
BRITISH
WASHINGTON
IDAHO
MONTANA
OREGON
NORTH DAKOT
SOUTH DAKOT
WYOMING
NEBRAS
NEVADA
UTAH
COLORADO
KANS
CALIFORNIA
ARIZONA
NEW MEXICO
OKLAHOM
TEXAS
MEXICO
LOWER CALIFORNIA
GULF OF CALIFORNIA
PACIFIC OCEAN
COLUMBIA
SNAKE
MISSOURI RIVER
COLORADO RIVER
RIO GILA
RIO GRANDE RIVER
RIO PECOS RIVER
ARKANSAS RIVER
CANADIAN
2-5
0-2
INCHES
IN DEPTH.
0-2
2-5
5-10
10-20
20+
121° 119° 117° 115° 113° 111° 109° 107° 105° 103° 101° 99°

F


APPROXIMATE
MEAN ANNUAL RUNOFF
OF THE RIVERS
OF THE
UNITED STATES
FROM
NEWELL
10 – 20
20+
LAKE SUPERIOR
LAKE MICHIGAN
L. HURON
LAKE ERIE
L. ONTARIO
GULF OF MEXICO
ATLANTIC OCEAN
MAINE
VERMONT
N. HAMP.
MASS.
CONN.
NEW YORK
PENNSYLVANIA
NEW JERSEY
MARYLAND
WEST VIRGINIA
VIRGINIA
NORTH CAROLINA
SOUTH CAROLINA
GEORGIA
FLORIDA
ALABAMA
MISSISSIPPI
TENNESSEE
KENTUCKY
OHIO
INDIANA
ILLINOIS
MICHIGAN
WISCONSIN
IOWA
MISSOURI
ARKANSAS
LOUISIANA
95° 93° 91° 89° 87° 85° 83° 81° 79° 77° 75° 73° 71° 69° 67° 65°
47° 45° 43° 41° 39° 37° 35° 33° 31° 29° 27° 25°

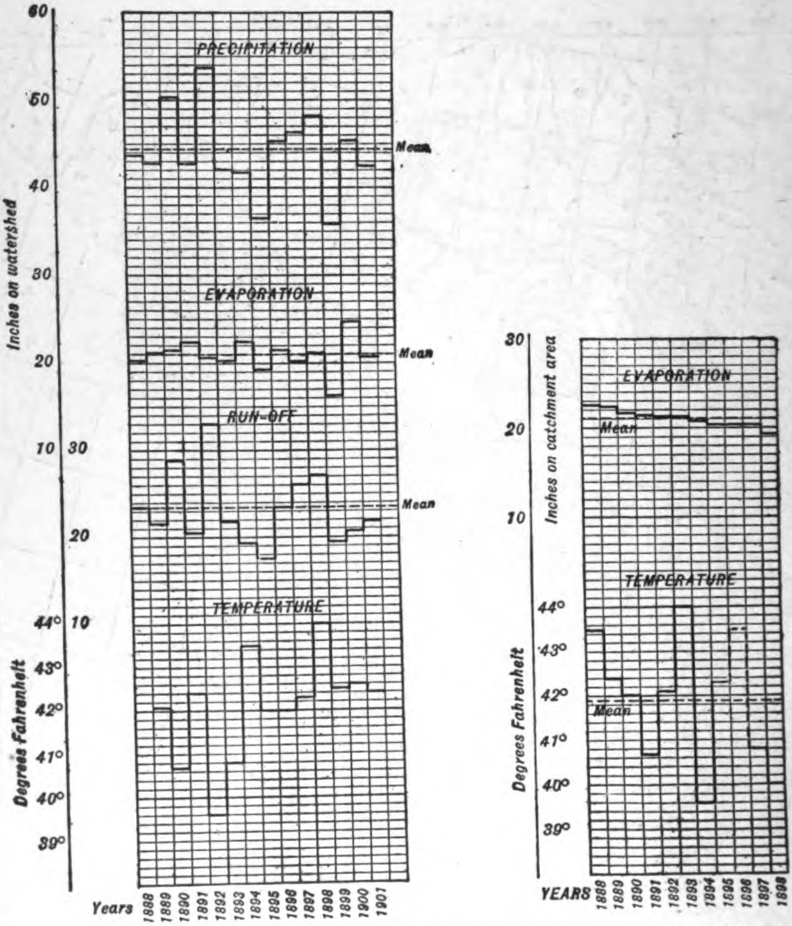


Fig. 75.—Relations Between Precipitation, Run-Off, Evaporation and Temperature on Upper Hudson River.

Years arranged according to regular order and decreasing evaporation.

rearranged by Mr. Rafter, and in Figure 73 the relation for the years has been arranged in the order of their evaporation, and compared with the mean temperature for the year. This figure serves to show that while temperature may, and unquestionably does, influence evaporation, yet the mean annual temperature has no controlling effect on the annual evaporation. It will be noted that for the year 1878, when the mean temperature was a maximum, the evaporation was considerably below the average for this drainage area. Similar relations for the Sudbury River basin are shown in Fig. 74 and for the Upper Hudson River basin in Fig. 75.

89. **Relations of Annual Rainfall and Run-Off.**—Figure 76 is a mean run-off map of the United States and should be compared with the map of average rainfall. The run-off as shown by this map is expressed in inches on the drainage area and similarly to the common expression for the amount of rainfall. The value of this map is comparative only. In this case, as in the cases of rainfall and evaporation, the mean conditions are subject to wide variations. A detailed study of local conditions is always necessary in order to fully understand and appreciate the influence of extreme conditions and of local factors.

The relation between the annual rainfall and run-off on various drainage areas is shown in Figures 71 to 75, inclusive, as previously described. The mean relations between these two factors on four selected drainage areas are, however, more clearly shown by the graphical diagrams Figs. 77, 78 and 79. From these diagrams a

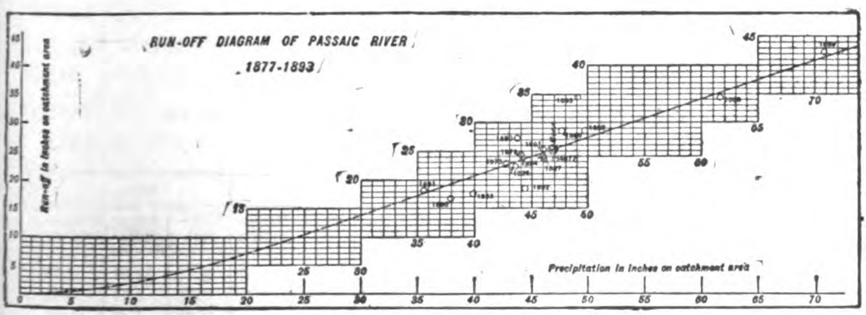


Fig. 77.

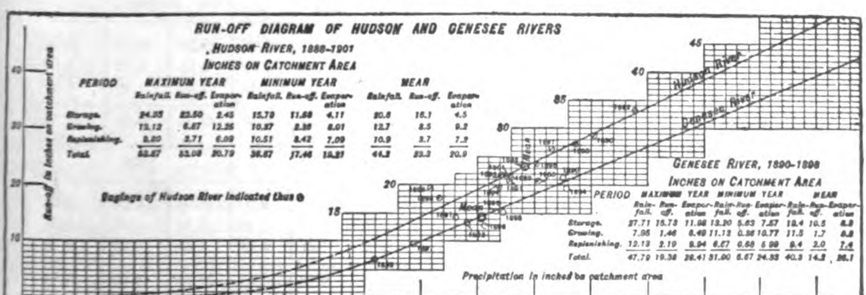


Fig. 78.

mean relation can be traced for each area from which, however, there are considerable departures in individual years. The study, therefore, of this subject on this basis will demonstrate the mean relation and the departure therefrom which must be expected on the area considered and other areas where physical conditions are similar.

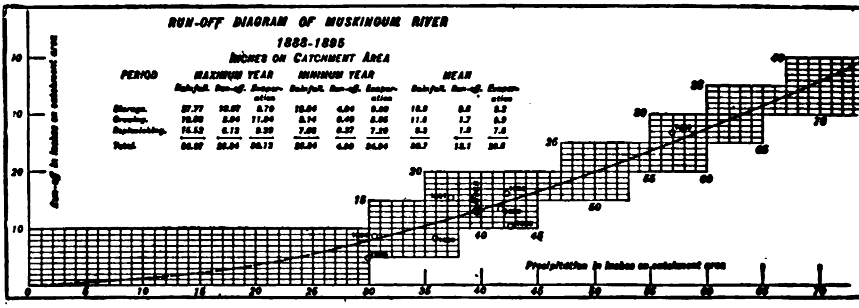


Fig. 79

Table XII.—*Muskingum River, 1888-1895, inclusive.*

[Catchment area=5,828 square miles.]

Period.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
	1888.			1889.			1890.		
Storage	17.16	5.17	11.99	13.52	6.02	7.50	27.77	18.07	9.70
Growing	14.31	1.77	12.54	12.12	1.24	10.88	13.68	2.64	11.04
Replenishing	11.14	3.39	7.75	10.24	.96	9.28	15.52	6.13	9.39
Year	42.61	10.33	32.28	35.88	8.22	27.66	56.97	26.84	30.13
	1891.			1892.			1893.		
Storage	16.72	12.42	4.30	20.39	9.06	11.33	25.04	14.13	10.91
Growing	13.56	1.77	11.79	16.54	3.65	12.89	8.31	1.22	7.09
Replenishing	7.08	1.37	5.71	4.81	.67	4.14	9.01	.85	8.16
Year	37.36	15.56	21.80	41.74	13.38	28.36	42.36	16.20	26.16
				1894.			1895.		
Storage				16.93	7.63	9.30	13.04	4.04	9.00
Growing				4.56	.66	3.90	9.14	.49	8.65
Replenishing				9.02	.41	8.61	7.66	.37	7.29
Year				30.51	8.70	21.81	29.84	4.90	24.94

90. **The Water Year.**—The relation of annual rainfall and annual run-off is more or less obscured by variations in the periodic distribution of the annual rainfall. A study of the relation of the periodic rainfall and the periodic run-off is therefore necessary.

For a comprehensive understanding of the relation of rainfall to run-off it is more convenient to refer to the water year instead of the calendar year. The water year is the annual division of time that represents the full annual cycle of change in hydrological conditions. It does not, as a rule, conform very closely to the calendar year, neither is the water year constant for each annual period in its beginning or end, but varies as meteorological conditions vary.

As previously stated, in the greater portion of the United States, the water year naturally divides itself into periods, beginning, approximately, with December, and ending, approximately, with the following November. The period from December to and including May is termed the "Storage" period; June, July and August constitute the "Growing" period, and September, October and November are termed the "Replenishing" period. Not only the year but these periods as well vary each year, and are not necessarily limited by our artificial division of calendar months and years.

During the storage period, the snows of winter and the rains of spring saturate the ground, and a large amount of water is held in storage in lakes, swamps, and forests, and in pervious soils, sands and gravels. The portions of this stored water tributary to a drainage area but not necessarily within the boundaries thereof, and at elevations above the level of the stream, are, when conditions demand, available to supply the stream flow, and are also available for the purpose of sustaining plant life. Such waters will feed a stream to an extent depending on their character and magnitude, regardless of the amount of the immediate rainfall, and will cause a stream to flow for several months, even without rain, if the pervious deposits and other storage resources are well developed upon the area. These relations vary widely with each individual area, and in areas not well provided with such deposits the streams often run dry through the warm days of summer.

Whenever the surface of the stream falls below the ground water gradient the ground water is affected and begins to supply the stream flow. This sometimes occurs early in May, and seldom later than the beginning of June. During June, July and August the rainfall is rarely sufficient to take care of the evaporation and growth of vegetation without something of a draft on the ground water, and the stream flow during this period is usually entirely dependent on the ground water, except during exceptionally heavy rainstorms. By the end of the growing period about August 31st the ground water is often so reduced as to be capable of storing several inches of rainfall. During the replenishing and storage periods of winter and spring the ground begins to receive its store of water, and, with favorable rainfalls, the ground becomes fully saturated by the end of April or May.

Table XIII.—*Hudson River, 1888–1901, inclusive.*

[Catchment area=4,500 square miles.]

Period.	1888.			1889.			1890.		
	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
Storage	20.40	17.06	3.34	17.10	14.04	3.06	24.75	19.28	5.47
Growing	10.25	2.05	8.20	15.05	4.26	10.79	13.50	2.85	10.65
Replenishing	13.27	4.53	8.74	10.81	3.41	7.40	12.10	6.81	5.29
Year	43.92	a23.64	20.28	a42.96	21.71	21.25	a50.35	28.94	21.41
	1891.			1892.			1893.		
Storage	20.69	16.59	4.10	24.95	22.50	2.45	19.33	15.20	4.13
Growing	13.49	2.07	11.42	19.12	6.87	12.25	13.87	3.12	10.75
Replenishing	8.78	1.90	6.88	9.80	3.71	6.09	8.98	3.59	5.39
Year	42.96	20.56	22.40	53.87	33.08	20.79	42.18	21.91	20.27
	1894.			1895.			1896.		
Storage	21.37	13.18	8.19	15.79	11.68	4.11	22.17	16.52	5.65
Growing	8.73	3.20	5.53	10.37	2.36	8.01	10.25	2.53	7.72
Replenishing	11.87	2.99	8.88	10.51	3.42	7.09	12.79	4.58	8.21
Year	41.97	19.37	22.60	36.67	17.46	19.21	45.21	23.63	21.58
	1897			1898.			1899.		
Storage	19.77	14.60	5.17	22.80	18.61	4.19	19.48	15.15	4.33
Growing	15.80	7.79	8.01	13.52	3.24	10.28	7.40	1.63	5.77
Replenishing	10.94	3.80	7.14	12.19	5.27	6.92	8.91	2.76	6.15
Year	46.51	26.19	20.32	48.51	27.12	21.39	35.79	19.54	16.25
				1900.			1901.		
Storage				21.13	16.12	5.01	18.47	14.84	3.63
Growing				12.11	2.30	9.81	15.09	4.02	11.07
Replenishing				12.17	2.25	9.92	9.02	3	6.02
Year				45.41	20.67	24.74	42.58	21.86	20.72

a Approximate.

Table XIV.—*Connecticut River, 1872–1885, inclusive.*

[Catchment area = 10,234 square miles.]

Period.	1872.			1873.[a]			1874.[a]		
	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-fall.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
Storage	14.92	13.30	1.62	18.16	21.80	3.64	23.08	23.04	0.04
Growing	18.96	6.29	12.67	10.11	2.71	7.40	14.37	6.62	7.75
Replenishing	12.42	6.64	5.78	15.04	5.22	9.82	7.76	2.15	5.61
Year	46.30	26.23	20.07	43.31	29.73	13.58	45.21	31.81	13.40
Period	1875.			1876.[a]			1877.		
Storage	17.51	15.47	2.04	22.50	24.74	− 2.24	18.09	12.68	5.41
Growing	14.55	3.80	10.75	12.51	3.35	9.16	14.00	2.91	11.09
Replenishing	11.36	3.60	7.76	10.57	2.28	8.29	13.08	5.27	7.81
Year	43.42	22.87	20.55	45.58	30.37	15.21	45.17	20.86	24.31
Period.	1878.			1879.			1880.		
Storage	21.88	18.02	3.86	23.19	21.49	1.70	18.29	14.78	3.51
Growing	13.59	3.45	10.14	16.07	2.92	13.15	11.82	2.45	9.57
Replenishing	10.56	3.06	7.50	9.48	2.93	6.55	11.58	2.62	8.96
Year	46.03	24.53	21.50	48.74	27.34	21.40	41.69	19.85	21.84
Period.	1881.			1882.			1883.		
Storage	20.83	16.02	4.81	[b]20.50	12.14	8.36	[b]12.85	8.73	4.12
Growing	11.30	2.93	8.37	[b]11.45	3.35	8.10	[b]13.50	2.51	10.99
Replenishing	11.38	3.39	7.99	[b]6.50	2.17	4.33	[b]6.20	1.37	4.83
Year	43.51	22.34	21.17	38.45	17.66	20.79	32.55	12.61	19.94
Period.				1884.			1885.		
Storage				21.42	20.20	1.22	18.58	13.63	4.95
Growing				12.14	2.79	9.35	14.82	3.20	11.62
Replenishing				8.51	2.61	5.90	11.76	5.61	6.15
Year				42.07	25.60	16.47	45.16	22.44	22.72

[a] Not included in mean. [b] Rainfall computed, approximate.

91. **Relation of Periodic Rainfall to Run-Off.**—For streams where the observations of flow have been made for a number of years, comparisons can readily be made of the relation of annual and periodic rainfall and run-off. Such investigations should be made by the water power engineer when considering a river relative to its availability for water power purposes. An analysis of such data for the Muskingum, Hudson, and Connecticut Rivers as made by Mr. Rafter, is shown in Tables XII, XIII and XIV (for ad-

ditional tabular data see Appendix). Graphical representations of the periodic relations of the rainfall and run-off on the Upper Hudson River basin are shown in Fig. 80, and the same relations for the Sudbury River basin are shown in Fig. 81.

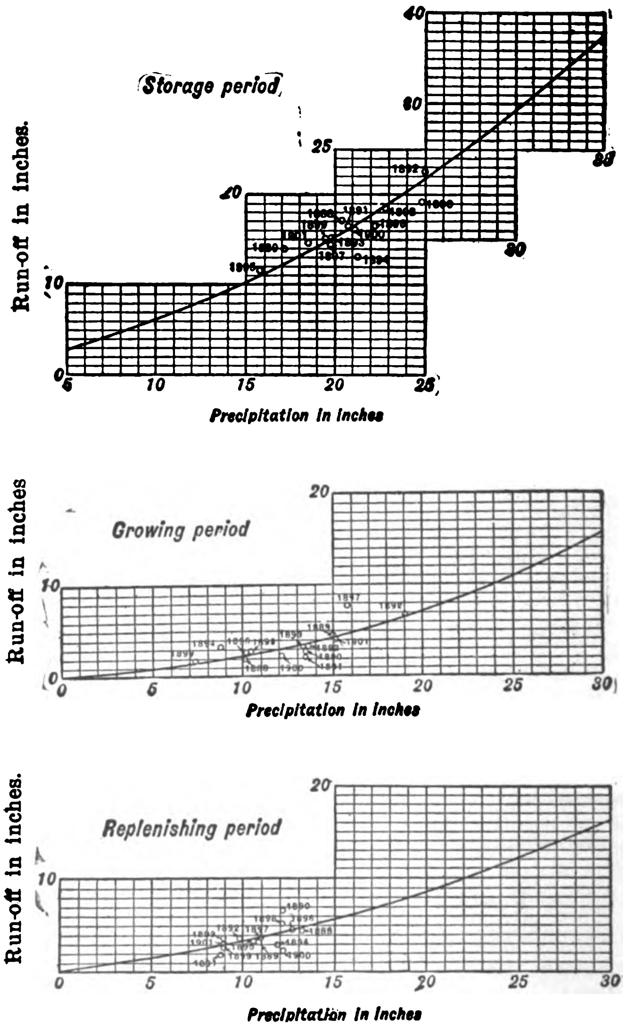


Fig. 80.—Rainfall and Run-Off of Upper Hudson River for Each Period of the Water Year.

[From W. S. and I. Paper No. 80 "Relation of Rainfall to Run-Off."]

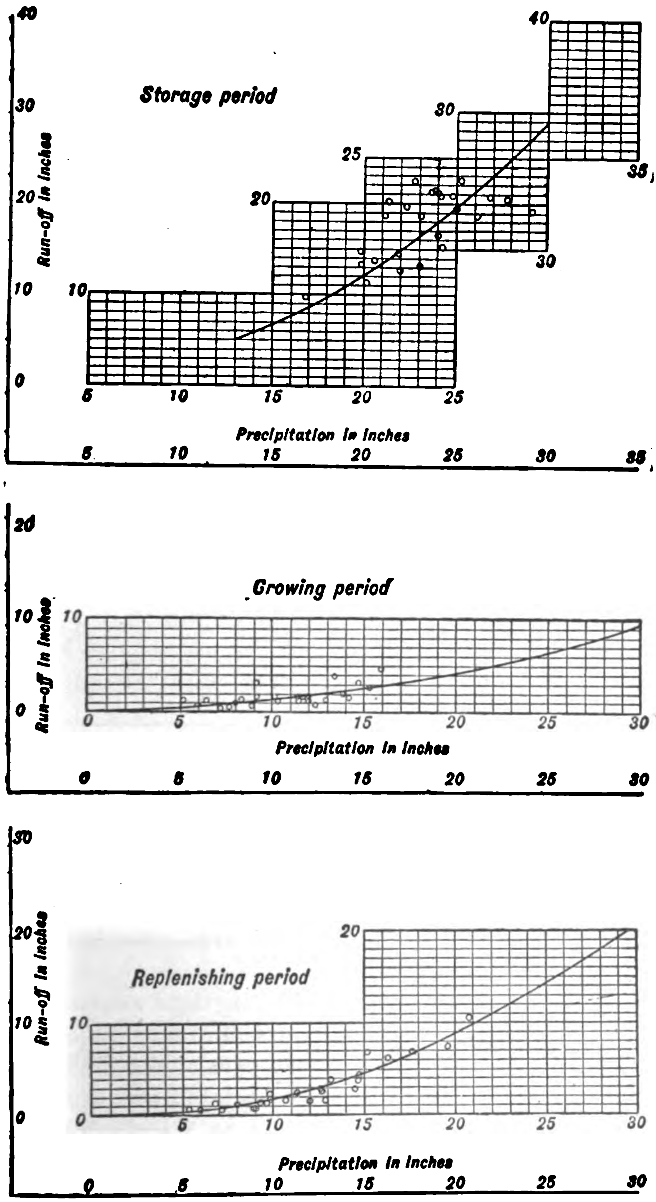


Fig. 81.—Rainfall and Run-Off of Sudbury River for Each Period of the Water Year.

[From W. S. and I. Paper No. 80 "Relation of Rainfall to Run-Off."]

92. Monthly Relation of Rainfall and Run-Off.—The relations of rainfall to run-off from month to month on a given drainage area are not usually as direct and definite as the annual and periodic relations. The mean and extreme relations can, however, often be established within somewhat wider limits, and such relations will permit of the formation of at least a general idea of the probable limits of the monthly run-off, under other rainfall conditions. The wide range of the possible error of such estimates will be shown by the divergence of independent observations from the normal. To establish accurately the maximum and minimum limits, it is probable that observations, at least as extended as those needed for accurate rainfall estimates, will be needed.

The observed relations between the monthly rainfall and the monthly run-off in various drainage areas are shown by Figs. 82, 83, 84 and 85.

On Fig. 82 are shown the relations of monthly rainfall and run-off for several Northern river basins, and on Fig. 83 are shown the same relations for several Southern river basins. An examination of these diagrams will show the marked effect of seasonal temperatures and conditions upon the quantity of run-off. The high percentage of run-off in the spring should be noted; also how the percentages of run-off in these rivers drop with the advance of the season and rise again in the fall.

On Fig. 84 are given the monthly relations of rainfall and run-off for thirty years on three small river basins in the immediate vicinity of Philadelphia. These drainage areas, being small, are more readily and directly affected by rainfall, hence the relations are much more marked and uniform than those that exist on larger rivers. The marked variation from normal due to the influence of other varying conditions on the drainage area, especially during the summer months, should be noted.

Figure 85 shows a set of monthly diagrams prepared by Emil Kuichling, C. E., for his discussion of the relation of rainfall to run-off in certain rivers in the Eastern part of the United States.

On these diagrams the figures not enclosed are numbers of observations from drainage basins Nos. 1 to 8 inclusive, of the following list. The figures enclosed in circles are the numbers of observations from drainage basins Nos. 1 to 28, inclusive.

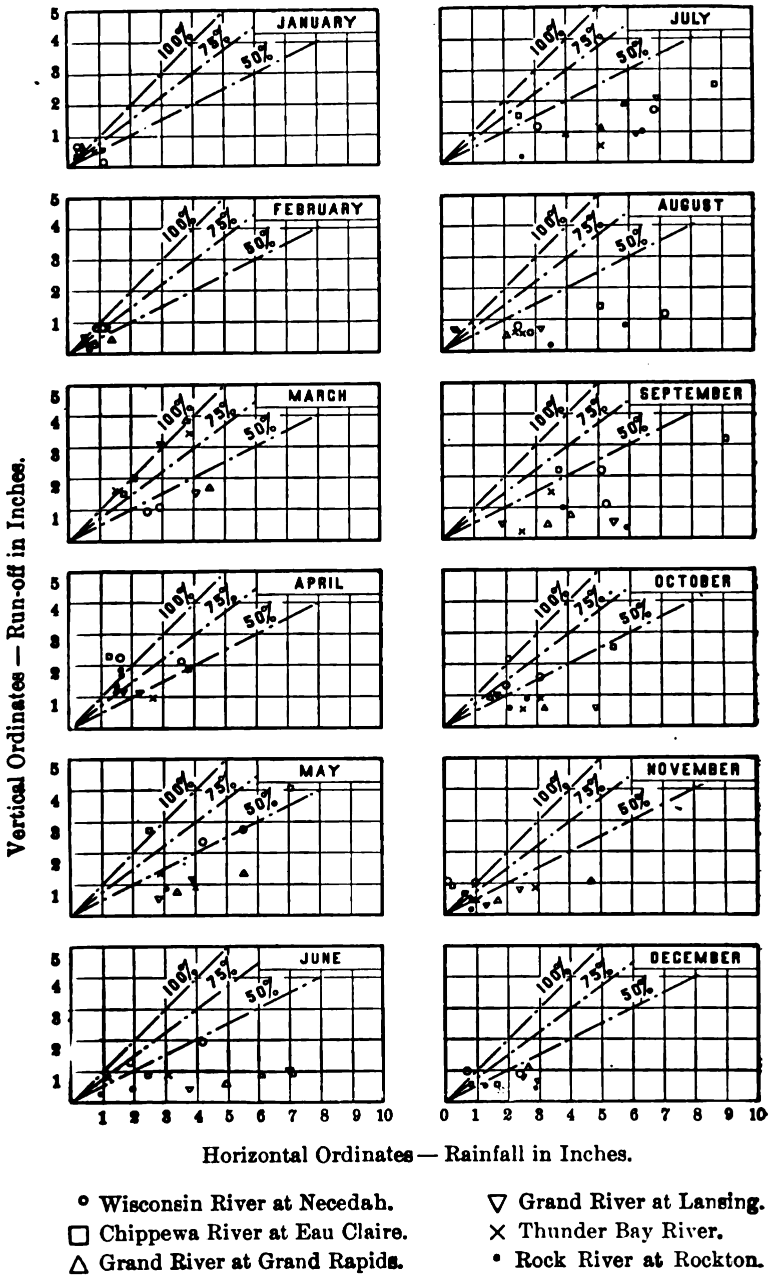
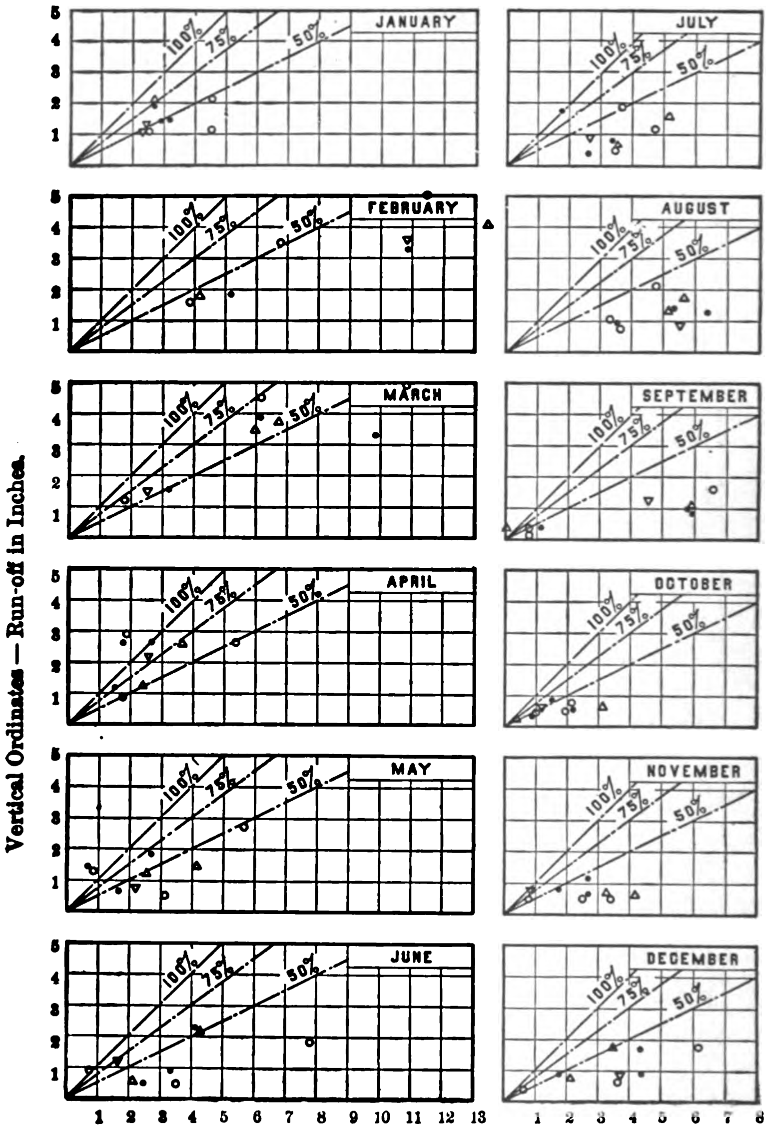


Fig. 82.—Monthly Rainfall and Run-Off—Northern Rivers.



Horizontal Ordinates — Rainfall in Inches.

● Talladega Creek, Watershed Area 156 Square Miles.
▽ Upadachee River, " " 440 " "
• Alcovy River " " 228 " "

Fig. 83.—Monthly Rainfall and Run-Off—Southern Rivers.

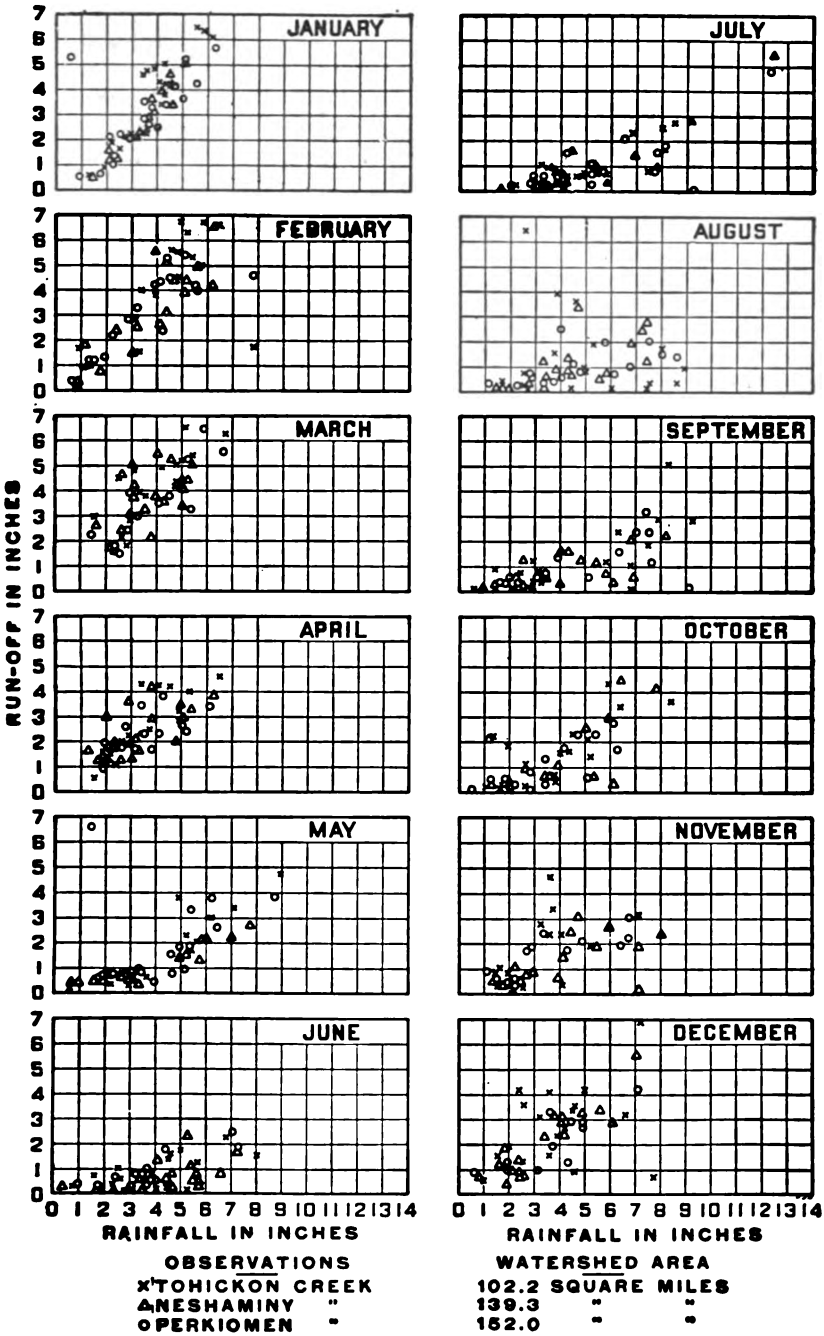


Fig. 84.—Relation between Rainfall and Run-Off on Tohickon, Neshaminy, and Perkiomen Creeks near Philadelphia, Pennsylvania.

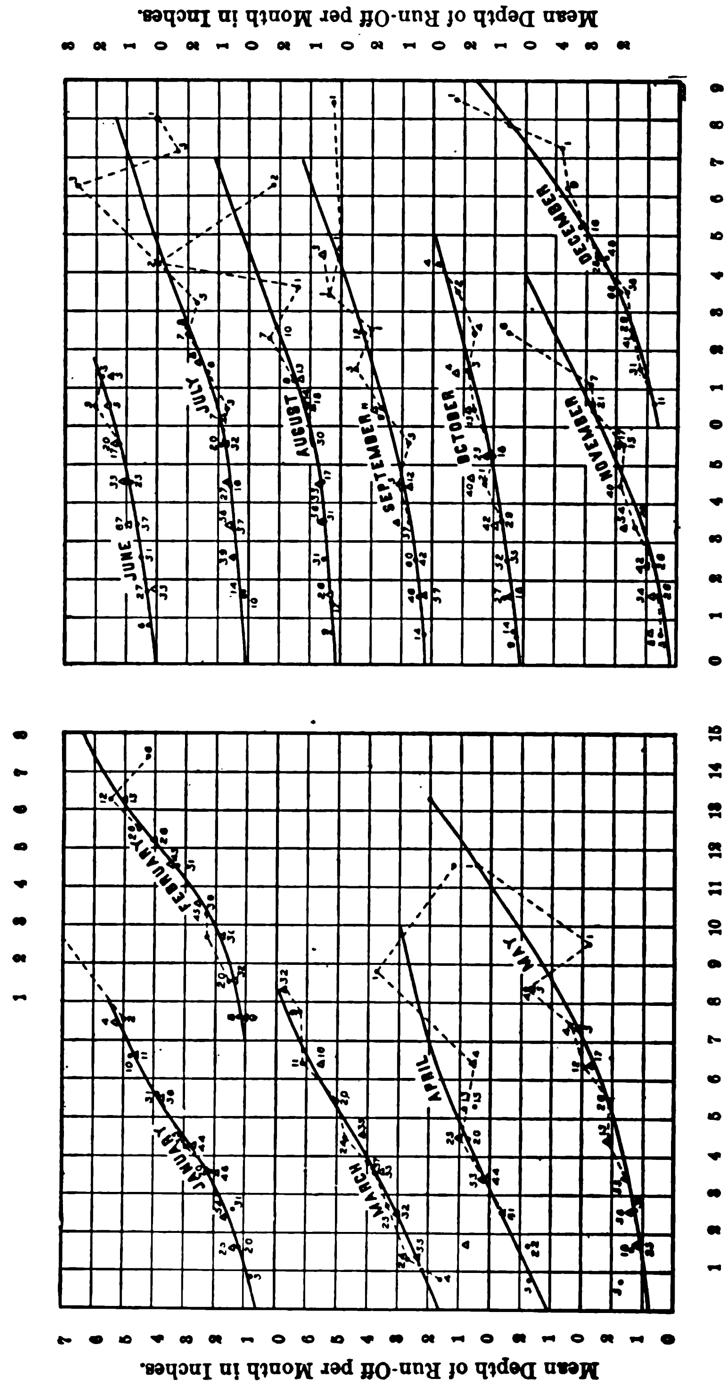


Fig. 85.—Depth of Rainfall per Month in Inches. Relations Between Monthly Depths of Rainfall and Run-Off.

[From Report of Emil Kuichling, C. E. New York State Canal Survey.]

Watersheds from which Observations were platted on Diagram 85.

No.	Name of Basin.	Area in Sq. Miles.	No. of Years Record.
1	Croton River, N. Y.	338.0	30
2	Perkiomen Creek, Pa.	152.0	13
3	Neshaminy Creek, Pa.	139.3	13
4	Tohickon Creek, Pa.	102.2	14
5	Sudbury River, Mass.	75.2	25
5	Hemlock Lake, N. Y.	43.1	12
7	Mystic Lake, Mass.	27.7	18
8	Cochituate Lake, Mass.	19.0	33
9	Cayadutta Creek, N. Y.	40.0	2
10	Saquoit Creek, N. Y.	51.5	2
11	Oneida Creek, N. Y.	59.0	2
12	Nine-Mile Creek, N. Y.	63.0	1
13	Garoga Creek, N. Y.	80.8	1
14	E. Branch Fish Creek, N. Y.	104.0	1
15	Oriskany Creek, N. Y.	144.0	2
16	Mohawk River, N. Y., at Ridge Mills	153.0	2
17	W. Branch Fish Creek, N. Y.	187.0	3
18	Salmon River, N. Y.	191.0	1
19	East Canada Creek, N. Y.	256.0	2
20	West Canada Creek, N. Y.	518.0	2
21	Schroon River, N. Y.	563.0	4
22	Passaic River, N. J.	822.0	17
23	Raritan River, N. J.	879.0	3
24	Genesee River, N. Y.	1070.0	7
25	Mohawk River, N. Y., at Little Falls	1306.0	2
26	Black River, N. Y.	1889.0	4
27	Hudson River N. Y., at Mechanicville, N. Y.	4500.0	12
28	Muskingum River, Ohio	5828.0	8

A continuous graphical record for ten years, showing the relations of rainfall to run-off on the Illinois River basin, based on observations of stream flow made at Peoria, Ill., is shown by Fig. 71.

93. **Maximum Stream Flow.**—In the construction of spillways, dams, and reservoirs, and the study of their effect on the overflow of embankments, levees, and lands, the question of maximum run-off becomes important.

Many formulas have been suggested by engineers for determining flood flows, each of which is based on more or less extended observations, and are applicable only when used under conditions similar to those on which they are founded. Very few of these formulas take into account the great number of conditions that modify the results. For this reason most of such formulas are of little use except for the purpose of rough approximation. None of these should be used without a knowledge of the conditions under

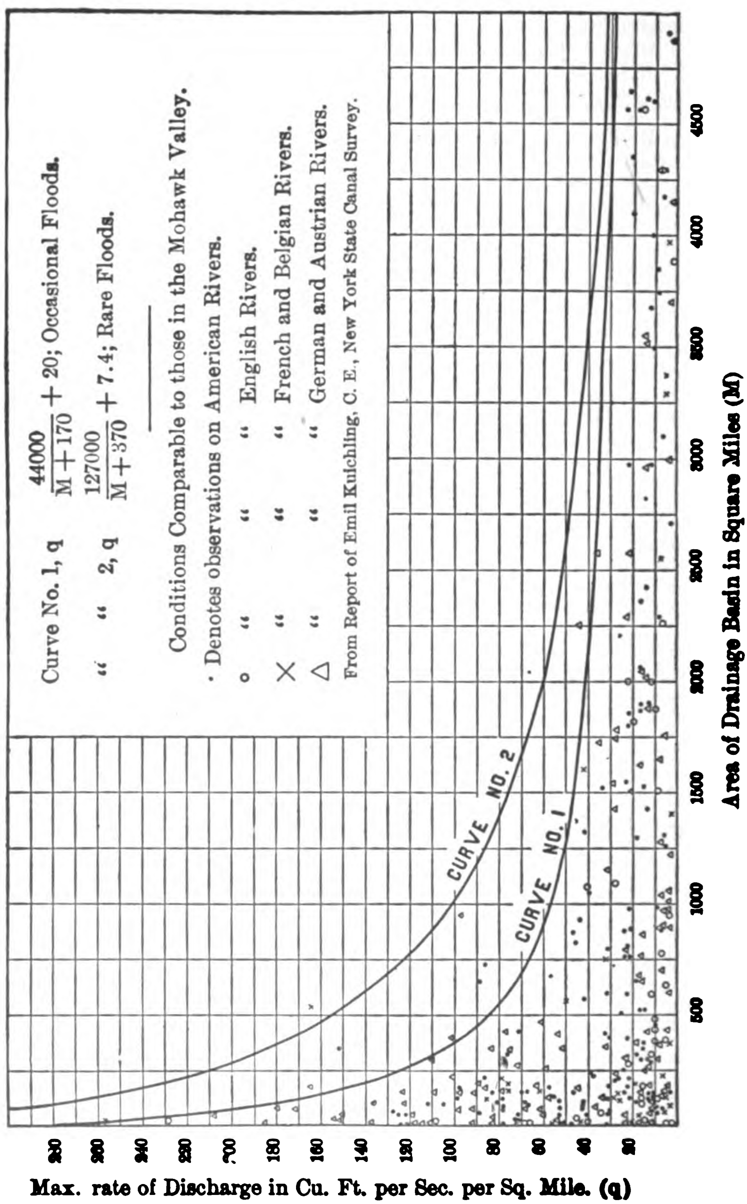


Fig. 86.—Rates of Maximum Flood Discharges of Certain American and European Rivers.

which they are applicable. Such calculations should, wherever possible, be based on the known ratio of actual maximum and minimum flows on the drainage areas, or on drainage areas adjacent and similar thereto, and the use of a factor of safety as great as the importance of the local condition will warrant. Such data serves as the best and most conservative guide for all calculations of this class.

A record of the maximum and minimum flows of various American and foreign streams from the report of Mr. Kuichling, to which reference has already been made, is contained in the Appendix.

Figure 86 shows a graphical representation of the actual rate of maximum flood discharge of these rivers and on this diagram is given the formulas, both graphically and analytically, for ordinary and occasional maximum floods as proposed by Mr. Kuichling. It is evident that Mr. Kuichling has endeavored to represent the maximum flood conditions that may occur on any river. In many localities, the results given are much larger than the actual conditions of flow will warrant.

In some cases the overflow of lands and property by floods, caused by back water or otherwise, may be prevented by the construction of levees and the installation of pumping plants for drainage purposes. Under such conditions both the extreme height of the flood and the length of its occurrence become important and can be determined only by gauge observation. A graphical study of such data affords the best means for its consideration. Figure 87 shows hydrographs of the high water conditions on the Fraser River at Mission Bridge, British Columbia. This stream is fed by the melting snows of the foot-hills, and the floods occur at essentially the same time each year within certain limits, as a rule reaching a maximum during May, June or July. The differences that occur from year to year are shown by the different hydrographs which represent, however, gauge heights in feet and not discharges. The highest record is that of the flood of June 5, 1894, of which, however, no hydrograph was obtained.

94. Estimate of Stream Flow.—For the purpose of estimating water power no safe deduction can be made from average run-off conditions, although a knowledge of such conditions is desirable. The information that is needed for the consideration of water power is a clear knowledge of the maximum and minimum conditions, the variations which occur between these limits and a knowledge of the length of time during which each stage is likely to occur

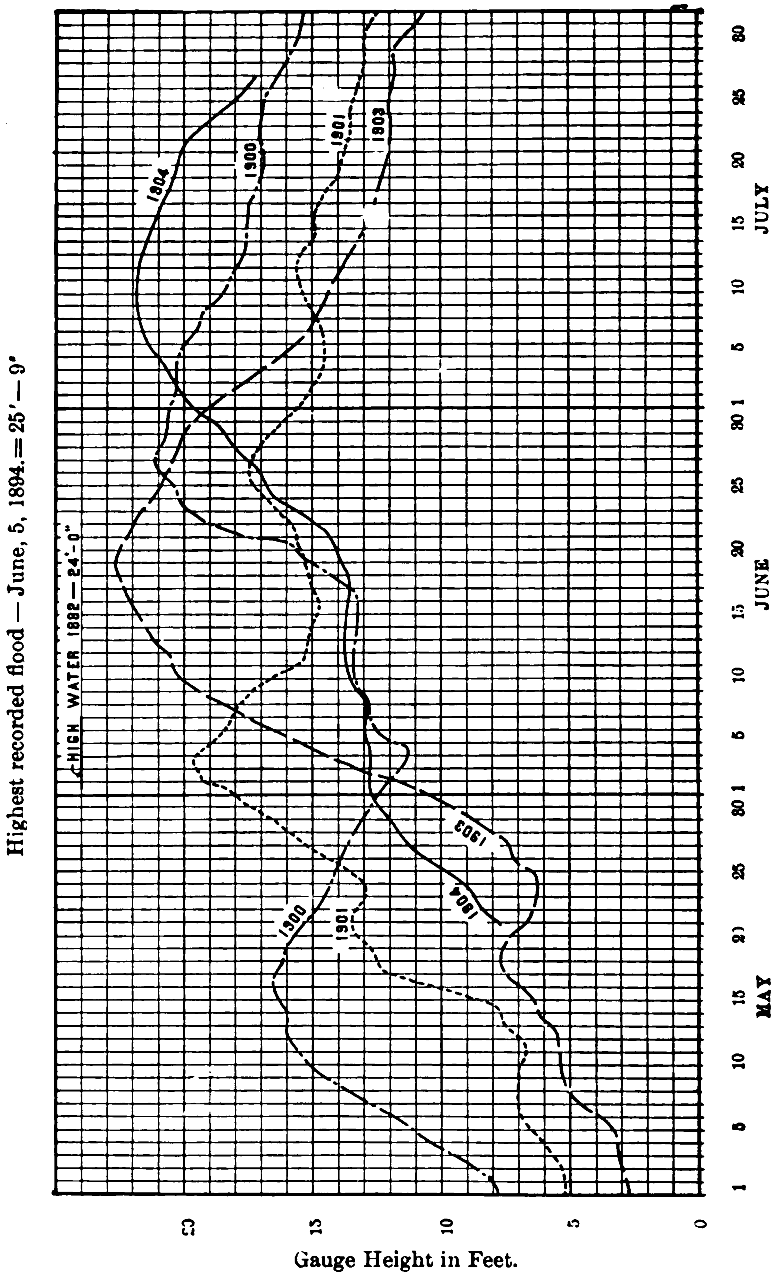


Fig. 87.—High Water—Fraser River at Mission Bridge, B. C.

throughout the year or throughout a period of years. As pointed out in the previous section, the extreme conditions are important in considering the height of flood as influenced by spillways and other obstructions in the river. The extreme and average low water conditions commonly control or limit the extent of the plant which should be installed.

By the illustrations already shown it is fully demonstrated that the run-off of any stream, either for the year, period or month, cannot be approximately expressed either as an average amount or as a fixed percentage of the rainfall. An expression showing the relation between rainfall and run-off necessarily assumes quite a complex form, from which large variations must be expected. Where average amounts of run-off are considered, care must be used to base the deduction on correct principles. In considering the variation in the monthly flow of a stream, the flows of such stream should be considered in the order of their monthly discharge rather than in their chronological order. For example: in Table XV, the mean monthly flows, of various streams, in cubic feet per second per square mile of drainage area are given. These flows are arranged in the chronological order of the months. The average monthly discharges of the streams are calculated therefrom, and are shown in the last column. An examination of this table will show that the minimum monthly flow of a stream does not always occur during the same month for each year. For the consideration of these streams for water power purposes, the better arrangement of the recorded flow is not in the sequence of the months, but by the monthly periods arranged in the relative order of the quantities of flow.

In Table XVI this data has been rearranged. In this arrangement the least flow for any month in a given year is placed in the first line and the flows for other months are arranged progressively from minimum to maximum. The average for each month will, by this arrangement, give a much better criterion of the average water power to be expected from each drainage area during each year than the average monthly flow as determined in Table XV.

TABLE XV.

***Mean Monthly Flows of Various Eastern Streams Arranged in Chronological Order.* (*In Cubic Feet per Second per Square Mile.*)**

Kennebec River at Waterville, Me.
Drainage Area 4380 sq. miles.

Year.	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
January	.60	.37	.46	.98	.81	.73	.53	.54	.73	.88	.92	2.22	.70	.80
February	.53	.40	.41	.64	.84	.77	.54	2.05	.57	.87	.88	.20	.60	.72
March	.95	.91	.45	2.98	.86	2.56	.73	2.07	1.10	6.67	4.42	.86	1.20	1.97
April	2.64	3.33	5.43	6.21	5.75	6.76	5.31	6.45	9.39	5.07	3.74	3.41	3.08	5.13
May	6.92	2.17	2.17	3.87	6.10	5.70	4.81	6.41	3.46	3.85	1.66	4.71	2.40	4.17
June	3.47	1.77	1.46	1.25	2.94	2.26	2.00	2.28	1.88	3.48	1.52	1.89	1.53	2.14
July	1.31	1.30	.80	1.21	2.96	.89	1.14	1.31	1.17	1.79	1.19	1.22	1.07	1.34
August	.51	.67	.61	.71	1.65	.71	.73	.95	.95	1.15	.88	1.07	.73	.87
September	.46	.62	.40	.77	1.04	.59	.43	.63	.64	.96	.57	.98	.68	.68
October	.53	.85	.28	.83	.60	.92	.28	.69	.67	1.20	.44	1.07	.40	.67
November	.51	.85	1.27	2.07	1.29	1.77	.46	1.44	.55	1.03	.33	.77	.52	.99
December	.36	.44	1.37	.62	1.21	.59	.51	.93	1.72	.99	.32	.60	.47	.78
Average	1.65	1.12	1.27	1.84	2.17	1.97	1.46	2.14	1.90	2.32	1.41	1.58	1.12	......

Merrimac River at Lawrence, Mass.
Drainage Area 4553 sq. mi.

Year.	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
January	1.53	2.92	1.87	.65	.66	.68	1.44	.75	1.62	1.73	.74	.72	2.24	.86	.57	.83	1.24
February	1.70	2.96	.94	1.10	.94	.51	2.00	1.01	1.71	1.07	3.62	.53	1.20	1.99	.63	.49	1 40
March	3.44	5.19	1.61	2.86	3.16	1.28	4.62	2.32	4.09	2.62	3.56	2.04	6.06	5.66	2.64	2.26	3 30
April	3 79	4.73	1.79	3.42	2.43	4.35	4.00	3.87	3.34	5.81	4.06	3.94	3.72	3.34	4.45	3.47	3.73
May	3.14	1.61	2.25	4.28	1.54	1.37	.98	2.22	2.42	2.09	2.21	4.04	2.18	0.94	3.74	1.12	2 26
June	1.73	1.00	1.28	.97	1.33	.67	.77	2.79	1.42	.65	87	1.61	1.13	2.21	1.00	.89	1.27
July	.69	.64	1.05	.52	.50	.57	.45	2.37	.58	.54	.40	.62	.93	1.00	.60	.57	.75
August	.75	.54	1.06	.57	.87	.48	.44	1.12	.83	.46	.41	.96	.81	.72	.55	.58	.67
September	1.84	.56	.87	.61	.40	.37	.67	.61	.64	.44	.83	.57	.74	.51	.62	1.64	1 33
October	2.70	.47	.47	.79	.50	.88	1 14	.48	1.41	.39	.55	.86	1.54	.79	.78	.70	.90
November	1.95	.54	1.43	.74	.78	2.10	1.46	1.28	2.17	.61	1.28	.65	1.23	.64	.58	.73	1.14
December	1.44	90	.86	1.17	.67	2.06	.96	2.28	1.93	.61	1.49	2.09	1.74	.80	.39	1.22	1.29
Average	2.06	1.84	1.29	1 43	1.11	1.27	1.58	1.76	1.85	1.42	1.63	1.53	1.96	1.62	1.38	1.21	......

Hudson River at Mechanicville, N. Y.
Drainage Area 4500 sq. mi.

Year.	'88	'89	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
January	1.41	2.41	2.50	1.84	4.19	.71	1.50	.86	1.51	.89	1.72	1.49	1.80	.69	....	1.56	1.29	1.35	1.60
February	.82	.84	1.74	2.59	2.06	1.02	1.07	.79	1.04	.87	1.50	1.17	2.77	.54	1.53	2.19	1.52	.79	1.38
March	1.52	1.84	2.47	3.94	2.41	1.97	3.28	.93	3.02	2.71	4.49	2.14	1.72	1.80	5.53	6.87	2.46	2.09	2.84
April	4.73	3.01	3.35	4.45	4.79	3.98	2.47	5.29	5.55	4.24	3.05	5.25	5 02	6.28	2.88	3.11	4.61	5.06	4.15
May	4.76	1.97	3.98	1.23	4.37	4.95	1.68	1.52	1.02	2.70	2.46	2.17	2.00	2.60	1.76	.78	2.96	1.82	2.48
June	1.09	1 52	1.64	.71	2.80	1.07	1.58	.63	1.05	2.63	1.17	.58	.91	1.73	1.40	1.88	1.50	2.12	1.45
July	.34	1.28	.43	.52	2.06	.56	.70	.57	.62	2.47	.57	.54	.52	.79	1.98	1.08	.58	1.54	.96
August	.38	.95	.45	.59	1.22	1.11	.55	.87	.54	1.83	1.14	.31	.60	1.03	1.40	1.81	1.39	1.25	.94
September	.63	.41	1 97	.45	.99	1.53	.42	.58	.64	.61	.86	.46	.42	.89	.81	.91	1.48	2.67	.93
October	1.02	.83	2.05	.33	.63	.86	.81	.58	.91	.56	1.75	.58	.47	.94	1.53	2.25	2.62	1.85	1.12
November	2.36	1.77	2.03	.91	1.69	.81	1.42	1.87	2.52	2.22	2.05	1.42	1.11	.83	1.41	1.28	1.03	....	1.57
December	2.22	2.93	.72	1.91	.93	1.60	.97	2.42	1.54	3.20	1.25	1.02	1 13	1 88	1.83	1.18	.87	....	1.62
Average	1.77	1.65	1.94	1.62	2.34	1.68	1.37	1.41	1.66	2.08	1.83	1 43	1 50	1.67	....	2.03	1.86	....	......

TABLE XV.—Continued.

Potomac River at Point of Rocks, Md.
9654 sq. mi.

Year.	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
January	2.40	1.95	.83	.57	1.81	1.78	.76	.89	1.38
February	.85	3.00	1.38	.37	3.37	2.30	1.81	.58	1.71
March	1.59	3.72	1.93	1.45	5.64	2.77	1.16	2.43	2.58
April	1.67	1.22	.96	4.07	2.99	2.99	.77	.68	1.92
May	1.89	1.20	.45	2.85	.32	.64	.97	.46	1.13
June	.42	.54	.86	2.01	.83	1.86	1.05	.68	.97
July	.26	.27	.31	1.11	.82	1.32	.47	1.06	.64
August	2.34	.25	.20	.87	.26	.50	.25	.60	.66
September	.26	.25	.14	.77	.15	.48	.17	.33	.32
October	1.45	.18	.14	.40	.29	.33	.12	.30	.40
November	.87	.83	.48	.48	.29	.22	.14	.24	.38
December	1.60	.42	.64	2.62	1.96	.30	.23	1.10	1.11
Average	1.30	1.11	.69	1.46	1.50	1.29	.66	.78	....

From Table XVI it will be seen that the average minimum monthly flow of the Hudson River at Mechanicville, N. Y., is .52 cubic foot per second per square mile, the smallest monthly minimum for any year during the period of the observations being .31 and the largest monthly minimum for any year being .81. On the Potomac River, with a somewhat greater total annual rainfall, the average minimum monthly flow is .21, the smallest monthly minimum for the year being .12, and the largest monthly minimum for any year being .37. These figures, it must be remembered, are averages for each month, and the actual minimum flow during the period is a much less quantity. These records show that the minimum flow of a stream cannot be based on the mean annual rainfall. This same

TABLE XVI.

Mean Monthly Flow of Various Eastern Streams Arranged in Order of their Magnitude. (In Cubic Feet per Second per Square Mile.)

Kennebec River at Waterville, Me.
4410 sq. mi. 4380 sq mi.

Year.	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
Minimum	.36	.37	.28	.62	.60	.28	.59	.54	.55	.87	.32	.20	.40	.46
	.46	.40	.40	.64	.81	.43	.59	.63	.57	.88	.33	.60	.47	.56
	.51	.42	.41	.71	.84	.46	.71	.69	.64	.96	.44	.77	.52	.62
	.51	.62	.45	.77	.86	.51	.73	.93	.67	.99	.57	.86	.60	.70
	.53	.67	.46	.83	1.04	.53	.77	.95	.73	1.03	.88	.98	.68	.78
	.53	.85	.61	.98	1.21	.54	.89	1.31	.95	1.15	.88	1.07	.70	.90
	.60	.85	.80	1.21	1.29	.73	.92	1.44	1.10	1.20	.92	1.07	.73	.99
	.95	.91	1.27	1.25	1.65	.73	1.17	2.05	1.17	1.79	1.19	1.22	1.07	1.26
	1.31	1.30	1.37	2.07	2.94	1.14	2.26	3.07	1.72	3.48	1.52	1.89	1.20	1.87
	1.64	1.77	1.46	2.98	2.96	2.00	2.56	2.28	1.88	3.85	1.66	2.22	1.53	2.23
	3.47	2.17	2.17	3.87	5.75	4.81	5.70	6.41	3.46	5.07	3.74	3.41	2.40	4.05
Maximum	6.92	3.33	5.43	6.21	6.10	5.31	6.76	6.45	9.39	6.57	4.42	4.71	3.08	5.73

TABLE XVI.—Continued.

Hudson River at Mechanicville, N. Y.
Drainage Area 4500 sq. mi.

Year.	'88	'89	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
Minimum ..	.34	.44	.43	.33	.63	.56	.42	.57	.54	.56	.57	.31	.42	.54	.81	.78	.58	....	.52
	.38	.88	.45	.45	.93	.71	.55	.58	.62	.61	.86	.46	.47	.69	....	.91	.87	.79	.66
	.63	.84	.72	.52	.99	.81	.70	.58	.61	.87	1.14	.54	.52	.79	1.40	1.08	1.03	....	.81
	.82	.95	1.64	.59	1.22	.86	.81	.79	.91	.89	1.17	.58	.60	.83	1.40	1.18	1.29	1.25	99
	1.02	1.28	1.74	.71	1.69	1.02	.97	.86	1.02	1.83	1.25	.58	.91	.89	1.41	1.28	1.39	1.35	1.18
	1.09	1.52	1.97	.91	2.06	1.07	1.07	.87	1.04	2.22	1.50	1.02	1.11	.94	1.53	1.31	1.48	1.35	1.34
	1.41	1.77	2.03	1.23	2.06	1.11	1.42	.93	1.05	2.47	1.72	1.17	1.13	1.03	1.53	1.56	1.50	1.54	1.48
	1 52	1.84	2.05	1.84	2.41	1.53	1.50	1.52	1.51	2.63	1.75	1.42	1.30	1.73	1 76	1.88	1.52	1.82	1.73
	2.22	1.97	2.47	1.91	2.80	1.60	1.58	1.54	1.54	2.70	2.05	1.49	1.72	1.80	1.83	2.19	2.46	2.09	2.00
	2.86	2.44	2.50	2.59	4.19	1.97	1.68	1.87	2.52	2.71	2.46	2.14	2.00	1.88	1.98	2.25	2.62	2.12	2.35
	4.73	2.93	3.35	3.94	4.37	3.98	2.47	2.42	3.02	3.20	3.05	2.17	2.77	2.60	2.38	3.11	2.96	2 67	3.12
Maximum..	4.76	3.04	3.98	4.45	4.79	4.95	3.28	5.29	5 55	4.24	4.49	5.25	5.02	6.28	5.53	6.87	4.61	5.06	4.85

Merrimac River at Lawrence, Mass.
4553 sq. mi.

Year.	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
Minimum	.69	.47	.47	.52	.37	.87	.44	.48	.58	.39	.33	.53	.74	.51	.39	.49	.48
	.75	.54	.86	.57	.40	.48	.45	.61	.64	.44	.40	.57	.81	.64	.55	.57	.59
	1.44	.54	.87	.61	.44	.57	.67	.75	.83	.46	.41	.62	.93	.72	.57	.58	60
	1.53	.56	.94	.65	.50	.57	.77	1.01	1.41	.54	.55	.65	1.13	.79	.58	.70	.88
	1.70	.64	1.05	.74	.50	.63	.96	1.12	1.42	.61	.74	.72	1.20	.80	.60	.73	.87
	1.73	.90	1.06	.79	.66	.67	.98	1.28	1.62	.61	.87	.86	1.23	.86	.62	.83	.96
	1.84	1.00	1.28	.97	.67	.88	1.14	2.22	1.71	.65	1.28	.96	1.54	.94	.63	.89	1.12
	1.95	1.61	1.43	1.10	.78	1.28	1.44	2.28	1.93	1.07	1.49	1.61	1.74	1.00	.78	1.12	1.42
	2.70	2.92	1 61	1.17	.94	1.37	1.46	2 32	2.17	1.78	2.21	2.04	2.18	1.99	1.00	1.22	1.88
	3.14	2.96	1.79	2.36	1.33	2.06	2.00	2.37	2.42	2.09	3.56	2.09	2.24	2.21	2.64	1.64	2.30
	3.44	1.73	1.87	3.42	2.43	2.10	4.00	2.79	3.34	2.62	3.62	3.94	3.72	3.34	3.74	2.26	3.20
Maximum...........	3.79	5.19	2.25	4.28	3 16	4.35	4.62	3.87	4.09	5.81	4.06	4.04	6.06	5.66	4.45	3.47	4.32

Potomac River at Point of Rocks, Md.
9654 sq. mi.

Year.	'98	'99	'00	'01	'02	'03	'04	'05	Ave.
Minimum	.26	.18	14	.37	.15	.22	.12	.24	.21
	.26	.25	.14	.40	.26	.30	.14	.30	.26
	.42	.25	.20	.48	.29	.33	.17	.33	.31
	.85	.27	.31	.57	.29	.48	.23	.46	.43
	.87	.33	.45	.77	.32	.50	.25	.58	.51
	1.45	.42	.48	.87	.33	.64	.47	.60	.66
	1.59	.54	.64	1.11	.62	1.32	.76	.68	.91
	1.60	1.20	.83	1.49	1.81	1.78	.77	.68	1.27
	1.67	1.22	.86	2.01	1.96	1.86	.97	.89	1.43
	1.89	1.95	.96	2.62	2.99	2.30	1.05	1.06	1.85
	2.34	3.00	1.38	2.85	3.37	2.77	1.16	1.10	2 25
Maximum...............	2.40	3.72	1.93	4.07	5.64	2.79	1.81	2.43	3.01

fact is more fully demonstrated by the tables on maximum and minimum run-off given in the Appendix. From the data in the Appendix it will be noted that the recorded minimum of some of the Southern streams is between .5 and .6 cubic feet per second per square mile, while numerous other streams will vary from .2 to .4; nevertheless a large portion of the streams shown have minimum flows of .1 and less.

CHAPTER IX.

RUN-OFF (Continued).

95. **Relation of Run-Off to Topographical Conditions.**—The relative run-off from a drainage area depends largely on its topographical condition. This is due to the fact that climatic condition depends on the elevation and slope of the drainage area, and also to the fact that the rapid removal of the water from steep slopes assures less activity in the other factors of rainfall disposal and consequently a greater run-off. Mr. F. H. Newell in a paper before the Engineering Club of Philadelphia (see Proceedings Engineering Club of Philadelphia, vol. 12, page 144, 1895) presents a diagram (see Fig. 88) which shows in a broad way, the influence of such conditions. In describing this diagram Mr. Newell says:

"The diagonal line represents the limit or the condition when all of the rain falling upon the surface, as upon a steep roof, runs off; the horizontal base, the conditions when none of the water

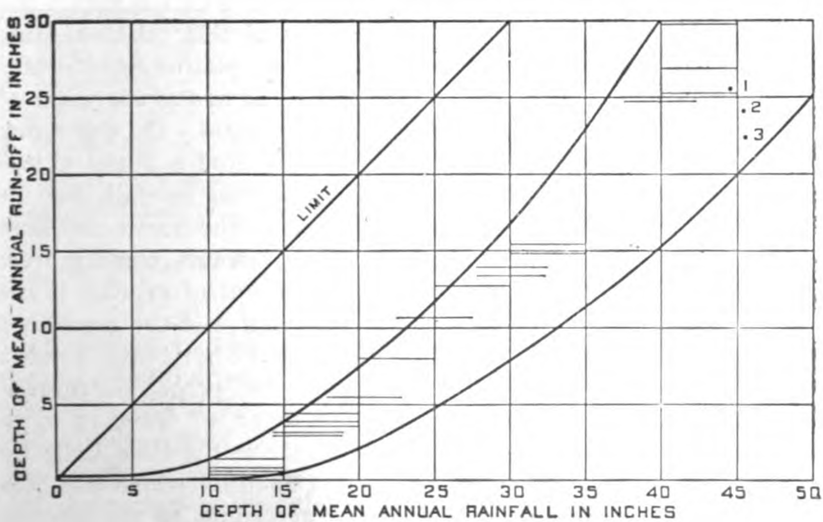


Fig. 88

flows away. Between these are the two curved lines, the lower representing the assumed condition prevailing in a catchment basin of broad valleys and gentle slopes, from which as a consequence there is relatively little flow, and the upper curve, an average condition of mountain topography, from which large quantities of water are discharged. For example, with a rainfall of 40 inches on an undulating catchment basin, about 15 inches is discharged by the stream, while from steep slopes 30 inches runs off. With less mean annual rainfall the relative run-off is far less, as for example, with 20 inches, about 7 inches of run-off is found in steep catchment basins, and about 3 inches on the rolling plains and broad valleys of less rugged topography. Following these curves down, it would appear that as the average yearly rainfall decreases the run-off diminishes rapidly, so that with from 10 to 15 inches no run-off may be expected on many areas, and from 2 to 4 inches from the mountains. There is an apparent exception to this, in that with very small annual rainfall the precipitation often occurs in what is known as cloudbursts, large quantities of water falling at a surprisingly great rate. Under these conditions the proportion of run-off to rainfall increases, as the water does not have time to saturate the ground."

"These curves should not be regarded as exact expressions, but as indicating general relationships and as showing graphically deductions based upon long series of observations of quantities not determined with exactness. Computations of this relation made in various parts of the country have, when platted graphically, fallen near or between these curves, according to the character of the country from which the water was discharged. On the figure are shown three average determinations, numbered 1, 2 and 3, representing respectively the relation of run-off to rainfall, for the Connecticut, Potomac and Savannah Rivers. The horizontal lines indicate determinations made for western streams coming from areas of small precipitation. The exact amount of rainfall is not known, as the observations are not representative of the conditions prevailing upon the mountains, and therefore the horizontal line has been used instead of a dot, as indicating the probable range of rainfall, as, for example, being from 10 to 15, or from 15 to 20 inches. The height of these short lines above the base indicates the average annual run-off of the basin, a quantity which has been determined with considerable accuracy according to the method just described."

Figure 88 is presented on account of the general principles illustrated thereby and should be used for such purpose only. While the limits given by Mr. Newell are sufficiently broad to include many of the conditions in the United States, they are too broad to give a sufficiently definite relation for most local conditions and too narrow to include all conditions which may occur in the United States. The latter fact is perhaps best illustrated by Fig. 89, reproduced from a paper by Messrs. J. B. Lippincott and S. G. Bennett on "The Relation of Rainfall to Run-Off in California", published in the Engineering News, vol. 47, page 467. This figure shows the annual and mean run-off from various California drainage areas based on several years' observations. The diagram shows both the Newell curves, illustrated in Fig. 88, and three mean curves for California conditions, also several mean and numerous annual run-off observations which can be studied in detail in the article above referred to. The general curve for large drainage areas is for areas of 100 square miles or over.

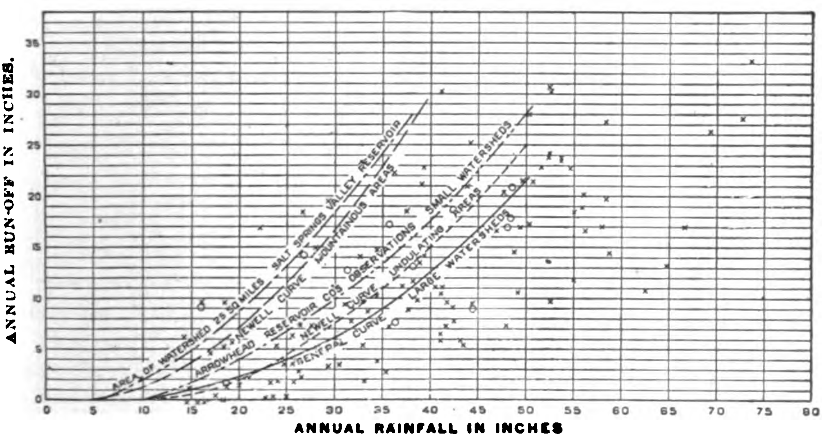


Fig. 89

96. Effects of Geological Condition on the Run-Off.—The geological condition of a drainage area has a marked effect on the run-off. The determination of the exact geological conditions of any drainage area, which control or modify the resulting run-off, is difficult or even impossible and can seldom be done with sufficient accuracy so that the results may be even approximated without actual observations on the drainage areas. The effects of these conditions, however, are important and they are here pointed out so that such effects may be realized and the fact appreciated that the run-off of streams otherwise similarly located may be materially different on account of difference in these conditions. A good example of the geological influence on run-off may be seen by comparing the stream flow of any of the Northern Wisconsin streams with that of the Rock River in the Southern portion of the state. Most of the Northern Wisconsin streams flow, in part, over pervious beds of sand-stone and a considerable amount of the water falling on their drainage areas is undoubtedly lost through absorption by the underlying strata. These losses undoubtedly affect the flow of the stream to a considerable extent. These streams, however, have no large under-flow through loose material which can absorb and transmit any considerable portion of the rainfall that would otherwise appear as surface run-off. The Rock River, on the other hand, follows for a considerable portion of its course through Wisconsin, its pre-glacial drainage valley which is filled to a depth of 300 feet or more with drift material consisting largely of sands and gravels through which a large amount of water doubtlessly escapes. The

TABLE XVII.

Comparative Mean Monthly Run-Off of the Wisconsin River at Necedah, Wisconsin, and the Rock River at Rockton, Illinois, in Cubic Feet Per Second Per Square Mile.

	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1903												
Wisconsin river	.45	.44	2.04	1.43	2.50	1.19	1.56	1.15	2.73	1.83	.86	1.34
Rock river	....	....	....	....	....	....	.91	.63	.91	.78	.44	.45
1904.												
Wisconsin river	....	....	....	2.21	2.63	1.96	1.02	.66	.90	2.34	.98	....
Rock river	.45	.77	2.80	1.76	.88	.39	.26	.24	.38	.50	.30	.41
1905.												
Wisconsin river	....	....	1.56	2.72	1.91	4.02	1.50	1.05	1.28	.99	.81	1.53
Rock river	.60	.53	2.10	1.63	1.10	1.06	.64	.41	.43	.39	.40	.44
1906.												
Wisconsin river	....	....	....	3.90	1.81	1.86	1.13	.90	.89	.83	1.17	1.41
Rock river	1.56	1.59	1.92	1.49	.58	.37	.38	....	....	.10	.21	.28

deposits of this old river bed have been quite extensively explored for water supply purposes and yield very large quantities of water for domestic and manufacturing supplies. Most of the under-flow, however, undoubtedly passes away to an unknown outlet as the modern river leaves the old valley near Rockford, Ill.

A comparison between mean monthly flows of the Wisconsin and Rock Rivers, as shown in Table XVII, will give an idea of the effect of these different conditions as shown by the run-off of these two rivers.

97. The Influence of Storage on the Distribution of Run-Off.—Favorable pondage conditions on a watershed have an important effect on the distribution of the run-off, and this effect is readily discernible in the records of flow from such areas.

Figure 90 is a hydrograph of the discharge of the various rivers draining the Great Lakes for the years 1882 to 1902. A general similarity is seen in the annual variations in these hydrographs and yet there is a considerable variation from the maximum to the minimum discharge in accordance with the rainfall and other conditions prevalent on the watershed. In every case, however, the minimum of the year is found to occur at about the same time, and the time of maximum height is also fairly constant. The ratios between maximum and minimum flow are very much less than those that obtain on other watersheds where the pondage area is much less.

In the St. Lawrence River the maximum mean monthly discharge is about 320,000 second feet, and the minimum is about 185,000 second feet, the maximum being not quite double the minimum. In the discharge of the Niagara River the maximum mean monthly discharge is about 260,000 cubic feet, and the minimum about 175,000, the fluctuation being still more moderate.

The mean monthly discharge of the St. Marys River shows a maximum of about 110,000 second feet, and a minimum of about 50,000. The ratio here is somewhat higher, because, in this case, Lake Superior and its drainage area being the source of supply, the relation of pondage to drainage area is less than in the combined lakes, and the effect is seen in the variation in the discharge of this river.

98. Effects of Area on the Run-Off.—The size of the drainage area of any stream has a marked effect on the distribution of the run-off. The hydrographs of small areas show the effects of heavy rains by an immediate and marked increase in the flow.

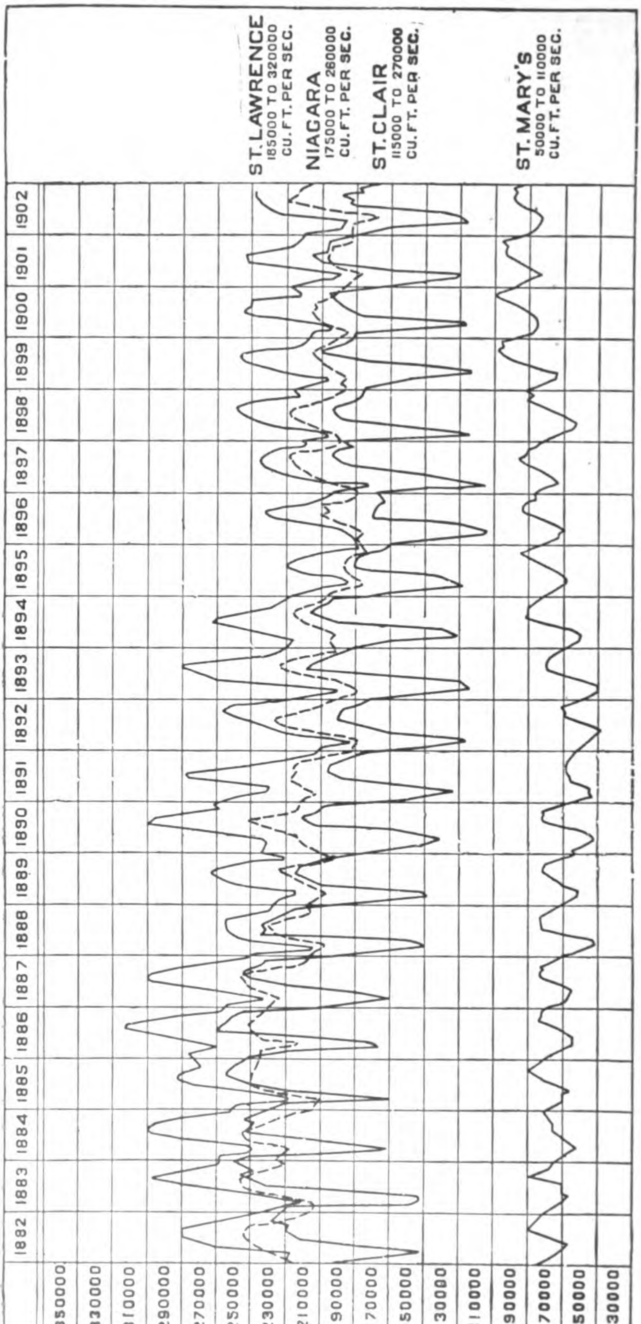


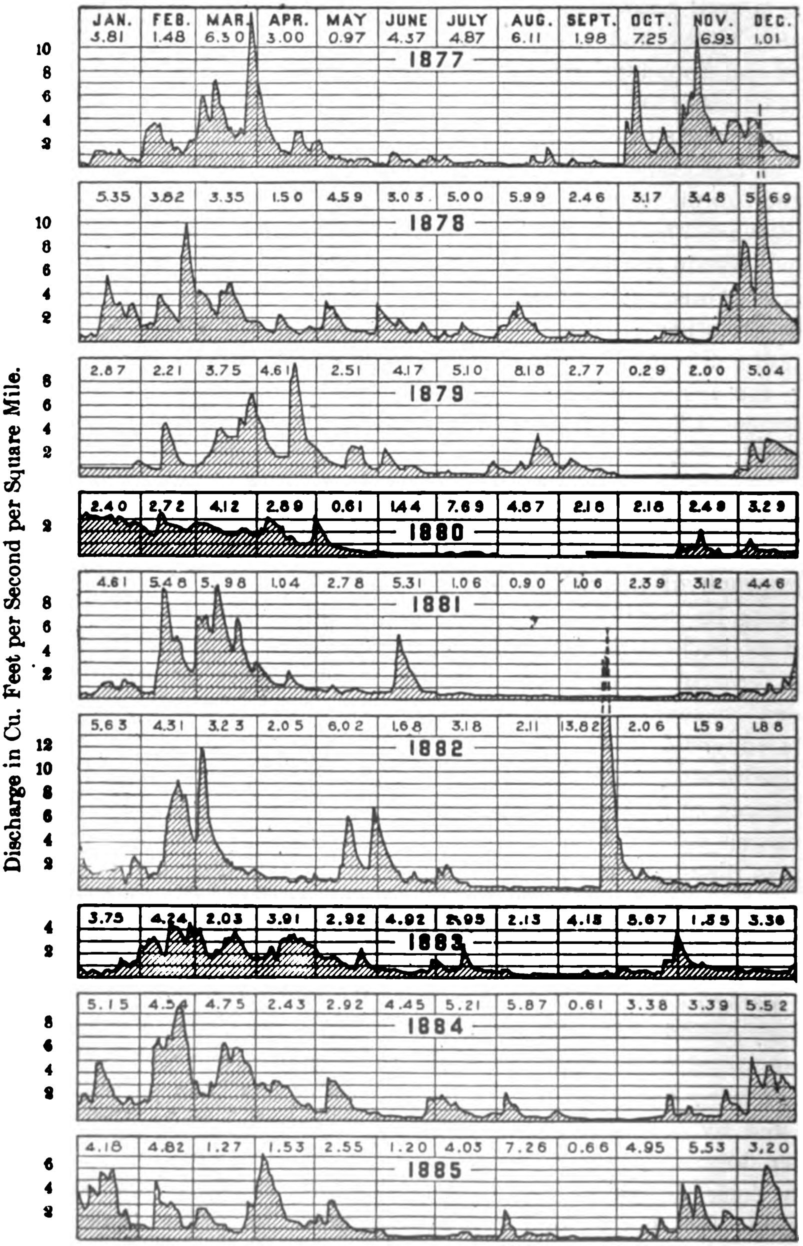
Fig. 90.—Hydrograph of discharge of Great Lakes.

This is well shown by a comparison of the hydrographs of Perkiomen Creek and the Kennebec River (Fig. 96), and of the Hood and Spokane Rivers (Fig. 99). On small streams where pervious deposits are largely developed, the rainfall is rapidly absorbed and does not so radically affect the run-off. Large streams do not feel the immediate effect of rainfall, on account of the time required for the run-off to reach the main stream. The flow of large streams is also modified by the fact that uniform conditions of rainfall seldom obtain on the entire area. On large drainage areas, conditions of rainfall may prevail on one or more of the tributaries only, while on other portions of the drainage area the conditions may be quite different. Such conditions may frequently be reversed, with the result that the larger the stream the less becomes the extremes of flow and the greater the uniformity of flow.

99. **The Study of a Stream From Its Hydrographs.**—The influences of various factors on the run-off, as above discussed, can be clearly seen from an analysis of the stream flow data, but they can best be appreciated by noting their effect on the hydrograph. The hydrograph of the actual flow of a stream is the best means of studying its manifold variations, but to fully comprehend the wide limit of such variations, hydrographs must be available for a long term of years. When the hydrographs are sufficiently extended to cover all of the usual variations in rainfall and other meteorological conditions, they afford a comprehensive view of the entire subject of the run-off of the stream.

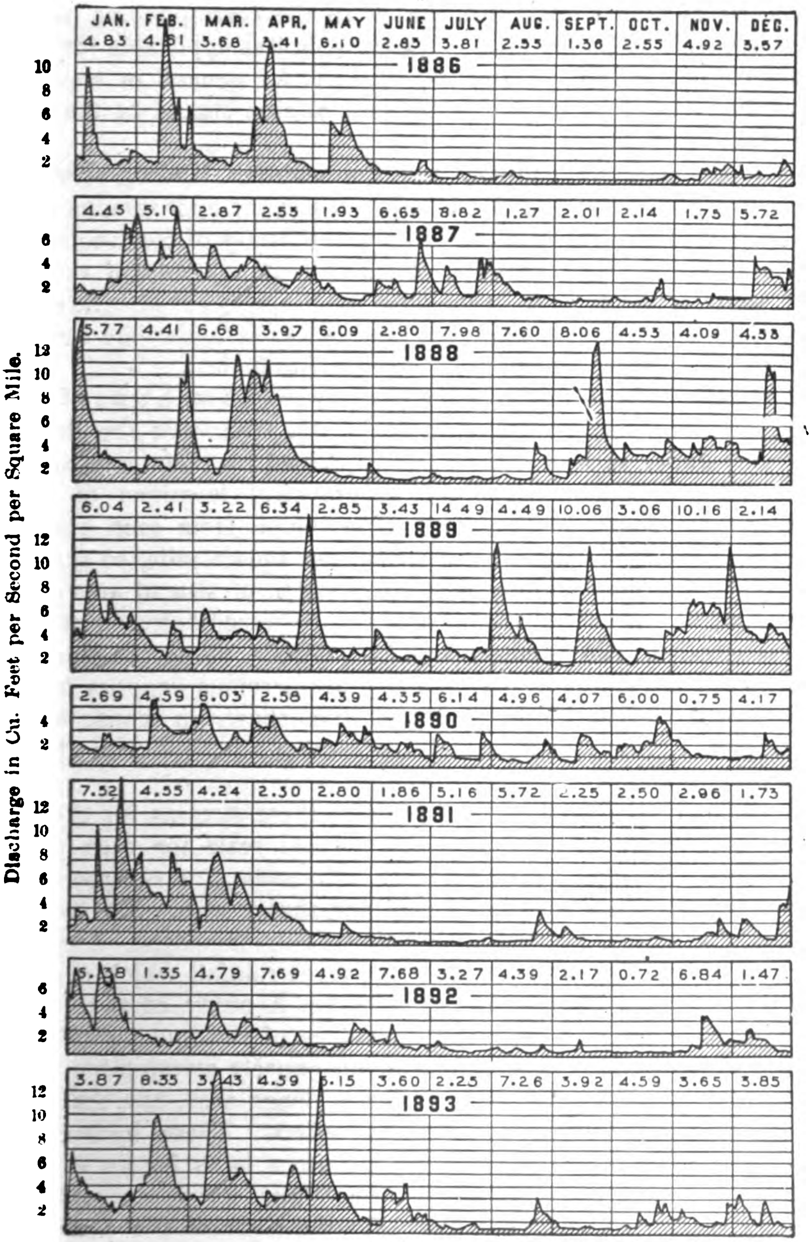
Figures 91 and 92 show hydrographs of the Passaic River for seventeen years. From these hydrographs the actual variations in flow as they have occurred on this drainage area during this period can be seen. The average monthly rainfall on the drainage area has also been shown on these diagrams and the effects of such rainfall on the run-off should be noted. It is important to note especially the marked effect of a limited rainfall during the months of the storage period, when the ground has previously become saturated, as compared with the effects of the same or greater rainfalls during the growing period, when the ground water has been partially exhausted by the demands of vegetation and the draft of the low water flow.

In these diagrams, and those following, the flows are shown in cubic feet per second per square mile, in order that their value for comparative purposes may be increased. The absolute discharge of a river in cubic feet per second gives no comparative



Figures near top of each diagram show total monthly rainfall.

Fig. 91.—Daily flow of Passaic River, Little Falls, N. J.



Figures near top of each diagram show total monthly rainfall.

Fig. 92.—Daily flow of Passaic River, Little Falls, N. J.

measure of discharge values, but when the corresponding area is also shown, the diagram becomes more or less applicable for comparative purposes to other areas. Hence, for general or comparative discussion, the discharge per unit of area should be the basis of consideration.

100. Comparative Run-Off and Comparative Hydrographs.—In studying and comparing all run-off data and the hydrographs based thereon it is important to note that a uniformity of conditions produces a uniformity of results. Such data is not only of value in the study of the river from which it is obtained, but also furnishes information regarding other streams that exist under the same or similar conditions, both physical and meteorological.

Table XVIII, which shows the monthly run-off for a term of years of certain Michigan streams, gives a comparison of the flow of streams under such conditions, as expressed by their comparative monthly run-off. The relative geographical locations of these streams are shown in figure 93. The run-off from each drainage area is given in cubic feet per second per square mile, so that the results are strictly comparable, the question of size of area being eliminated. A general resemblance can be traced between most of these streams. The Manistee and Au Sable Rivers, in the Northern portion of the state, have sand and other pervious deposits largely developed on their drainage areas, and show, in consequence, greater uniformity of flow and a greater mean flow than that of the other streams.

Comparative hydrographs of some of these streams for the year 1904 are shown in Fig. 94. The vertical scale for each of the hydrographs shown on the diagram is the same, and represents the discharge in cubic feet per second per square mile. The relative flows of the different streams are thus easily compared. On these diagrams has also been shown the average rainfall which occurred on each drainage area for each month. A study of the rainfall record in connection with the flow lines of the hydrograph, will show that the difference in flow is not entirely attributable to the prevailing rainfall conditions on the drainage area, but that other physical influences have a material effect. These hydrographs were originally prepared in order to form a basis for an estimate of the probable horse power on the White River, on which no gauge readings had been taken. On the right of the diagram is shown a horse power scale from which the probable power of the White River, with a given fall and drainage area, and on the basis

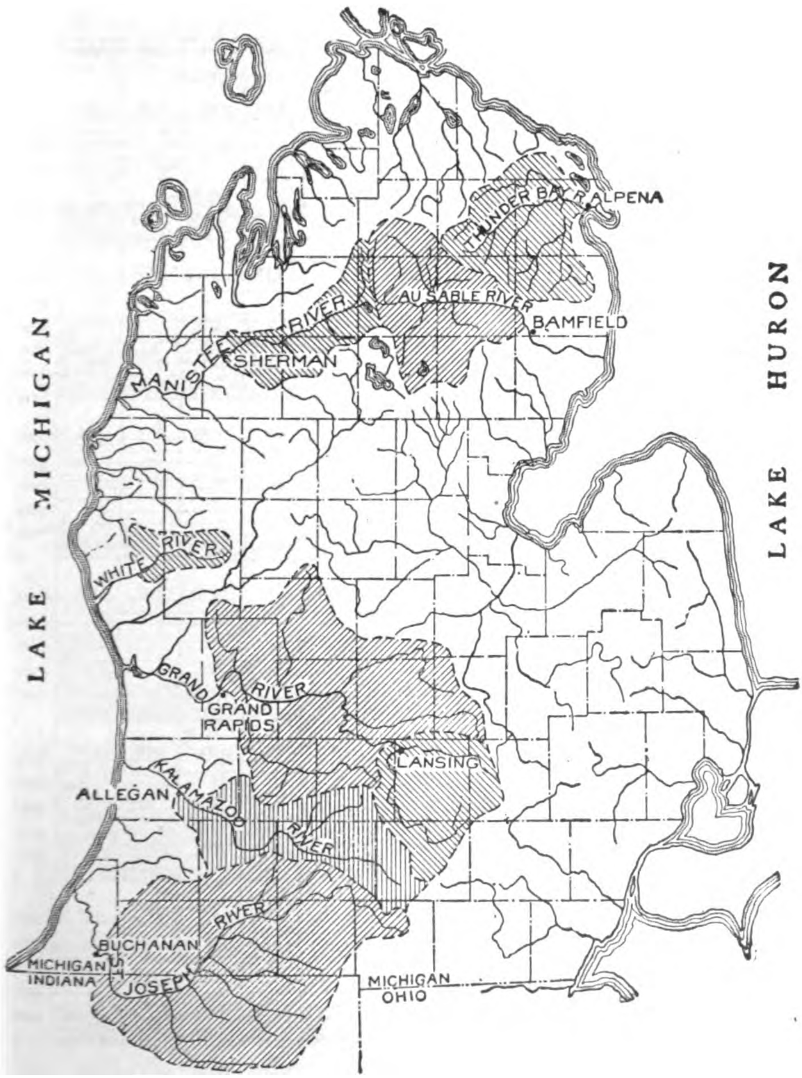


Fig. 93.—Map showing location of various Michigan drainage areas.

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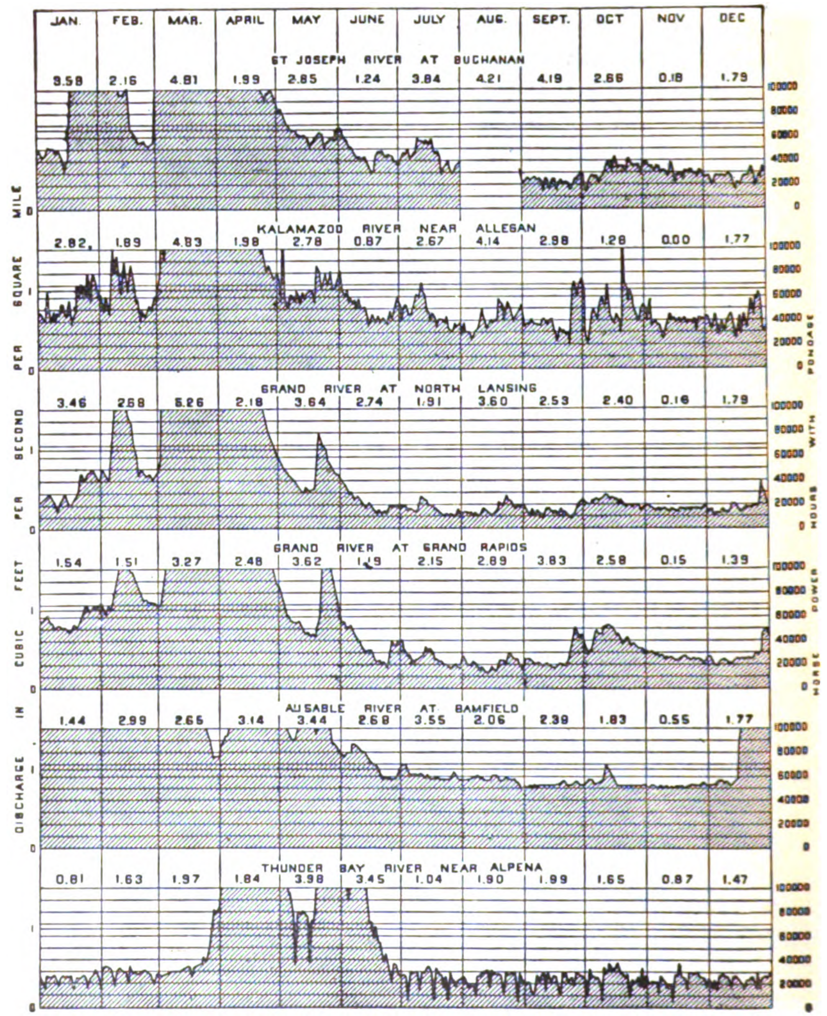

Fig. 94.—Comparative Hydrographs of Various Michigan Rivers for the year 1904.

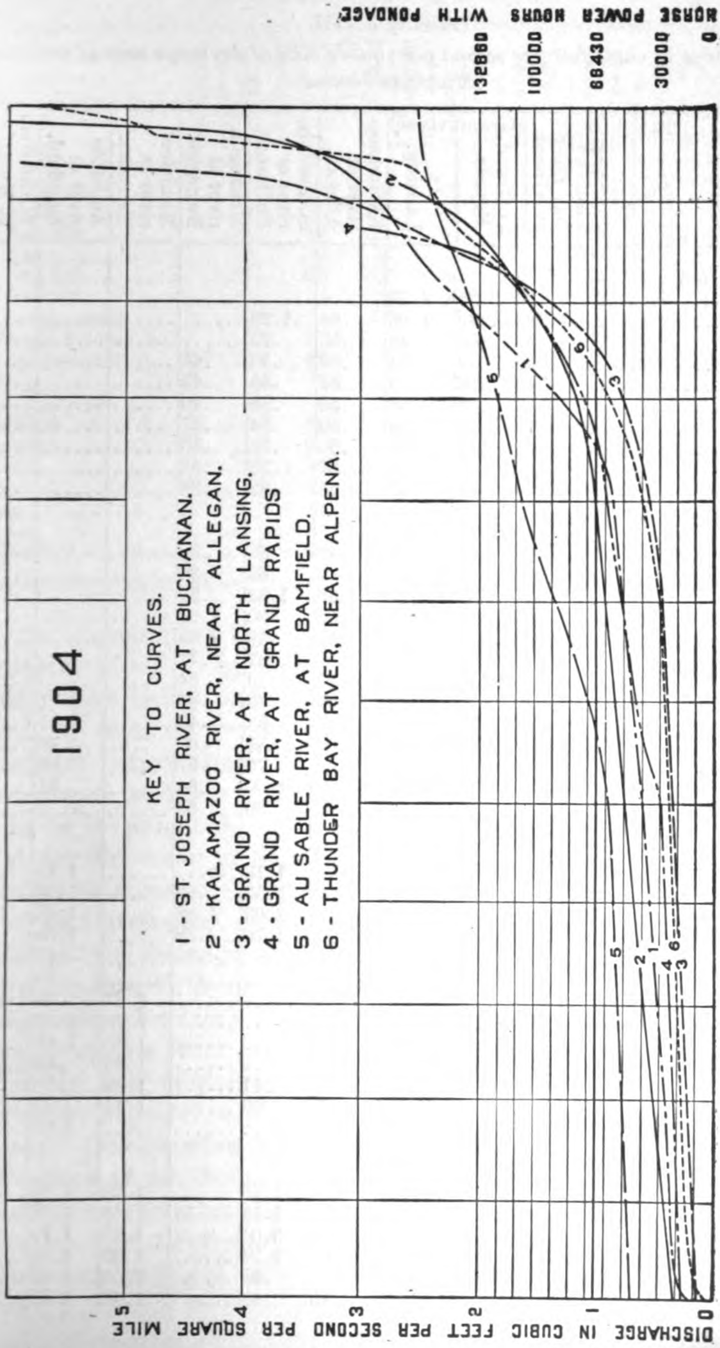

Fig. 95.—Duration Curves of various Michigan rivers.

TABLE XVIII.

Discharge in cubic feet per second per square mile of drainage area of various Michigan rivers.

	Thunder Bay river at Alpena	Grand river		Kalamazoo river at Allegan.	St. Joseph river at Buchanan.	Muskegon river at Newaygo.	Manistee river at Sherman.	Au Sable river at Bamfield.	White river at Moran's Bridge.
		At Grand Rapids.	At Lansing.						
1901									
March	......	3.25	2.73	......	......	......	......	......	......
April	1.49	1.39	1.06	.64	1.29	......	......	......	......
May	1.18	.66	.48	.53	.76	......	......	......	......
June	.63	.49	.34	.57	.53	.45	......	......	......
July	.74	.92	.78	.51	.45	.45	......	......	......
August	.51	.38	.58	.52	.38	.40	......	......	......
September	1.31	.39	.44	.50	.54	.37	......	......	......
October	.70	.47	.51	.50	.71	.50	......	......	......
November	.41	.42	.35	.57	.70	.38	......	......	......
December	.32	.65	.66	.54	.82	.38	......	......	......
1902									
January	.40	.46	.55	.46	.69	.30	......	......	......
February	.29	.40	.43	.46	.62	.33	......	......	......
March	1.31	1.41	1.26	.58	1.32	.57	......	......	......
April	.91	1.03	1.02	.55	.90	1.03	......	......	......
May	.78	1.15	1.09	.55	.98	1 34	......	......	......
June	.74	.70	.88	.56	.92	.77	......	......	......
July	.40	1.57	1 78	.62	1.10	.64	......	......	......
August	.46	.53	.57	.54	.60	.47	......	.71	......
September	.21	.57	.50	.52	.58	.46	......	.71	......
October	.48	.79	.84	.61	.84	.67	......	.75	......
November	.77	.95	.66	.63	.79	.57	......	.95	......
December	.34	.96	.62	.64	1.00	.54	......	.91	......
Yearly mean	.59	.88	.85	.56	.86	.68	......	......	......
1903									
January	.44	1.53	.83	.93	1.13	......	......	1.48	......
February	.55	2.26	1.36	1.20	1.52	.67	......	1.18	......
March	1.67	2.13	2.69	1.84	2.05	1.58	......	1.43	......
April	1.16	2.04	2.45	1 63	1.76	1.38	......	1.38	......
May	.62	.68	.52	.76	......	.91	......	.97	......
June	.44	.53	.46	.69	......	......	......	.78	......
July	.48	.45	.53	.62	......	......	1.22	.79	......
August	.83	.52	.79	.69	......	......	1.40	1.03	......
September	.79	1.06	1.04	.92	......	......	1.40	1.01	......
October	.68	1.15	.62	.81	......	......	1.35	.86	......
November	.43	.54	.43	.68	.41	......	1.28	.78	......
December	.38	.62	.33	.72	.66	......	1.41	1.13	......
Yearly mean	.71	1.05	1.00	.93	......	......	......	1.07	......
1904									
January	.38	......	.48	.82	1.49	......	1.28	1.94	......
February	.46	......	1.07	.98	1.48	......	1.18	2.35	......
March	.64	......	3.05	3.44	3.07	......	1.29	1.79	......
April	3.48	2.90	2.22	2.08	2.24	......	2.35	1.89	......
May	1.79	1.00	.69	1.03	.95	......	2.00	1.49	......
June	1.17	.52	.33	.73	.68	......	1.42	1.05	......

TABLE XVIII.—Continued.

	Thunder Bay river at Alpena.	Grand river At Grand Rapids.	Grand river At Lansing.	Kalamazoo river at Allegan.	St. Joseph river at Buchanan.	Muskegon river at Newayga.	Manistee river at Sherman.	Au Sable river at Bamfield.	White river at Moran's Bridge.
July	.36	.35	.25	.66	.67	......	1.22	.90	.46
August	.36	.29	.24	.55	......	.45	1.18	.85	.66
September	.34	.35	.21	.61	.33	.33	1.11	.77	.68
October	.38	.59	.32	.71	.47	.45	1.19	.82	.93
November	.35	.37	.24	.55	.47	.42	1.09	.76	.79
December	.35	......	.26	.56	.40	.39	1.08	1.37	.94
Yearly mean	.84	......	.78	1.06	......	......	1.36	1.33	......
Mean for last 5 or 6 months	.35	.41	.25	.61	.47	.41	1.14	.91	.74
1905									
January	......	.63	......	......	......	......	1.20	1.31	1 34
February	......	.67	......	......	......	......	1.31	1.97	1.51
March	......	1.94	......	......	......	......	1.52	1.19	1.55
April	......	1.47	......	......	......	......	1.81	1.07	1.55
May	......	1.46	......	......	......	......	1.51	1.14	1.47
June	......	2.76	......	......	......	......	1.29	.98	1.84
Mean for 6 mos.	......	1 49	......	......	......	......	1.61	1.11	1.54

of the comparative flows of various Michigan rivers, could be estimated. In Fig. 95 these hydrographs have been re-drawn, the daily flows being platted in the order of their magnitude. This form of diagram represents the best basis for the comparative study of stream flow for power purposes where storage is not considered, and where the continuous power of the passing stream is to be investigated.

A careful study of Figs. 94 and 95 will show that the run-off is similar in streams situated under similar geographical, topographical, and geological conditions, and having equal, or similar, rainfalls on the drainage area. The departure of the various streams here considered, from the average of all, gives a very clear idea of the errors which may be expected in estimating the flow of any particular stream from the hydrographs of other adjacent streams, or from the flow of streams more remote, and which are located under different physical conditions.

101. Comparative Hydrographs From Different Hydrological Divisions of the United States.—The hydrographs of streams differ widely in character, both in accordance with their geographical location and the diverse physical character of their drainage areas. Their geographical location affects their climatic, geological and

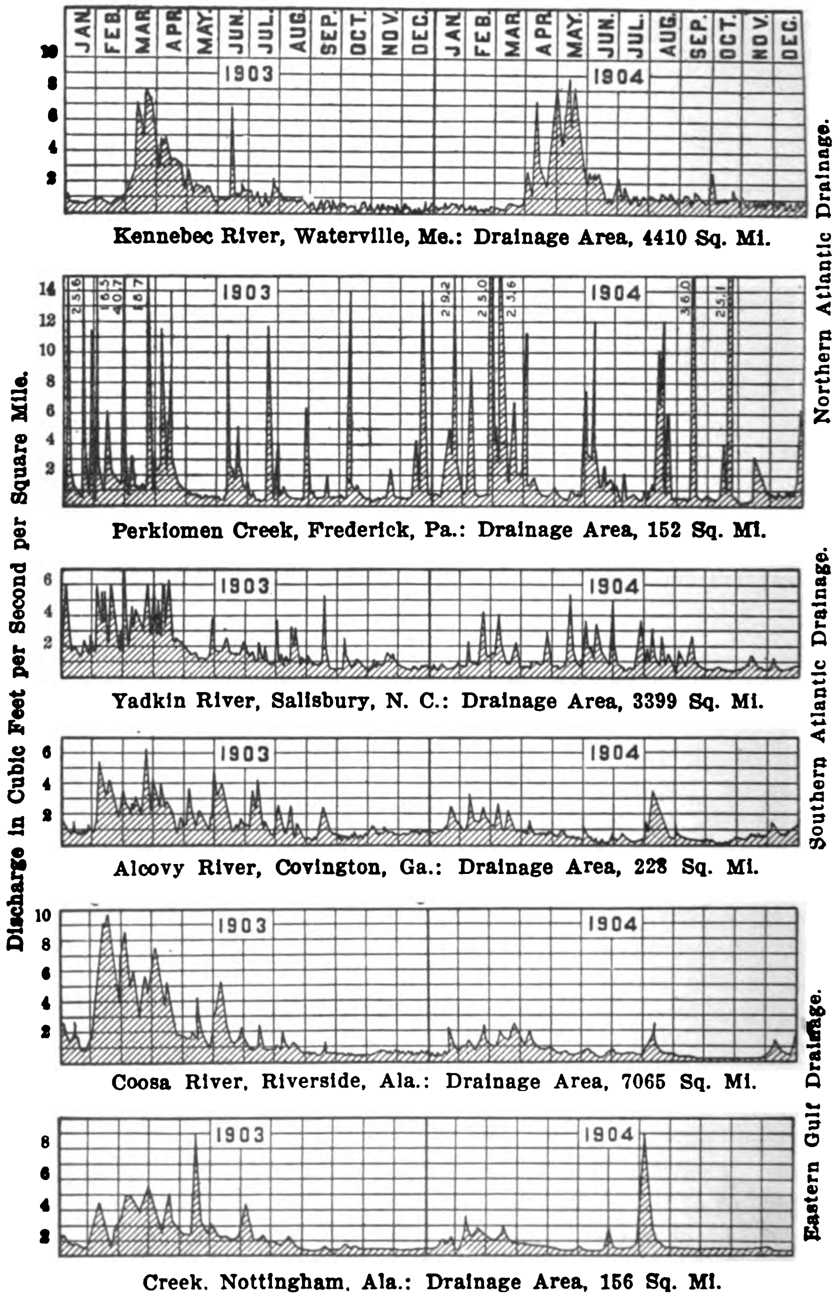

Fig. 96.—Hydrographs of Atlantic and Eastern Gulf Drainage.

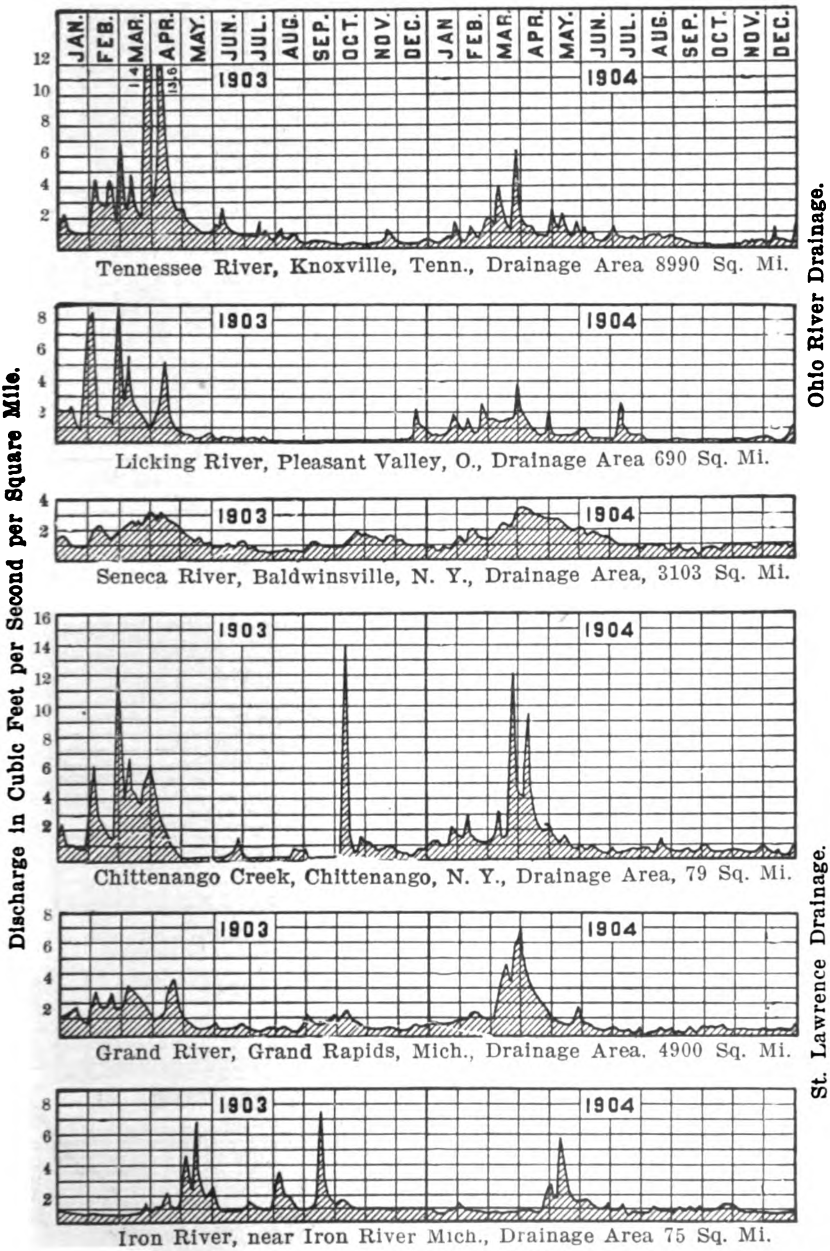

Fig. 97.—Hydrographs of Ohio Valley and St. Lawrence Drainage.

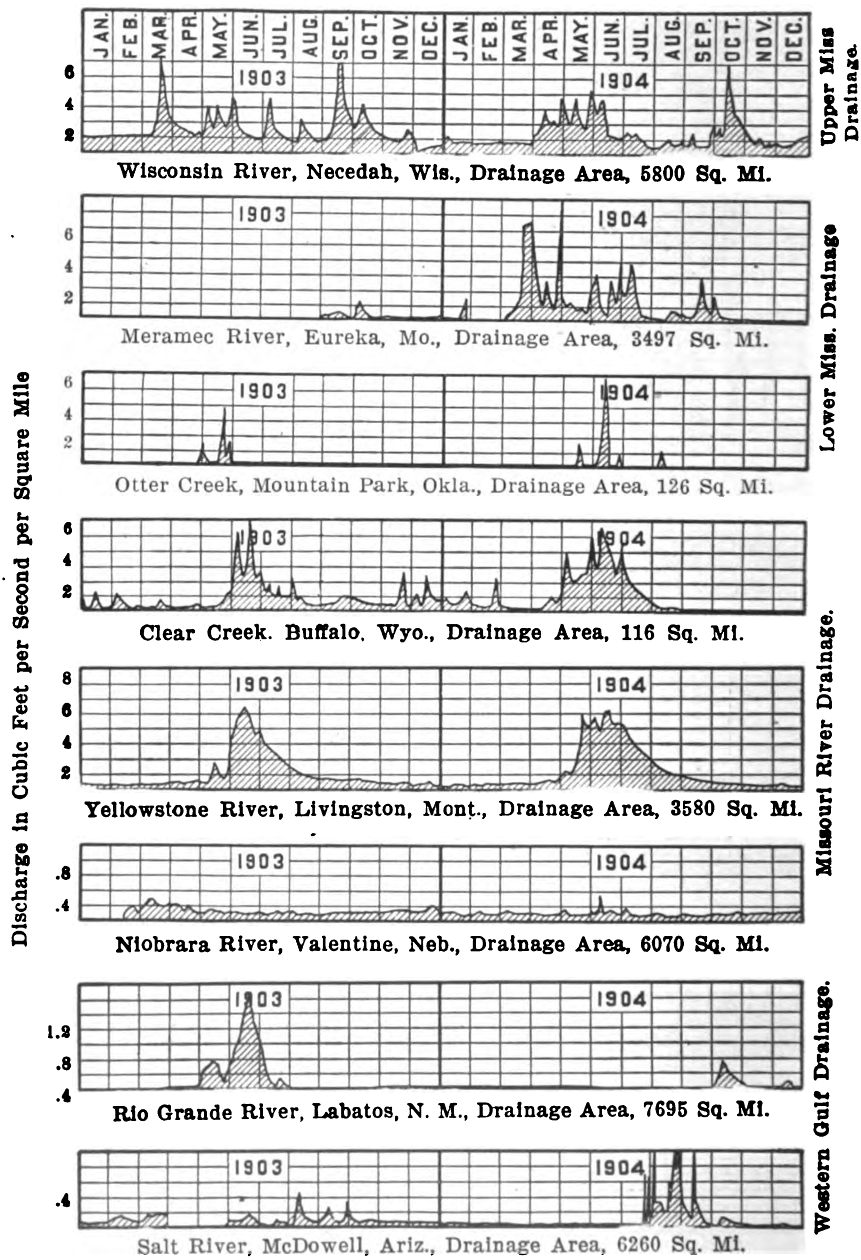

Fig. 98.—Hydrographs of Mississippi Valley and Gulf Drainage.

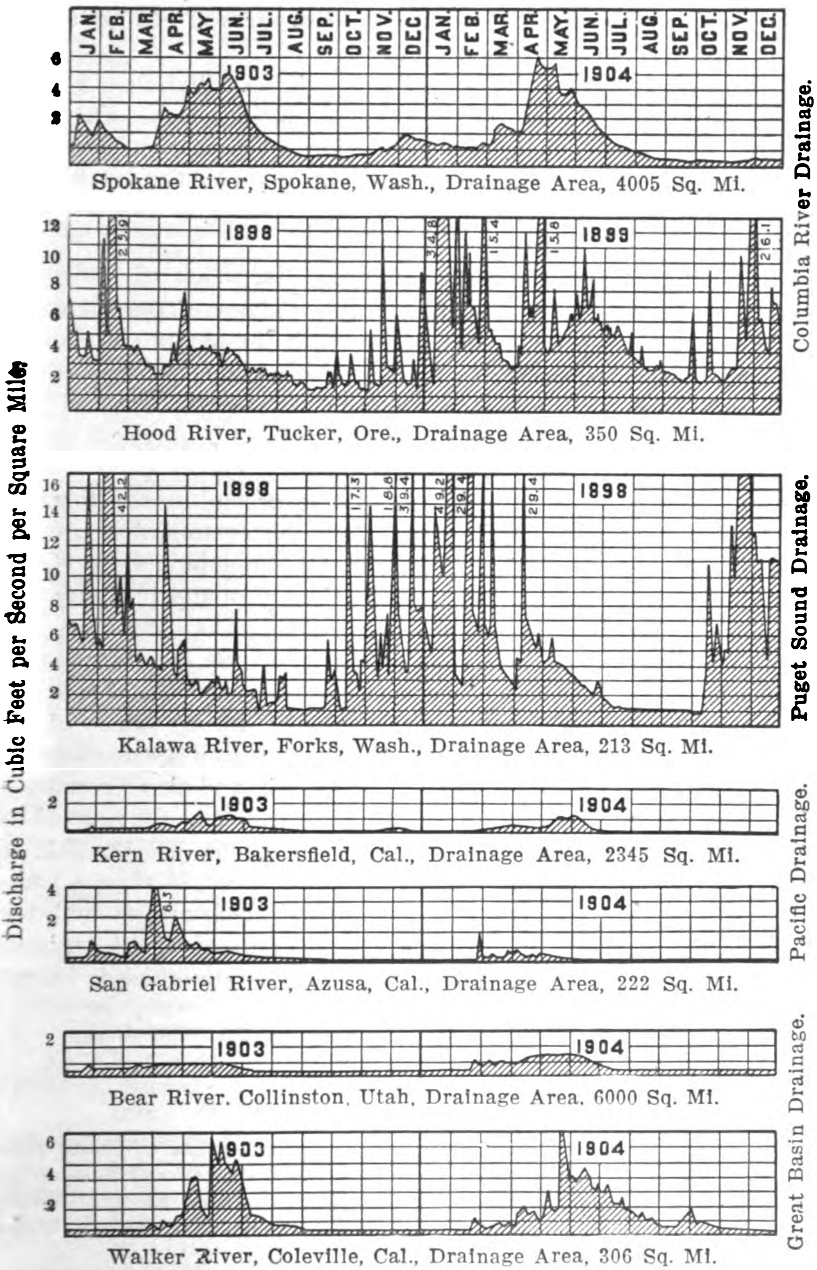

Fig. 99.—Hydrographs of Western Drainage.

topographical conditions, and results in a material difference in the distribution and quantity of run-off.

Hydrographs from the various hydrological divisions of the United States are shown by Figs. 96 to 99, inclusive. For each drainage area hydrographs for two years are shown in order to eliminate, partially at least, the effect of any peculiar conditions which might have obtained during a single year, and to show that the hydrographs are characteristic.

102. General Conclusions.—A complete discussion of run-off is impossible in the space available in this volume. Attention has been called to the general laws upon which the amount of run-off depends, and to the similarity in flow that obtains on watersheds which are physically similar, also to the variations in run-off that occur on different watersheds due to differences in physical conditions.

Each stream presents peculiarities of its own, and in investigating stream flow the data available is seldom the same and is always found to be much too limited for a complete understanding. Only general suggestions can be offered for the study and investigation of these subjects. Attention has been directed, as clearly as possible, to the errors which are likely to arise in the investigation of water power conditions by comparative study. From a knowledge of such errors the engineer will realize the limiting values of his conclusions, and hence should so shape his design as to effect as safe a construction as the condition will permit, and also a construction which will bear out fairly well his conclusions at the time of its inception. It is evident that no exact conclusions are possible in these matters, and that an element of uncertainty is always present. A knowledge of the extent of these uncertainties and the probable limits of exact knowledge are as important to the engineer as his ability to draw correct conclusions from data which is known to be correct.

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CHAPTER X.

STREAM FLOW.

103. Flow in Open Channels.—The discussion of the flow of water in open channels in Chapter III includes only such channels as have uniform cross sections, alignment, and gradient and a bed of uniform character throughout the length considered. Such conditions are closely approximated in artificial channels in which the quantity of water flowing is under control. In such channels, and with a steady flow,—that is with the same quantity of water passing every cross section in the same time,—it is shown that:

(1) $$v = c\sqrt{\frac{ah}{pl}} = c\sqrt{rs} \text{ and that}$$

(2) $$q = av = ac\sqrt{\frac{ah}{pl}} = ac\sqrt{rs}$$

In natural water courses no two cross sections are the same but may differ in area, a, and wetted perimeter, p; and the fall, h, in any length, l, usually differs considerably from reach to reach. The quantity, q, of water flowing in any such stream is also constantly changing. There every condition of uniform flow is lacking and can only be approximated for selected reaches of such streams and during periods when stream flow is fairly steady.

104. Changes in Value of Factors with Changes in Flow.—From an examination of equation (2) it is evident that in any channel as the quantity of water flowing, q, changes, there must be a corresponding change in some or all of the factors on the other side of the equation.

For steady flow in a uniform channel, s remains constant and all changes are confined to the values of a, c and r. The laws of change in the values of c are given by Kutter's and Bazin's formulas, but are best illustrated and understood by reference to Fig. 22, which is a graphic expression of the formula of Bazin.

In variable flow a change in all of the factors usually accompanies a change in the value of q, each factor changing in accordance with the physical conditions of the channel.

The changes in the value of c, in an irregular channel, do not always seem to follow Bazin's law. In some cases c is even found to decrease as r increases. The law of simultaneous increase in c and r presupposes a channel of uniform character and condition. If an increase in the hydraulic radius, r, in any channel is accompanied by a radical change in the character of its bed the law will not hold. It is evident that under such conditions the values of c for different values of r are not fairly comparative. No more uniform law of change can be expected under such conditions than would occur in the comparison of the relation of c and r for entirely different channel sections.

In Fig. 100 are shown the observed values of c and r for certain reaches of the Wisconsin River above Kilbourn, Wis. It will be

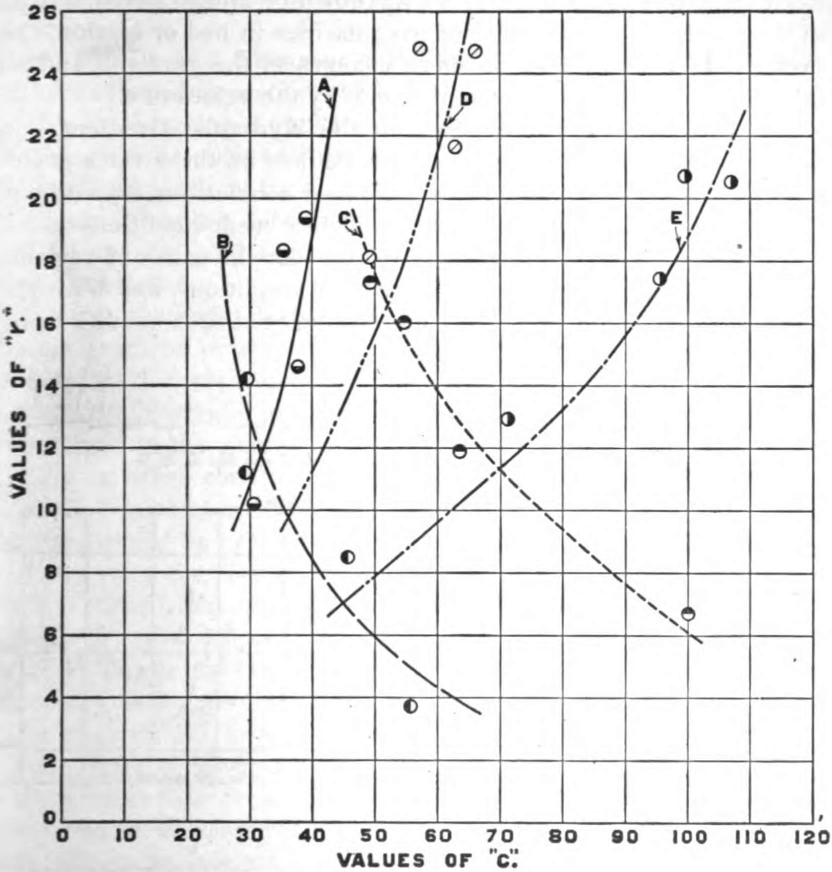

Fig. 100.—Relations of Coefficient to Hydraulic Radius in Certain Reaches of the Wisconsin River.

noted that the value for reaches A, D and E follow in general the law as established by Bazin. These are fairly uniform. On the other hand the values of c and r for reaches b and c seem to follow an entirely different law, a condition due to irregularities in the cross section of the reach.

Where the values of a, p and r vary radically from section to section and differ materially from the values in the sections considered and on which calculations are based, the value of c will be found to differ radically from that which the character of the bed and the entire section would indicate. Absurd values of c are a clear indication that the sections selected are not representative. The calculated value of c is modified by all unknown or unconsidered factors of the reach. The influences of irregularities in bed or section, the presence of unconsidered bends or changes in the gradient, and all other irregularities in the channels, modify the values of c.

105. Effects of Variable Flow on the Hydraulic Gradient.—In order to understand the effect of variable flow on the surface gradient of a stream, and in order to realize how conclusions drawn from the laws of uniform flow must be modified to meet conditions found in natural streams, it is necessary to consider the cause of variable flow in a stream, the variation in channel conditions, and both the effect of flow on such conditions and the effect of such conditions on the flow of a stream.

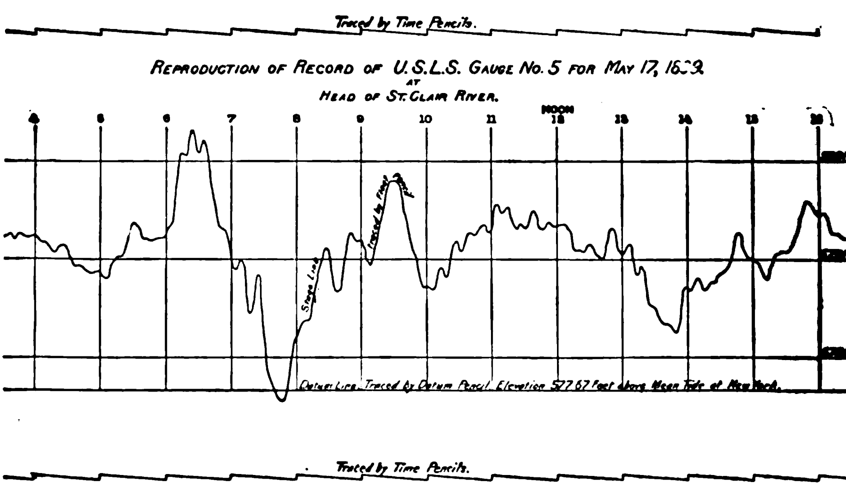

Fig. 101.—Variations in Gauge Height of the St. Clair River.

The surface of a stream is constantly fluctuating, not only on account of the variation in flow, but also on account of wind, barometric pressure and changes in the hydraulic gradient. Such changes occur from hour to hour, and even from minute to minute. Larger rivers, fed directly by great lakes, are more susceptible to these changes on account of the broad lake area, giving wind and barometric pressure greater opportunity to act. Every stream is, however, more or less susceptible to these changes, and gauge readings taken daily, therefore, show only in an approximate way the true height of the surface of the river at the point of observation. This is well shown by Fig. 101, which is reproduced from the autographic record of a gauge at the head of the St. Claire River.

106. Effects of a Rising or a Falling Stream on Gradient.—In a channel of uniform section, the bed of the channel AB (see diagram A, Fig. 102) having a uniform slope, all cross sections, such as Aa and Bb, will be alike and the wetted perimeters and the hydraulic radii will be identical for all sections. The fall, bx, will be uniform in all equal lengths, l, of the channel, and such uniform conditions will be maintained for all regular discharges after regular flow is once established.

In such channels, during changes in the stages of flow, the hydraulic gradient or slope will change until uniform flow is established. In all cases illustrated in Figs. 102 and 103, the line ab represents the hydraulic gradient which will obtain if uniform flow is maintained in the channel and if there be no change in the channel section or other conditions. The actual water surface, caused by variable flow, is in each case shown by the line a'b. In each case, the fall, bx, would be necessary to produce uniform flow from A to B and to assure the flow of the normal quantity of water passing the section Bb as in diagram A. In diagram B and C, Fig. 102, the conditions of variable flow in a uniform channel are graphically represented. The actual flow is greater or less than the normal quantity, according as the gradient is increased or diminished.

In diagram B, the conditions with a rising stream are shown. Under these conditions the quantity of water passing the section Aa is greater than the quantity passing the section Bb, by the quantity of water necessary to fill up the channel of the stream to a new and uniform surface gradient. The head needed to produce the flow past the section, Aa, is represented by the height, xx'. The total fall between A and B is therefore greater than that required for the

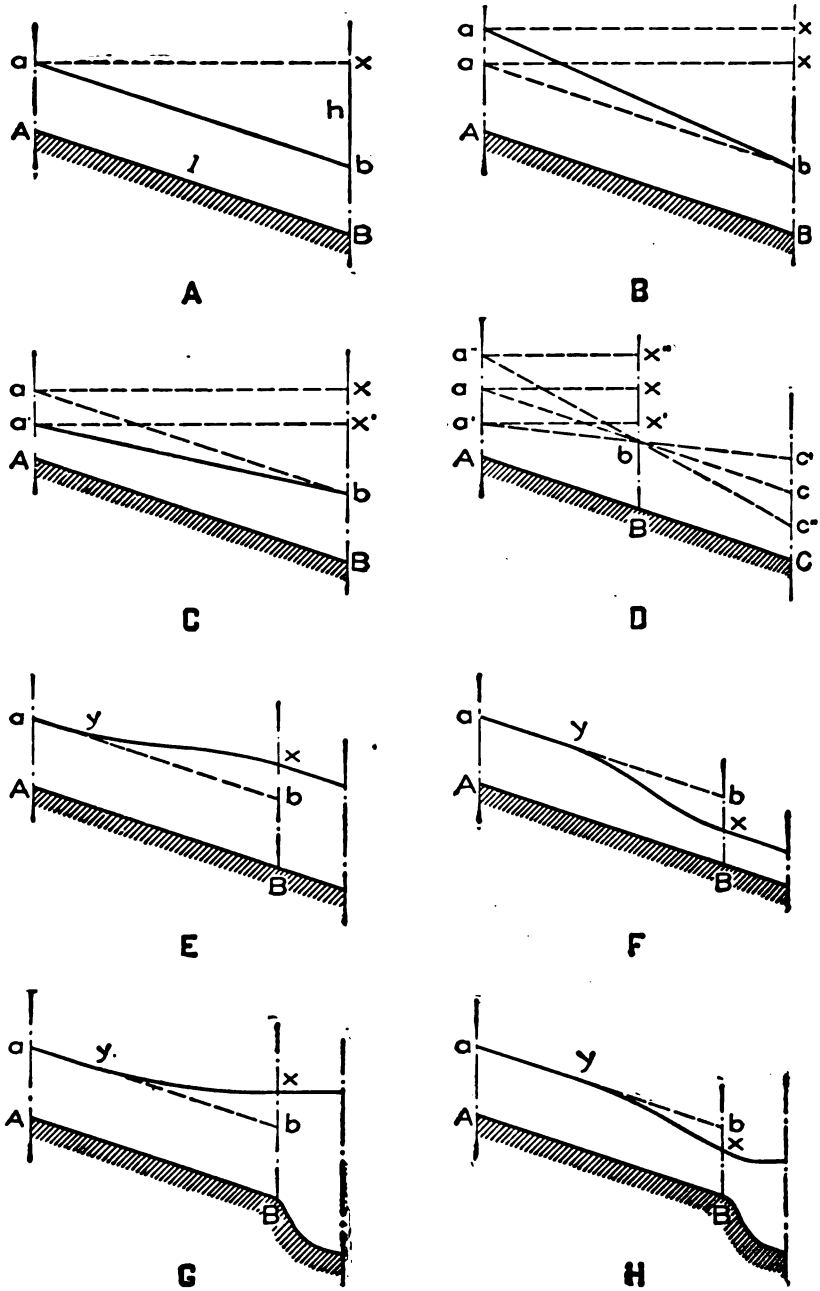

Fig. 102.—Effects of Variable Flow on the Hydraulic Gradient of a Stream.

uniform flow as represented by the head bx'. This produces not only a greater flow at Aa, but also a flow greater than would be normal at section Bb.

In diagram C, Fig. 102, the conditions of a falling stream are represented. In this case, the head at section Bb at the moment of observation would, if the flow was uniform, produce a normal flow which would require the fall, bx, to maintain it. With a falling stream, the section AB is emptying and the quantity of water passing the section Aa is less than the quantity of water passing the section Bb, which in turn is also less than the normal flow for the existing head. A less fall is therefore required to produce the flow passing Bb, which, with the lower slope and the same cross section, is less in quantity than would be the case under conditions of uniform flow. This fall is represented by the height, bx', which is less than the height bx, required for uniform flow by the height xx': consequently the slope of the river is a'b.

From the above considerations it will be seen (see diagram D, Fig. 102) that a given gauge height, Bb, may not always represent the same flow, for the discharge, Q, is a function not only of the cross section, a, but also of the slope, s. A single gauge height may therefore represent a considerable range of flows depending on the hydraulic gradient which may pass through the point with a uniform, a rising or a falling stream. It is obvious that the flows represented by the hydraulic gradient, a' bc', abc and a''bc'', while producing the same gauge height at Bb, nevertheless represent three different conditions of flow.

In the establishment of the relations between gauge heights and flow, it is therefore important that the observed flow corresponding to a given gauge reading be taken during a period of essentially uniform flow, for, from the above considerations, it will be seen that any determination or observation made with a rising or a falling stream must necessarily be more or less in error. It will also follow that, after a rating curve and rating table have been established, the gauge height taken during changes in the conditions of flow will be more or less in error, although such errors will equalize to a considerable extent and will, in the main, prove unimportant.

107. **Effects of Channel Condition on Gradient.**—The flow of water in a natural channel is far from being uniform and it is important for the engineer to realize this lack of uniformity and the effect of such conditions upon the flow of the stream. In any channel of uniform gradient, as AB in diagram E (Fig. 102), if at the

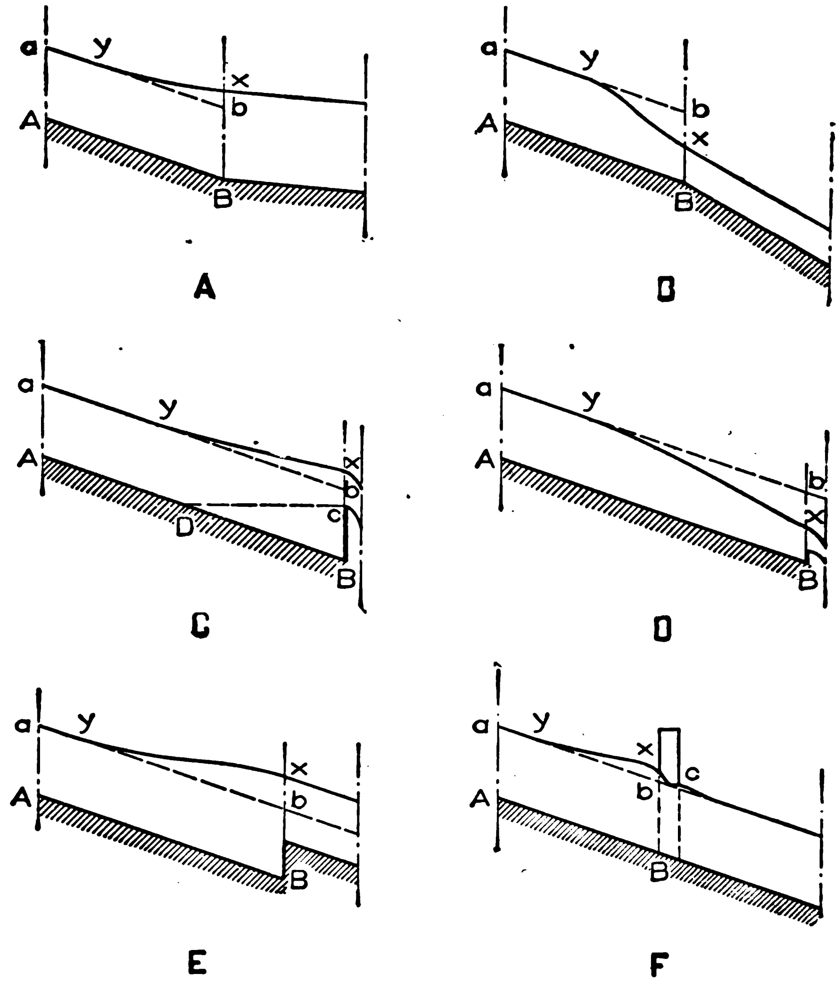

Fig. 103.—Effects of Channel Grade and of Obstruction on the Hydraulic Gradient of a Stream.

section Bb the coefficient c is decreased on account of increased roughness in the bed of the stream, or if the area of the channel, a, is contracted, a change in the hydraulic gradient will follow. The normal gradient with uniform flow would take the position ab, but on account of the change in conditions at Bb, the depth must increase to keep q a constant; a must increase to offset the decrease in c or c must increase to offset the decrease in a if q remains constant. The surface must therefore rise to the point x and a new hydraulic gradient will be established and maintained until other changes in the channel condition again modify the same. Between the new and old gradients, a transition curve will be established extending both above and below the point at which the change in condition takes place to some point, y, frequently a long distance upstream.

The opposite condition is shown by diagram F, Fig. 102. In this diagram the effect of an increase in the coefficient, c, of the bed or in the area, a, of the stream is represented. If c increases, a less section will be required below that point and again the surface is lowered; or if the width of the stream increases, the depth will diminish in order that ca may remain constant.

Variable flow is also caused by a sudden enlargement in the river section or by a discharge of the stream into a larger stream or into a lake or pond. Such conditions are shown by diagrams G and H, Fig. 102. The character of the transition curve in such cases will depend on the height of the surface of the water into which the stream is discharged. If the water surface of the lake is above b, the curve will be concave upward (see diagram G) and if the surface is below b, the curvature will be downward (see diagram H).

108. Effect of Change in Grade and of Obstructions.—Variable flow may also be caused by changes in the slope of the stream bed as shown by diagrams A and B, Fig. 103. The area of the stream must increase as the bed slope is decreased, or must decrease as the slope of the bed is increased in order to fulfill the conditions of equation (2).

It is evident that uniform slope may be maintained even with changed conditions if the changes that occur give rise to equal and opposite effects. For example, uniform slope may be maintained if the area of section a is reduced and the coefficient c is increased to such an extent that the product ac remains constant at each section of the channel.

Variable flow is also caused by the passage of the stream over weirs or dams and the effect on the gradient will vary as shown by diagram C and D, Fig. 103. Variations may also be caused by a change in the bed (see diagram E, Fig. 103), or by local contractions, submerged weirs or other obstructions as shown by diagram F, Fig. 103.

In all of the above described cases it is obvious that if the slope of the stream is measured on any of these transition curves, a false idea of slope will obtain and a false relation will be established for

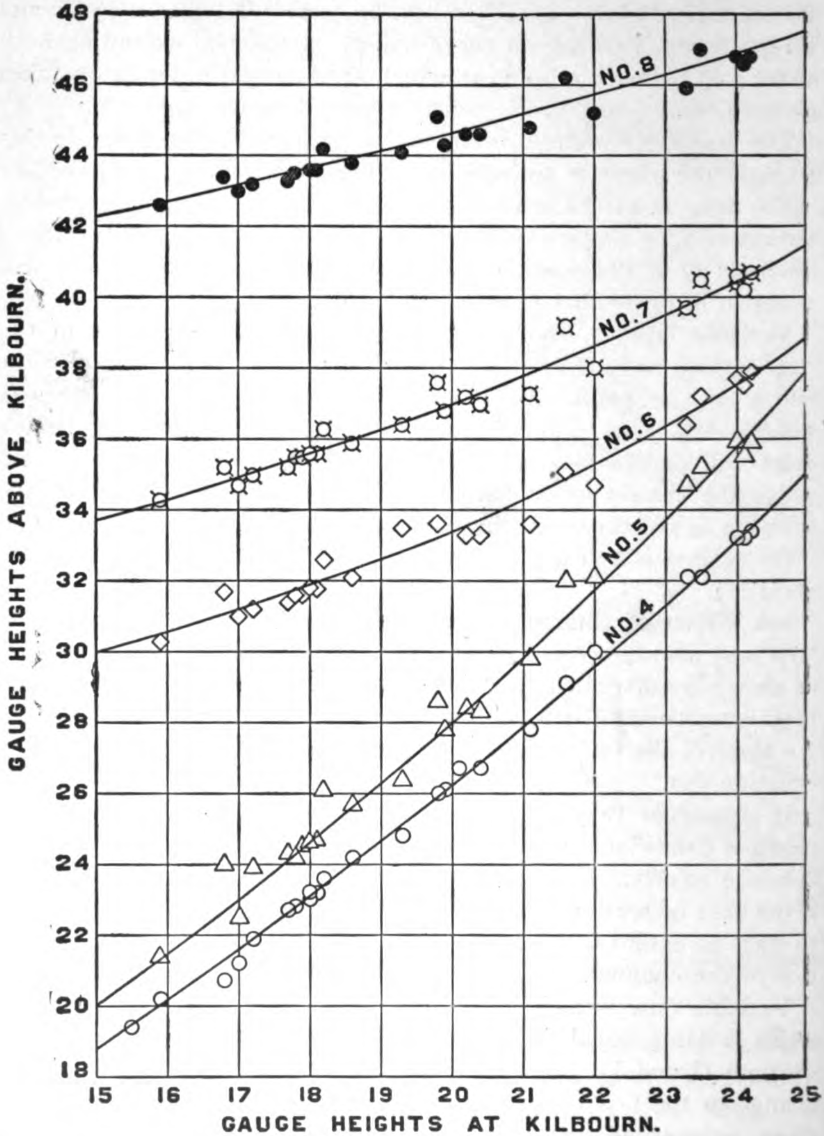

Fig. 104.—Relations of Guage Heights at Various Stations on the Wisconsin River.

the condition of stream flow. It is therefore essential in any measurement of a stream or in the establishment of any gauging station that the location for such observations be carefully selected on a reach of the stream where conditions of essentially uniform flow prevail and that all observations be taken during stages where the flow of the stream is practically constant. If gauges are established at various points along the course of a river and are read simultaneously, and if the flow is uniform and no falls, rapids or tributaries intervene, the same differences in elevation should always obtain with the same stage of water.

A system of gauges as described above was recently established at Kilbourn on the Wisconsin River in order to determine the river slopes near that place. A large number of practically simultaneous readings were taken in order to determine the relations between the gauge heights at the various points compared with the Kilbourn gauge.

Fig. 104 shows the results of the gauge readings at the various stations compared with the gauge readings at Kilbourn. It will be noted from the diagram that the slope of the river was far from uniform at different times during these readings, and, in a number of cases, the same gauge reading at Kilbourn was accompanied by readings at other gauges that differed from each other by more than a foot. For example, compare the gauge readings at Kilbourn with the readings at gauge No. 5. With a gauge reading of 17 ft. at Kilbourn, the normal gauge reading at No. 5 should be 23 feet, and with a normal flow, the fall between gauge No. 5 and the Kilbourn gauge would be 5 ft. From the diagram it will be seen that during a certain stage of flow in the river the gauge reading at gauge No. 5, with a 17 foot reading at Kilbourn, was about 22½ ft. Under these conditions the fall between gauge No. 5 and the Kilbourn gauge was only 4½ ft. The slope being reduced, the quantity of water actually passing the Kilbourn gauge under these conditions was less than the normal flow for the 17 ft. gauge height. On two other occasions where the gauge reading at Kilbourn was approximately 17 feet, the actual gauge reading at gauge No. 5 was about 24 feet. During these conditions the actual fall in the river between gauge No. 5 and the Kilbourn gauge was 5 feet, or one foot more than normal. Hence the quantity of water flowing by the Kilbourn gauge at this time was more than the normal quantity indicated by the Kilbourn gauge.

Readings of other gauges compared with the Kilbourn readings

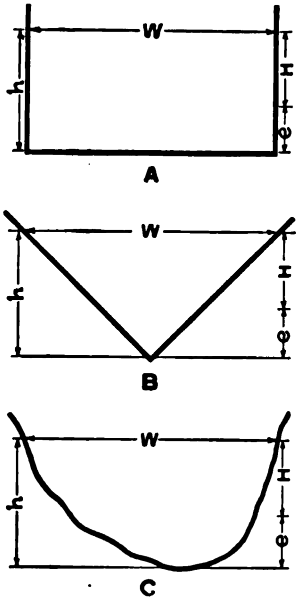

Fig. 105.

will show that at certain times the flow was normal and at other times the river must have been rising or falling and that consequently the gauge at Kilbourn at the time of such reading, was not accurately representing the quantity of water flowing by the Kilbourn section. The above example taken of the variation in slope between the Kilbourn gauge and gauge No. 5 indicated practically the maximum abnormal conditions. The actual variation in flow at Kilbourn during these conditions was not determined and is not definitely known.

109. Relation of Gauge Heights to Flow.—The area of any cross section equals the product of the height of the section into some function of its width:

$$a = h \times f(w) \tag{3}$$

In a rectangular cross section f=1, (see A, Fig. 105). In a triangular section, f=.5 (see B, Fig. 105). In all cases of regular section f can be mathematically expressed, and for irregular sections (see C, Fig. 105) the relation may be obtained by measurement. If the height of the surface is referred to a gauge height, H, the zero of the gauge may or may not correspond with the bottom of the channel. If H = the gauge height, then h=H+e, in which e is the distance from the bottom of the channel to the bottom of the gauge. Substituting, therefore, the value of h in equation (3) it becomes:

$$a = (H + e) \times f(w) = Hf(w) + ef(w), \tag{4}$$

And substituting this value in equation (2) it becomes:

$$Q = Hf(w)\, c \sqrt{rs} + ef(w)\, c \sqrt{rs} \tag{5}$$

With this equation, and with the flow in a fixed and uniform channel, if the relation can be established between r, s, c, e, w and f for each gauge height, H, the corresponding value of Q can be determined. As these relations are mathematically expressed for uniform flow by the above equation, they can also be represented graphically by a curve which will show the relation between Q and H for all conditions of uniform flow that obtain in the given channel. Such a curve is called a discharge or rating curve. This equation (5) can be readily solved when f is a regular variable and when c, r and s can be determined. Where the function, f, is an irregular variable, no mathematical solution is practicable but the relations may be determined experimentally and can be expressed by a rating table or graphically by a rating curve. Such a rating table and curve can be constructed for every fixed channel or section of a stream for condition of uniform flow, no matter how irregular the section or how the values of the function of the section may vary for different gauge heights.

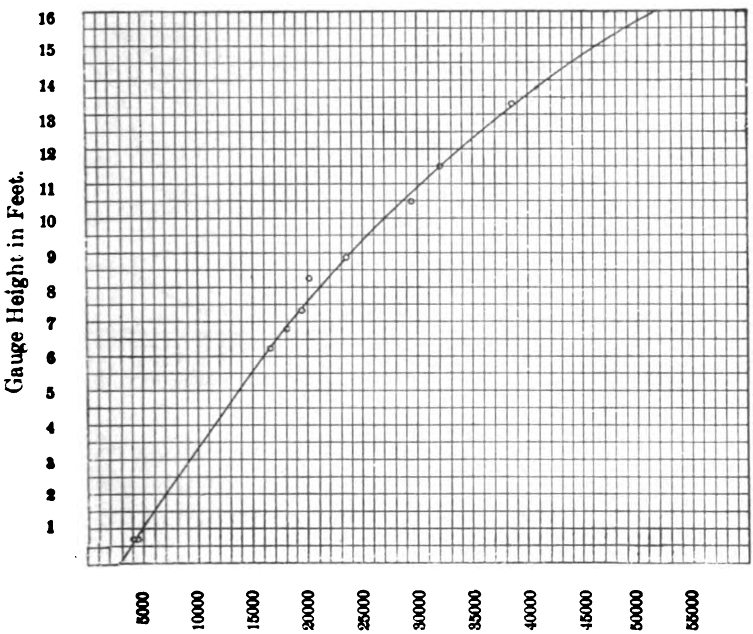

Fig. 106.—Rating Curve for Wisconsin River at Kilbourn, Wis.

Fig. 106 shows a rating curve established for the Wisconsin River at Kilbourn, Wis. The small circles show the flow relative to gauge height at the time the observations were made. They were carefully made in a fairly satisfactory section and fall fairly well on a smooth curve drawn from this data to represent the relation of gauge height to flow at similar or intervening heights.

The character of the rating curve for regular and irregular sections is shown by Fig. 45, page 95. Whenever the section remains similar for different gauge heights, the rating curve will be a smooth curve, but when irregularities occur in the section, the curve becomes broken more or less according to the extent of the irregularity.

It has already been pointed out that any change in the cross section of the stream after a rating curve has been established will necessitate the establishment of a new curve. The variation in rating curves under variation in channel conditions is shown in Fig. 46, page 96.

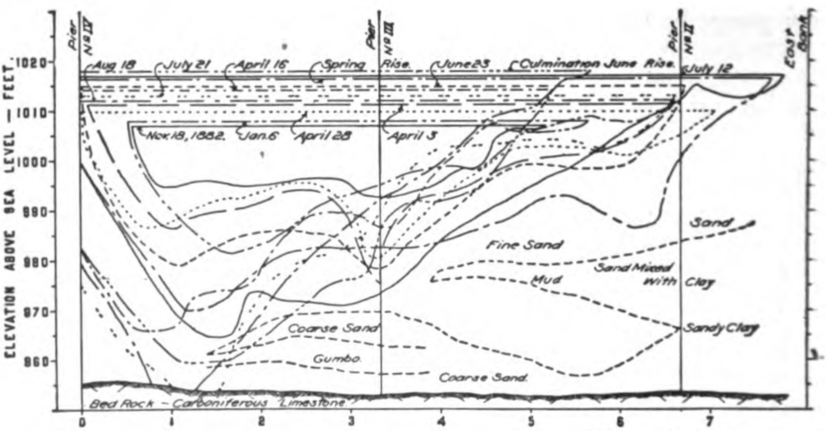

Fig. 107.—Variations in the Cross-section of the Missouri River near Omaha, Neb.*

The actual change in channel conditions that affects the relation of head and flow is well illustrated by Fig. 107 which shows the changes that actually took place in the cross section of the Missouri River near Omaha, Nebraska.

110. Variations in Velocity in the Cross-section of a Stream.— The velocity of flow of a stream varies greatly at different points in any cross section. In any channel the friction of the sides and bed reduces the velocity of that portion of the stream in contact and adjacent to them. If the bed at different points of the cross-section is not uniform, as is always the case in the beds of natural streams, the retarding effects on different portions of the stream varies, and a consequent variation in velocity results. The distribution of the velocities in the cross section of the St. Clair River is shown in Fig. 108, both by lines of equal velocity and by figures giving the velocity as actually measured. In this figure the effect of the friction

*Todd. Bull. 158 U. S. Geol Surv.

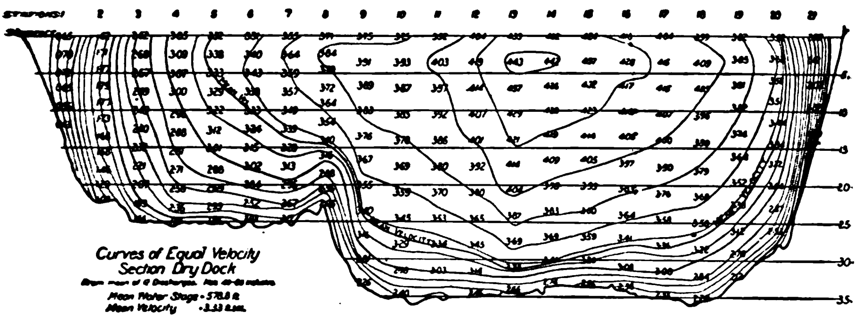

Fig. 108.

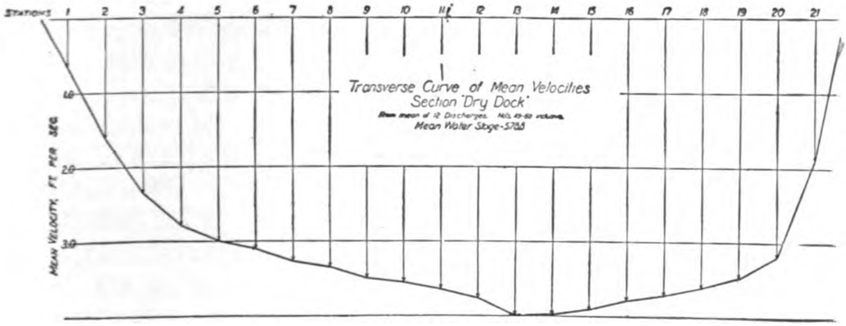

Fig. 109.

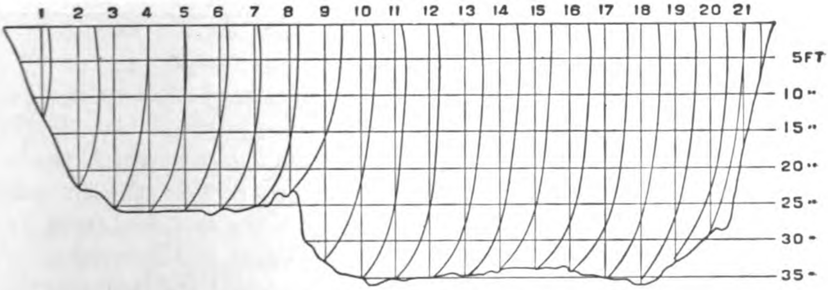

Fig. 110.—Vertical Velocity Curves, Section Dry Dock.

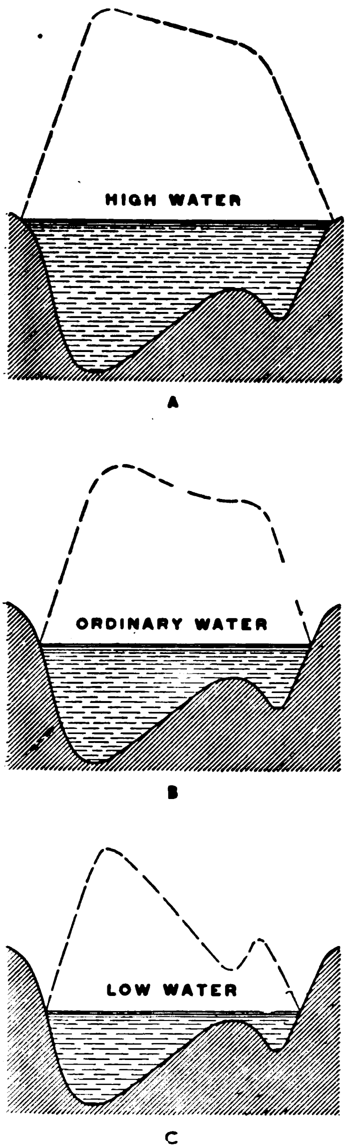

Fig. 111.

of the bed and banks is clearly shown. The friction between the stream surface and the atmosphere is also shown by the fact that the maximum velocity is not at the surface but is a short distance below the surface. The surface velocity may be modified radically by the direction and velocity of the wind.

Fig. 109 shows the transverse curve of mean velocities in this section. The distribution of velocities in each vertical section is shown in Fig. 110. The velocities here shown are relative only as compared with each vertical. The velocity at the bottom of each curve is that shown by figures in Fig. 108.

The distribution of velocities in any section is not the same under all conditions of flow but differs materially with the stage of the river. This is illustrated by Fig. 111 in which is shown three sections of the same stream illustrating conditions of low, medium and high water. Above each section is shown a corresponding transverse curve of mean velocities of flow. The change in the distribution of velocities as the stream increases should be noted.

The distribution of velocity is also affected by bends in the stream above the point of observation which tends to throw the current of the stream toward the concave side, and to cause a transverse slope in the section of the stream at the curve. Such a condition (see Fig. 112) creates cross currents and eddies and produces conditions of variable flow.

From Fig. 108 it will be seen that in any vertical line in a given section, the velocities will vary with the condition of the bed, and

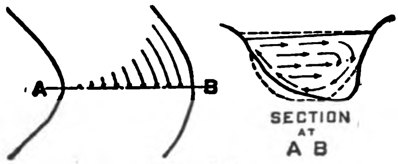

Fig. 112.

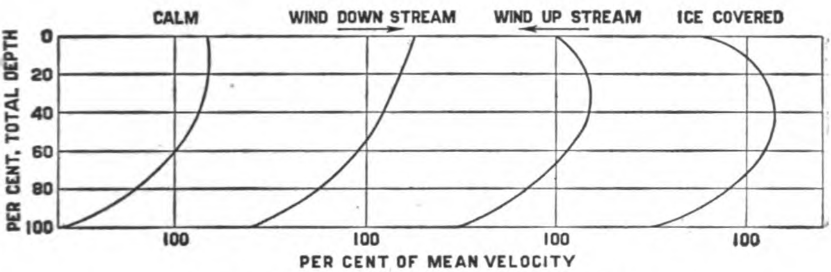

Fig. 113.—Ideal Vertical Velocity Curves.

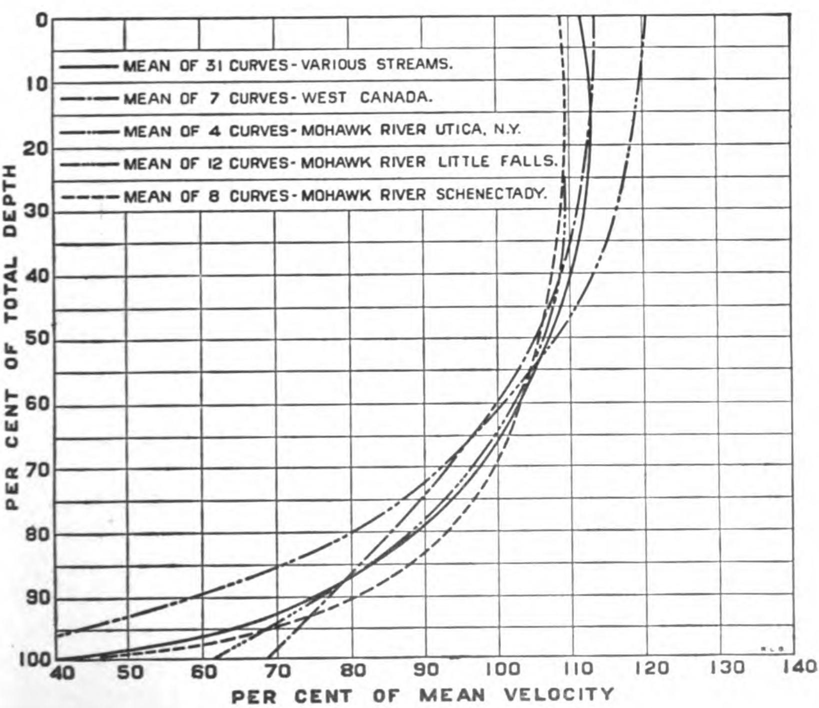

Fig. 114.—Mean Vertical Velocity Curves.

the influence of air current or ice at the surface. These conditions have an influence on the velocities in each section considered. Variations in the vertical velocities can be better studied by means of the vertical velocity curve, which can be obtained by means of velocity observations taken in a vertical line from the surface to the bed of the stream. Ideal curves under various conditions are illustrated by Fig. 113. Figs. 114, 115 and 116 are reproduced from the report of the State Engineer of New York for the year 1902. These diagrams show comparisons between the mean vertical velocities of streams having different classes of beds. From these illustrations it will be noted that there is a general similarity between the various velocity curves which aids materially in the measurement of stream flow. It will be noted, for example, that the mean velocity, in any vertical velocity curve from an open channel, lies near the point of .6 total depth but that with varying conditions this position may vary from 55 per cent. to about 75 per cent. of the depth. The velocity at .6 depth is found to average nearly 100 per cent of the mean velocity, but may actually vary from 95 per cent. to 105 per cent. of the mean

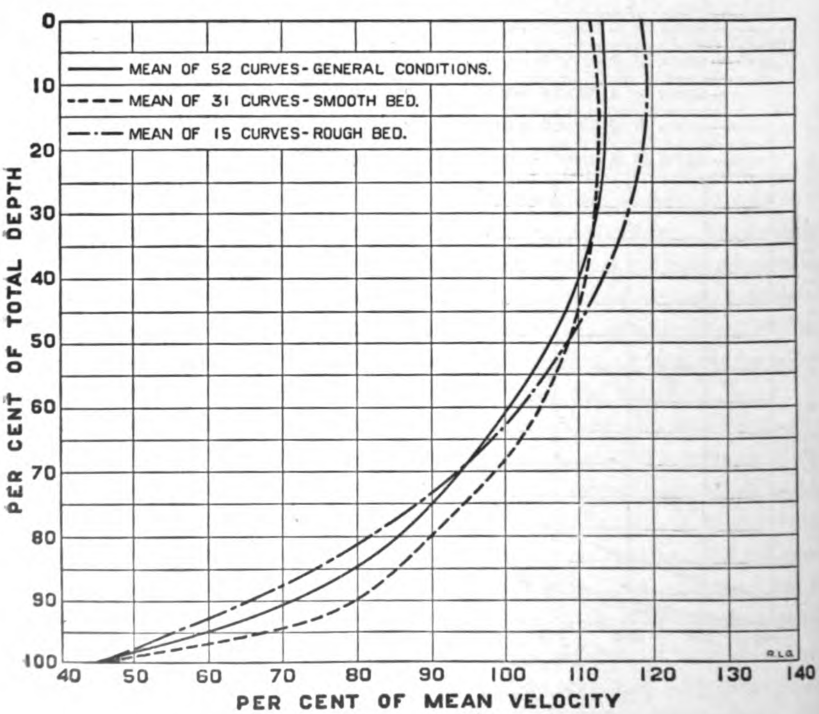

Fig. 115.—Mean Vertical Velocity Curves.

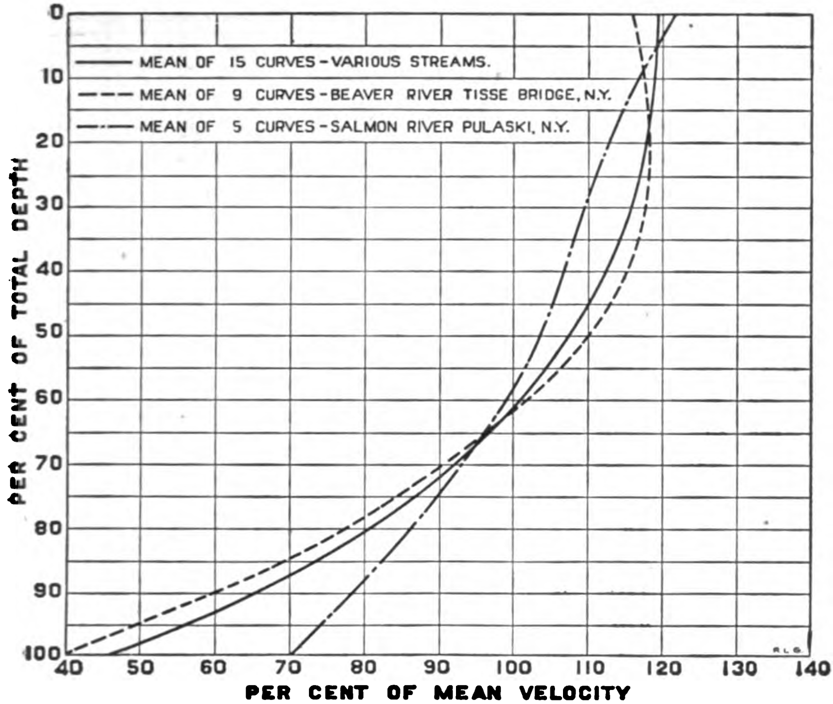

Fig. 116.—Mean Vertical Velocity Curves.

velocity. The velocity at the surface is subject to the external influence of atmospheric currents and is not so constant in its relation to the mean velocity. The surface velocity will average about 110 per cent of the mean velocity of the vertical curve, but is found to vary with the variations in conditions from 105 per cent. to 130 per cent. of such velocity.

111. **Effects of Ice-Covering on the Distribution of Velocities.**—The effect of the formation of an ice sheet over a stream is to materially increase the surface friction and results in a rearrangement of velocities in the cross section. As the ice sheets form in winter, the conditions will vary from that of an open stream to that of a closed channel. The velocities are gradually affected as the ice begins to form, until the entire surface is affected where the stream becomes entirely covered. As the ice sheet thickens more of the cross section of the stream is occupied by the ice sheet, and greater friction results. Fig. 117 shows two vertical velocity curves, one for an open and one for an ice-covered channel. These may be regarded

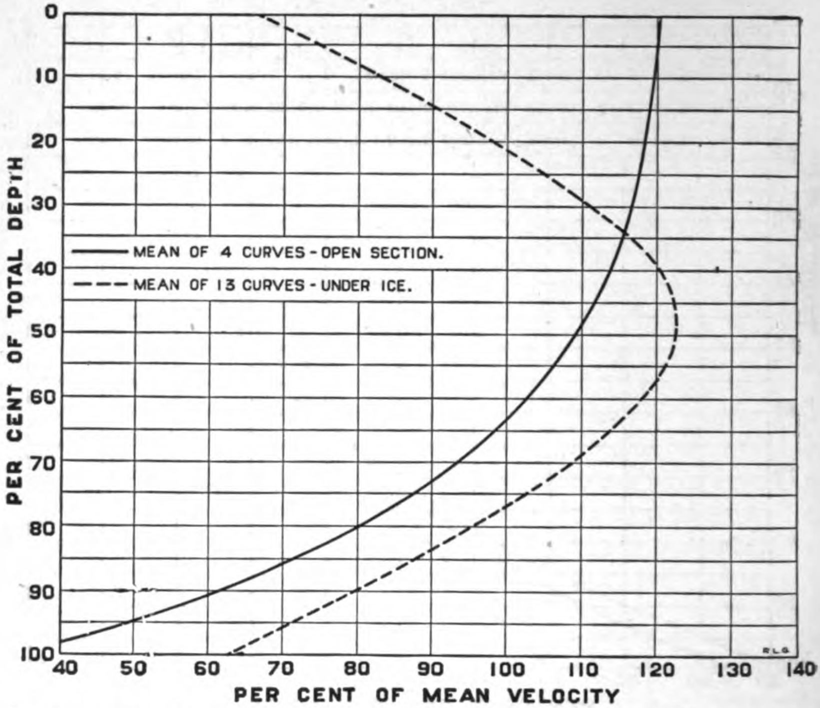

Fig. 117.—Comparative Mean Vertical Velocity Curves for Open and Ice Covered Section.

as typical of open and closed conditions between which the actual velocities will vary with the conditions of the ice.

The change in the distribution of velocities results in an entire change in the relation between gauge height and flow so that the rating curve for an open section will not apply to the river under ice conditions.

If therefore the stream flow is to be accurately determined during such condition, it becomes necessary to establish the new relation between gauge height and flow.

As before noted, such relations vary somewhat with the conditions of the ice sheet but may be regarded as fairly constant when the section is fairly clear and deep. The relations between the rating curves for this open channel and for ice conditions as determined by the United States Geological Survey for the Wallkill River at Neupaltz, N. Y. is shown in Fig. 118.

Table XXI, from an article by F. A. Tillinghast (see Engineering News, May 11th, 1905), shows the relations of maximum and

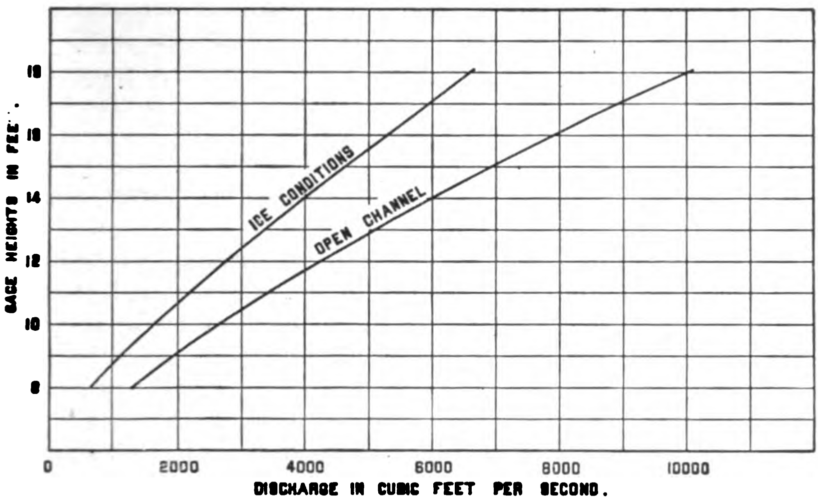

Fig. 118.—Rating Curve for Wallkill River at Newpaltz, N. Y.

mean velocities in the verticals. It should be noted that there are two points of mean velocity under ice conditions that average 11 per cent. and 71 per cent. of the total depth below the surface. The point of maximum velocity is at an average depth of 36 per cent. of the total depth of the stream and averages 119 per cent. of the mean velocity.

TABLE XXI.

Position of Mean and Maximum Velocities in a Vertical Plane Under Ice.

Stream and Place	Depth from Under Surface of Ice Feet	Number of curves	Depth of Mean Velocity		Depth of Maximum Velocity	Coefficient to reduce Max. to Mean
Wallkill at Neupaltz, N. Y.....(a)	4 to 12	20	0.12	0.71	0.38	0.85
Wallkill at Neupaltz, N. Y.....(b)	4 to 19	26	0.13	0.74	0.38	0.86
Esopus at Kingston, N. Y.......(a)	2.3 to 7.4	16	0.08	0.68	0.36	0.80
Esopus at Kingston, N. Y.......(b)	5 to 8	8	0.11	0.73	0.37	0.85
Rondout at Rosendale, N. Y...(a)	4 to 8	5	0.08	0.68	0.35	0.82
Rondout at Rosendale, N. Y...(b)	5 to 7	8	0.13	0.21	0.35	0.86
Connecticut at Orford, N. H...(c)	2.5 to 7.7	18	0.11	0.69	0.35	0.85
Mean.....................	....	......	0.11	0.71	0.36	0.84

Notes: a. By F. H. Tillinghast.
b. By W. W. Schlecht.
c. By C. A. Holden.

CHAPTER XI.

THE MEASUREMENT OF STREAM FLOW.

112. **Necessity for Stream Flow Measurements.**—In order to ascertain the value of a stream for water power purposes, it is necessary to determine the amount and variations in its continuous flow either by comparison with the flow of other streams or by the direct observation of the flow of the stream itself. As has already been shown, the latter method is by far the most satisfactory as the determination of the actual flow of the stream eliminates all errors of comparison, and the necessity for any allowances or modifications on account of differences in geological, geographical, topographical or meteorological conditions on the drainage area.

The Hydrographic Division of the United States Geological Survey has undertaken the gauging of a large number of streams in the United States and has established numerous gauging stations at which observations have been made for a number of years. This data, references to which are given in the list of literature appended to Chapter IX, is of great value for comparative purposes. It is seldom, however, that, when a stream is to be investigated for water power purposes, flow data, at the particular point under consideration, is available. One of the first duties of the engineer, therefore, usually consists in making measurements of the stream flow and establishing stations at which the daily flow can be observed and recorded.

The methods in use by the United States Geological Survey are the result of much study and investigation and probably represent the most practical methods for making such observations with a fair degree of accuracy. Many of the methods and suggestions in this chapter are based on the methods and conclusions of the Survey as modified by the experience and practice of the writer.*

* These methods are described in detail in Water Supply and Irrigation Papers No 94, entitled, "Hydrographic Manual of the United States Geological Survey," and No. 95, entitled "Accuracy of Stream Measurements." See also "River Discharge" by J. C. Hoyt and N. C. Grover,—John Wiley and Sons, 1907.

113. Methods for the Estimate or Determination of Flow in Open Channels.—There are three general methods of estimating or determining the flow of water in streams with open channels.

First—By the measurement of the cross section and slope and the calculation of flow by Chezy's formula, together with Kutter's or Bazin's formulas for estimating the values of the coefficient.

Second—By means of weirs or dams of such form that the coefficient of discharge is known, and

Third—By the measurement of the cross section area and the velocity of current passing through the same.

The method which should be selected for any particular location depends on the physical conditions of the problem, the degree of accuracy required, the expense which may be permissible and the length of time during which the record is to be continued.

114. Estimates from Cross-section and Slope.—Chezy's formula,

$$v = c\sqrt{rs}$$

together with the formulas of Kutter and Bazin, for the determination of the flow of streams, has already been discussed in Chapters III and X. Much information is now available in regard to the value of the coefficient c, but this value varies greatly in different streams, in accordance with the conditions of the beds, and in the same stream under various conditions of flow. The results obtained from the application of these formulas are therefore necessarily very approximate. The method, however, is of considerable value in estimating the flood discharge of streams and in obtaining an approximate knowledge of flow under other conditions where other methods are not available or are difficult of application.

In using this method two or more cross sections of the stream should be measured on reaches of the river where the cross section and other conditions are fairly uniform and can be readily determined and at a time when the flow is steady. It is also important that the stream in which the flow is to be estimated shall be comparable in cross-section, depth, and other conditions, on which the value of the coefficient c depends, with other streams on which the value of c has been determined.

115. Weir Measurement.—Where dams are so located that they can be utilized for weir measurements, and are so constructed that such measurements are reasonably accurate, or where suitable weirs can be constructed from which such measurements can be made, such dams and weirs afford the best practicable method for measurements of the flow of a stream. In order to assure accurate results in weir measurements, the following conditions must be fulfilled:

First—The dam or weir must have sufficient height so that back water will not interfere with the free fall over the same; otherwise the dam will be available for purposes of measurement only during stages when no such interference exists.

Second—The dam or weir body must be so constructed that no leak of appreciable size will occur during the time when it is utilized for measuring purposes.

Third—The abutments of the dam or sides of the weir must be so constructed as to confine the flow over the dam at all stages: otherwise the weir will be useless for measurements during flood conditions.

Fourth—the crest of the weir must be level and must be kept free from obstructions caused by floating logs or ice.

Fifth—The crest of the dam or weir must be of a type for which coefficients for use in the ordinary weir formula have been determined. (See Chapter III.)

Sixth—If the dam has an adjustable crest, great care must be used to prevent leakage along such crest and to keep a complete and detailed record of the condition of the crest during the time of the observations.

Seventh—If water is diverted around the dam, which is usually the case when a dam is built for power purposes or for navigation, the diverted water must be measured or estimated and added to the amount passing over the dam. Such diverted water can sometimes be measured by a weir or current meter. When such water is used in water wheels, an accurate record of the gate opening of the wheels can be kept, from which the amount of water used in the wheels can be estimated if the wheel's discharge has been calibrated or if the wheel is of some well known type.* The conditions for the accurate determination of weir discharge should be such as not to involve the use of low heads of less than 6″ over broad crested dams.

Measurements by means of a weir or dam have the general advantage of continuity of record during the periods of ice and flood and the disadvantage of uncertainty of the coefficient to be used in the weir formula, of complication by the diversion of water around the dam, and the interference of flow by the occasional lodgement of material, or of injury to the crest.

* See Water Supply and Irrigation Paper No. 180,—Turbine Water Wheel Tests and Power Tables—by R. E. Horton.

116. Measurement of Flow by the Determination of Velocity.— The discharge of a stream, or the quantity of water flowing past a certain section of the stream in a given time, is the product of two factors: first, the area of the cross section; and second, the mean velocity of flow through said section.

If the flow in the cross-section of the stream were uniform the measurement of the flow would be a simple matter. A surface float, timed between given stations, or a current meter placed at any point in the cross-section, would then indicate the average velocity. Such conditions, however, never obtain. It is therefore necessary to ascertain the mean velocity of flow in the section which is a much more difficult matter.

Two methods of measuring the velocity of a stream are in use: first, by the use of a current meter, and second, by the use of floats. Each of these methods has advantages peculiar to itself, which must be known and appreciated in order that intelligent measurements may be made.

117. The Use of the Current Meter.—The current meter (Fig. 119) is an instrument designed to revolve freely with the current so that by determining the number of its revolutions the velocity of

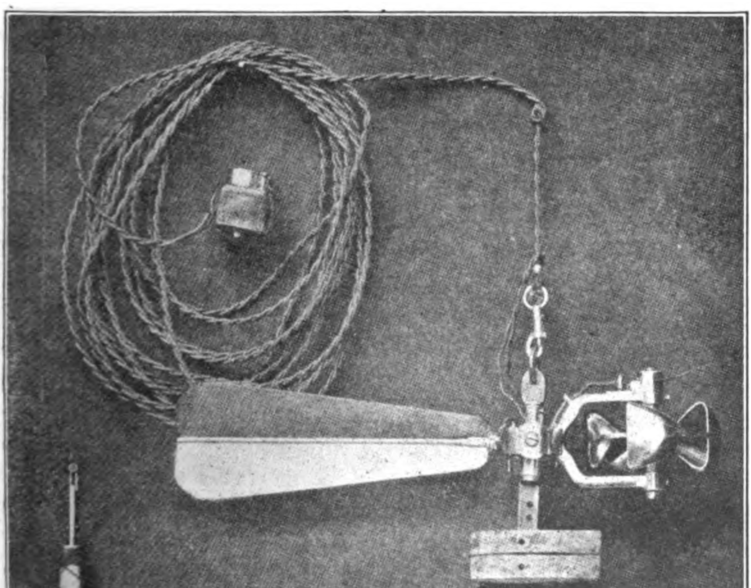

Fig. 119.—Price Electric Current Meter with Buzzer.

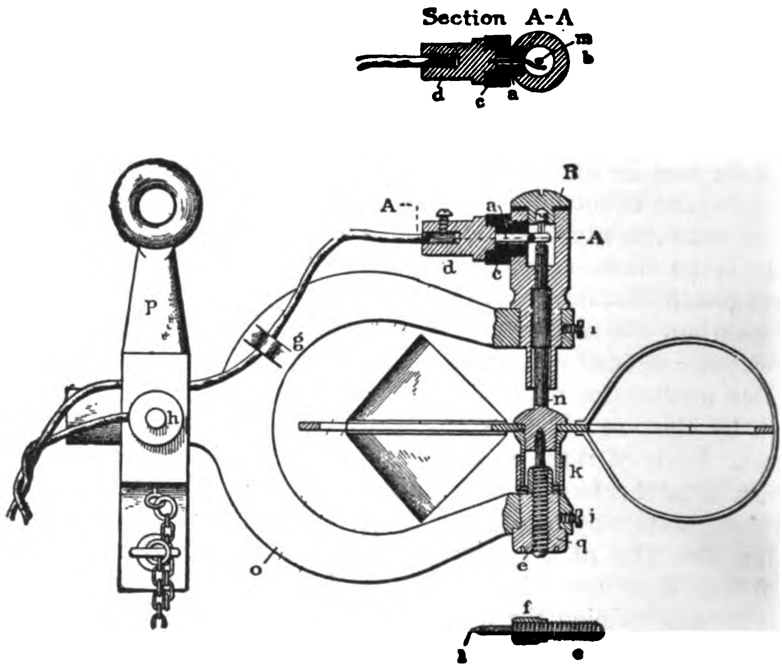

Fig. 120.—Cross Section of small Price Electric Current Meter, Showing details.*

the current will be known. A well made current meter carefully maintained and frequently rated is reasonably accurate when properly used under conditions to which it can be applied. As the friction of operation is rarely constant, the relation of current velocities to number of revolutions is not always strictly proportional and it is necessary to determine the relation between the revolutions of the meter and the corresponding velocity of water. This is accomplished by rating the meter, which is usually done by passing it through still water at known velocities and noting the results. It is assumed that the same relation will exist between the revolutions of the meter and its longitudinal velocity through still water and between its revolution and the velocity of flowing water when this meter is held in a similar position in a stream. The meter should be rated under conditions as nearly similar as possible to those under which it was, or is to be, used. The meter when being rated is usually at-

*From W. S. & I. Paper No. 94 Hydrographic Manual, by E. C. Murphy. J. C. Hoyt and G. B. Hollister.

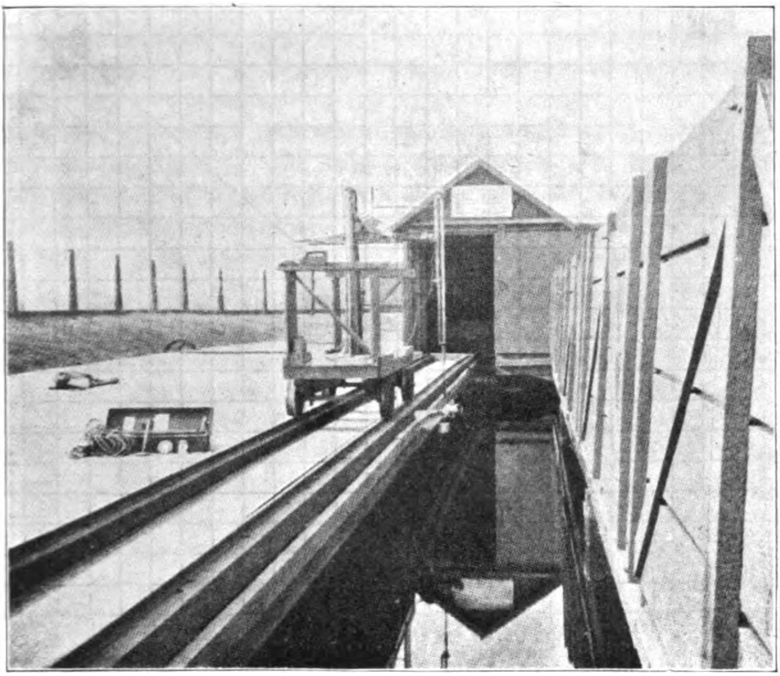

Fig. 121.—Current Meter Rating Station at Denver, Col.*

tached to some movable device (see Fig. 121) such as a carriage or boat which is propelled by hand or machinery at a known rate over a fixed distance. Observations of the revolutions of the meter at various rates of speed are noted and the relation is then established between the velocity of the meter and the revolutions of the meter wheel. This data may be platted upon cross-section paper or so arranged in tabular form that the corresponding velocity may be immediately ascertained when the revolutions of the meter are known. (See Fig. 122.) Experiments have shown that with velocities less than one-half of a foot per second little or no dependence can be placed upon the meter observations and that for velocities below one foot per second, the meter usually under registers. Where such low velocities obtain, float measurements are believed to be more accurate.

118. Current Meter Observations and Computation.—On account of the great variation in velocity at different points in the cross-sec-

*From Hydrographic Manual.

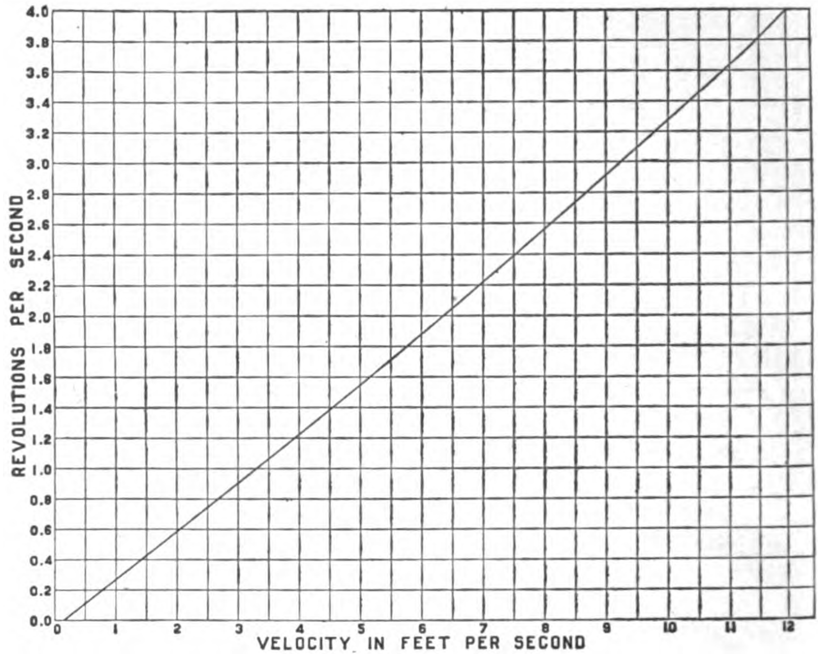

Fig. 122.—Current Meter Rating Curve.

tion, the flow through any unit of area may vary more or less from the flow through other similar areas. On this account it is desirable, in order to systematically survey the velocities in a cross-section, as well as for ease in calculation, to divide the cross-section area into parts, both horizontally and vertically, and determine the actual velocity of each of said parts. As a basis for the work, the cross-section of the stream should first be obtained by sounding. The vertical sections, chosen for the purpose of water observation, are usually five feet or more apart but the horizontal divisions are usually somewhat less as the variations in the vertical velocities are usually much greater than in horizontal velocities. The size of both horizontal and vertical division depends on the irregularity of the distribution of velocity in the cross-section as well as on the accuracy required in the determination of flow. The greater the care used in the determination of the velocities in the unit areas and the greater the number of such sub-divisions of the cross-section, the more accurate will be the work.

The meter readings may be made in one of four ways:

First—By determining the velocity at frequent, definite intervals of depth and then ascertaining the point and amount of average velocity in each vertical section.

Second—By what is known as the integration method, which consists in lowering and raising the meter with uniform motion from the surface to the bottom of the vertical section and noting the average velocity determined by this method.

Third—By making a point measurement at the depth corresponding to the thread of mean velocity as determined in the first method.

Fourth—By determining the velocity at some other point of observation and deducing the mean velocity from the known relation of the point measured to the point of mean velocity. The last two methods can be safely used where the vertical velocity curve has been determined with sufficient accuracy, and are fairly approximate at other sections where the conditions are not of an unusual nature.

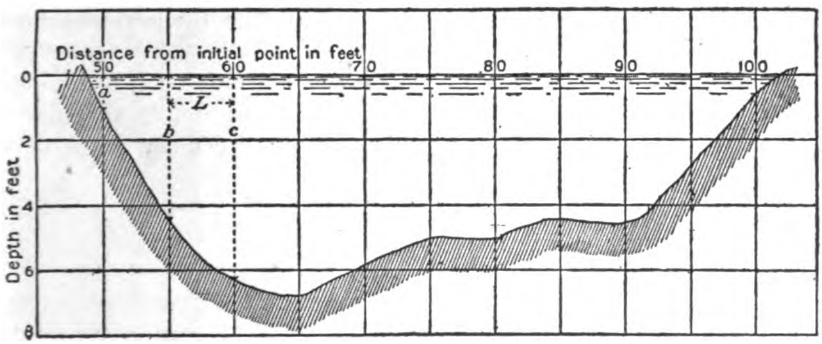

Fig. 123.—Cross-section of Saline River at Guaging Station near Salina, Kans.

"Fig. 123 shows the cross section of the Saline River near Salina, Kan., on September 30th, 1903, while the discharge measurements recorded in Table XXII were being made. The soundings were taken at each 5 feet of width from the initial point and the velocity was observed at 0.6 depths below the surface in each of these verticals.

The discharge through each 5-foot strip might be computed separately, but the computations are shortened by finding the discharge through each double strip at a time."

* From Water Supply and Irrigation Paper No. 94,—Hydrographic Manual by E. C. Murphy, J. C. Hoyt and G. B. Hollister. See page 46 et seq.

Let d'_m = mean depth for double strip;
V'_m = mean velocity for double strip;
a, b, c are three consecutive depths, L feet apart;
V_a V_b V_c are observed velocities in the verticals a, b, c
L = the width of a single strip;
Q′ = the discharge through double strip.

"The mean depth and the mean velocity for the double strip of width 10 feet are found from the formula:

$$(1) \qquad d'_m = \frac{a + 4b + c}{6}$$

$$(2) \qquad V'_m = \frac{V_a + 4V_b + V_c}{6}.$$

The discharge through the double strip is.

$$(3) \qquad Q' = d'_m\ V'_m\ 2L = \left(\frac{a + 4b + c}{6}\,2L\right) \left(\frac{V_a + 4V_b + V_c}{6}\right)$$

Formulas (1) and (2) are based on the assumption that the stream bed is a series of parabolic arcs, also that the horizontal velocity curves are parabolic arcs, both of which assumptions are approximately true at good current-meter stations.

In computing the discharge and the mean depth through a single strip near the stream bank or a pier the mean velocity is found from the formulas:

$$(4) \qquad V_m = \frac{V_0 + V_a}{2}$$

$$(5) \qquad d = \frac{a' + a}{2}$$

where either V_0 or V_a and a′ or a may be "0".

Velocity is computed to two places of decimals, mean depth, area, and discharge to one place of decimals for streams of ordinary size; for small streams with hard, smooth botton, where depth can be measured to hundredths foot, the mean depth and area should be computed to two places of decimals and the discharge to one place."

These observations can be taken in shallow streams by wading or from a cable car (see Fig. 124), boat or bridge as the circumstances and conditions permit. A rope or cable, marked into suitable divisions and stretched across the stream, offers the best means of locating the horizontal points at which observations in the vertical planes are to be made.

119. **Float Measurements.**—Where a single or only an occasional measurement of the flow of a stream is to be made, the use of floats

TABLE XXII

Gaging made September 30, 1903, by E. C. Murphy. Meter No. 388, on Saline River near Salina, State of Kansas.

[Gage height: Beginning 7.96 ft., ending 7.96 feet. River stationary. Total area, 233 sq. feet. Mean velocity, 0.82. Discharge, 190 second-feet.]

Dist. from initial point.	Obervations.				Velocity computatoins.				Computations of—		Area of sec.	Dis-cha'ge of section.	Remarks. (On condition of channel, wind, equipment, gage, boat, cable, methods, accuracy. Use cross-section pages in back of book for sketches.)
	Depth.	Depth of observation	Time in seconds.	Revolutions.	Total number of revolutions.	Revolutions per second.	Velocity per second.	Mean velocity per second.	Width.	Mean depth.			
49	0.9	......	......	....	......	......	0.0	0.20	1	0.6	0.6	0.1	Length of gage wire measured and found to be 35.47 feet.
50	1.1	0.7	50	7 and 8	15	......	0.40	......	......	......	......	......	Clear. No wind.
55	4.5	2.7	50	16 and 17	33	......	0.82	0.79	10	4.2	42.0	33.2	
60	6.3	3.8	50	21 and 22	43	......	1.06	......	......	......	......	......	
65	6.8	4.1	50	19 and 18	37	......	0.92	0.95	10	6.6	66.0	62.7	
70	5.9	3.5	50	19 and 20	39	......	0.97	......	......	......	......	......	
75	5.0	3.0	50	23 and 24	47	......	1.15	1.11	10	5.2	52.0	57.7	
80	5.0	3.0	50	22 and 23	45	......	1.10	......	......	......	......	......	
85	4.4	2.6	50	14 and 15	29	......	0.73	0.73	10	4.5	45.0	32.9	
90	4.5	2.7	50	6 and 6	12	......	0.33	......	......	......	......	......	
95	2.8	1.7	50	0 and 0	0	......	0.00	0.17	5	3.6	18.0	3.1	
100	0.6	0.4	50	0 and 0	0	......	0.00	0.00	7	1.3	9.1	0.0	
102	0.0	......	......	....	......	......	......	......	......	......	......	......	
......	......	......	......	....	......	......	......	......	......	......	232.7	189.7	

Computed by E. C. Murphy. Checked by E. C. Murphy.

is believed to be preferable, as under such conditions the calibration of the current meter and the exercise of necessary skill in its use are not apt to receive proper attention. Under such circumstances, therefore, float measurements are believed to be more accurate.

In the use of floats the writer usually selects round soft wood one to two inches in diameter and in various lengths, varying by about 6″. These are weighted at the lower end, usually by attaching pieces of lead pipe so that they will float with only about one to three inches of the rod exposed. To the exposed end is usually attached small red or white streamers so that they may be readily seen and yet not be seriously affected by wind.

A point for the gauging is selected where the stream is fairly straight and uniform in section, and ropes, wires, or cables are

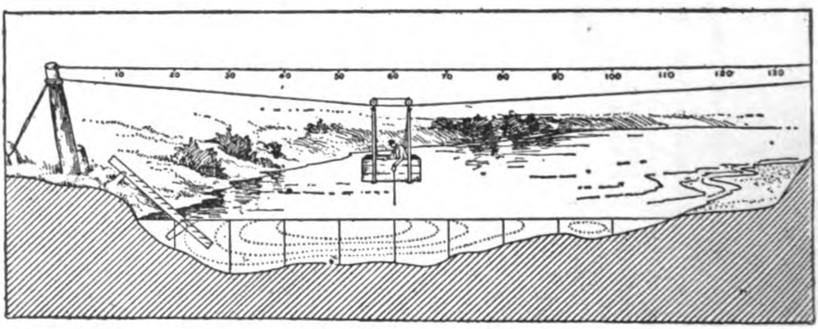

Fig. 124.—Cable Station, Car Guage, etc.

stretched tightly across the stream, parallel to each other and 25, 50 or 100 feet apart, as the location and velocity of the stream seem to demand. The ropes or wires should be tagged at intervals of 5, 10 or 25 feet, as the conditions seem to warrant, beginning at zero on the straight bank.

In starting the work a float is selected that will reach as near the bottom as possible without touching and should be about .9 depth. The float is started 5 to 10 feet above the upper line and so placed that it will pass as nearly as possible under one of the tags. The point at which it actually passes under the line is noted and recorded, also the point and time at which it passes the lower line. If the float should touch the bottom or a snag in its passage, the next shorter length should be used until the float passes both lines freely. Floats should be run at frequent intervals across the stream usually at each of the tagged stations.

Extensive experiments were made by Francis at Lowell, Mass., in 1852 to determine the accuracy of rod float measurements.*

He found that discharge measurements based on the determination of velocities by floats were nearly always large as compared with measurements by a standard weir. This was due to the fact that the rod, on account of not reaching the bottom, was not affected by the low velocity near the stream bed and hence indicated too great a velocity. He found that the effect could be corrected by multiplying the discharge as obtained by the floats by a coefficient as follows:

(6) $Q = CQ_1$ in which

Q = actual discharge

Q_1 = discharge as determined by floats.

C = coefficient $= 1 - 0.116\,(\sqrt{D} - 0.1)$ and

D = ratio $\dfrac{\text{distance of bottom of float from bottom of stream}}{\text{depth of stream.}}$

It will be observed that this coefficient C is always less than unity except where D is less than 0.01 which condition could not be possible in any natural stream.

The Francis experiments were made in a channel of rectangular cross section and floats of uniform length were used. In a natural stream the depth will vary at different points in the cross section and floats of different lengths must be used. In such cases D will vary widely for the various floats used and to apply the correction, the velocity as determined by each float should be reduced by its particular consatnt, C.

Experiments made at the Cornell Hydraulic Laboratory in 1900 by Kuichling, Williams, Murphy and Boright confirmed Francis' conclusion that rod float measurements are too large, only two out of thirty being smaller than measurements made by a standard weir. No attempt was made, however, to verify Francis' formula for the correction of such observations.*

In calculating the discharge from these measurements the average cross-section, in square feet, of each division is calculated and multiplied by the average velocity for the same in feet per second and the product will represent the discharge in cubic feet per second of the section represented by that float and the sum of the sections of all the floats will give the total discharge of the stream.

* See "Lowell Hydraulic Experiments" by James B. Francis, pp. 146–208.

* See W. S. & I. Paper No. 95, Accuracy of Stream Measurements. p. 54.

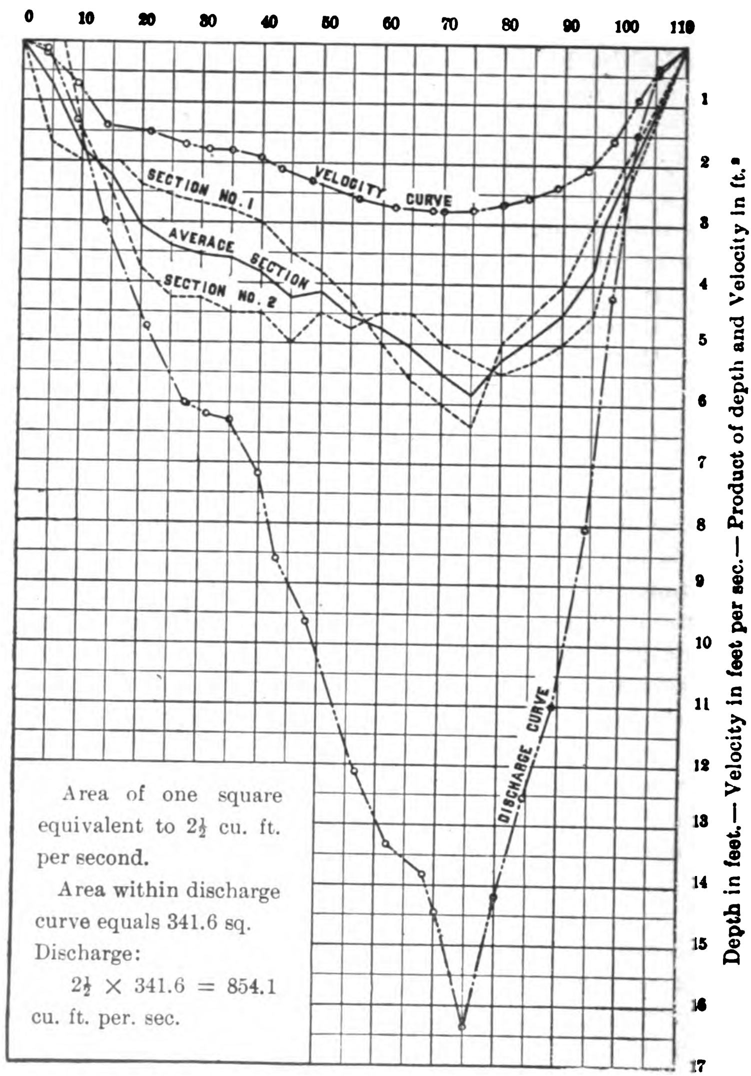

Fig. 125.—Graphic Determination of Stream Flow From Measurements.

It is frequently desirable to calculate the discharge graphically, which may be done as shown by Fig. 125. This is done by plotting the two sections at the tag lines over each other and drawing in an average section between them. It is frequently desirable to draw in the floats in their true length and average position so that it may be seen at once how well the section was covered by the floats.

Under each float is laid off the velocity as determined by the same, to a selected scale, and a mean velocity curve is drawn through these points. By multiplying the ordinate of the velocity curve by the ordinates of the mean section, a quantity is obtained on the discharge curve which, when fully constructed, gives a discharge polygon, the area of which represents at the correct scale the discharge in cubic feet per second of the stream.

120. The Application of Stream Gaugings.—A single measurement of stream flow is of comparatively little value as a basis for estimating the continuous character of the flow of the stream, as will be seen by examination of any of the hydrographs previously shown. The flow of a stream, while it may appear to the casual observer uniform, is actually subject to many and violent fluctuations and the flow may vary several hundred per cent. from minimum to maximum within a few days.

It has already been pointed out that in order to study the flow of a stream intelligently it is necessary to know the variations in flow that take place from day to day for a long term of years during which the effect of the extreme of all of the factors controlling stream flow may have made themselves manifest.

The actual measurement of the flow of a stream by current meter or floats is usually accomplished with considerable difficulty, and it would be practically impossible to repeat such measurements daily for the length of time for which records are desired. It has already been pointed out that under many conditions it is possible to establish a discharge or rating curve which will show the relation of the height of the water surface to the flow through orifices over weirs or through channels of various forms. In the establishment of such relation it is assumed that the raising of the water surface to a given height is always accompanied by the same flow of water through the section. In order to assure accuracy in the observations based on such a rating curve, sections must be selected where the conditions assumed are correct. Such stations should be selected, where possible, on a fairly long uniform reach of the stream

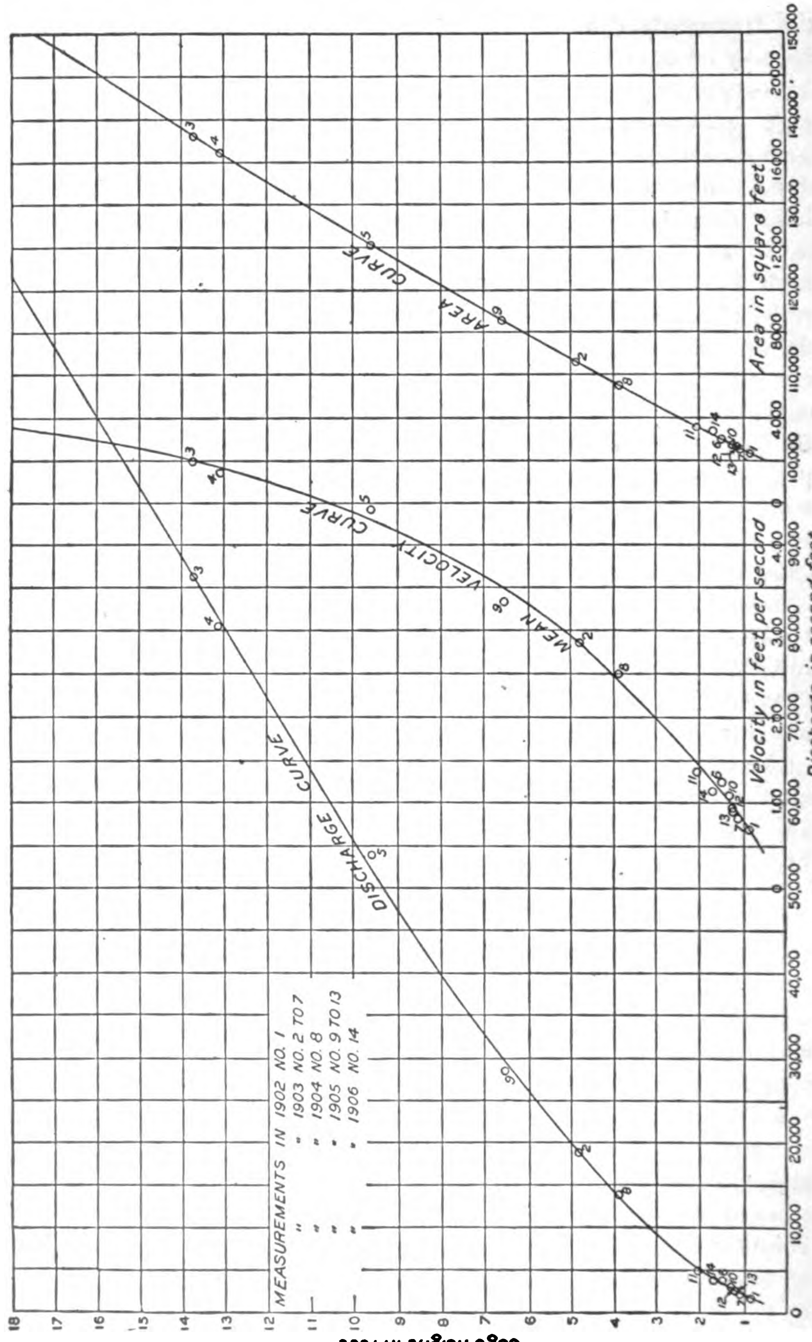

Fig. 126.—Discharge, Velocity and Area Curves for the Potomac River at Point of Rocks, Md. (U. S. G. S.)

and beyond the influences of the back water from large rivers or dams.

After gaugings of the stream have been made under a considerable range of conditions and a rating curve is established therefrom, it is not necessary thereafter to measure the daily flow but only to note daily the gauge height. It has been determined by many observations that under constant conditions a fixed relationship exists between gauge height and the discharge of a stream, subject to the errors due to variable flow as described in Chapter X. If the section and other conditions of the stream flow remain unchanged, the rating curve will remain constant and hence the daily gauge height can be quickly read and recorded and will give at once, by reference to the rating curve or table, the quantity of water flowing in the stream at all times.

From the soundings and levels made to determine the cross section, an area curve can be constructed showing the variation of area with gauge height. The float or current meter observations furnish the necessary data for the construction of a curve of mean velocities. The product of the area and mean velocity, as shown by these two curves, for any given gauge height, must equal the discharge and must equal the reading of the discharge curve for the same gauge height. The construction of these curves, and a consideration of their properties, furnishes a check on the construction of the discharge curve and aids materially in correcting any apparent irregularities therein.*

Fig. 126 shows the discharge, mean velocity and area curves for the Potomac River at Point of Rocks, Md.

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CHAPTER XII.

WATER WHEELS.

121. Classification of Water Wheels.—Water wheels include most of the important hydraulic motors that are adaptable to large hydraulic developments. They may be divided into three classes, viz:

First—Gravity wheels.

Second—Reaction wheels.

Third—Impulse wheels.

In gravity wheels the energy of the water is exerted by its weight acting through a distance equal to the head.

In both reaction and impulse wheels the potential energy due to the weight of the water under the available head is first converted into kinetic energy. This kinetic energy does work in the reaction wheel through the reactive pressure of the issuing streams upon the movable buckets from which they issue.

In the impulse wheel the nozzles or guides are stationary and the energy of the issuing streams is utilized by the impulsive force which they exert in impinging against movable surfaces or buckets.

Figs. 127, 128 and 129, which illustrate the various types of wheels included in the above classes, are adapted, with many modifications from Reuleaux's "Constructor." *

122. Gravity Wheels.—Fig. 127 shows the various types of gravity water wheels or those wheels that are driven by the weight of the water. At moderate velocity, these motors are practically operated by gravity only, although under some conditions the impulse due to the velocity of the entering water may have an appreciable effect. In Fig. 127, A is an undershot water wheel; B is a half-breast wheel (see also Figs. 3 and 4), and C is a high breast wheel. D is an overshot wheel. In C and D the buckets should be so designed as to retain the water until they reach the lowest point in the revolution of the wheel. E in this Figure illustrates Dup-

* "The Constructor." F. Reuleaux—trans. by H. H. Suplee, Philadelphia. Pa., 1893.

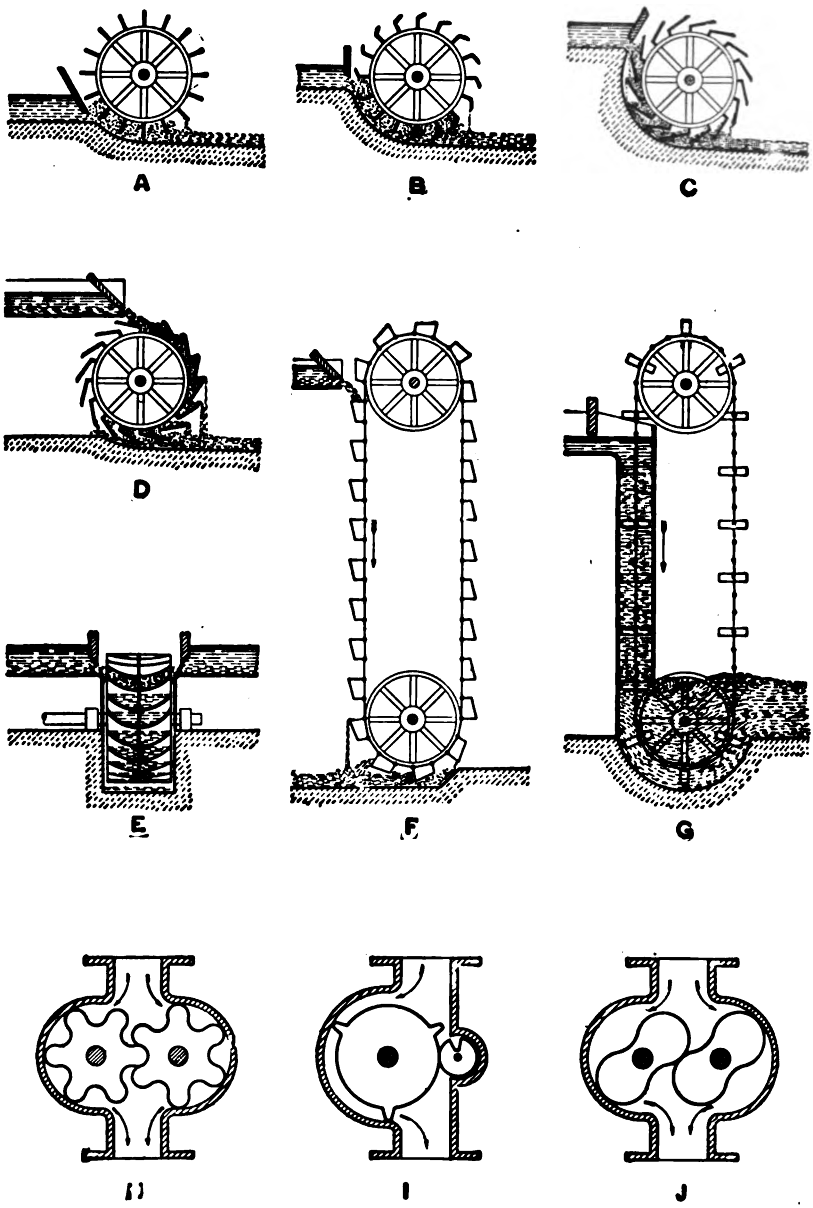

Fig. 127.—Diagram of Gravity Wheels.

pinger's side-fed wheel. F illustrates an endless chain of buckets which is essentially the same in principle as the overshot wheel. G is a similar arrangement using discs running with as small a clearance as possible in a vertical tube. When the water acts only by gravity, the wheels represented by A to E, inclusive, are only practicable when the wheel can be made as large or larger in diameter than the fall of the water. Where small diameters must be used, the arrangements shown in F and G are available. Very small wheels acting under high pressures may be employed by making use of the so-called chamber wheels, illustrated in H, I and J.

123. **Reaction Wheels.**—The wheels illustrated by the diagrams in Fig. 128 are of the second class or reaction wheels. Diagram A illustrates Barker's Mill of the form known as the Scotch turbine illustrated also by Fig. 8. This form of turbine is known in Germany as the Segner wheel. The water enters the vertical axis and discharges through the curve arms. B represents a screw turbine which is entirely filled with water. C shows a Girard current turbine which has a horizontal axis and is only partially submerged. D is Cadiat's turbine with central delivery. It resembles the Fourneyron turbine except that there are no guides to direct the flow into the buckets. E is Thompson's turbine with circumferential delivery and horizontal axis. The discharge from this turbine is about the axis at both sides.

In diagrams A, B, C, D and E the column of water is received as a whole and enters the wheel undivided. The remainder of the forms illustrated in Fig. 128 show wheels in which the flow is divided into a number of separate streams by guides interposed in the streams before the water enters the wheel. Diagram F illustrates the Fourneyron turbine which acts with central delivery. The guide vanes are fixed and the discharge of the water is at the circumference of the wheel. The ordinary vertical form of the Fourneyron turbine is illustrated in Fig. 128. Diagram G, also in Fig. 128, is a modification of the Fourneyron turbine in which the water is being delivered upward from below. This form is sometimes called the Nagel's turbine. Diagram H is the Jonval or Henschel turbine. (See also Fig. 135.) The guide vanes in this turbine are above the wheel which is entirely filled by the water column. Diagram J is the Francis turbine in practically its original form. (See also Fig. 14.) Diagram I illustrates the present American form or modification of the original Francis turbine. K is the Schiele turbine, a double wheel with circumferential delivery and axially directed discharge.

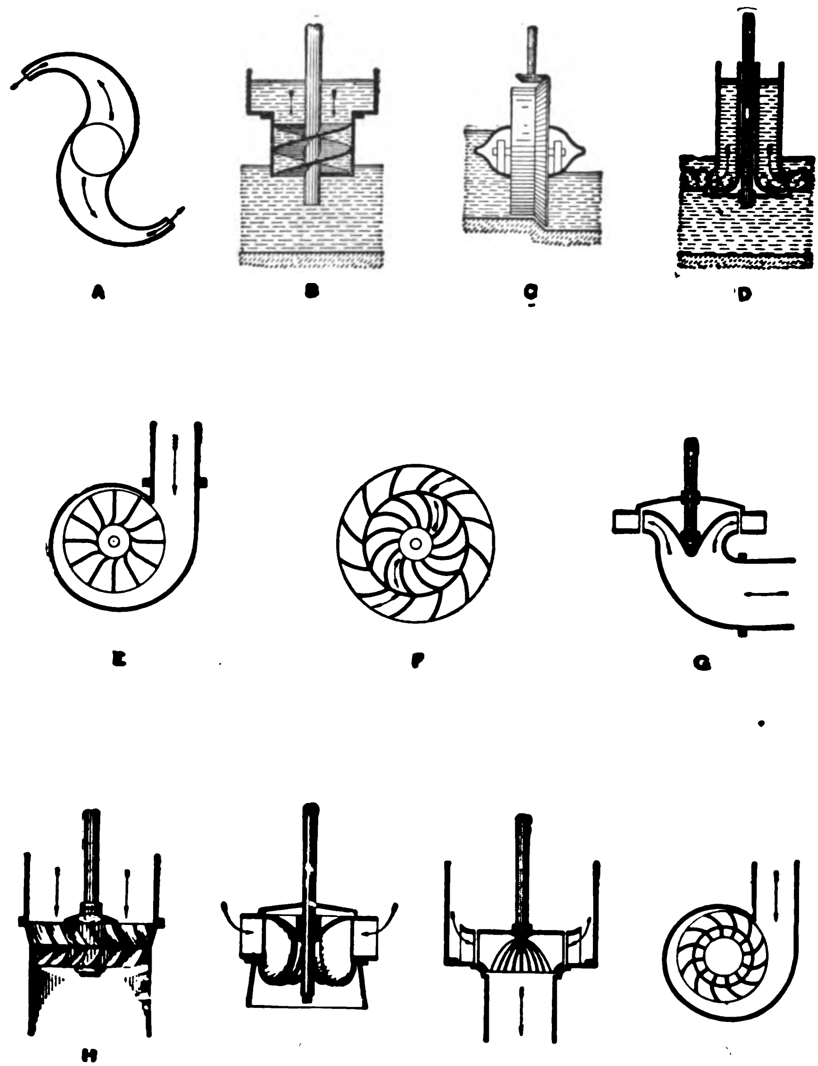

Fig. 128.—Diagrams of Reaction Wheels.

In forms H, I, J and K, a draft tube may be used below the wheel to utilize any portion of the fall which occurs below the level of the bottom of the wheel.

In all reaction turbines, the water acts simultaneously through a number of passages around the entire circumference of the wheel. In the impulse or action turbine, the water may be applied to all of the buckets simultaneously or to only a portion of the circumference at a time.

124. **Impulse Wheels.**—The wheels illustrated in Fig. 129 are the third class of wheels which are driven by the impulse due to the weight of water acting through its velocity. Of these wheels, A is the current wheel or common paddle wheel. The paddles are straight and either radial or slightly inclined toward the current, as in the illustration. (See also Figs. 1 and 2.)

Diagram B is Poncelet's wheel. (See also Fig. 5.) The buckets run in a grooved channel and are so curved that the water drives upward and then falls downward, thus giving a better contact.

Diagram C shows an externally driven tangent wheel. The buckets are similar to the Poncelet wheel but with a sharper curve inward. The discharge of the water is inward. D is an internally driven tangent wheel similar to the preceding but with an outward discharge.

E is the so-called hurdy-gurdy or tangential wheel. The water is delivered through a nozzle and the wheel is practically an externally driven tangent wheel of larger diameter and with a smaller number of buckets.

Diagrams F, G and H illustrate three types of impulse wheels with inclined delivery. (See also Figs. 6, 7, 9 and 10.) Diagram F shows a crude form of vertical wheel similar in form to the Indian wheel, Fig. 6. It is used on rapid mountain streams and is probably the original conception from which the turbine has been developed. Diagram G is the Borda turbine and consists of a series of spiral buckets in a barrel-shaped vessel. Diagram H is a Danaide turbine which has spiral buckets enclosed in a conical tube. This is an old form of wheel formerly used in France.

125. **Use of Water Wheels.**—Almost all water wheels in practical use are modifications of some of the above forms and by a study of these forms a wheel may be classified and a clearer understanding obtained of the principles of its operation. Many of the forms of wheels shown in Figs. 127, 128 and 129 are practically obsolete or are used only in minor plants or for special conditions

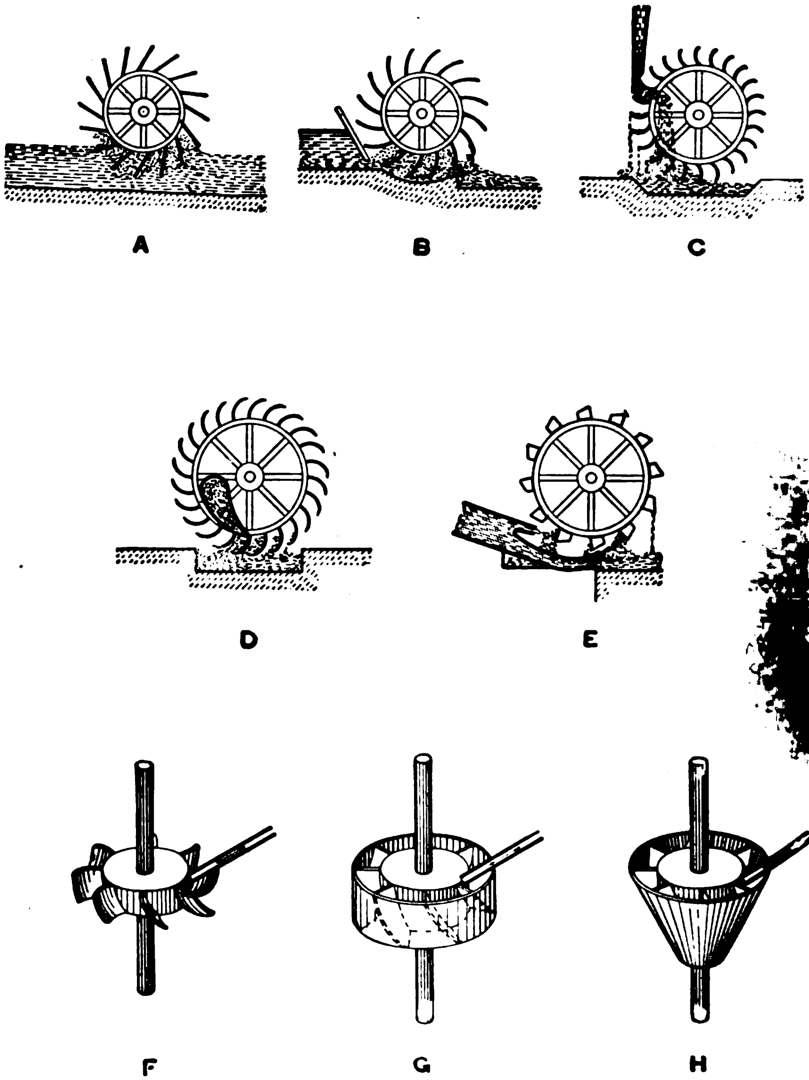

Fig. 129.—Diagrams of Impulse Wheels

that make them of only general interest in the study of water power.

While gravity wheels are still occasionally used their application is entirely to the smaller water power plants. In many cases the turbines purchased for such installations are of cheaper make, poorly designed, constructed and selected, and often improperly set and, consequently, inefficient. In such cases, and where the question of back water and the interference of ice is not important, the

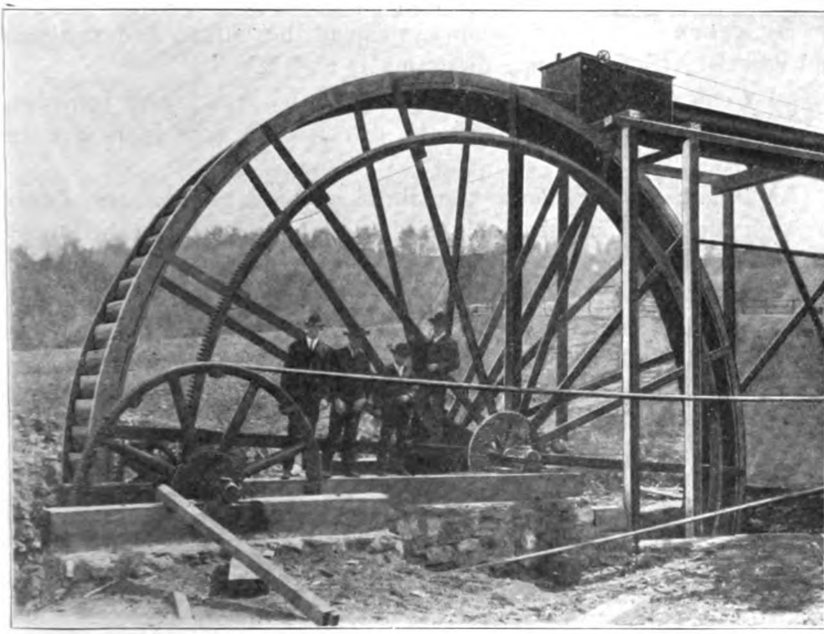

Fig. 130.—"Overshot" Water Wheel. Manufactured by Fitz Water Wheel Co.

gravity wheel may be more efficient and quite satisfactory. Well designed and well constructed gravity wheels are said to give efficiencies of 85 per cent. and above. (See Frontispiece and Fig. 130). With such plants the engineer has usually little to do and consequently they will not be further considered here. The types of wheels now most largely used for moderate and large water power developments are the reaction and impulse turbines.

126. **Classification of Turbines.**—All moder turbines consist of a wheel to which buckets are attached and which is arranged to revolved in a fixed case having attached to it a nozzle, guide or

series of guides. The guide passages or nozzles direct the water at a suitable angle onto the buckets of the wheel. The revolving wheel contains curved buckets or passages whose functions are to receive the water, utilize its energy and discharge or waste it as nearly devoid of energy as possible.

Turbines may be classified in various ways:

First.—In accordance with the action of the water on the same.

(A) *Reaction or pressure turbines,* such as the Fourneyron, Jonval, Francis, etc. (See Fig. 128, G, H, I and J.)

(B) *Action or impulse turbines,* such as the Girard and tangential wheels. (See Fig. 129, diagrams D and E.)

(C) *Limit turbines,* which may act either by reaction or impulse.

Second.—In accordance with the direction of flow in reference to the wheel.

(A) *Radial flow turbines.* In these turbines the water flows through the wheel in a radial direction. These may be subdivided into—

(a) *Outward radial flow turbines,* such as the Fourneyron and Cadiat. (See Fig. 128, diagrams F and D.)

(b) *Inward radial flow turbines,* or wheels in which the water flows inward in a radial direction such as the Francis and Scheile turbines. (See Fig. 128, J and K.)

(B) *Axial flow turbines* in which the general direction of the water is parallel to the axis of the wheel such as the Jonval and Girard wheels of similar design. (See Fig. 128, H.)

(C) *Mixed flow turbines,* or turbines in which the flow is partially radial and partially axial as in turbines of the American type. (See Fig. 128, diagram I; also Figs. 143 to 158 inclusive).

Third.—In accordance with the position of the wheel shaft.

(A) *Vertical* (See Figs. 132, 134, 135, 151, etc.).

(B) *Horizontal* (See Figs. 140, 152.)

Fourth.—In accordance with the arrangement of nozzles or guides.

(A) *Complete turbines* with guides surrounding the entire wheel.

(B) *Partial turbines* with guides partially surrounding the wheel in one or more groups.

The re-action turbine is a turbine with restricted discharge which acts through the reactive pressure of the water. Under some conditions the energy of the water may be exerted, at least in part, by its impact or momentum. The impulse turbine acts principally through the momentum of the moving mass of water although, when the current reverses, some reactive pressure may be recognized. The limit turbine may act entirely as a reaction or as an impulse turbine according to the conditions under which it operates.

127. **Condition of Operation.**—These wheels operate under the following conditions:

REACTION OR PRESSURE TURBINES.

Guides complete.

Buckets with restricted outlets.
Buckets or wheel passages completely filled.
Energy most largely developed through reactive pressure.
Discharge usually below tail water or into a draft tube.

ACTION OR IMPULSE TURBINES.

Guides partial or complete.

Buckets with outlets free and unrestricted.
Wheel passage never filled.
Energy entirely due to velocity.
Discharge must be above tail water.
No draft tube possible, except with special arrangement which will prevent contact of tail water with wheels.

LIMIT TURBINES.

(A) Buckets so designed that the discharge is unrestricted when above tail water.

Buckets in this case are just filled. Act without reactive effect.

Discharge above tail water.

(B) If tail water rises to buckets, the discharge is restricted and reaction results.

In this case the full bucket admits reaction and discharge may be below tail water.

128. **Relative Advantage of Reaction and Impulse Turbines.**—The reaction wheel is better adapted for low and moderate heads, especially when the height of the tail water varies and where the amplitude of such variation is a considerable percentage of the total head. Such a wheel, which is designed to operate with the buckets filled, can be set low enough to utilize the entire head at

all times and will operate efficiently when fully submerged. The reaction wheel can therefore be set to utilize the full head at time of low tail water and when the quantity of flow is limited. For low head developments this is an important factor. The impulse turbine, on the other hand, must have a free discharge and must therefore be set far enough above the tail water to be free from back water if it is to be operated at such times.

Another difference between the reaction and the impulse turbine is the higher speed with which the former operates. This is often a distinct advantage, for direct connection with high speed machinery, and with low and moderate heads. On the other hand, with high heads the slower speed of the impulse wheels is frequently of great advantage, especially in the form of the tangential wheel when the diameter can be greatly increased and very high heads utilized with moderate revolutions. In such cases the height of the back water is usually but a small percentage of the total head, and the loss due to the higher position of the wheel is comparatively small.

The speed of a wheel for efficient service is a function of the ratio of the peripheral velocity of the wheel to the spouting velocity of water under the working head. This ratio will vary from .65 to .95 in reaction turbines, according to the design of the wheel. In impulse turbines this ratio varies from .40 to .50.

129. Relative Turbine Efficiencies.—The impulse turbine has the further advantage of greater efficiency under part gate,—that is, at less than its full capacity. When, as is usually the case, a wheel must operate under a variable load it becomes necessary to reduce the discharge of the wheel in order to maintain a constant speed with the reduced power required. (See Fig. 131). This is accomplished by a reduction in the gate opening which commonly greatly affects the economy of operation.

The comparative efficiencies of various types of the turbines are shown in Fig. 131. The maximum efficiency of turbines when operated at the most satisfactory speed and gate will be about the same for every type, if the wheel is properly designed and constructed and the conditions of operation are suitable for the type used. This maximum efficiency may vary from 75 to 85 per cent., or even between wider limits, but, with suitable conditions, should not be less than 80 per cent. In order to make the curves on the diagram truly comparative, the percentage of maximum efficiency

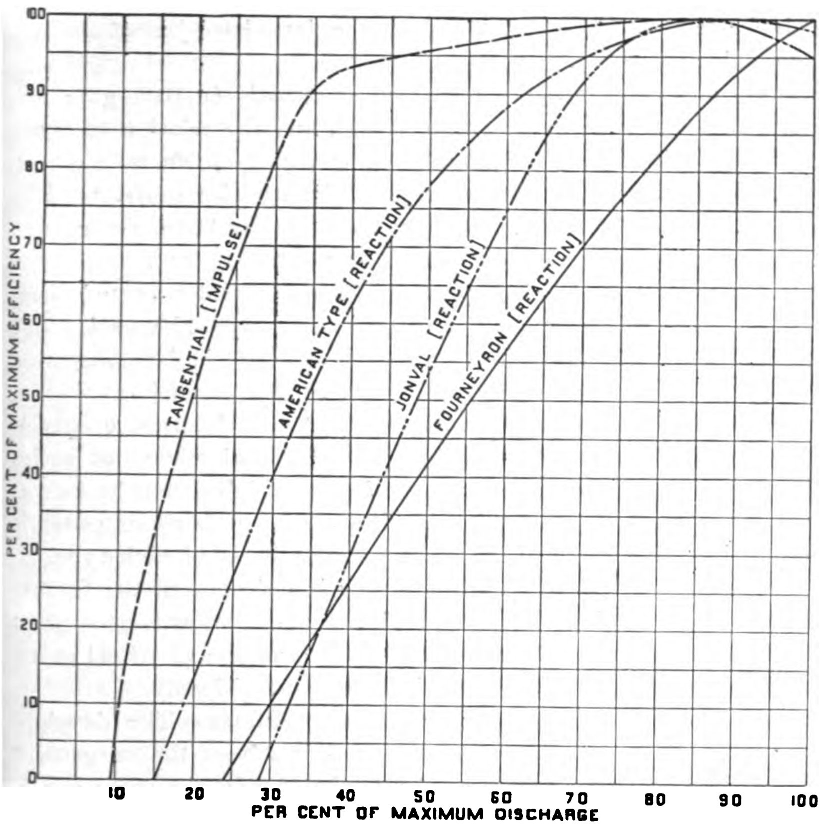

Fig. 131.—Comparative Efficiencies of Various Types of Turbines.

and of maximum discharge are plotted instead of the actual efficiencies and actual discharge.

The Fourneyron turbine usually shows very poor efficiencies at part gate as shown in Fig. 131. The curve for this turbine is drawn from Francis' test of the Tremont (Fourneyron) turbine (see Fig. 132, also Table LXI) and is substantiated by efficiency curves shown by various tests by James Emerson.*

The Jonval turbines usually show better part gate efficiencies than the Fourneyron but are not as efficient, under such conditions, as turbines of the inward flow or Francis type. The Jonval curve, shown in Fig. 131, is plotted from the test made in 1884 at the

* See "Hydrodynamics" by James Emerson.

Holyoke testing flume * of a 30-inch regular Chase-Jonval turbine. (See Table LXXVI).

The American-Francis turbine varies greatly in part gate efficiency according to the details of design and the relation of speed and head under which it operates. The curve shown in Fig. 131, representing this type, is from the test of a wheel manufactured by J. & W. Jolly of Holyoke, Massachusetts, similar but not the same as that illustrated by the characteristic curve Fig. 249.

The impulse wheels when properly designed and operated show a higher part gate efficiency than any other type of wheel. The curve shown in Fig. 131 is from a test of a 12″ Doble tangential wheel in the laboratory of the University of Wisconsin.†

As already indicated, the design of the wheel has a great influence on its efficiency at part gate. Individual wheels or series of wheels of any type may therefore depart widely from the curves above shown, which are intended only to show as fairly as possible the usual results obtained from well made wheels of each type.

It should be noted also that efficiency is only one of the factors influencing the choice of a wheel and that many other factors must be weighed and carefully considered before a type of wheel is selected as the best for any particular set of conditions.

130. **Turbine Development in the United States.**—The development of the turbine in the United States has been the outgrowth of some seventy years of practical experience. In the early settlement of the country the great hydraulic resources afforded facilities for cheap power and numerous water powers were developed under low and moderate heads. These developments created a corresponding great demand for water wheels and stimulated invention and manufacturing in this line. American inventors have devised many different forms of wheels which were patented, constructed, tested and improved to meet the prevailing conditions. When a successful wheel was designed, it was duplicated in its original form and its proportions increased or diminished, to conform to the desired capacity. As wheels of greater capacity or of higher speed have been required, modifications have been made and improved systems have resulted.

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* See page 44 of 1897 catalogue of Chase Turbine Manufacturing Co.. Orange, Mass.

† From "Test of a 12″ Doble Tangential Water Wheel," an unpublished thesis by H. J. Hunt and F. M. Johnson.

The best American water wheel construction began with the Boyden-Fourneyron and Geylin-Jonval turbines of improved French design, but modern American practice began to assume its characteristic development with the construction of the Howd-Francis turbines, already described. Moderate changes in the form and arrangement of buckets and other details gave rise to the earlier forms of "Swain," "Leffel" and "American" wheels each of which consisted of an inward flow turbine modified from the earlier designs of Howd and of Francis as the experience of the inventor seemed to warrant. In all of these cases the wheels discharged inward and essentially in a radial direction and had to be built of sufficient diameter to provide an ample space for receiving the discharging waters. This necessitated slow speed wheels of comparatively low capacity (see Table I, page 13). In order to secure higher speed, the diameters of the wheels were reduced thus reducing the power. This reduction was, however, more than counterbalanced, in the later wheels, by an increase in the width of the bucket in an axial direction. It was found also that the capacity of the wheels could also be materially increased, with only small losses in efficiency, by decreasing the number of buckets. Wheels were gradually reduced in diameter and the buckets increased in breadth until, in many cases, they reached very nearly to the center of the wheel. This necessitated a downward discharge in the turbine and resulted in the prolongation of the buckets in an axial direction in many cases to almost double the width of the gate. From this development has resulted the construction of a series of wheels known as the "American turbines" having higher speed and greater power than has been reached in European practice.

The entire line of development has, until within the last fifteen years, been toward the increase of speed and power for low and moderate head conditions. It is only within this period that a considerable demand has been felt in this country for turbines having other characteristics and adapted for higher heads.

The American type of turbine, in its modern form is not designed or suitable for high heads its origin being the result of entirely different conditions. About 1890 came a demand for turbine wheels under comparatively high heads which manufacturers of wheels of the American type were therefore poorly equipped to meet. The first of such wheels supplied were therefore of European types, which apparently better suited such conditions. Recognizing, however, the importance of meeting such demands, the American manufacturer found that the wheels of essentially the original Francis type were well suited for this purpose. The narrow wheel and numerous buckets of the earlier types reduced the discharge of water, and, increasing the diameter, reduced the number of revolutions. Such types of wheels of high efficiency can now be obtained from the leading manufacturers in the United States, and, while many manufacturers still prefer to furnish simply their stock designs, which are only suited for the particular conditions for which they were designed, still, other manufacturers are prepared to furnish special wheels which are designed and built for the particular conditions under which they are to be used.

The systems of wheels offered by American manufacturers, which can be readily and quickly duplicated at a much less expense than would result from the design of special wheels for each particular customer, has resulted in the ability of American manufacturers to furnish water wheels of a fairly satisfactory grade and at a cost which would have been possible in no other way. In the United States the cost of labor has been comparatively high and special work is particularly expensive, much more so than in Europe where skilled mechanics receive a compensation for labor which is but a small fraction of that of their American competitors. Average American practice, at the present time, leaves undoubtedly much to be desired and considerable advance may be expected from the correction of designs, resulting from practical experience and by the application of scientific analysis.

131. The American Fourneyron Turbine.—As noted in Chapter I, one of the first reaction turbines developed in the United States was the Boyden wheel of the Fourneyron type.

In these wheels (see Fig. 132) the water entered from the center, guided by fixed curve guides, g, (Fig. 133) and discharged outward through the buckets, B. The use of these wheels gradually spread and they rapidly replaced many of the old overshot and breast wheels used up to that time, and soon became the foremost wheel in New England.

The manufacture of the Fourneyron turbine has, for common use, been discontinued on account of the competition of other cheaper wheels which were found to be more efficient at part gate

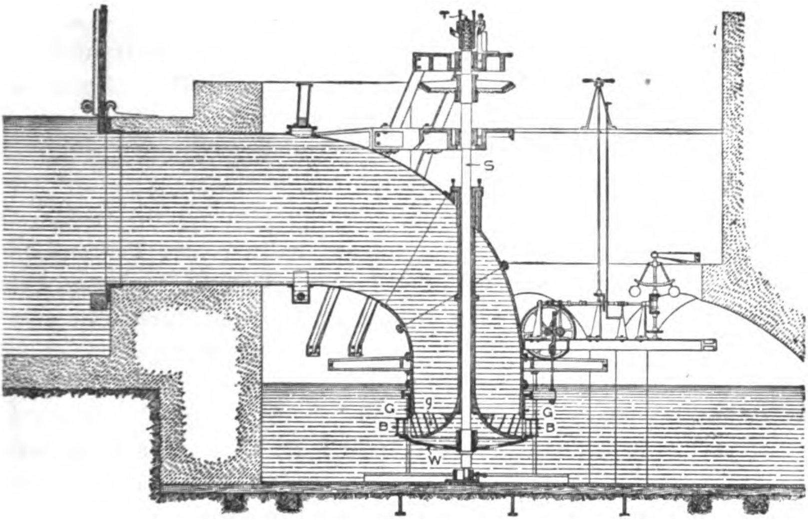


Fig. 132.—Tremont (Boyden-Fourneyron) Turbine (after Francis).

Fig. 133.—Guides and Buckets of Tremont (Boyden-Fourneyron) Turbine.

and more generally satisfactory under ordinary conditions of service.

The Fourneyron turbine, when well designed and constructed, is a turbine of high full gate efficiency. This wheel is adapted for high heads where a comparatively slow speed is desired,—and it is now frequently used for high grade and special work where its peculiarities seem best suited to such conditions.

One of the modern applications of the Fourneyron turbine is that in the power plant of The Niagara Falls Water Power Company. Fig. 134 shows vertical and horzontal sections of one of the double Fourneyron units used by this company in their first plant. These wheels discharge 430 cubic feet per second and make 250 revolutions per minute; at 75 per cent. efficiency each wheel will develop 5,000 horse power. The buckets of these wheels are divided vertically into three sections or stories in order to increase their part gate efficiencies. These wheels are of Swiss design by the firm of Faesch and Picard and were built by The I. P. Morris Company of Philadelphia. The wheels are vertical and connected by vertical shafts, each with one of the dynamos in the station above. The shaft is built of three-quarter inch steel, rolled into tubes 38 inches in diameter. At intervals the shafts pass through journal bearings, or guides, at which points the shafts are reduced to 11 inches in diameter and are solid. The speed gates of these wheels are plain cylindrical rims which throttle the discharges on the outside of the wheels and which, with the co-operation of the governor, keeps the speed constant within two per cent under ordinary conditions of operation. Another wheel of this type is that manufactured and installed at Trenton, Falls, N. Y., by the same firm. (See Fig. 311.)

132. The American Jonval Turbine.—The Jonval turbine, originally of French design, was introduced into this country about 1850 and became one of the most important forms of turbine of early American manufacture. In the tests of turbines at Philadelphia in 1859–60 (see page 360) a Jonval turbine developed the highest efficiency and the type was adopted by the city for use in the Fairmount Pumping Station. Like the Fourneyron turbine, these wheels, while highly efficient at full gate, have largely been superceded by other cheaper and more efficient part gate types,—except for special conditions.

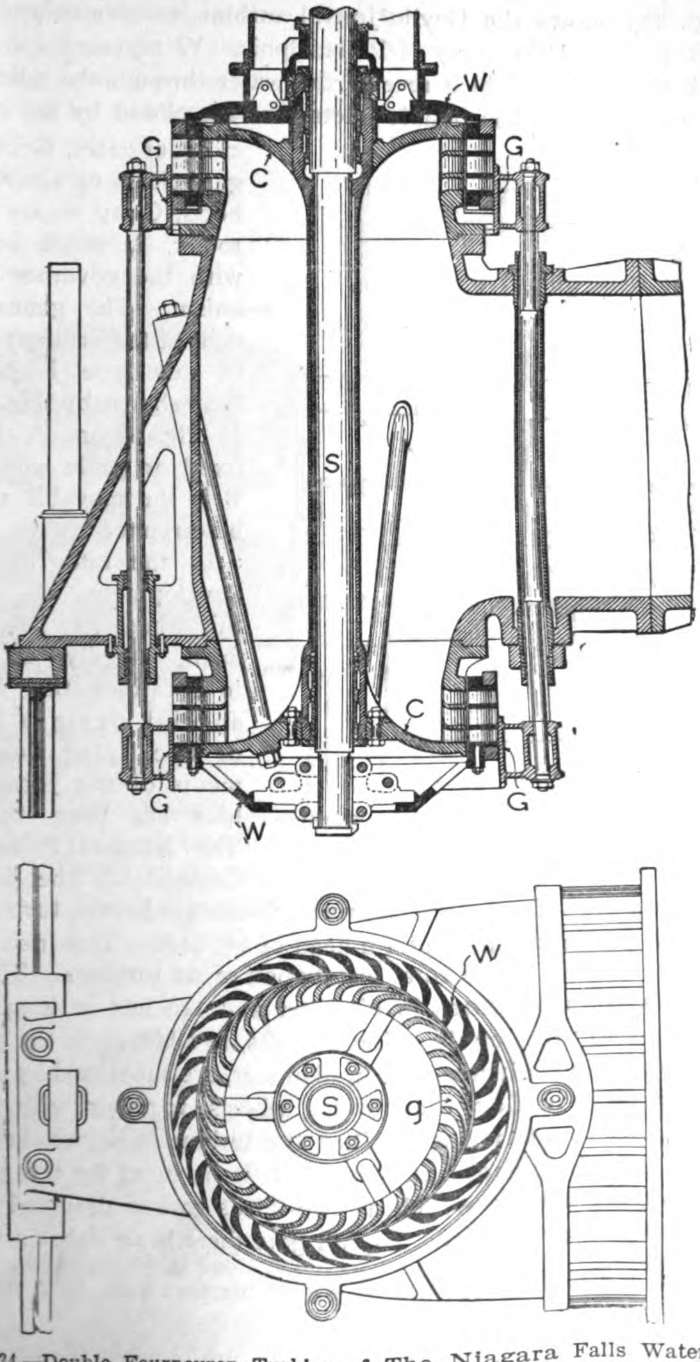


Fig. 134.—Double Fourneyron Turbine of The Niagara Falls Water Pow[er] Company. (Designed by Faesch & Picard; built by I. P. Morris & C[o.]

Fig. 135 shows the Geylin-Jonval turbine as manufactured by the R. D. Wood Company of Philadelphia. W represents the runner, B the buckets which receive the water through the guides, g. The wheel shown has double inlets that are closed by the double cylinder gates, GG. This gate closes up against the hood, C, by means of the rod r, r, which connect with the governor mechanism. The general design of the ordinary wheel of this type is perhaps best shown by Fig. 136.* In this figure A is the fixed or guide wheel and B is the movable or turbine runner.

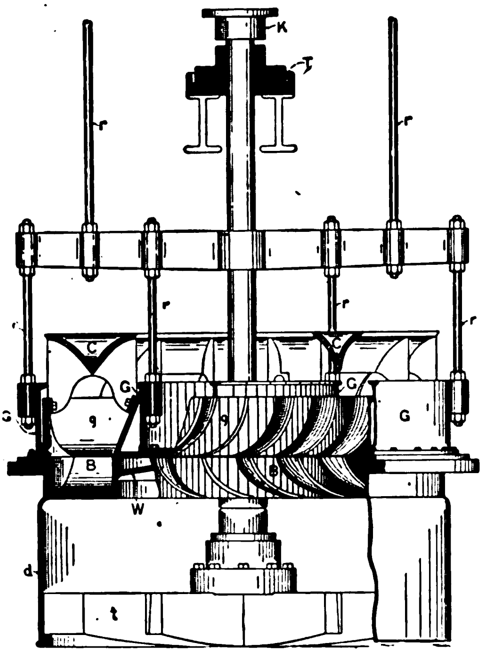


Fig. 135.—Vertical Geylin-Jonval Turbine (Manufactured by R. D. Wood & Co.).

In the later hydraulic developments the use of this wheel has been confined, largely at least, to locations that require special designs. One of the later developments of the Jonval turbine has been that for The Niagara Falls Paper Company. The first installation consisted of three upward discharge Jonval turbines of 1,100 horse power each, under a head of 140 feet. The installation provided, however, for a total installation of six turbines. The vertical shafts are 10 inches in diameter and 140 feet in length and weigh about 19 tons each. These shafts, in addition to the weight of the wheels,—which are 4′ 8″ in diameter, are supported by marine thrust bearings, under the beveled wheels, together with a step bearing under the turbine. When the turbine is in use, however, the weight of the wheel and the shaft is balanced by the upward pressure of the water which at two-thirds gate is designed to exactly balance this weight. At full gate there is an unbalanced up-

* See page 7, 1877 catalogue, J. L. & S. B. Dix, Glen Falls, N. Y.

ward pressure, and, at less than two-thirds gate, an unbalanced downward pressure; these pressures are, however, only the difference between the weights and the water pressure and are easily cared for by the bearings above described.

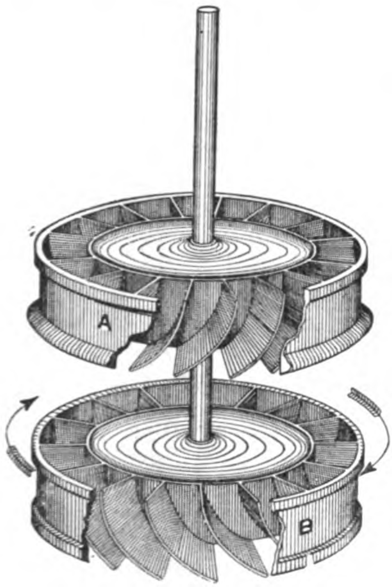


Fig. 136.—Jonval Turbine as Manufactured by J. L. & S. B. Dix.

These wheels have thirty openings and operate at 260 revolutions per minute. The gates are provided with sleeves (cylinder gates) each weighing 2,800 pounds and slide outside the guide wheels to the hood. These sleeves are guided by four rods which extend above the turbine casing about 10 feet to a yoke which is counterbalanced. A sectional view of one of these turbines is shown in Fig. 137 and the general arrangement of the plant is shown in Fig. 138.

A still more recent type of the Jonval turbine is the double, horizontal wheel, built for The Niagara Falls Hydraulic Power and Manufacturing Company and installed in 1898. (See Figs. 139, 140). These wheels have a common, central intake and quarter-turned draft tube which turns down to and is sealed in the tail race below the floor. The speed control is effected by a register gate through which the water passes before it reaches the guide ring. This is said to give a somewhat lower efficiency at part gate than does a gate interposed between the guide tubes and runner bucket. Economy of water at part gate is said to be no particular object in this plant and reduced efficiency is, in fact, an advantage in that it reduces the gate movement and retains a velocity in the penstock, with a given change of load, and consequently reduces the inertia action and aids the speed regulation. This turbine is rated at 2,500 H. P. at 250 revolutions per minute, under the normal head of 210 feet.*

* See "The Electrical World," January 14, 1899.

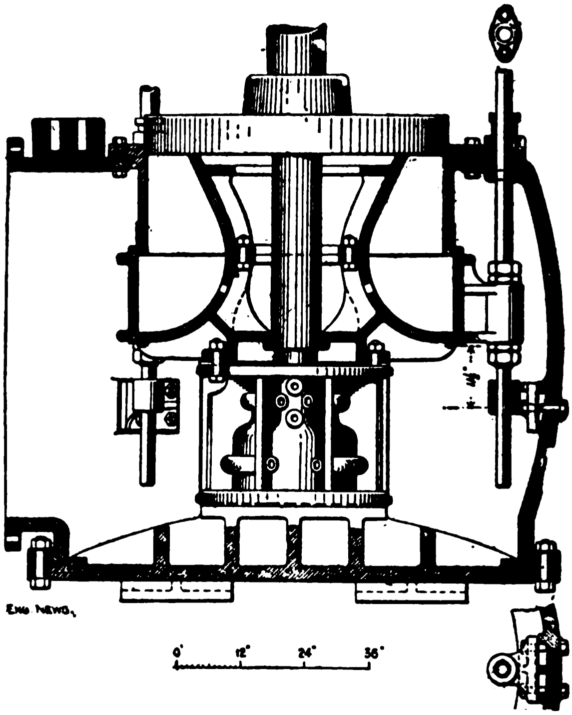


Fig. 137.—Geylin-Jonval Turbine of Niagara Falls Paper Mill Co. **Manufactured by R. D. Wood & Co. (From Eng. News, Apr. 5, 1894.)**

133. **The American Type of Reaction Turbine.**—The Howd Wheel (Fig. 13) from which the idea of the Francis inward flow wheel (Fig. 12) was derived, was invented in 1838 and acquired a considerable market throughout New England. From these wheels originated the American inward and downward or mixed flow turbines.

The early wheels of American manufacture were designed very much after the style of the Francis wheel with changes, more or less radical, in the shape and details of the buckets. The demand for wheels of greater power, and higher speed, has resulted in a gradual development of other and quite different forms.

The development of the turbine in the United States is well illustrated by that of the "American" turbine of Stout, Mills & Temple, now The Dayton Globe Iron Works Co. This wheel was

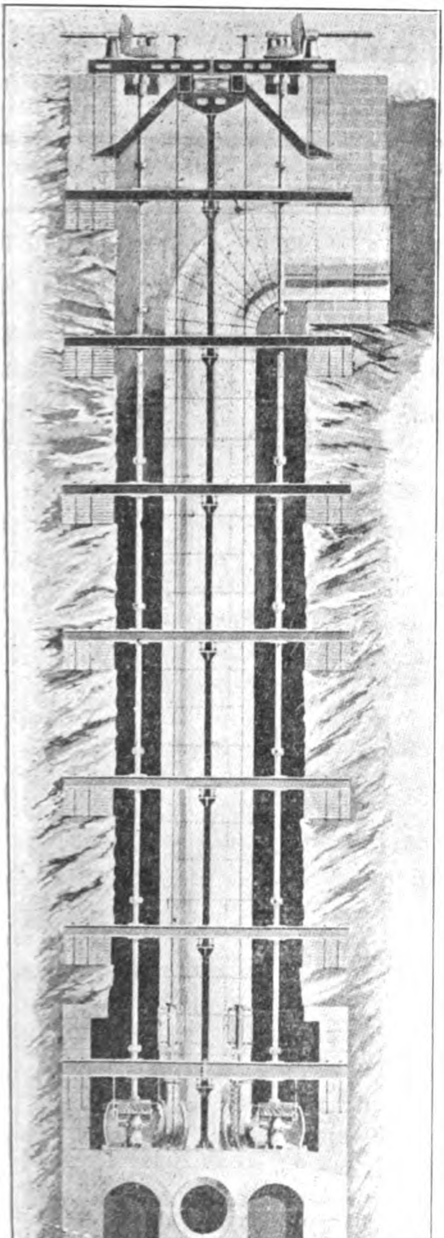


Fig. 138.—Plant of the Niagara Falls Paper Co. Showing Installation of Jonval Turbines. (From Cassier's Magazine, Nov., 1904.

designed in 1859 and was called the American Turbine. The general form of the original turbine wheel is shown in Fig. 141.

This was followed (1884) by the design of what is known as the "New American" turbine, illustrated by Fig. 142. In this wheel the buckets are lengthened downward and have a partially downward as well as inward discharge.

This wheel was followed in 1900 by the "Special New American" illustrated in Fig. 143, having a great increase in capacity and power.

The fourth and most recent type (1903) is the "Improved New American" illustrated in Fig. 144. The comparative power and speed of these various wheels is shown in the tables on pages 258 and 259.

Table XXIII is misleading to the extent that while the diameter of each wheel is given as 48" such diameters are not strictly comparative. Part of the additional capacity and power of the "Special New American" and of the "Improved New American" is due to the cutting back of the buckets (see Figs. 141 to 144) which, while it reduces the diameter at the point of measurement, gives a discharge which would be fairly comparative with wheels of the older type of perhaps three or four inches larger diameters. (See Sec. 140.)

TABLE XXIII

Development of "American" Turbines.—Capacity, Speed and Power of a 48-inch Turbine under a 16-foot Head.

	Year brought out.	Discharge in cu. ft.	Rev. per min.	Horse power.
American	1859	3271	102	79.1
Standard New American........	1884	5864	102	141.8
New American	1894	9679	107	234.0
Special New American..........	1900	11061	107	267.0
Improved New American........	1903	13234	139	325.0

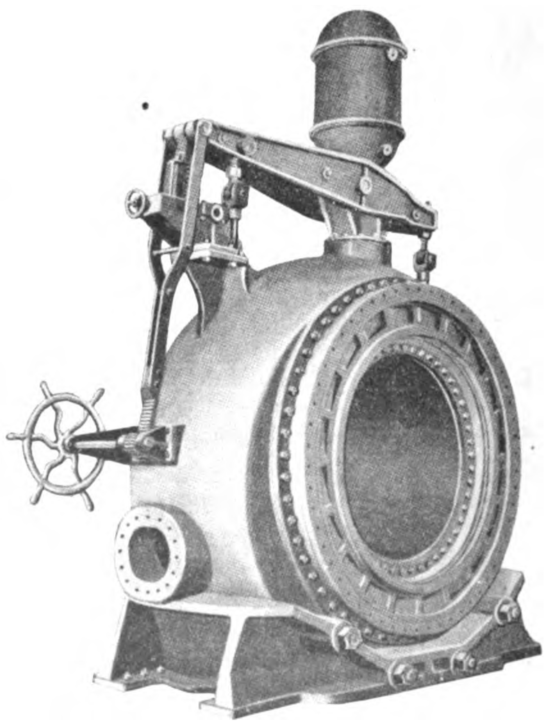


Fig. 139.—Horizontal Geylin-Jonval Turbine of Niagara Falls Hydraulic Power & Manufacturing Co. Showing Guide Chutes.*

*** Cuts 139 and 140 reproduced from Electrical World, Jan. 14, 1899. Turbines manufactured by R. D. Wood & Co.**

The development of turbines may also be illustrated by a comparison of the size and speed of turbines of various series required to develop essentially the same power. (See Table XXIV.)

TABLE XXIV

Increase in Speed of "American" Turbines for Same Power (16-foot head).

	Size of wheel.	Horse power.	R. P. M.
American	48	79.1	102
New American	36	81.5	136
Special New American	27½	87.3	186
Improved New American	25	87.5	267

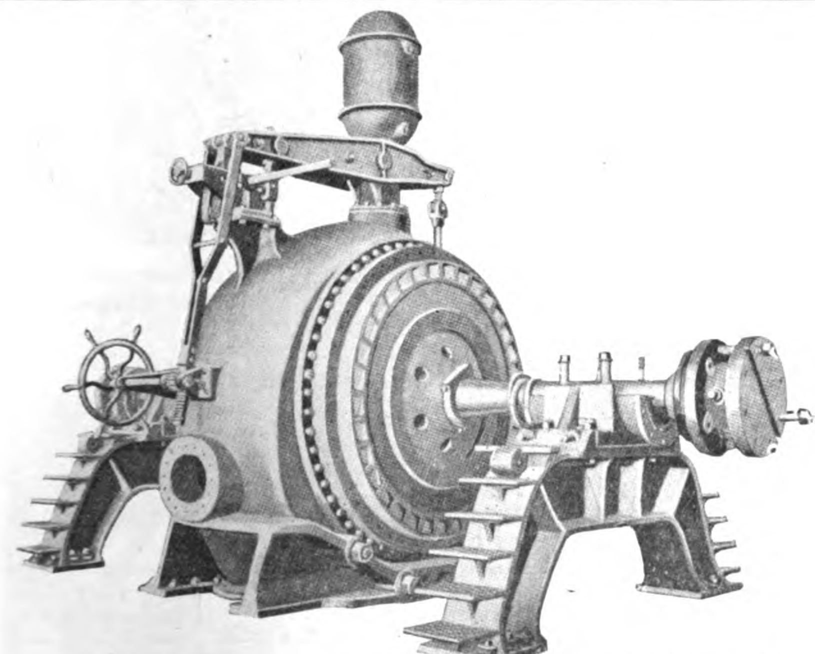


Fig. 140.—Horizontal Geylin—Jonval Turbine Showing Bucket Ring.*

Figs. 145 and 146 show a vertical and a horizontal half plan, half section of a vertical Improved New American turbine. W is the crown and hub of the wheel; B, the buckets; G, G, are the wicket

*See foot note page 258.

gates that control the admission of water to the wheels and which are operated by means of the ring Gr, which is moved by an eccentric and rod, r, connected with the governor through the shaft, P.

The inner edges of the bucket are spaced some distance from the shaft and the main discharge is inward and downward, though a portion of the bucket will admit of a slightly outward discharge.

134. The Double Leffel Turbine.—Perhaps the greatest departure of American inventors from the lines of the original Francis

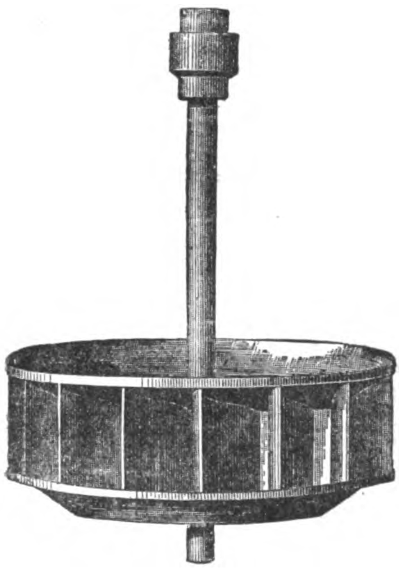


Fig. 141.—American Turbine Runner.*

Fig. 142.—New American Turbine Runner.

TABLE XXV.

Development of "Leffel" Wheel.—Capacity, Power and Speed of 40-inch Wheel Under 16-foot Head.

	Year brought out.	Discharge.	Rev. per minute.	Horse power.
Standard	1860	2547	138	64½
Special	1870	3672	138	93
Samson	1890	6551	158	155
Improved Samson	1897	8446	163	207

* Manufactured by The Dayton Globe Iron Works Co.

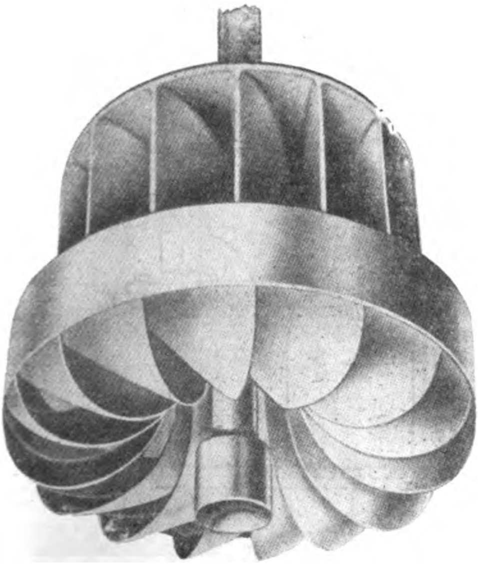


Fig. 143.—Special New American Turbine Runner.*

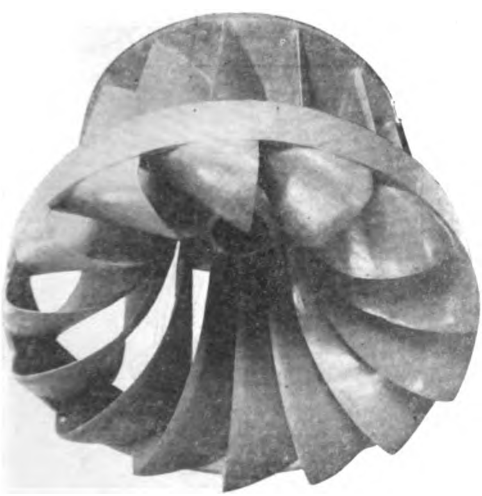
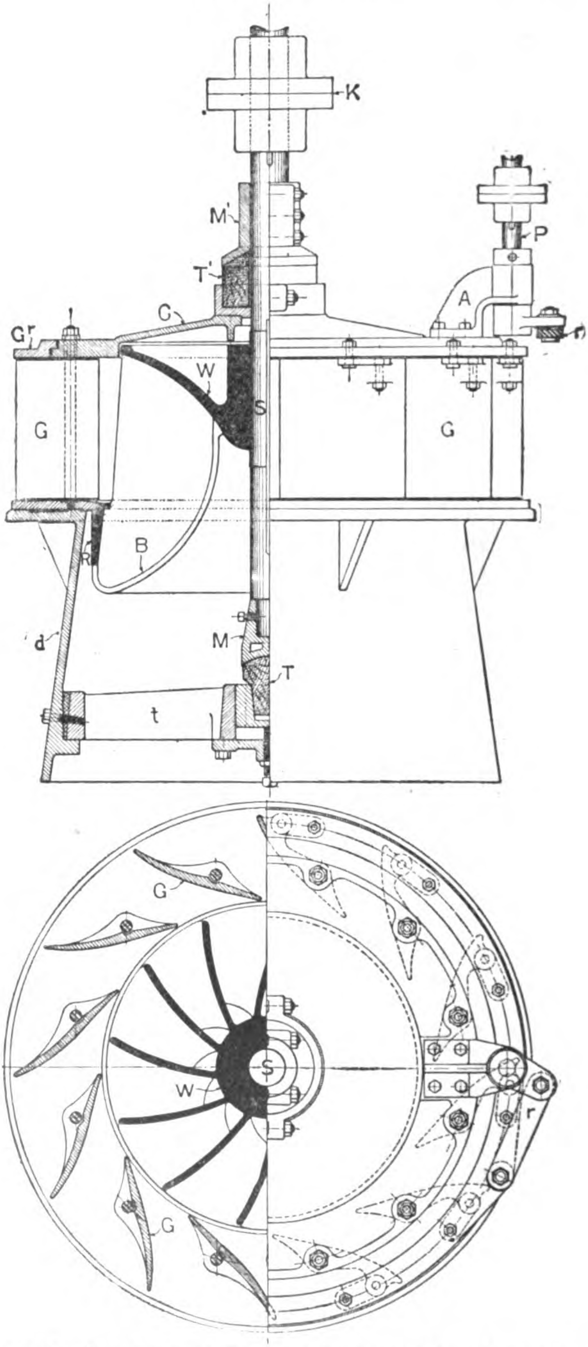


Fig. 144.—Improved New American Turbine Runner.*

type of turbine was that of James Leffel. In this wheel was combined a double runner, the upper half being a radial inflow runner of the Francis type and the lower half consisting of a runner with inward radial admission and axial discharge, essentially on the line of the later development of the American type of wheels. The wheel, as originally designed, had the narrow bucket, slow speed and low power of all early American wheels. In its later development the buckets have been extended inward and downward and these wheels have found their best modern development in the Samson-Leffel wheel, illustrated in Figs. 147 to 151.

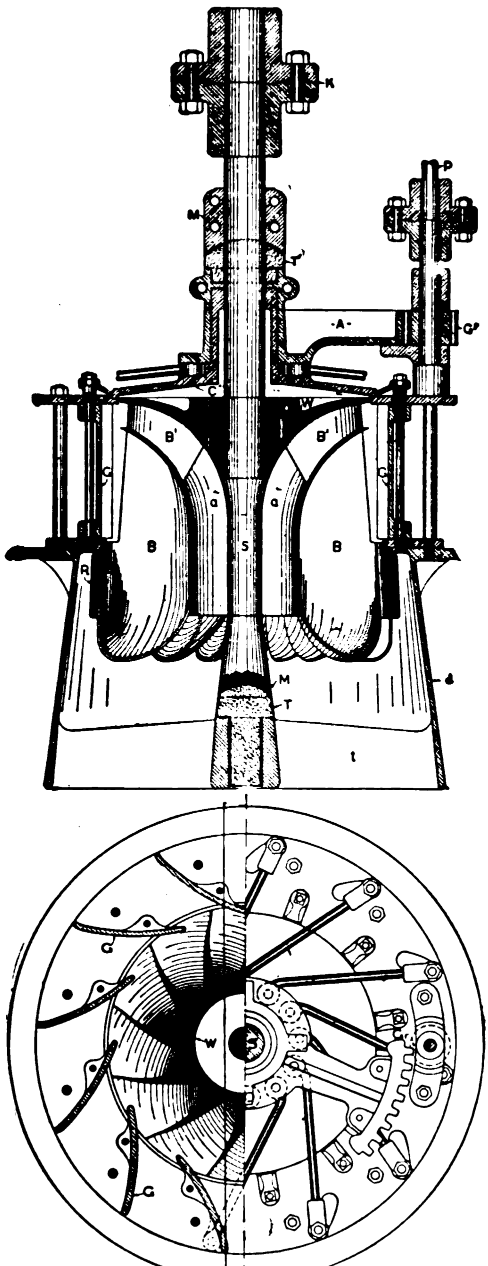
In Fig. 147, W represents the hub and crown of the wheel which is securely keyed to the shaft, S. B′ B′ are the upper buckets that discharge inward and downward through the passage aa. The lower buckets, BB, it will be noted, have the same lines as other modern wheels of the American type. They receive the

* Manufactured by The Dayton Globe Iron Works Co.



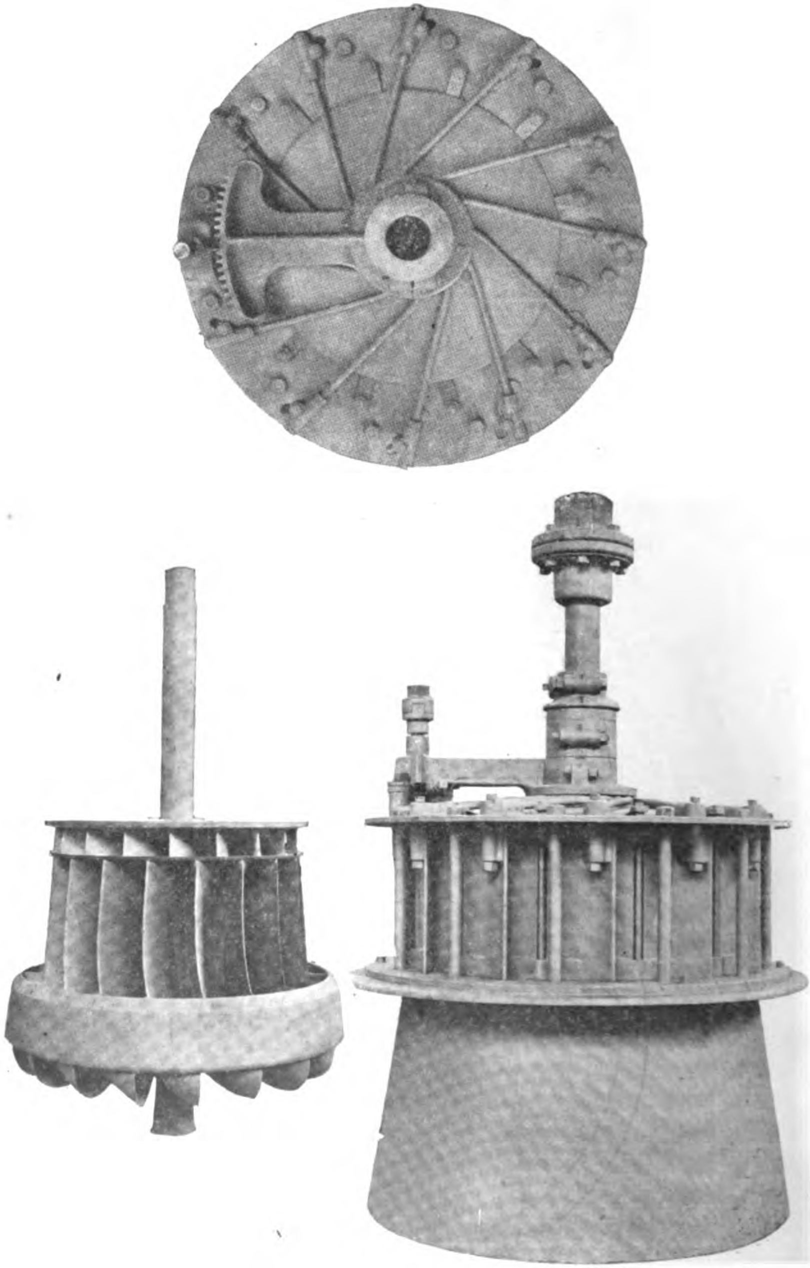
Figs. 145 and 146.—Section and Plan of Improved New American Turbine.*

* Manufactured by The Dayton Globe Iron Works Co.



Figs. 147 and 148.—Section and Plan of Samson Turbine.*

* **Manufactured by The James Leffel & Co.**



Figs. 149, 150 and 151.—Top View, Runner and Outside View of Samson Turbine.*

* Manufactured by The James Leffel & Co.

water inward and discharge it downward, outward and inward with the general purpose of distributing it over the cross-section of the turbine tube. The gates, G, are of the wicket type and are connected by rods with an eccentric circle which is operated through the arm, A, and the gearing, Gr, by the governor shaft, P. The gate gearing is well shown by reference to the section-plan, Fig. 148, and the top view, Fig. 149.

The Samson turbine runner is illustrated in Fig. 150, and Fig. 151 shows an outside view of one of the vertical, turbine units.

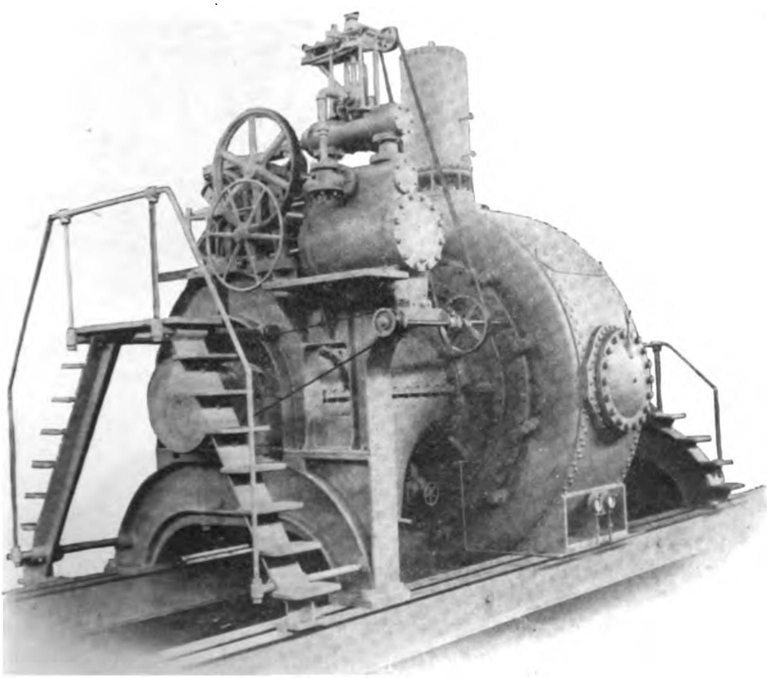


Fig. 152.—Double Horizontal Leffel Turbine of The Niagara Falls Hydraulic Power & Manufacturing Co. Manufactured by The James Leffel & Co.

The development of this wheel is illustrated by Table XXV. This table is fairly representative of the growth of this turbine as the diameter is, in all cases, the maximum diameter of the wheel. (See Sec. 140.)

The adaptability of the earlier turbine designs to the later moderate head developments is well illustrated in the design of the wheels for The Niagara Falls Hydraulic Power and Manufacturing Company, installed by The James Leffel Company about 1892. These turbines have the single narrower buckets, smaller discharge and relatively slower speed of the earlier designs. The runners are double discharge, horizontal, seventy-four inches in diameter and operate at a speed of 250 revolutions per minute under a head of 215 feet, and each wheel develops about 3,500 horse power.

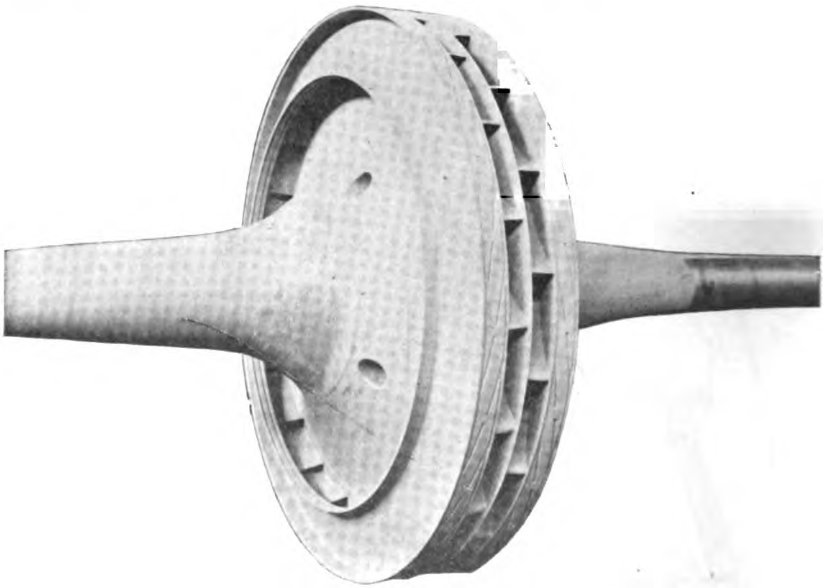


Fig. 153.—Leffel Double Runner of The Niagara Falls Hydraulic Power & Manufacturing Co. Manufactured by The James Leffel & Co.

Fig. 152 shows one of these units complete. Fig. 153 is a view of the runner. For a test of this wheel, made December 1903, see page 381.

135. **Other American Wheels.**—The development of modern American wheels could, perhaps, have been equally well illustrated by the growth of various other American turbines. The development of all American wheels up to the present time has been on the line of increasing both the speed and the power of the wheel for low head, with a return to the earlier type for wheels to be used under the moderate heads.

Fig. 154 illustrates a runner of the well-known McCormick pattern. Mr. J. B. McCormick, who had previously become familiar with certain wheels of large capacity designed and patented by Matthew and John Obenchain, re-designed and improved these wheels, about 1876, and secured high efficiencies together with increased power far beyond any other wheels of that period. McCormick wheels in their original or modified form are now made by a large number of American manufacturers and these wheels have had a marked effect on the design of almost all modern American water wheels. The runner in the illustration is the Hunt-McCormick runner as manufactured by The Rodney Hunt Machine Company, but is very similar to the McCormick wheels of various other manufacturers.

Fig. 154.—Hunt-McCormick Runner of The Rodney Hunt Machine Co.

The Smith-McCormick runner is manufactured by The S. Morgan Smith Company. This company has also recently brought out a new wheel called the "Smith Turbine," of greater power and higher speed, the runner of which is illustrated by Fig. 155. Fig. 156 represents the Victor runner or "type A" runner of The Platt Iron Works Company, designed for low heads.

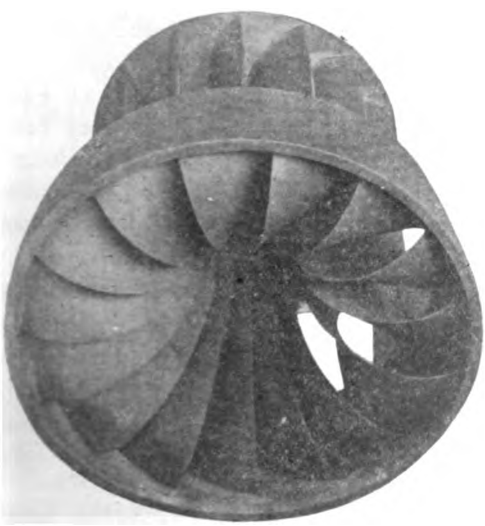


Fig. 155.—Smith Runner of S. Morgan Smith Co.

Fig. 157 is the "type B" runner, of the same Company, designed for medium heads. This runner again illustrates the tendency to return to the earlier forms of runner for medium head wheels. This latter type has also been adopted by other manufacturers of turbines, as may be seen by reference to Fig. 158 which shows the Hunt runner manufactured for moderate heads by The Rodney Hunt Machine Company.

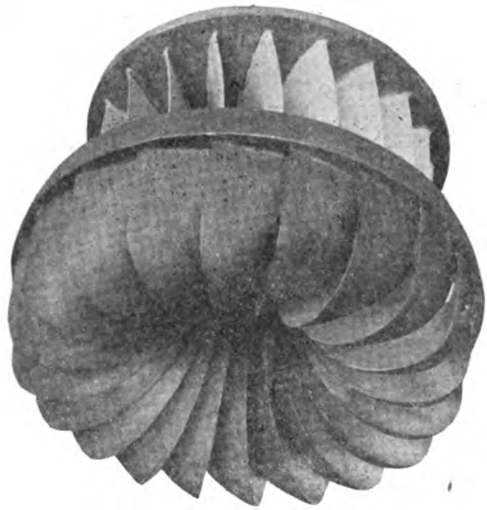


Fig. 156.—Victor or "Type A" Runner of The Platt Iron Works Co.

Fig. 157.—High Head or "Type B" Runner of The Platt Iron Works Co.

Fig. 159 is from a shop photograph of the Shawinigan Falls turbine manufactured by the I. P. Morris Company. This is one of the largest turbines ever constructed and develops 10,500 horse power under a head of 140 feet. It is a double mixed inflow type with spiral casing and a double draft tube through which the water discharges outward from the center. The diameter of the casing at the intake is 10½ feet and the sectional area gradually diminishes around the wheel in proportion to the amount of water flowing at each point. The wheel complete is 30 feet in height and weighs 182 tons. The runner, which is of bronze, is shown in Fig. 160.

Figs. 161 and 162 show two sections of a single turbine of the Francis inflow type built for the Snoqualmie-Falls plant of The Seattle & Tacoma Power Company by The Platt Iron Works Company. The turbine has a capacity of about 9,000

H. P. under 270-foot head at 300 R. P. M. The runner is 66 inches in diameter and has a width of 9½ inches through the buckets.* This is believed to be the largest capacity single discharge wheel yet constructed.

For further details see Figs. 183, 189 and 190.

136. Early Development of Impulse Wheels.—As already pointed out (see Chapter I, Figs. 6 and 7), water wheels of the impulse type were among the earlier forms used. In the practical construction of water wheels for commercial purposes in this country, the reaction turbine was, however, the earliest form of development. This was because the reaction turbine was best suited for the low heads first developed. As civilization advanced from the more level country into the mountainous regions the conditions were found to radically differ. In the former location large quantities of water under low heads were available; in the latter, the streams diminished in quantity but the heads were enormously increased. These conditions demanded an entirely different type of wheels for power purposes and the demand was met by the construction of the tangential wheel now so widely and successfully used in the high head plants of the West.

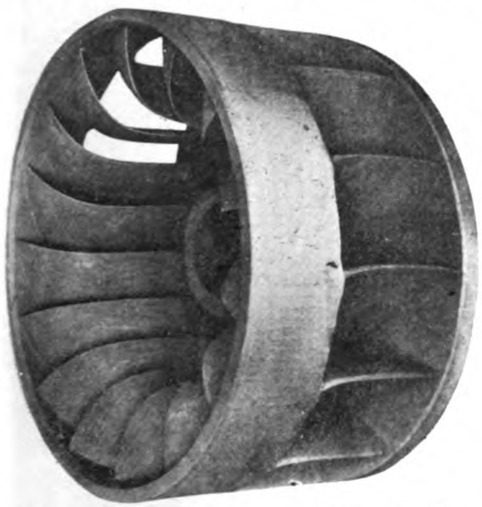


Fig. 158.—Hunt Runner of The Rodney Hunt Machine Co.

The earliest scientific consideration of impluse wheels in this country was by Jearum Atkins who, apparently, anticipated the design of the wheels of the Girard type in Europe by his design of such a wheel in 1853.† (See Fig. 163.)

*See "Engineering News," March 29, 1906.

† See "Tangential Water Wheels" by John Richards, Cassier's Magazine, vol. V, p. 117.

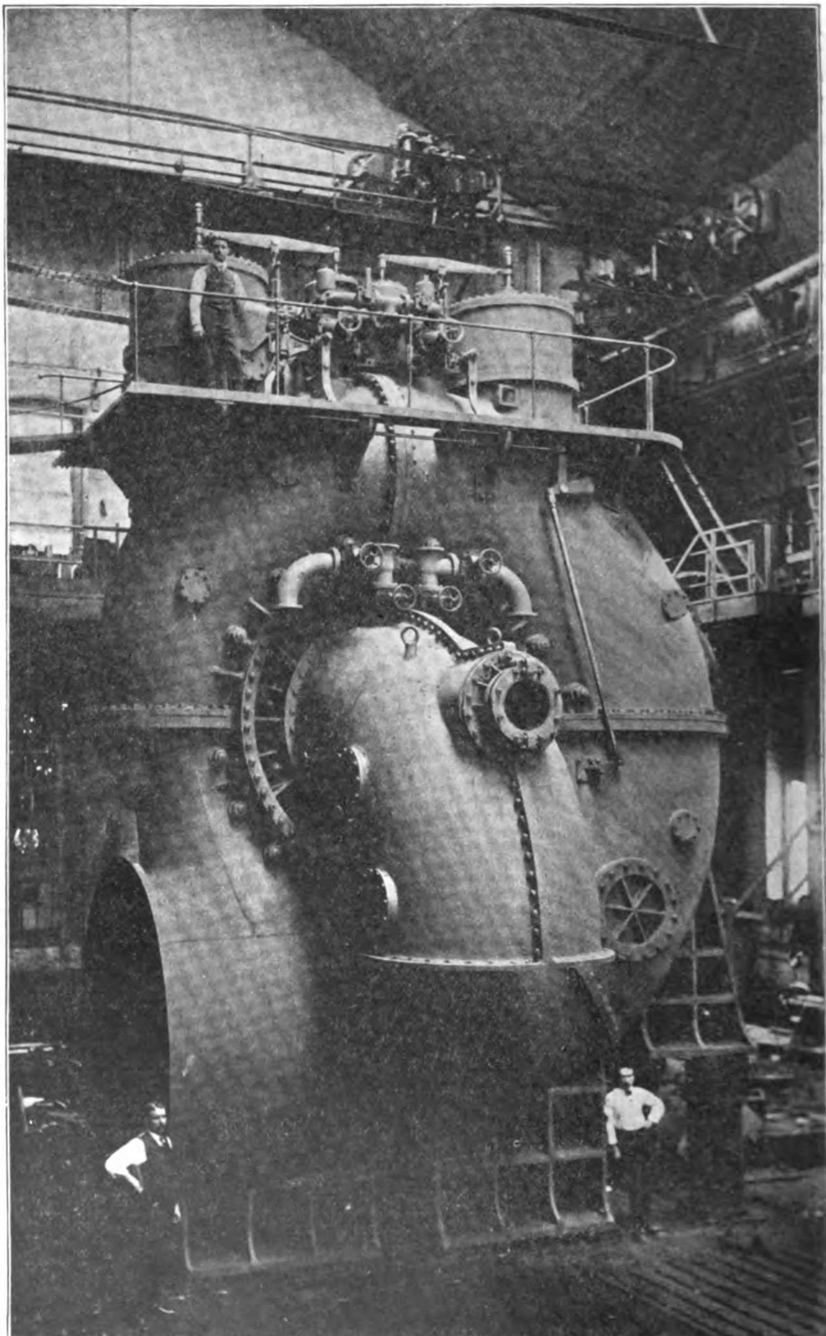


Fig. 159.—Shawinigan Falls Turbine, Manufactured by I. P. Morris Co.

In Atkins' first application for a patent (in 1853) he shows a clear conception of the principles of the impulse wheel.

After describing the mechanical construction of his wheel, Mr. Atkins says: "The important points to be observed in the construction of this wheel and appendages, are: First, that the gearing * * * should be so arranged as to allow the wheel's velocity at the axis of the buckets to be equal to one-half the velocity of the water at the point of impact, * * *

"As the power of water, * * * is measured by its velocity, * * * it is obvious that in order that the moving water may communicate its whole power to another moving body, the velocity of the former must be swallowed up in the latter. This object is

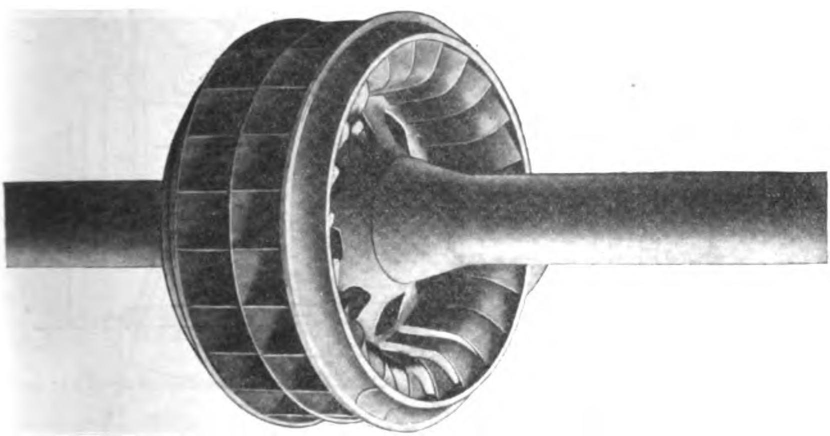


Fig. 160.—Shawinigan Falls Turbine Runner.

effected by the before-described mode of applying water to a wheel in the following manner, the velocity of the wheel, as before stated, being one-half that of the water.

"Let us suppose the velocity of the water to be twenty-four feet per second; then the velocity of the wheel being twelve feet per second, the relative velocity of the water with respect to the wheel, or the velocity with which it overtakes the wheel, will be twelve feet per second. Now it is proved theoretically, and also demonstrated by experiment, that water will flow over the entire surface of the semi-circular buckets of the wheel with the same velocity with which it first impinged against them, or twelve feet per second. Then, as the water in passing over the face of the buckets has described a semi-circle, and as its return motion on leaving the wheel is in an opposite direction from that of the wheel, its velocity with respect to the wheel being twelve feet per second, and as the wheel has an absolute velocity of twelve feet per sec-

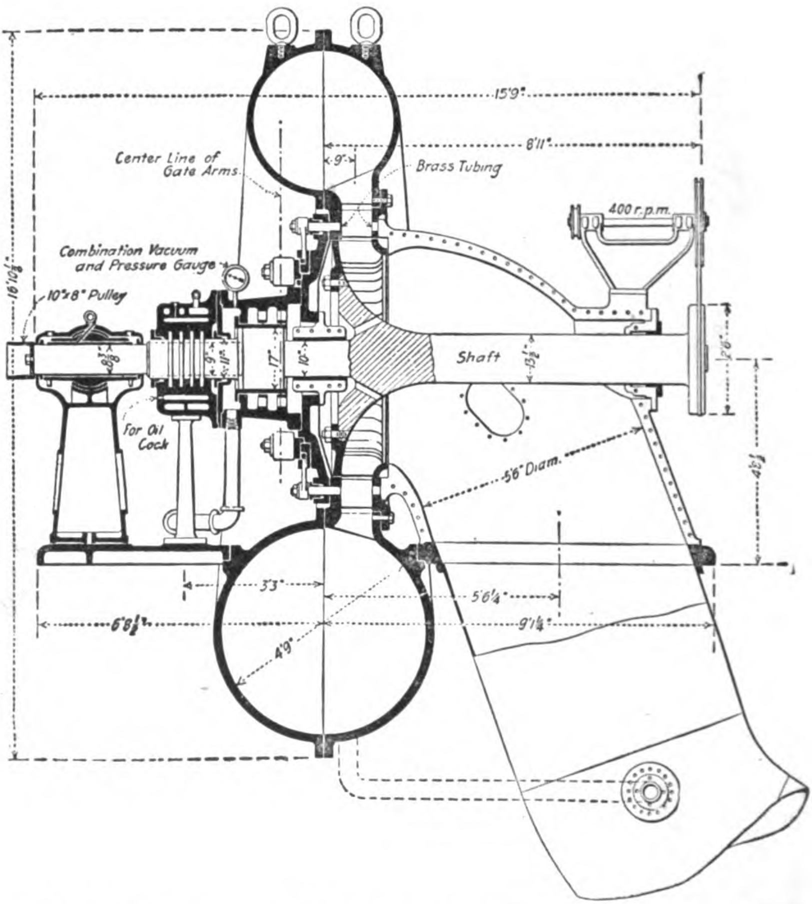


Fig. 161.—Section Snoqualmie Falls Reaction Turbine. The Platt Iron Works Company.

ond, it is obvious that the absolute velocity of the water with respect to a fixed point is entirely suspended at the moment of leaving the inner point of the buckets, its whole velocity, and consequently its whole power, having been transmitted to the wheel."

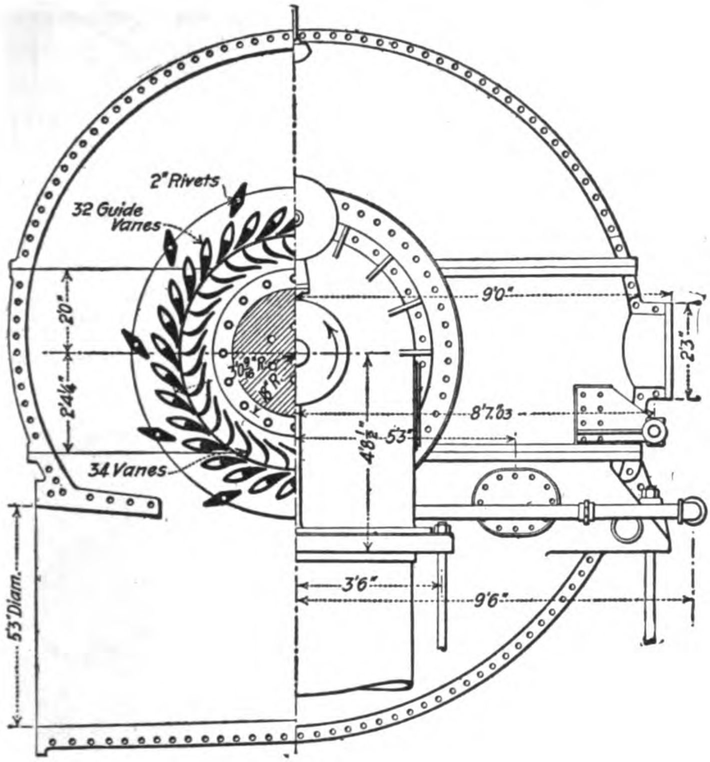


Fig. 162.—Section-Elevation Snoqualmie Falls Reaction Turbine (The Platt Iron Works Co.).

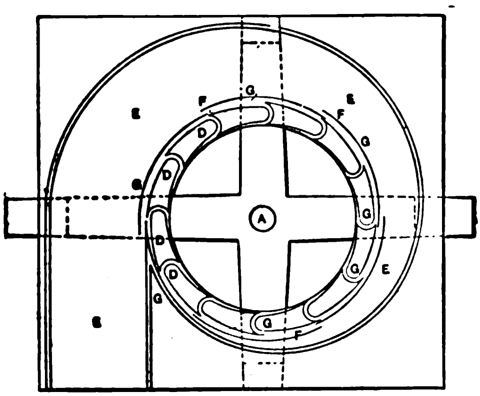
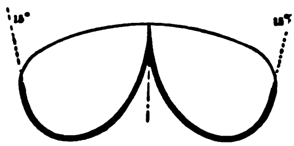
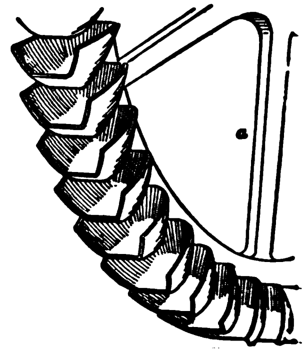


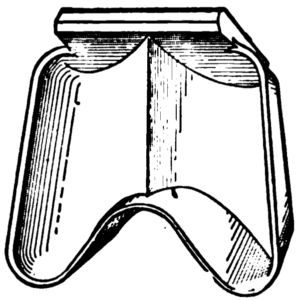
Fig. 163.—Plan of Atkins Wheel and Wheel Case (1853). From Cassier's Magazine, Vol. v, p. 119.



a. Moore bucket, 1874.

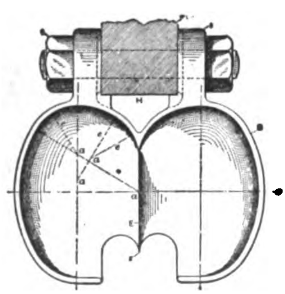


b. Knight buckets, 1870.

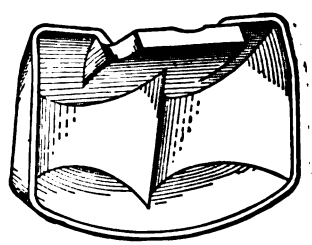


c. Dodd bucket, 1889.

d. Hug bucket, 1897.



e. Doble Ellipsoidal bucket, 1889.



f. Pelton bucket, 1880.

Fig. 164.—Buckets of Tangential or Impulse Water Wheels. (Trans. Am. Inst. Mining Eng. 1899.

Mr. Atkins' first application for a patent was rejected. After a long illness, from which he finally recovered, he again applied for a patent which was finally granted in 1875. The Atkins' patents are simply of historical interest as his inventions have had little effect on the practical development of the impulse wheel.

137. American Impulse Wheels.—The impulse wheel found its earliest practical development in California where the conditions for the development of power made such a wheel necessary. The early tangential wheel, used on the Pacific Coast, was quite simple in construction and the development of the buckets, which began with the simpler flat and curved forms, was very largely based on the experimental method used for the development of the reaction

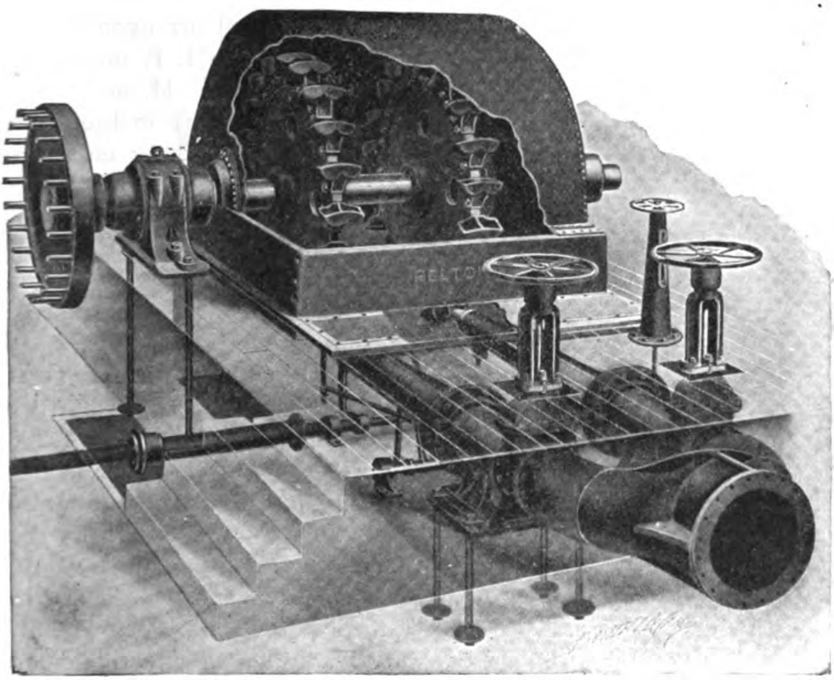


Fig. 165.—Telluride Double Tangential Wheels. 2000 H. P. 500 Foot Head. (Pelton Water Wheel Co.)

turbine in the East. Experiments were made at the University of California, by Mr. Ralph T. Brown, as early as 1883, and the bulletin, published by the department was the earliest literature on tangential wheels published in this country.

With the early development of the tangential bucket are connected the names of Knight, Moore, Hesse, Pelton, Hug, Dodd and Doble, and many other inventors, whose wheels have become well-known and widely used. The most extensive early development of this wheel was by The Pelton Water Wheel Company whose work has been so widely known and used as to make the name "Pelton Wheel" a common title for all wheels of the tangential type.

Some of the many forms of American buckets used are shown in Fig. 164 with the approximate date of their invention or design.

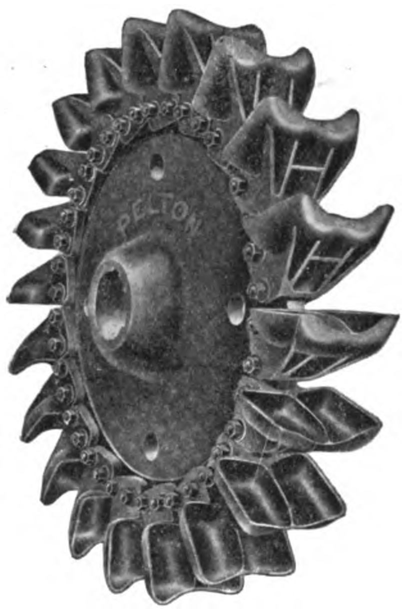


Fig. 166.—Pelton Tangential Water Wheel Runner. Designed for 5000 H. P. at 865 foot head and 225 R. P. M. (Pelton Water Wheel Co.)

The general arrangement of a double 2000 H. P. unit, running at 200 R. P. M. under 500 foot head is shown in Fig. 165. This is one of three units installed by The Pelton Water Wheel Company for The Telluride Transmission Plant of Colorado.

The wheels are of cast steel fitted with steel buckets, held in position by turned steel bolts. They are connected by a flexible coupling to a 1,200 H. P. generator.

Fig. 166 shows the runner of an impulse wheel made by the same company. This is 9' 10" in diameter, and is designed to develop 5,000 H. P. at 225 R. P. M. under an effective head of 865 feet.

Fig. 167 shows the runner of an impulse wheel manufactured by the Abner Doble Company. This runner was from the Doble Water Wheel Exhibit at the St. Louis Fair and developed 170 H. P. at 170 R. P. M. under a head of 700 feet and generated direct current for use on the intramural railway.

In addition to the tangential wheels already described, a few manufacturers have developed wheels of the Girard type. One such wheel, designed and built by The Platt Iron Works Company, is illustrated in Figs. 168 to 171, inclusive. Fig. 168 is a section-elevation showing the arrangement and design of the guides and buckets of the wheel. Fig. 169 shows a section through the wheel and on the line of the shaft. In these figures W represents the runner; BB the buckets; g, the inlet guides, and G, the gate by which all or a portion of the guide passages may be closed and the power of the wheels reduced. The gate, G, is connected by the gearings, Gr, with the rod, r, which is connected through the rocker

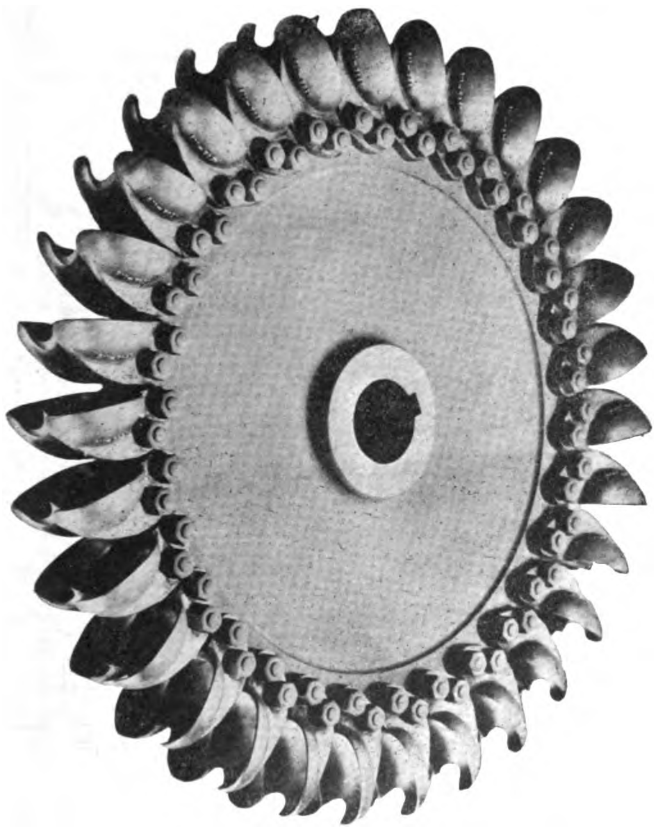


Fig. 167.—Doble Runner. (Abner-Doble Co.)

arm with the governor mechanism. The wheel or runner of this turbine is shown by Fig. 170, and a general view of the wheel is shown by Fig. 171.

138. Turbine Development in Europe.—Modern European turbine practice has been the development of the last twenty years. European manufacturers have approached the subject more on the basis of theoretical analysis than has been done in America. The conditions of development have also been largely special and not under such uniform conditions as in America. The result has been the development of special designs for special locations and the rapid accumulation of a considerable experience under a wide range

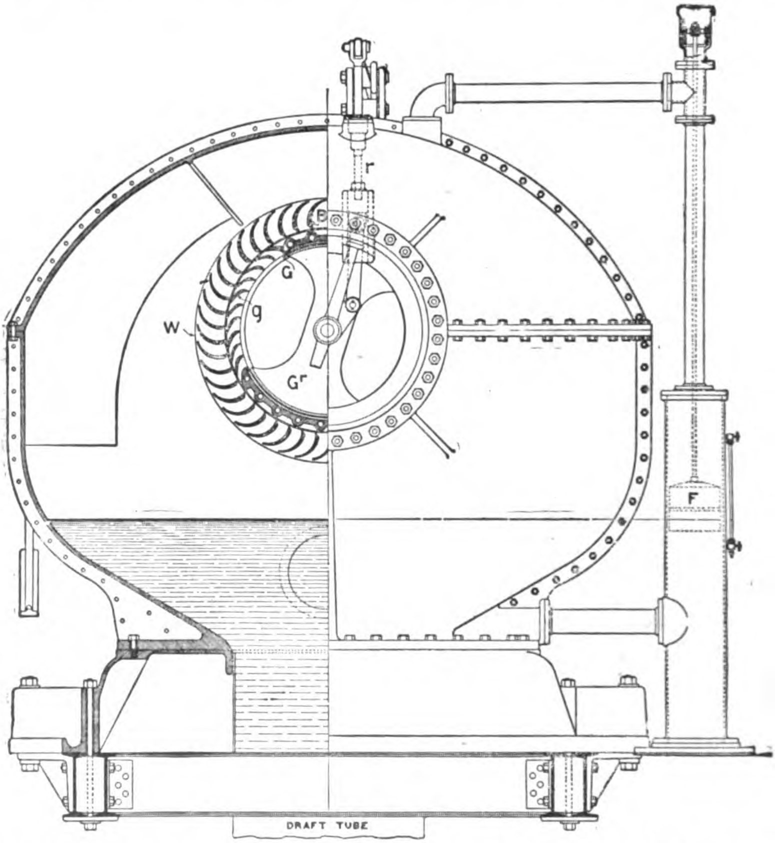


Fig. 168.—End Section and Elevation, Girard Impulse Turbine with Draft Tube. (Platt Iron Works Co.)

of conditions. While the radial flow turbines were the earlier type developed, European practice has been largely centered on the axial flow wheels of the Jonval type for complete turbines, and axial flow and radial flow wheels of the Girard type for partial turbines under high heads.

The axial flow turbine while simple in construction and low in cost is difficult to regulate and hence the demands of electrical development for close regulation has given rise to a variety of modern designs which are summarized by Mr. J. W. Thurso essentially as follows: *

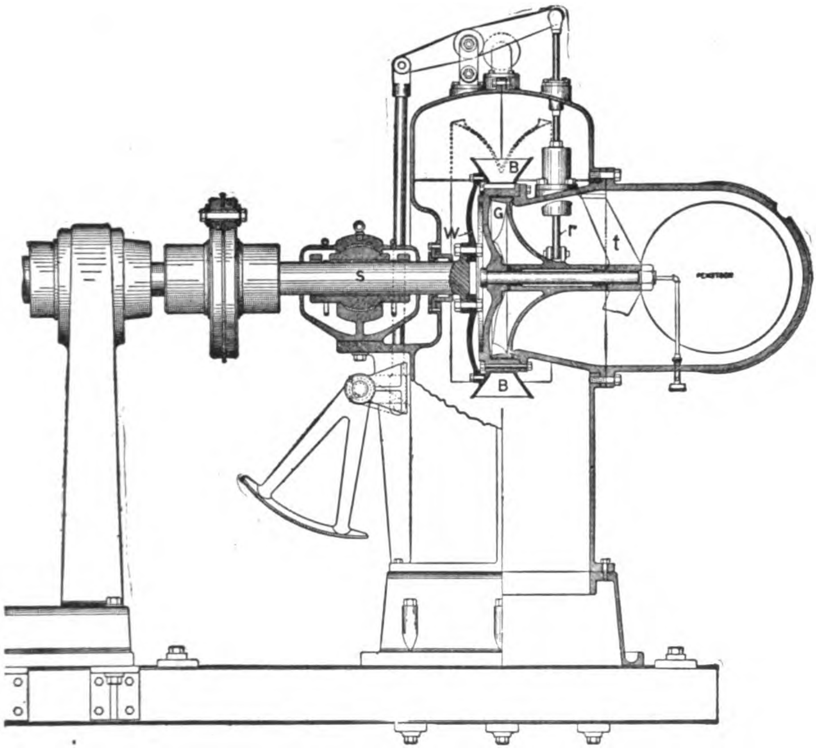


Fig. 169.—Longitudinal Section Girard Impulse Turbine. (Platt Iron Works Company.)

1st. For low heads to 20 feet. Radial inward flow, reaction turbines with vertical shafts and draft tubes.

2nd. For medium heads, 20 to 300 feet. Radial inward flow reaction turbines with horizontal shafts and concentric or spiral cases and draft tubes.

3rd. For high heads over 300 feet. Radial outward flow, full or partial action turbines (of the Girard type) with horizontal shafts,

* See "Modern Turbine Practice" by J. W. Thurso.

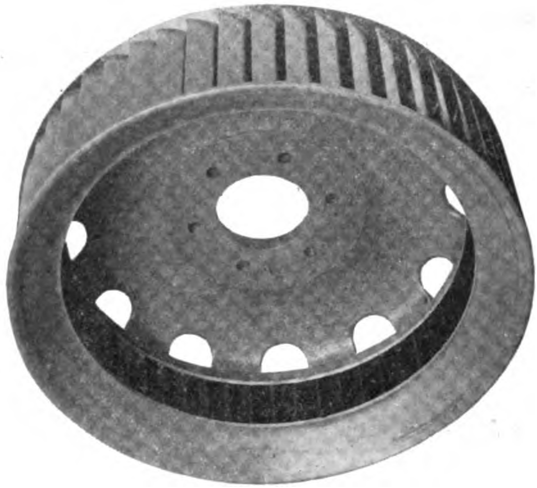


Fig. 170—Runner of Girard Turbine. Type C, High-Pressure Runner. (Platt Iron Works Co.)

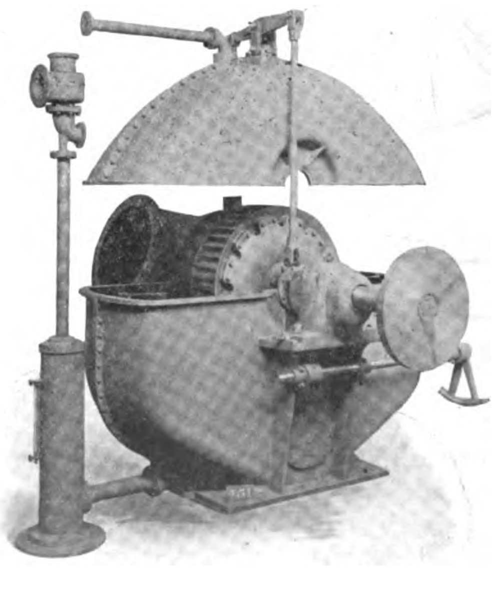


Fig. 171.—General View of Girard Turbine with Cover Raised. (Platt Iron Works Co.)

often with draft tubes; also, modified impulse wheels of a tangential type.

The types of turbines for low and moderate heads are modifications of the Francis inward flow turbine.

Earlier European practice is perhaps well represented by Fig. 172 which represents one of eight turbines installed by Messrs. Escher, Wyss & Co. for the City of Geneva, Switzerland. These wheels are of the Jonval type and operate under heads sometimes as great as 12 feet but during high water the heads decrease to about five and one-half feet. The turbines consist of three annular rings or buckets and are so designed that the water is admitted to as many buckets as may be required for economical operation under the very great differences in the condition of supply. The width of the inner and intermediate rings are each seventeen and three-quarters inches, and the outer ring is eleven inches, all measured radially. The outside diameter of the wheel is thirteen feet, eleven inches. The outer ring of guides is not provided with means for excluding the water from the buckets but the intermediate or inner rings can be entirely and independently closed. The

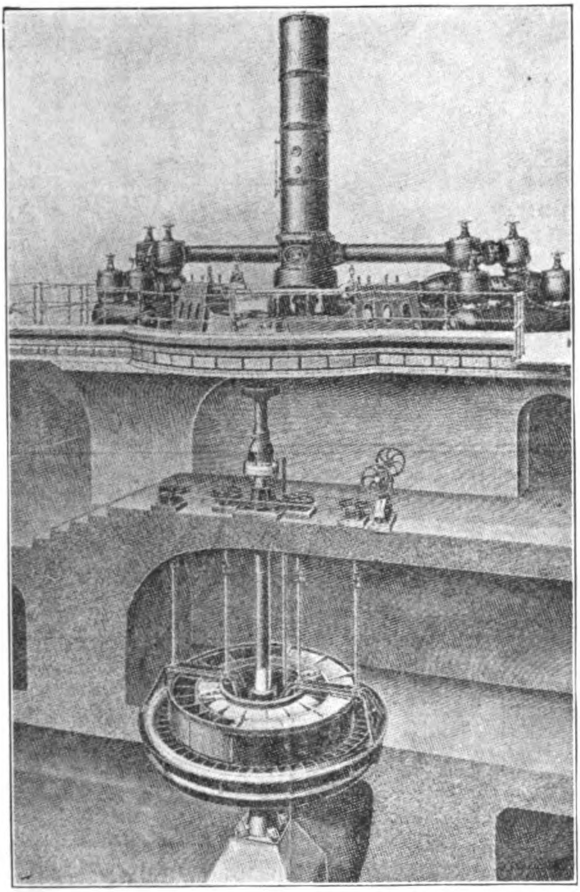


Fig. 172.—One of the seventeen 210 H. P. Jonval Turbines at the Geneva Water Works. Built by Escher, Wyss & Co.

gates for closing the intermediate and inner rings consist of a flat plate in the form of a half ring, which lies on the top of the crown and a vertical curtain which hangs from the end of the plate and completes the closure of the other half of the bucket the openings of which are on the side of the same, the water entering the buckets by a quarter-turn.

These turbines are used to operate the pump that furnishes the water supply for the city of Geneva for domestic and maunfacturing purposes.

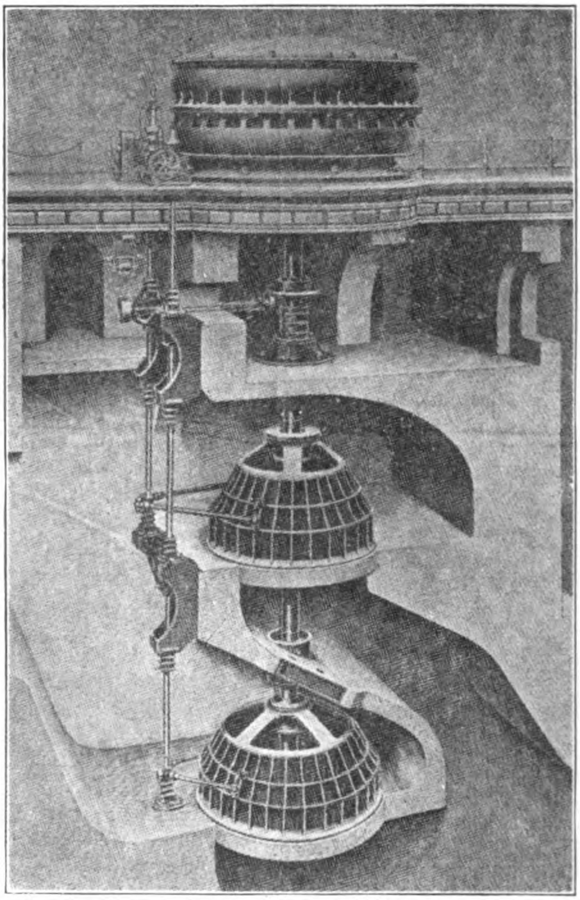


Fig. 173. The 1200 H. P. Double Turbine at Chivres near Geneva. Escher, Wyss & Co. (Cassier's Magazine, October, 1897.)

Fig. 173 shows a pair of vertical turbines furnished by the same company for Chivres near Geneva. Here the fall in summer is 15 feet and in winter 28 feet. The lower turbine will develop 1,200

H. P. at 80 R. P. M. under the higher head, and under the lower head the turbine above works with the lower one.

Each turbine is cone shaped and divided into three compartments in order to maintain the efficiency of the wheels at the same revolutions under the wide range in heads.

Rapid advancement is now being made in turbine design both in this country and in Europe and the progress can best be known and appreciated by reference to the current technical press.

CHAPTER XIII.

TURBINE DETAILS AND APPURTENANCES.

139. **The Runner—Its Material and Manufacture.**—The runners of most reaction turbines (see Figs. 136, 142 to 149, 151, 154 to 159, 161) consist of hubs, crowns and rings, to which the buckets are attached. The wheels are sometimes cast solid, and sometimes built up. In built-up wheels the buckets are first cast, or otherwise formed, after which they are placed in a form or moulded, and the crowns, hubs and rings are cast to them. Turbine water wheels for low heads are usually made of cast iron or of cast iron with steel buckets. Wheels for high heads are frequently made of cast bronze or of cast steel. (See Figs. 158 and 159.)

Probably the majority of cast wheels manufactured at the present time are cast in one solid casting of buckets, rings, hubs, and crowns. The buckets are formed by carefully prepared cores and in such manner as to leave them uniform in spacing and thickness, and smoothly finished so as to admit of the passage of water through or between them without excessive friction. With wheels so cast, no material finishing or smoothing of the surfaces of the bucket is practicable, and the casting must come from the sand with a satisfactory surface. In wheels cast solid, great care is necessary in order to prevent serious shrinkage strains. This is partially overcome by the use of soft iron, which results, however, in increased wear of runners subject to the action of sand-bearing waters.

With buckets cast separately, a higher surface finish of the bucket is possible; but when separate buckets are made and afterwards united, the runner must be strongly banded in order to give it the necessary strength. Buckets of sheet steel, forged or bent to the desired shape, present a uniform and satisfactory surface, and when punched at the edges before casting, form a solid and substantial wheel.

The runners of Girard impulse wheels (see Fig. 171) are made in the same manner as reaction runners.

The runners of tangential wheels are usually made with separate buckets and body. (See Figs. 167 and 168.) The bodies are made according to the severity of the service, of cast iron, semi-steel, forged steel, etc. The buckets, dependent on the conditions of service, may be of cast iron, cast steel, gun metal, bronze, etc. The buckets, in the best wheels, are cast, shaped and polished and carefully fitted to the wheel body. The bolt holes are then carefully drilled and reamed and the buckets are bolted in position by carefully turned and fitted bolts.

140. Diameter of the Runner.—The diameters of reaction runners are measured at the inlet, and, when the buckets at the inlet are parallel and of one size, the determination of the turbine diameter is a simple matter. (See Fig. 174, diagram A.) In order to give the runner greater speed and capacity, the buckets are sometimes cut back at a point opposite the bottom of the gate opening (see diagram B), and the diameter of the runner opposite to the gates is reduced below that of the lower diameter. In such cases the edges of the buckets are sometimes made parallel with the shaft but are usually inclined upward. In the latter case, the diameter of the wheel at its top may be considerably reduced over its diameter at the offset. In such cases the cutting back of the runner may be one or more inches at the bottom line of the gate with an inch or more inclination to the top of the buckets, and the diameter of the wheel at D and D‴, diagram B, may differ from two to six inches or even more.

With wheels so constructed, there is considerable difference in the practice of different manufacturers in measuring and listing the diameter of the wheels made by them. In some cases, the inside diameter, from ring to ring, D, diagram B, of the runner, is given as the list diameter. In other cases, the diameter is taken at the inner angle of the offset as D′. In a number of cases the diameter is measured at about the center of the gateway, D″, and in other cases, the diameter is measured at the upper and smaller diameter of the runner, D‴. This variable practice leads to a considerable difference in the nominal diameter of the various turbines as listed in the catalogues, and frequently a runner listed as of a certain diameter by one manufacturer may be two to six inches larger than the runner of another manufacturer which is listed as of the same diameter. This discrepancy in the method of measuring and listing the diameter of turbine runners accounts, in some degree, for the apparent greater capacity, higher speed or greater power of the wheels of one manufacturer over those of another.

The practice of some of the American manufacturers of turbines, in measuring and listing the diameters of their wheels, is shown in Table XXV. In this table, all runners which are not cut back and with edges parallel to the shaft, are classified as Style A, even where they differ widely from the form shown in diagram A, Fig. 174.

All runners with buckets cut back are classified as Style B, even where the bucket edges are parallel with the shaft.

The diameters of tangential runners are usually measured between the centers of buckets or on the diameter of the circle on which the center of the jet impinges on the buckets.

TABLE XXV.

Practice of Various American Manufacturers in Measuring and Cataloging the Diameter of Turbine Water Wheels.

Manufacturer.	Name of Runner.	Style.	Point of measurement.
Dayton Globe Iron Works...	American	A	D
	New American.............	A	D
	Special New American.....	B	D''
	Improved New American[1]..	B	D'
Rodney Hunt Machine Co...	McCormick[2]...............	B	D
	Hunt......................	A	D
The James Leffel & Co.......	Standard Leffel............	A	D
	Special Leffel..............	A	D
	Samson....................	B	D
	Improved Samson..........	B	D
Platt Iron Works Co.........	Type A....................	B	D'
	Types B and C.............	A	D
S. Morgan Smith Co.........	McCormick[3]...............	B	D'
	Smith.....................	B	D'
The Trump Manufacturing Co.	Standard Trump[4]..........	B	D''
	High Speed Trump.........	B[5]	D
Wellman, Seaver, Morgan Co.	Jolly-McCormick	B	D'

[1] Fillet at angle. Diameter measured just above.

[2] Diameter of Hunt-McCormick runners as measured at the crown which projects beyond the tips of the buckets and is essentially the same in diameter as at D'

[3] Diameter of the Smith-McCormick runners is measured at the crown which projects beyond the tips of the buckets and is essentially the same in diameter as at D'.

[4] Diameter at D is 20% greater than at D''.

[5] Bucket of of high speed runner has parallel edges but is cut back as shown in B.

141. The Details of the Runner.—The reaction runner will vary in design with the conditions under which it is to operate and the experience and ideas of its designer. In American practice the manufacturer usually constructs a series of runners of similar homogeneous design; that is to say, each wheel of the series has all of its dimensions proportional to that of every other wheel of the series, and is of similar design in all of its parts.

On account of demands for considerable variations in speed or power, or on account of improvements which have been found desirable by reason of the demands of his trade, the manufacturer often designs and constructs several series of wheels, each of which is particularly adaptable to certain conditions which he has had to meet. (See Tables XXII and XXIV.) In such cases each series is best suited for the particular condition for which it was designed, and is not necessarily obsolete or superseded by the later series.

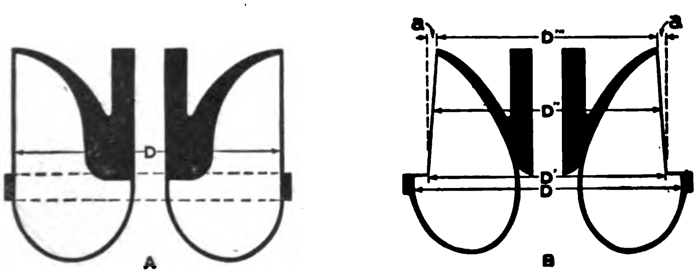


Fig. 174.

The curves of the runner buckets (see Figs. 13, 14, 133, 134, 136, 146–148, 175) must be such as to receive the jet of water from the nozzle or guides without shock, permit it to pass along the surface of the buckets or through the passages in the runner with minimum friction, and discharge it as nearly devoid of velocity as practicable.

To accomplish this, the relative position and relation of the curves of guides and buckets must be carefully arranged. As the jet of water is always directed forward in the direction of the revolution of the wheel, the jet has an original velocity in that direction, and, since the bucket must be so shaped as to give a continued contact, as the jet progresses and the wheel revolves, the portion of the bucket farthest away from the guides must be curved backward, and terminate at such an angle as shall permit the jet to pass away from the wheel with free discharge. (See Figs. 175 and 128.)

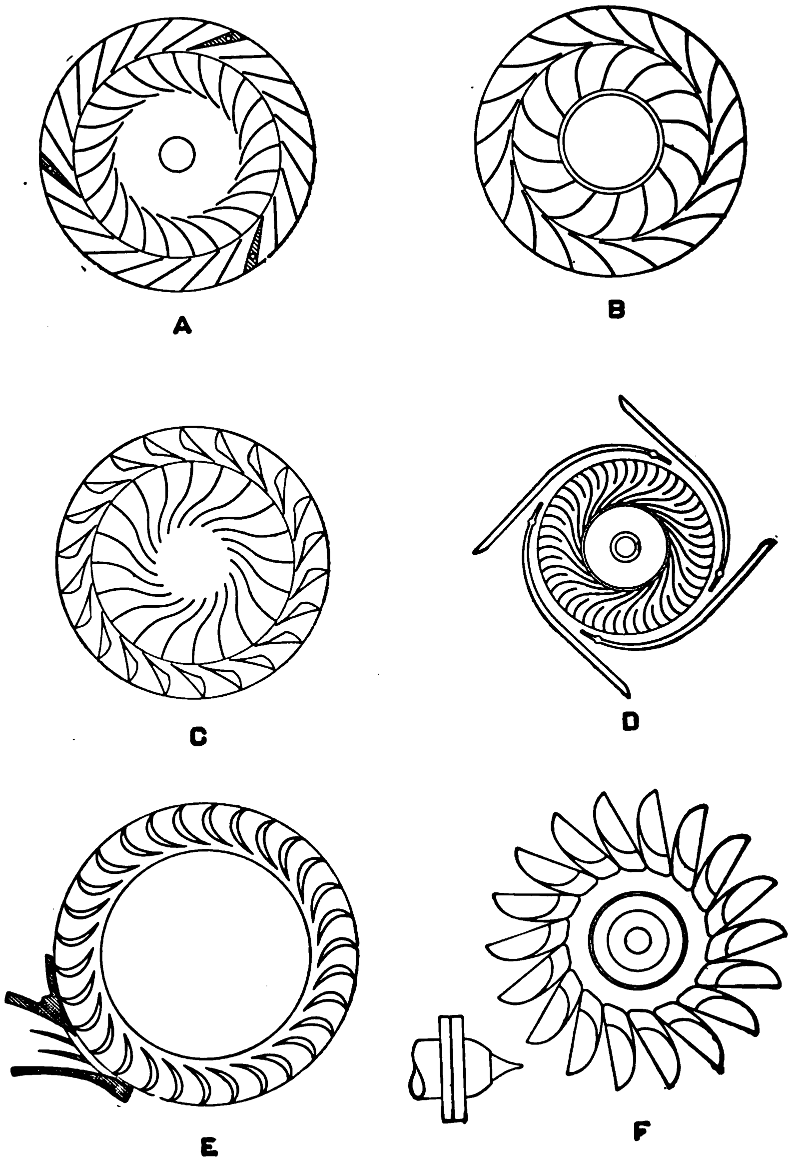


Fig. 175.—Curves of Buckets and Guides in Turbine Wheels.

Reaction runners are made either right or left handed as desired. When looking at the top of the runner, if the wheel is designed to move in the direction of the hands of a watch, it is called a right handed wheel, and if it moves in the other direction, it is called a left handed wheel. (See Fig. 176.)

The buckets, hub, crown, and ring of the reaction runner must be of sufficient strength to receive the impact or pressure of the moving column of water under the working head, and to transmit the energy to the shaft through which it is to be transmitted to the machinery to be operated.

A heavy ring is usually desirable, both to give strength and support to the outer edge of the buckets and also, under some circumstances, to give the effect of a fly-wheel in order to materially assist in maintaining uniform speed. Floating blocks or other material, in spite of the use of trash racks, sometimes reach the turbine, and when caught between the buckets and the case are apt to cause serious injury to the buckets.

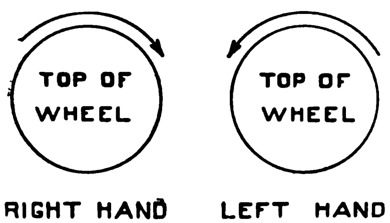


Fig. 176.—"Hand" of Water Wheels.

The runner is attached to a shaft passing through the hub, to which it should be closely fitted and strongly keyed to prevent its becoming loosened by vibration and the strain of operation. This is especially necessary in vertical wheels, for if, under these conditions, the wheel becomes loosened and drops from the shaft, it is apt to be practically destroyed. Impulse runners acting under high heads are subject to heavy shocks and must be especially substantial.

142. **Vertical Turbine Bearings.**—In all turbines where the discharge is axial and only in one direction, there is a reaction in the other direction that tends to unbalance the wheel and to cause a thrust in the direction opposite to the discharge. The leakage into the space back of the runner frequently produces a thrust in the opposite direction which may be wholly or partially relieved by openings left in the runner, usually close to the axis. In large units an attempt is made to balance these various pressures with some form of thrust bearing to sustain the difference in pressure which will occur under different conditions of operation.

In most single vertical turbines a simple step bearing is used. The bearing itself in American turbines usually consists of a lignum vitae block, turned to shape, and centered in a bearing block which is held firmly and centrally in place by the cross trees. The bearing block is shown by T, and the cross trees by t, in Figs. 146, 147 and 185. The bearing on the shaft itself is usually a spherical sector, or some other symmetrical curve of similar form. In some cases this bearing is cut directly in the shaft itself. (See Fig. 147.) In others, a cast iron shoe is provided and attached to the shaft. (See M, Figs. 145 and 184.) Above the turbine a second bearing is also provided (see T′, Figs. 145 and 147) to keep the shaft in vertical alignment. This bearing in American wheels is usually

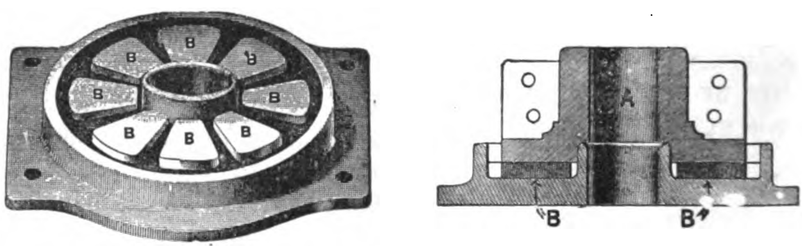


Fig. 177.—Geylin (Patent) Glass Suspension Bearing (R. D. Wood & Co.).

of the type shown in Fig. 182, except that it is adapted to its vertical position.

In the Geylin-Jonval turbine, manufactured by R. D. Wood Company, a patent glass suspension bearing is used. (Fig. 177.) This bearing is attached above the wheel (see T, Fig. 135) and has the advantage of being readily accessible. The turbine is here suspended on a circular disc composed of segments of glass, B. B. Fig. 177, arranged with depressed divisions which form a continuous space around each segment of which the disc is composed, allowing, while the turbine is in motion, a perfect, free circulation of the lubricating matter with which the space is filled.* The bearing is a true metallic ring, A, firmly secured to the turbine shaft which revolves on these stationary glass segments.

In most European vertical turbines the step bearing is simply a guide, the main bearing being above the turbine and more readily accessible than in the American form.

* Catalogue of R. D. Wood & Co., 1901, p. 107.

Figs. 178 and 179 represent vertical bearings of this kind. In these bearings C is a spherical sector so arranged as to take up any slight error in the vertical alignment of the shaft. Fig. 178 is a ball bearing; the hardened steel balls, AA, revolve between the special bearing plate, B and B_1.

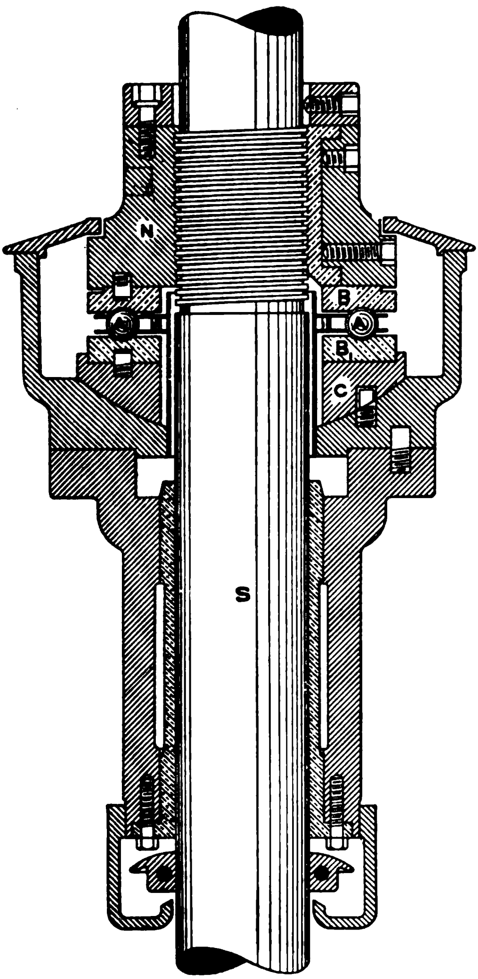


Fig 178.—Vertical Suspension Ball Bearing.*

In Fig. 179 oil is pumped under pressure through the inlet, pipe OE, into the space A. By its pressure the bearing plate, B, is raised from its companion plate, B, and the oil escaping between the plates lubricates them and overflows through the overflow pipe, OO.

In both Figs. 178 and 179 the height of the shaft is adjusted by the nut, N, which, after adjustment, is fastened securely in such position.

At the power plant of The Niagara Falls Power Company a thrust or hanging bearing of this disc type, somewhat similar to Fig. 179, is used (See Fig. 180). In this bearing the shaft is suspended to a revolving disc carried on a stationary disc. The discs are of close-grained charcoal iron of 25,000 pounds tensile strength and of 14″ inside, 34″ outside diameter. The lower or fixed disk is dowelled to a third disk with a spherical (3′ 4″ radius) seat. This

* Wasserkraftmaschinen von L. Quantz.

is to provide for an automatic adjustment for slight deviations from the vertical due to uneven wear of the discs and other causes.

The bearing surfaces between the discs are grooved to allow a circulation and distribution of the oil over the surface.

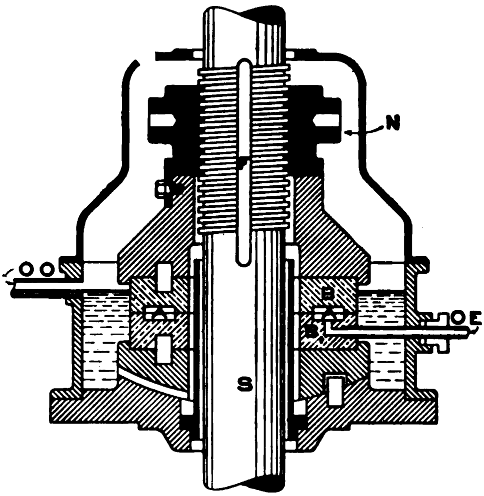


Fig. 179—Vertical Suspension Oil Pressure Bearing.*

Three methods of lubrication,—forced, self, and a combination system, were experimented with and the combination system finally adopted. In the system of forced lubrication, the oil enters the fixed disc at two diametrically opposite points and is forced between the discs under 400 pounds pressure. Self-lubrication is accomplished by oil supplied at the inner circumference of the disc and thrown outward by centrifugal force.

The disc bearings are enclosed in a case provided with sight holes through which the condition of the bearing as well as the temperature of the oil can be observed. A thermometer and an incandescent light are suspended in the casing for this purpose. The oil is cooled by water circulating pipes inside the casing.

The shaft is provided with a balancing piston (see Fig. 181) supplied with water from a pipe entirely independent of the penstock and under a head of 136 feet. This piston carries the greater part of the load, less than 2 per cent. of the load being left to be carried by the oil-lubricated disc bearing described above.

143. **Horizontal Turbine Bearings.**—In horizontal wheels various forms of bearing may be used according to the conditions and circumstances of their operation. When practicable the bearings should not be submerged and should otherwise be made as accessible as possible. In such cases the forms of bearings may be the same as those used on other machines subject to similar strains.

* Wasserkraftmaschinen von L. Quantz.

In many horizontal American wheels, where submerged bearings are necessary, lignum vitae bearings are used similar in type to the upper vertical bearing before mentioned (see T', Figs. 145 and 147). Such a bearing is shown in detail in Fig. 182. In this bearing the shaft, S, is sustained in position by the blocks, TT, which fit the

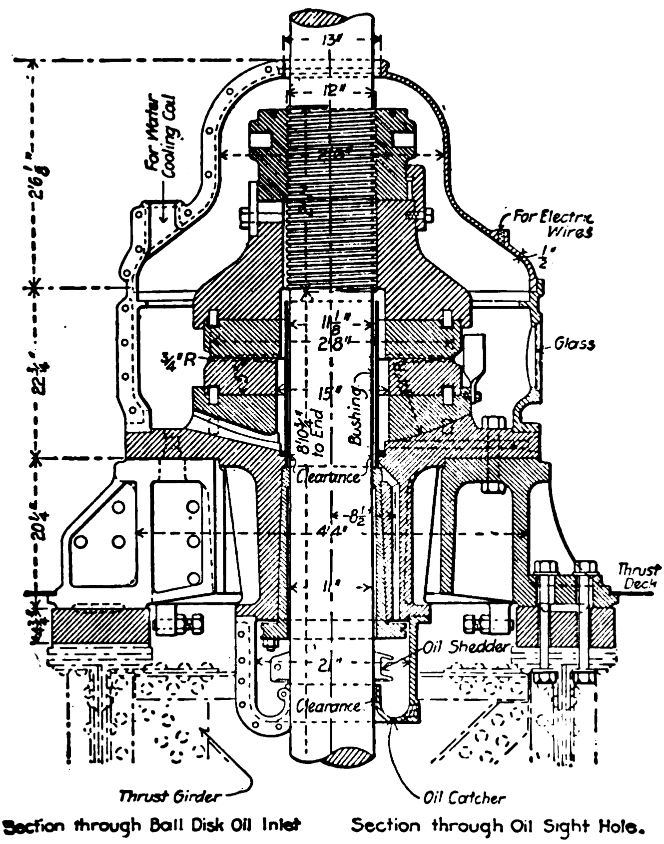


Fig. 180.—Vertical Thrust or Hanging Bearing of the Niagara Falls Power Co. (See Eng. Record, Nov. 28, 1903.)

recesses of the cast iron bearing block, K, which in turn is attached to a cross tie in the case or to a pedestal, P. The blocks, TT, are adjusted by means of the screws, BB, which, after adjustment are locked in position by the lock nuts, LL. Such submerged bearings are sometimes lubricated by water only, in which case opportunity must be given for the free circulation of the water. In other cases the boxes are made tight and flow into them along the shaft is prevented by stuffing boxes at each end of the main box, the boxes being lubricated by forced grease lubrication.

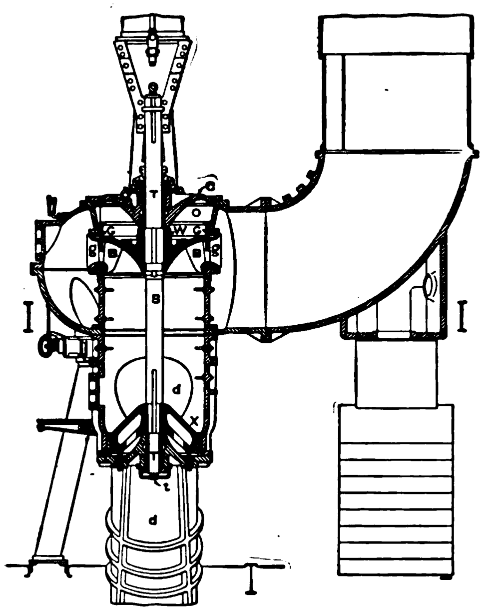


Fig. 181.—Section of Turbine used in new Power House of The Niagara Falls Power Company, showing Balancing Hydraulic Piston used to sustain Turbine and Shaft. (Eng. Record, Nov. 28, 1903.)

Bronze boxes of the types used for other high grade machines are sometimes used for submerged bearings. In such cases great care is necessary to prevent the entrance of grit-bearing waters. Such bearings are lubricated by forced oil or grease.

In forced lubrication it is desirable that both a force and return pipe be used so as to give visible evidence that the lubricant is actually reaching the bearing. In some cases bearings that would be otherwise submerged are made accessible at all times by metallic tubes (see Fig. 322) used as manholes.

Where the turbine is placed horizontally, gravity can no longer offset the thrust caused by the reaction of the turbine when the discharge is in one direction, and the thrust must therefore be overcome by the use of some form of thrust-bearing. Where other conditions permit, it is quite common practice to install two turbines on a single horizontal shaft, having their discharges in opposite directions, in which case the thrust of each turbine is overcome by the thrust of its companion (see Figs. 153, 160 and 316). In many cases, however, the arrangement, size and capacity of the wheels to be used are not such as will permit the use of twin turbines and thrust-bearing, and other means of taking up the thrust must be provided.

144. Thrust-Bearing in Snoqualmie Falls Turbine.—In the Snoqualmie Falls Turbine, manufactured by The Platt Iron Works Company (see Figs. 161 and 162), the device for taking up the thrust is thus described by the designing engineer, Mr. A. Giesler:*

"Single-wheel horizontal-shaft units are relatively infrequent in turbine practice, especially in large sizes, where the thrust of a single runner is large enough to require careful consideration. The thrust is made of two parts: (I) that due to the static pressure or effective head of water at the various points of the runner surface; and (2) that due to the deflection of the water from a purely radial

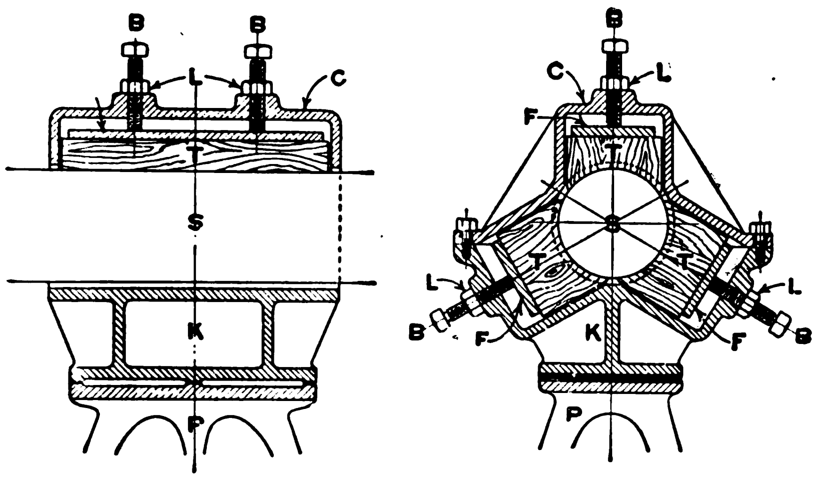


Fig. 182.—Horizontal Lignum Vitae Bearing as Used in American Turbines.

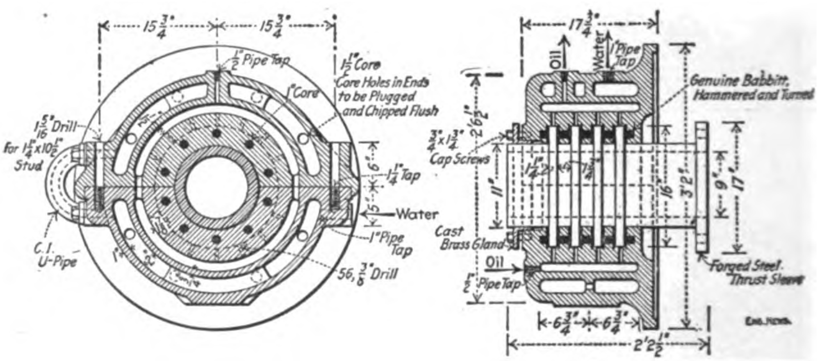
path through the wheel. As concerns the first part, the front face of the wheel is pressed upon by a pressure varying from the supply head at the outer circumference to the discharge pressure (vacuum) at the inner edge of the vanes, which latter extends over the whole central area of the runner (and shaft extension). The rear face of the runner is subjected to the pressure of water leaking through the radial air-gap between casing and runner, substantially equal to the supply head. This greatly over-balances the pressure on the front face, and the resultant thrust is to the right in Fig. 161 (toward the draft tube). The discharge ends of the vanes, being curved transversely, also have a pressure component directed to-

* See "Engineering News" of March 29, 1906.

ward the right. The velocity effect produces a thrust directed toward the left, but this is very small and does not materially reduce the pressure thrust.

"By far the larger part of the pressure thrust is eliminated by venting the space back of the runner into the discharge space. Six holes through the wheel near the shaft, indicated in Fig. 161, have this function. The water leaking in through the air-gap is continuously discharged through these vents into the draft-tube, and the accumulation of any large static pressure back of the wheel is thereby avoided.

"The average pressure on the front of the runner, however, is always lower, and the resultant thrust is therefore toward the draft-



Cross Section. **Longitudinal Section.**

Fig. 183.—Thrust-Bearing Snoqualmie Wheels.

tube, though its amount varies considerably, being greatest for full gate opening. This thrust is taken up by the balancing piston immediately back of the rear head of the wheel case, and the ultimate balance and adjustment of position is accomplished by the collar thrust-bearing behind the balancing piston.

"The balancing piston is a forged enlargement of the shaft, finished to a diameter of 17 inches, which works in a brass sleeve set in a hub-like projection on the back of the wheel-housing. The inside of the sleeve has six circumferential grooves, each one inch wide and one-quarter inch deep, as water packing. The chamber in front of the piston communicates by a pipe (containing a strainer) with the supply casing of the water-wheel, and therefore receives the full pressure of the supply head. The chamber back of the piston is drained to the draft-tube, so as to carry off any leakage past the piston. The device thus produces a constant thrust on the piston. directed toward the left. By throttling the pressure pipe this thrust can be adjusted as desired.

"The thrust-bearing shown in Fig. 161 and in detail in Fig. 183 consists of a group of four collars on the shaft, working in a babbitted thrust-block which is bolted to the back of the wheel-housing. The collars are formed on a steel sleeve which fits over the shaft and is bolted to the rear face of the balancing piston; this makes it possible, when the collars are worn out, to renew the bearing by dismounting the thrust-block and placing a new sleeve. The thrust-bearing is lubricated by oil immersion. An oil chamber is cored in the block and communicates by numerous oil holes with the bearing faces; a constant flow of oil is maintained by means of oil-supply and drain-pipes. Concentric with the oil chamber and outside of it a water chamber is cored in the block. Cooling water is supplied to this chamber by a pipe from the pressure side of the turbine, and drains from the top of the bearing through a drain-pipe to the draft-tube. A U-pipe attached at one side of the bearing forms connection between the water chambers of the upper and lower halves of the block. This detail avoids making the connection by a hole through the joint face, which would allow leakage of water into the oil-space and into the bearing.

"The balancing piston is so proportioned and the pressure supply pipe is throttled to such a point as to give exact balance (i. e., with zero thrust in the thrust-bearing) at about half to five-eighths the full output of the wheel. At larger power there will be an unbalanced thrust to the right, and at smaller output to the left, which are taken by the thrust-bearings. The maximum thrust on the collars is about 25,000 lbs. The collars are 2½ inches high (2⅜ inches effective) by 13½ inches mean diameter, giving a total effective bearing area on four collars of 418 sq. inches. The maximum collar pressure is thus about 60 lbs. per sq. in."

145. The Chute Case.—The chute case (see Figs. 146, 147 and 184) consists of the fixed portion of the turbine to which are attached the step and bearings of the wheel (T), the guide passages (g) which direct the passage of the water into the turbine bucket, and the gates (G) which control the entrance of the water, and also the case cover (C). The case cover keeps the wheel from contact with the water except as it passes through the guide and gates. To the chute case is usually attached the apparatus and mechanism for manipulating or controlling the position and opening of the gate. (A. P. Gr., etc.) In vertical turbines a tube, d, is usually attached to the lower ring, forming a casing in which the lower portion of the turbine revolves and on which the bridge tree, t, holding the step bearing is attached. When this tube is no longer than one diameter it is usually called the turbine tube; but when it is considerably extended, it is termed a draft tube.

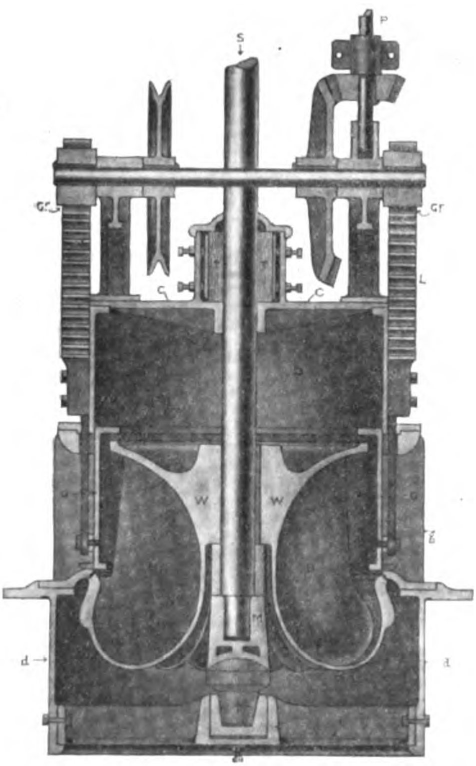


Fig. 184.

The design of the turbine tube depends largely on the character of the wheel. Some wheels discharge downward and inward, some almost entirely downward, some downward and outward, and in some cases, the wheel discharges in all three directions. For the best results the tube should be so designed that the water from the wheel shall be received by it with no radical change of velocity and so that the remaining velocity will be gradually reduced and the water discharged at the lowest practicable velocity.

The chute case and its appurtenances should be so designed that the water will enter the bucket with the least possible shock or resistance at all stages of gate and with a gradual change in velocity, and will discharge from the buckets into the turbine tube with as little eddying as possible and be evenly distributed over the cross section of the tube so as to utilize the suction action of an unbroken column of water. The case must also be designed of sufficient strength to sustain the weight of the turbine wheel and so that the step bearings are accessible and can be readily replaced or adjusted. The arrangement of the case must also be such that the openings between the wheel and the case are as small as practicable and the line of possible leakage will be as indirect as possible so as to avoid leakage loss.

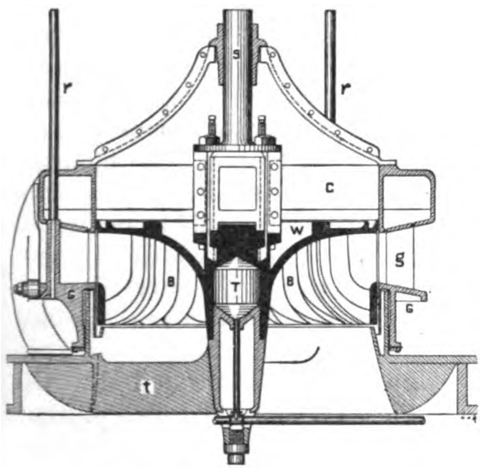


Fig. 185.—Section Swain Turbine.

Most chute cases are either cast or wrought iron. Cast iron usually lends itself to a more satisfactory design for receiving and passing the water without sudden enlargement and opportunities for losses by sharp angles and irregular passageways. Wrought iron, while not always lending itself readily to designs which eliminate all such losses, possesses much greater strength for a given weight which is a great advantage under some conditions.

146. Turbine Gates.—Three forms of gates are in common use for controlling the admission of water into reaction turbines. The cylinder gate consists of a cylinder closely fitting the guide that by its position admits or restricts the flow of water into the buckets. Fig. 184 is a section of a turbine of the McCormick type, manufactured by the Wellman-Seaver-Morgan Company, having a gate of this type, GG, between the guides and runners, which is shown closed in the cut. The gate is operated by the gearing, Gr., which raises it into the dome, O, through connection with the governor shaft, P. This same type of gate is used over the discharge of the Niagara-Fourneyron turbine (see GG, Fig. 134), over the inlet of the Geylin-Jonval turbine, GG, Figs. 135 and 137, and between the guides and buckets of the Niagara turbine. shown in Fig. 101.

A modified form of the cylinder gate is that used by the Swain Turbine Company (see Fig. 185), which is lowered instead of being raised into the dome as in Fig. 184.

A similar modification, called a sleeve gate by its designer, J. W. Taylor, is shown in Fig. 186.

When partially closed the cylinder gate causes a sudden contraction in the vein of water which is again suddenly enlarged in entering the runner after opening the gate. (See Fig. 188.) These conditions produce eddying which results in decreased efficiency at part gate. (See Figs. 185 and 186.)

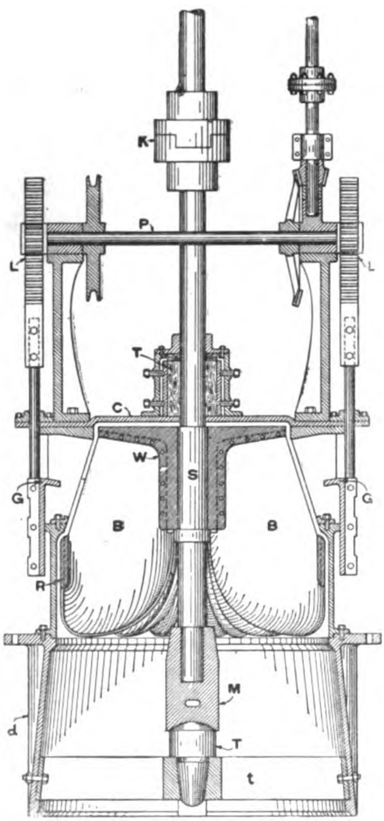


Fig. 186.—Section Taylor Sleeve Gate.

The wicket gate, when well made, is perhaps the most satisfactory gate, especially for moderate or high heads. It can be readily balanced and should be made with perhaps a tendency to drift shut, so that should the governor mechanism break or become disabled, the gates will drift shut. These gates are illustrated by GG, Figs. 147 and 148, which illustrate the wicket gate of the Samson turbine of The James Leffel & Company, and Fig. 187 which shows the wicket gate of the Wellman-Seaver-Morgan Company. In both cases the wickets are connected by rods with the eccentric circle and through an arm and section with the gearing Gr.

Figs. 145 and 146 show the wicket gate of the Improved New American, and Figs. 161 and 162 show the wicket gates of the Snoqualmie Falls turbine, manufactured by The Platt Iron Works. In both the New American and Snoqualmie wheels, the gates are moved by a gate ring (see Gr. Fig. 145). Figs. 189 and 190 show the details of the wicket gates and connection of the same to the gate ring of the Snoqualmie Falls Turbine.

The tendency to produce eddying is much reduced in well designed wicket gates, although the sudden enlargement of the reduced vein at part gate undoubtedly reduces the efficiency of the wheel. (See Fig. 191, A and B.)

The register gate (see G, Fig. 192) consists of a cylinder case with apertures to correspond with the apertures in the guides, g, and is so arranged that, when in proper position, the apertures register and freely admit the water to the wheel, and is also so constructed that when properly turned the gate cuts off the passage completely or partially as desired.

Considerable eddying is produced by the partially closed register gate, with a consequent decrease in part gate efficiency. (See Fig. 193.) The cylinder gate is usually the cheapest and most simple form of gate, but the wicket gate, if properly designed and constructed seems to admit of the entrance of water into the bucket with least possible resistance and eddying, and in the most efficient manner. This form of gate is the most widely used in high-grade turbine construction at the present time, although the cylinder gate is largely in use and has given satisfactory results.

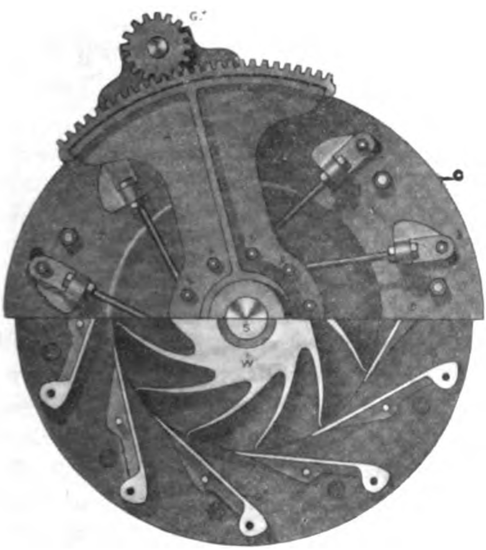


Fig. 187.—Wicket Gate of the Wellman-Seaver Morgan Co.

In some cases the passage of water is restricted or throttled by the use of a butterfly valve, either in the inlet or in the turbine tube. This throttles the inlet or discharge and regulates the head in a very inefficient manner, but may be reasonably satisfactory where economy of water is unnecessary.

In impulse wheels the gates are usually so arranged that the guide passages are opened one at a time instead of all opening partially, as in part gate conditions with the reaction wheel. This results in less loss in the eddyings caused by part gate. Fig. 194 shows the type of gate used by The Platt Iron Works in their Girard turbines where the guide passages are arranged symmetrically in three groups about the wheel. In the tangential wheel, where a single nozzle is used, the most efficient method found for reducing the opening is with the needle as illustrated in Fig. 195. This figure shows a cross section of the Doble needle nozzle, a form which gives a high velocity coefficient under a very wide range of opening. The character of the stream from a needle nozzle when greatly reduced is shown by Fig. 196 where the clear and solid stream gives evidence of high efficiency. If the flow of water through the nozzle is regulated by throttling the water with a valve before it reaches the nozzle, a very low efficiency results.

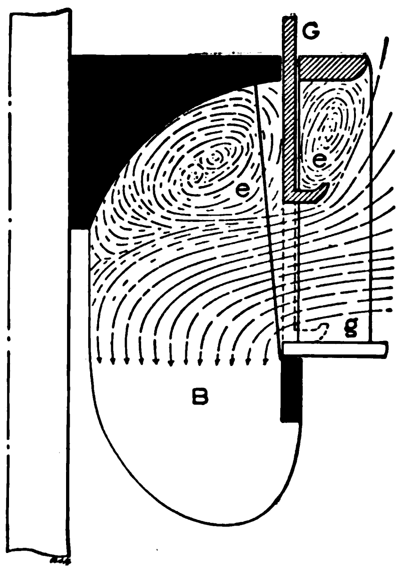


Fig. 188.—Showing Cylinder Gate Partially Open and Eddies Caused by Sudden Contraction and Enlargement of Entering Vein of Water.

147. **The Draft Tube.**—The reaction wheel is of particular advantage under low heads on account of the fact that it can run efficiently under water, and therefore, under backwater conditions, can be made to utilize the full head available. It is not necessary, however, to set the reaction wheel low enough so that it will be below water at all times for the principle of the suction pipe can be utilized and the wheel set at any reasonable distance above the tail water and connected thereto by a draft tube which, if properly arranged, will permit the utilization of the full head by action of the draft or suction pull exerted on the wheel by the water leaving the turbine through the tube from which all air has been exhausted. The water issuing from the turbine into a draft tube, which at the starting is full of air, takes up the air in passing and soon establishes the vacuum necessary for the draft tube effects. The function of the draft tube is not only to enable the turbine to utilize by suction that part of the fall from the wheel discharge to the tail water level, but it should also gradually increase in diameter so as

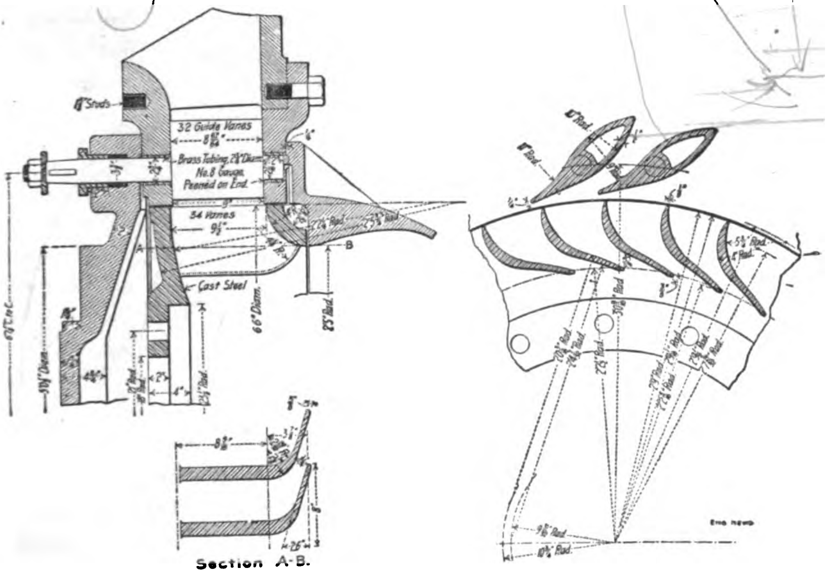


Fig. 189.—Showing Relations of Gate Guides and Buckets in Snoqualmie Falls Turbine (Platt Iron Works Co.).

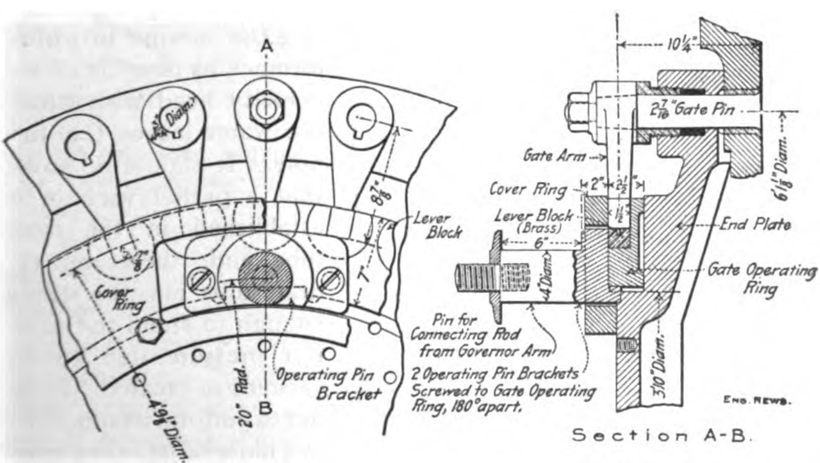
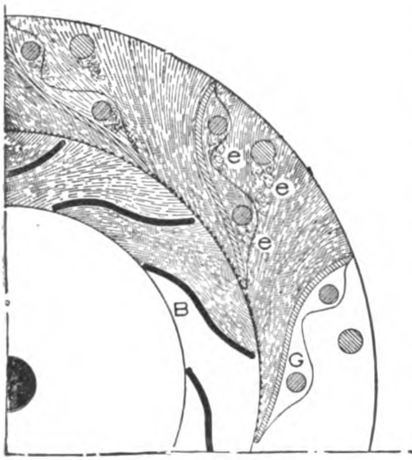
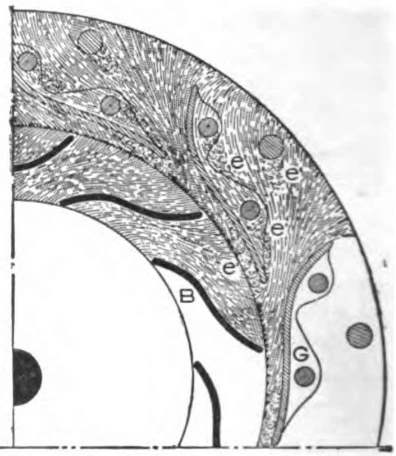


Fig. 190.—Showing Rigging for the Operation of Wicket Gate in Snoqualmie Falls Turbine (Platt Iron Works Co.).



A. Gate wide open.



B. Partial gate.

Fig. 191.—Showing Condition of Flow Through Open and Partially Closed Wicket Gates.

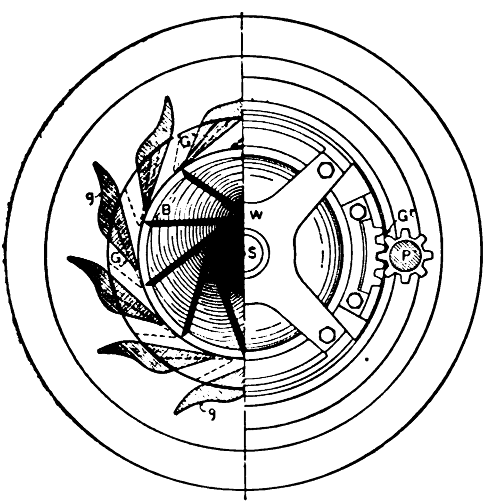


Fig. 192.—Register Gate (Platt Iron Works Co.).

to gradually decrease the velocity of the water after it is discharged from the turbine wheel, thus enabling the turbine to utilize as much as possible of the velocity head with which the water leaves the turbine. It should be noted that a partial vacuum is established in the draft tube and, therefore, the draft tube must be strong enough to stand the exterior pressure due to the vacuum so created. In order to perform its functions in a more satisfactory manner, it must also be made perfectly air tight.

One of the great advantages in the use of the draft tube is the possibility, by its use, of setting the wheel at such an elevation above the tail water that the wheel and its parts can be properly inspected, by draining the water from the wheel pit. Otherwise it would be necessary to install gates in the tail race and pumps for pumping out the pit in order to make the wheel accessible. Theoretically, the draft tube can be used of as great length as the suction pipe of a pump, and this is probably true of draft tubes for very small wheels. Practically, the draft tube should seldom be as long as 20 feet, especially for large wheels, for its success in the utilization of the head depends on the maintenance of an unbroken column of solid water, which is difficult to maintain in large tubes. As the size of the wheel increases the difficulties of maintaining a vacuum increase and the length of the draft tube should correspondingly decrease. It is practically impossible to maintain a working head with large turbines through long draft tubes with the turbine set at great distances above the water. Long draft tubes should, as a rule, be avoided and in all cases where draft tubes are used, they should be as straight and direct and as nearly vertical as possible. It is the principle of the draft tube that permits horizontal shaft wheels to be utilized, as otherwise, with this type of machinery, only a small portion of the head could be used to advantage under normal conditions, for such wheels being often direct connected to the machinery are, of necessity, placed above the tail water. The draft tube is commonly of iron or steel, but in plants where concrete construction is used the draft tube may be formed directly in the concrete of the station or wheel foundations.

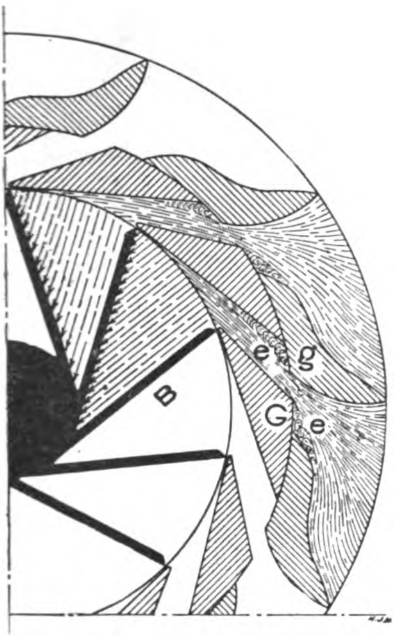


Fig. 193.—Showing Eddying Caused by Partial Closure of Register Gate.

On the Fourneyron turbine Boyden used what he termed a diffuser. (See Fig. 197.) The main purpose of the diffuser, and of the conical tube as well, is to furnish a gradually enlarged passage through which the velocity of the water as it leaves the wheel is

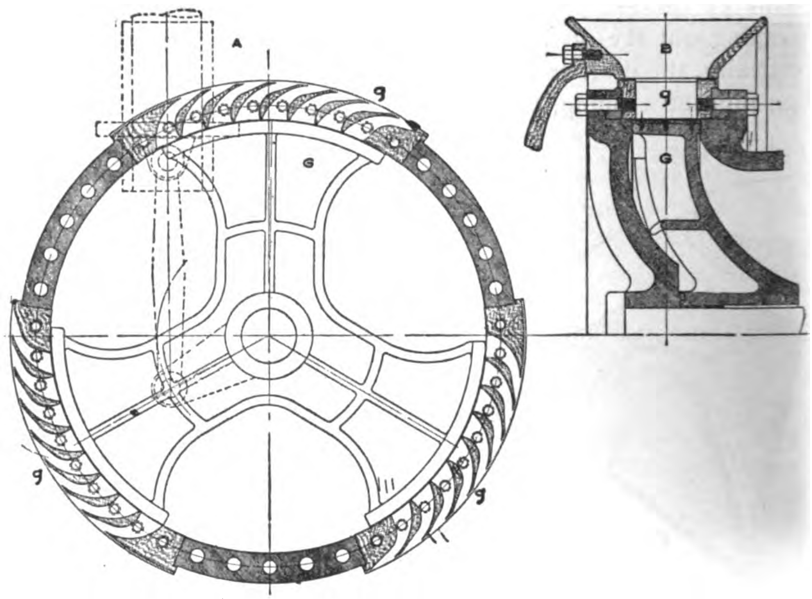


Fig. 194.—Gates and Guides of Girard Impulse Turbine. (Turbine Design as Modified for Close Speed Regulation, G. A. Buvinger, Proc. Am. Soc. M. E., Vol. XXVII.)

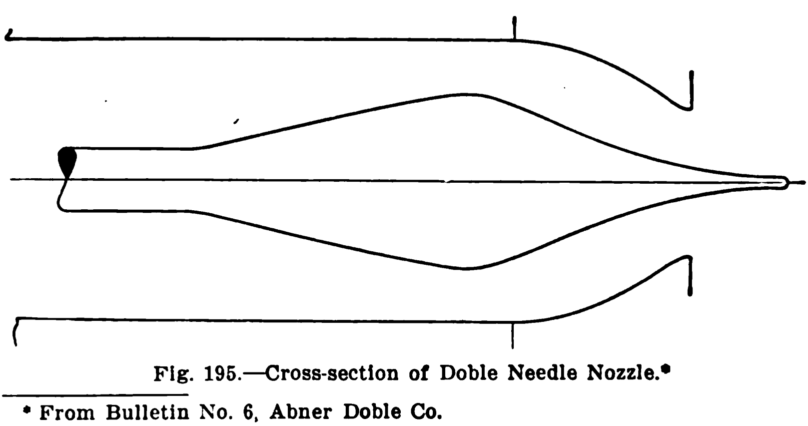


Fig. 195.—Cross-section of Doble Needle Nozzle.*

* From Bulletin No. 6, Abner Doble Co.

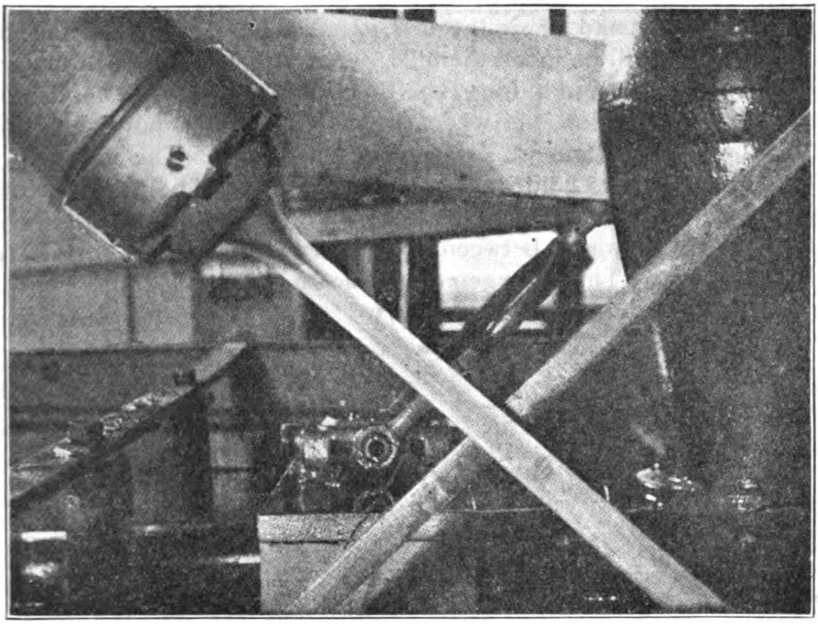


Fig. 196.—Stream from Doble Needle Nozzle.*

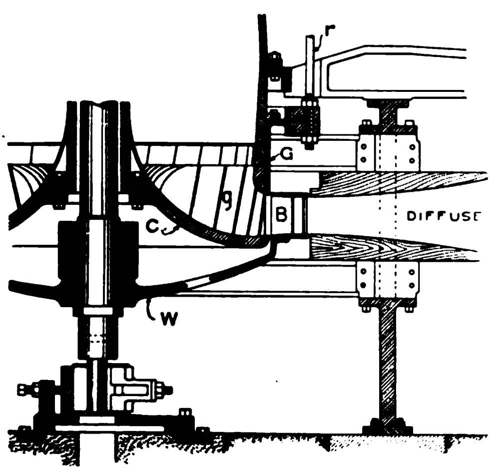


Fig. 197.—Boyden Diffuser.

so gradually reduced as to enable the velocity head to be utilized in the wheel, thus saving head which would otherwise be lost.

It has already been noted that impulse wheels of the Pelton and Girard types cannot operate satisfactorily submerged, and must be set at such positions that they will be above the tail water at all times. In many localities where the variation in the surface of tail waters is considerable, this means a large relative loss in the head utilized and that this type of wheel will therefore not be practicable except under high

* From Bulletin No 6, Abner Doble Co.

head conditions and where the loss entailed by the rise and fall of the tail water will be inconsiderable. An attempt has been made, however, to so design a draft tube that a vacuum will be established and maintained below the wheel, in such a manner, however, that the water will not come in contact with the wheel. The vacuum is so maintained as to hold the water at an established point just below the wheel, thus permitting the wheel to utilize the full head except for the small clearance between the wheel and the water surface in the draft tube. This arrangement is shown in Figs. 168 and 171, as applied by The Platt Iron Works Company to a Girard turbine.

CHAPTER XIV

HYDRAULICS OF THE TURBINE.

148. Practical Hydraulics of the Turbine.—It is not the purpose of this chapter to consider mathematically and at length the principles of hydraulic flow in relation to the curves of guides and buckets and the effects of such curves on the power and efficiency of the turbine. These relations are expressed by long and involved equations of considerable interest to the engineer who is to design and construct the turbine but of little practical value to the engineer who is to select and install it in a water power plant. Few of the designers of American wheels have given much attention to the involved mathematics of hydraulic flow in the turbine and the designs of most American wheels are based on the results of experiment and broad practical experience. The designs of Swiss and German wheels are, to a much greater extent, based on mathematical analysis. It is an open question whether the best work of either American or foreign manufacture shows any marked superiority in comparison with the other. The results actually attained in the manufacture of wheels in this country seem to show that the American practice in wheel design will give equal and even more uniformly satisfactory results than the European methods,—at least as carried out by foreign engineers under American conditions.

Correct theory must be the basis of all successful work. The theory of the experienced man may be unformulated and unexpressed, but correct design has always a correct theory as its basis even if unrecognized as such, and such a theory properly applied will lead to correct results. On the other hand, formulated theory will lead to correct results only as far as the theory is correct and takes into account all controlling or modifying factors and is properly applied. A correct theory, carefully formulated and properly applied, cannot fail to be of great service to the engineer in extending his experience to wider fields. Scientific study and mathematical analysis of the turbine, based on wide experience and careful experiments, can but lead to the accomplishment of better results than have yet been attained.

An understanding of certain laws of flow through turbines as confirmed by both theory and practice is essential to a proper comprehension of the principles which should govern the selection and installation of such wheels and these laws are considered in this chapter.

149. Nomenclature used in Chapter.—In the discussion in this chapter the letters and symbols used have the following significance:

a = Area of gate orifice or orifices.
α = Angle of deflection of jet.
β = Supplement to angle of deflection = $180° - \alpha$.
D = Diameter of wheel in inches.
E = Energy in foot pounds per second.
F = Force producing pressure or motion.
g = Acceleration of gravity.
h = Effective head at the wheel.
n = Number of revolutions per minute.
n_1 = Number of revolutions per minute for head h_1.
π = Ratio of circumference to diameter = 3.1416
P = Horse powers of turbine at any given head.
P_1 = Horse power of turbine at head h_1.
q = Discharge in cubic feet per second at any given head.
q_1 = Discharge in cubic feet per second at head h_1.
r_1 = Internal radius of wheel.
r_2 = External radius of wheel.
S = Space passed through by force acting.
u_1 = Velocity of wheel at gate entrance.
u_2 = Velocity of wheel at point of discharge.
v = Theoretical spouting velocity due to head = $\sqrt{2gh}$
v' = Velocity of the periphery of the impeller, in feet per second.
v_1 = Absolute velocity of water entering the wheel.
v_2 = Absolute velocity of water leaving the wheel.
v_r = Relative velocity of water entering the wheel.
v_R = Relative velocity of water leaving the wheel.
v_a = Average velocity.
W = Total weight per second.
w = Weight of unit of water = 62.5 lbs.
φ = Ratio peripheral velocity of wheel to spouting velocity of water $= \frac{v'}{v}$

TURBINE CONSTANTS.

C = Coefficient of discharge of gate orifice or orifices.
$\triangle$ = Constant relation of turbine diameter and speed.
K = Constant relation of turbine diameter to discharge.
K_2 = Constant relation of turbine diameter to power.
K_3 = Constant relation of peripheral velocity.
K_4 = Coefficient of relation of turbine speed and discharge.
K_5 = Coefficient of relation of turbine power and speed. (Specific speed.)

150. First Principles.—In the utilization of water for power purposes it is the first principle of design that *the water should enter the wheel without shock and leave it without velocity.* This should be interpreted to mean that the approaches of the water to the wheel must be such as to cause no loss by undue friction or by sudden contractions or enlargements (inducing eddies and other sources of lost energy), and that all shocks should be confined as far as possible to the action on the wheel buckets leaving the full amount of energy, and consequently the velocity, to be entirely converted to power therein.

In gravity wheels, illustrated by the various overshot wheels formerly so extensively used for water power purposes, the water should enter the wheel at the lowest practicable velocity and should be retained in the buckets until the buckets have made the greatest possible descent from the nearest practicable approach to the elevation of head-water, to the nearest practicable approach to the elevation of the tail water. Part of the velocity of approach to the wheel may be utilized by impact on the buckets but the entire energy remaining in the water as it falls or flows away from the wheel is lost, and cannot be further utilized in the wheel.

The greater the reduction in velocity, the greater the proportion of energy that can be utilized, but there comes a limit beyond which it is not practicable to go. This limit varies with different conditions and may be the result of too great expense in the building of raceways or in the construction of the machine itself. A point will be reached where the friction expended in the large machine needed to reduce the velocity will consume more energy than would be lost in inducing a higher velocity. These losses must be equalized. In practice it is found that about two or three feet per second are satisfactory velocities at which to reject or discharge the water used by motors. These velocities represent heads of from .062 to .014 feet, or from three-quarters to slightly less than two inches. The velocity of discharge must, however, be fixed for each individual case and after all conditions are fully understood and considered.

151. Impulse and Reaction.—A jet of water spouting freely from any orifice will acquire a velocity (see Eq. 9, Chap. II).

$$v = \sqrt{2gh} \tag{1}$$

and will possess energy in foot pounds per second (see Eq. 10, Chap. II) as follows:

$$E = \frac{Wv^2}{2g} = \frac{qwv^2}{2g} \tag{2}$$

The energy of the jet leaving the orifice is the product of a force, F, which acting on the weight of water, qw, for one second gives it the velocity v.

The space passed through by the force in one second, in raising the velocity from 0 to v is (see Eq. 6, Chap. II)

$$S = v_a t = \frac{v}{2} \tag{3}$$

and the work done in foot pounds is therefore

$$E = FS = \frac{Fv}{2} \tag{4}$$

From Equations 2 and 4 therefore

$$\frac{Fv}{2} = \frac{qwv^2}{2g} \text{ and therefore} \tag{5}$$

$$F = \frac{qwv}{g} \tag{6}$$

The force, F, is exerted, by reaction on the vessel of which the orifice is a part and may produce motion in that vessel if it be free to move, or it may produce motion in another body by impulse through the extinction of the momentum of the jet in impinging against it.

These equal and opposite forces are well shown:

1st. By the force required to sustain a hose nozzle against the reaction of a fire stream, and

2nd. By the force of the jet, from the nozzle so sustained when exerted against any object in its course.

These conditions are illustrated by Figs. 198 and 199.

The force, F, which may be exerted by a jet impinging against a surface depends on the momentum of the moving stream of water and is directly proportional to its velocity. It is also a function

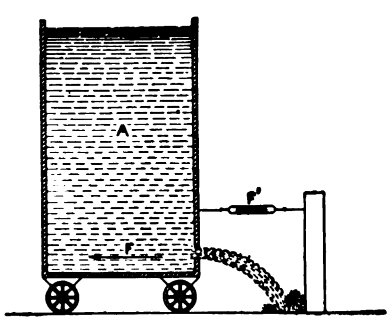


Fig. 198.

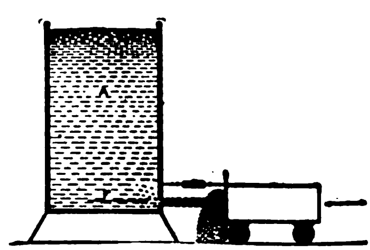


Fig. 199.

of the angle through which the jet is deflected. If friction be ignored, the stream will be deflected without change in velocity, and the force exerted against the surface in the original direction of the jet will be equal to the momentum of the original stream less

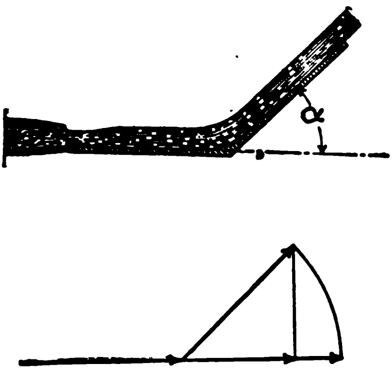


Fig. 200.

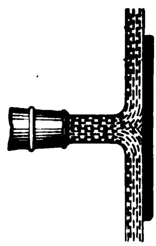


Fig. 201.

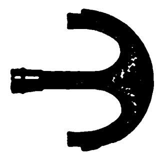


Fig. 202.

the component, in the original direction, of the momentum of the diverted jet. (See Fig. 200).

$$F = \frac{qwv}{g} - \frac{qwv}{g}\cos\alpha = \frac{qwv}{g}(1 - \cos\alpha) \quad (7)$$

If the jet impinges against a flat surface (see Fig. 201)

$$\alpha = 90^\circ, \text{ Cos } \alpha = 0 \text{ and}$$

$$F = \frac{qwv}{g} \quad (8)$$

If the jet is deflected 180° by means of a semi-circular bucket (see Fig. 202)

$$\text{Cos } 180^\circ = -1, \text{ and therefore}$$

$$F = 2\frac{qwv}{g} \quad (9)$$

152. **The Impulse Wheel.**—Impulse water wheels utilize the impulsive force of a jet impinging against buckets attached to the circumference of the wheel. The bucket must move under the impulse in order to transform the energy of impact into work and the ratio of v′, the velocity of the periphery of wheel, to the velocity v of the jet is indicated by ϕ

$$\varphi = \frac{v'}{v} \text{ and } v' = \varphi v \quad (10)$$

In determining the force, F, exerted upon the moving bucket, the relative instead of the actual velocity of the jet must be considered and it is readily seen that the value of the relative velocity v_r will be as follows:

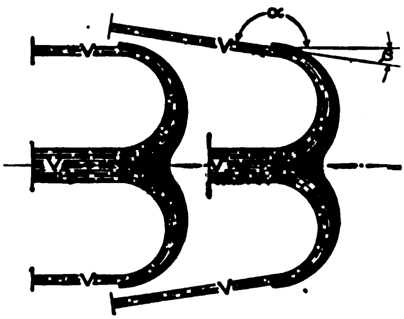


Fig. 203.

$$(11) \qquad v_r = v - \varphi v = (1 - \varphi) v$$

The relative weight of water that strikes a single bucket per second will also be less on account of the movement of the buckets, but as new buckets constantly intercept the path of the jet the total amount of water effective is equal to the total discharge of the jet. Hence from equations (7 and 11)

$$(12) \qquad F = (1 - \cos \alpha) \frac{qwv}{g} (1 - \varphi)$$

The work done upon the buckets per second is equal to the force, F, times the distance ϕ v through which it acts, i. e.

$$(13) \qquad E = F \varphi v = (1 - \cos \alpha)\,(1 - \varphi) \frac{qwv}{g} \varphi v$$

This is a maximum when $(1-\phi)\,\phi$ is a maximum the solution of which gives $\phi = .5$

Substituting $\phi = .5$ and $\alpha = 180°$, in equation (13), there is obtained

$$(14) \qquad E = \frac{qwv^2}{2g}$$

That is, E equals the entire energy of the jet (see equation 2), and hence the theoretical efficiency when $\phi = 0.5$ is 100 per cent.

Another criterion for maximum efficiency is that the absolute velocity of the water in leaving the bucket must be zero.

When $\alpha = 180°$, the absolute velocity with which the water leaves the bucket is evidently the velocity relative to the bucket minus the velocity of the bucket or

$$(15) \qquad v_a = (1 - \varphi) v - \varphi v = v - 2\varphi v = 0$$

This gives

$$\varphi = 0.5$$

153. **Effect of Angle of Discharge on Efficiency.**—In an impulse wheel it is not practicable to change the direction of the water through 180° as it would then interfere with the succeeding bucket. α must hence be less than 180° and the absolute velocity of the water in leaving the buckets cannot be zero. The loss from this source is small as α may differ considerably from 180° without much effect on the bucket pressure and hence on the efficiency. For example,—the ratio of actual pressure when α is less than 180° to maximum possible pressure with α = 180° is (see Fig. 203).

$$(16)\qquad \frac{(1-\cos\alpha)\,qw\,(1-\varphi)\dfrac{(1-\varphi)\,v}{g}}{(1-\cos 180^\circ)\,qw\,(1-\varphi)\dfrac{(1-\varphi)\,v}{g}} = \frac{1-\cos\alpha}{2} = \frac{1+\cos\beta}{2}$$

$$\text{If } \beta = 8^\circ,\quad \alpha = 172^\circ, \text{ and } \frac{1-\cos\alpha}{2} = .995$$

showing only 0.5 per cent. reduction. The effect on the efficiency is in the same ratio.

Fig. 204 illustrates the flow of the water in entering and leaving the bucket with all velocities given relative to that of the bucket. The jet leaves the bucket as shown with a relative velocity of $(1-\phi)$ v. If this velocity is combined graphically with the velocity of the bucket, ϕv, the true absolute residual velocity v_R of the water will be obtained. The efficiency is evidently maximum when ϕ has a value which makes v_R a minimum. This condition can readily be shown to maintain when the triangle is isoceles or when

$$(17)\qquad \varphi\, v = (1-\varphi)\, v$$

which gives

$$\varphi = 0.5$$

as obtained by two other methods and here shown to be independent of the angle β.

The absolute path of the water in space is shown by ABCD Fig. 204, and the magnitude of this velocity is shown below in curve EF where ordinates are absolute velocities along the tangent lines to curve ABCD at the point directly above. These curves are based on the assumption that ϕ=0.5 and the bucket is semi-circular in cross section as shown.

The theoretical considerations thus far discussed are modified by the frictional resistance which the bucket offers to the flow of water over its surface and by the spreading of the original jet from its semi-circular section to a wide thin layer in leaving the bucket.

Further loss no doubt takes place as a result of the fact that the bucket is in its assumed position at right angles to the direction of the jet only at one instant during its rotation. Upon entering

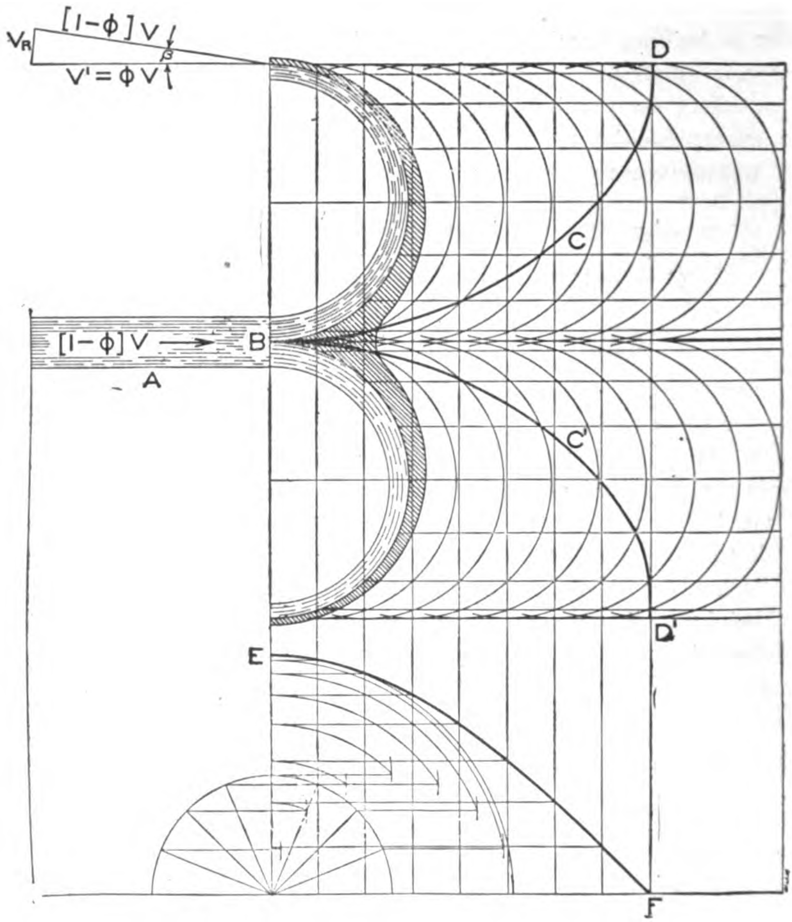


Fig. 204.

and leaving the jet it is inclined considerably to this direction and doubtless operates less efficiently. These conditions result in a much greater drop in efficiency than the above analysis would seem to indicate.

154. Reaction Wheel.—The flow of water through the buckets of a reaction wheel is less easily analyzed than in the case of the impulse wheel. The chief difference in the two types of wheels arises from the fact that the reaction wheel is "filled" and hence the velocity of the water relative to the buckets at any point does not remain constant but varies inversely as the cross sectional area of the passageway.

The path described by a particle of water in passing through the

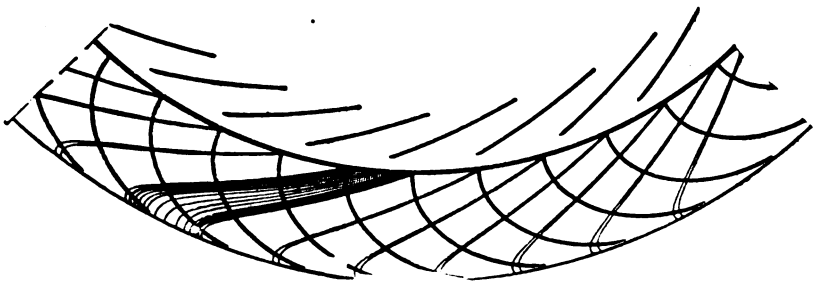


Fig. 205.

wheel has been investigated by Francis,* by a method based upon the assumption that "every particle of water contained in the wheel, situated at the same distance from the axis, moves in the same direction relative to the radius and with the same velocity." This assumption becomes more accurate as the number of buckets increases.

Fig. 205 shows the path, resulting from the application of this assumption, of the water through the "Tremont" Fourneyron wheel and Fig. 206, through the center vent wheel at the Boott Cotton Mills. The former indicates, since the jet of water is carried forward in the direction of rotation, that the water resists the rota-

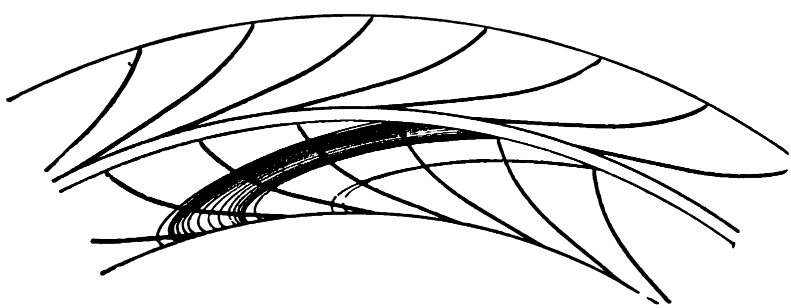


Fig. 206.

* See "Lowell Hydraulic Experiments," p. 39.

tion of the wheel until nearly to the circumference when it is suddenly deflected and leaves the wheel, as it should, in a direction nearly normal to the wheel.

The jet of water in the Boott wheel (Fig. 206), on the other hand, shows a continual backward deflection of its path from the point where it leaves the guides, and hence a continual delivery of its energy to the wheel. This seems to indicate a more logical condition and a better shaped bucket than that of the Fourneyron. It will be noted that the actual path of the water in this case is very similar to that in the impulse wheel shown in Fig. 204.

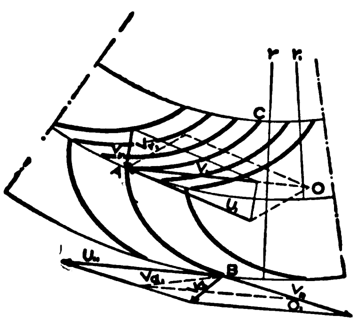


Fig. 207.

For the economical operation of the reaction wheel the following principles must be observed:

1st. In order that the jet of water may enter the wheel without shock the resultant of the velocity of the water as it leaves the guides and the velocity of the periphery of the runner must have a direction parallel to the bucket blades at this point, and a magnitude equal to that which will produce the required discharge through the cross sectional area of the passageway.

2nd. The relative velocity of the bucket and of the water relative to the bucket at the point of discharge must be such that the water leaves the buckets with the minimum practicable absolute velocity.

3rd. Such residual velocity as may remain in the discharging water must be conserved and utilized as far as practicable by the proper arrangement of the draft tube.

4th. In all wheels it is also essential by proper design to reduce losses from friction, eddying, etc., as greatly as possible.

The first requirement is illustrated in Fig. 207 where AB is one of the runner buckets of an outward flow wheel. The guides, AC, direct the water into the buckets with an absolute velocity, v_2. The velocity of the runner at point A, where the water enters, is u_2. The two velocities combined graphically give a resultant, v_r, which must be tangent to the curve of the bucket and equal to

(18) $$v_r = \frac{q_2}{a_2} \text{ where}$$

q_2 = required discharge through the passageway, and
a_2 = area of cross section of the passageway at point of entrance, A.

This requirement does not enter into the design of an impulse wheel since the jet impinges against the edge of the wedge-shaped partition in the bucket always in a direction tangent to the bucket curve at that point regardless of the relative speeds of runner and jet. Further, since the discharge is "free" and the buckets not "filled," no sudden change of velocity occurs.

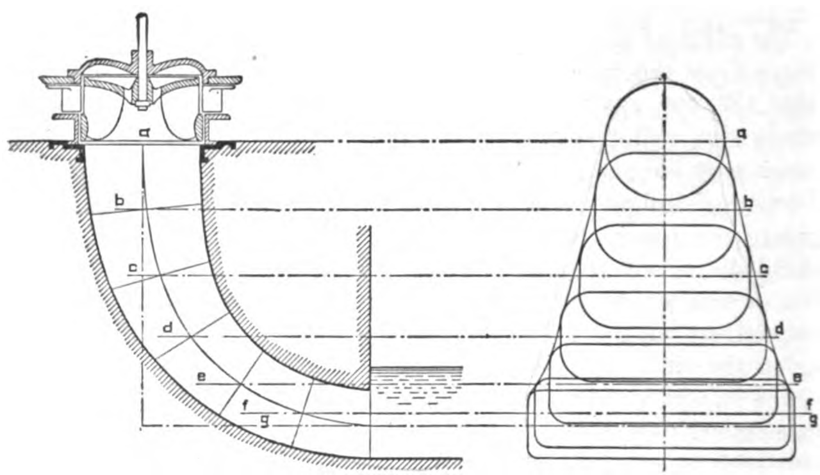
The effect of part gate conditions upon the first requirement depends upon the type of speed gate and may best be studied from Figs. 188, 191, 193 and 207. A change in either direction or magnitude of v_1 will change v_r unless the two effects tend to neutralize which may happen in some instances. In all reaction wheels the velocity of inflow, v_1, through the guides is increased by partly closing the gate, while the velocity, u_1, of the wheel remain unchanged. v_r will therefore change, and a change in either its direction or magnitude will produce an impact or sudden enlargement respectively as the water enters the runner, and therefore a loss, unless the direction of the guides is changed to correspond.

The wicket gate, when carefully designed, has given rise to part gate efficiencies more nearly approaching those of impulse wheels than with gates of any other type (see Figs. 131 and 236).

The second requirement, that of minimum residual velocity of the water in leaving the buckets, is shown graphically in Fig. 207. v_R is the velocity of discharge of the water relative to the bucket and is, of course, tangent to the curve of the bucket. u_2 is the peripheral velocity of the runner. The resultant of two velocities is the absolute velocity with which the water is discharged from the wheel, and is shown in magnitude and direction by line v_2. Now, at part gate the quantity of water discharged is less than that at full gate and hence v_R must also be less since the cross section of the passage must be filled. u_2 remains unchanged and hence the resultant v_2 will be increased with a corresponding waste of energy and loss in efficiency. This is an unavoidable loss in a wheel operating under part load and makes it impossible to maintain full efficiency of operation by any design whatever of the regulating gates. This loss does not appear in the impulse wheel since the velocity with which the water leaves the bucket is theoretically at least not influenced by the quantity.

The third requirement is partially satisfied by gradually expanding the draft tube from the wheel to the point of discharge. This will recover only the component of the residual velocity in the axial direction. The larger component of the residual velocity however tends to produce a rotation of the water column in the draft tube, and is not recovered by any present design.

The fourth requirement is evident.



Figs. 208–209.—Reaction Wheel with Concrete Draft Tube.*

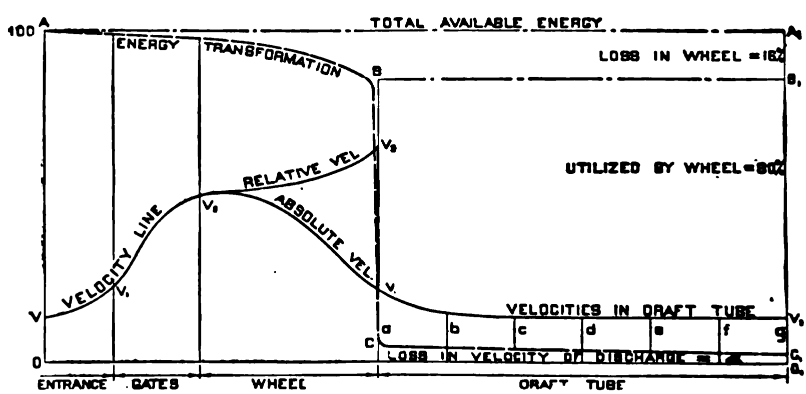


Fig. 210.—Graphical Relation of Velocity and Energy in the Flow Through a Reaction Turbine with Draft Tube.

* Turbinen and Turbinenanlagen, Viktor Gelpke, page 61.

155. Graphical Relation of Energy and Velocity in Reaction Turbine.—The relations of the changes in velocity and in energy in the passage of water through a reaction turbine and its draft tube are graphically shown in Fig. 210.

Fig. 208 shows the cross section of a radial inward flow reaction turbine with a concrete draft tube. The cross sections of the draft tubes at various points are shown in Fig. 209 from which it will be seen that the draft tube of this turbine gradually changes form and increases in cross section in order that the velocity of flow may be gradually decreased from the point of discharge of the turbine to the end of the draft tube.

The changes in absolute velocity in the passage of water into and through the turbine and draft tube are shown by line V, V_1, V_2, V_4, V_5; the height of the ordinates at these points shows the approximate absolute velocities at such points in the flow. The absolute velocity is a maximum at or near the point where the water enters the runner and is decreased as greatly as possible at the point of its discharge into the draft tube. By gradually increasing the area of the draft tube, an additional reduction in velocity is obtained, the water finally issuing with a velocity V_5. The maximum velocity, measured by the ordinate V_2, is, in reaction wheels, considerably below the spouting velocity ($\sqrt{2gh}$).

In its flow through the wheel, the velocity of the water relative to the bucket increases and becomes a maximum at the outlet of the wheel. This increase in relative velocity is shown by the line V_2, V_3.

The energy transformation which takes place during the change in velocity is illustrated by the dotted line marked "Energy transformation" which begins at a maximum of 100 per cent. at the entrance of the wheel; is decreased by friction, leakage, shocks, etc., by about 16 per cent. under full gate conditions. The energy is transformed into useful work in the wheel by the reaction at the point of discharge and utilizes about 80 per cent. of such energy, the remaining 4 per cent. being rejected in the discharge from the draft tube with a slight recovery of velocity energy as before described.

156. Turbine Relations.—In all water wheels the quantity of discharge, the power, speed, efficiency and effective head on the wheel are closely related and vary in accordance with certain definite laws modified by the design of the turbine and the conditions under which it is operated. The conditions of operation must be adapted to the type of machinery used, or the machinery must be selected in accordance with the conditions under which it must operate, in order that the best results may be attained.

If a jet or stream of water, with a velocity, v, acts on the moving surface of a motor bucket, this bucket, if the friction of the wheel is negligible, may acquire a velocity essentially equal to that of the jet, i. e., to the theoretical velocity due to the head. In actual practice the velocity of the bucket will always be less by the amount of velocity lost in overcoming the friction of the wheel. The velocity of the wheel here considered must be measured at the center of application of the forces, i. e., at the point of application of the resultant of all the forces of all the filaments of water that act on the wheel. Under conditions where the resultant velocity of water and bucket are the same, it is evident that the water will produce no pressure on the bucket and the motor can deliver no power. As soon as resistance occurs, the speed of the wheel is reduced. Under reduced speed the momentum of the jet, or the reactive pressure of the water, according to the circumstances of design, is converted into power. This impact or pressure increases as the speed or velocity of the bucket decreases until the maximum impact or pressure results with the bucket at rest, in which case also no work is done. At some speed, therefore, between these extremes the maxi-

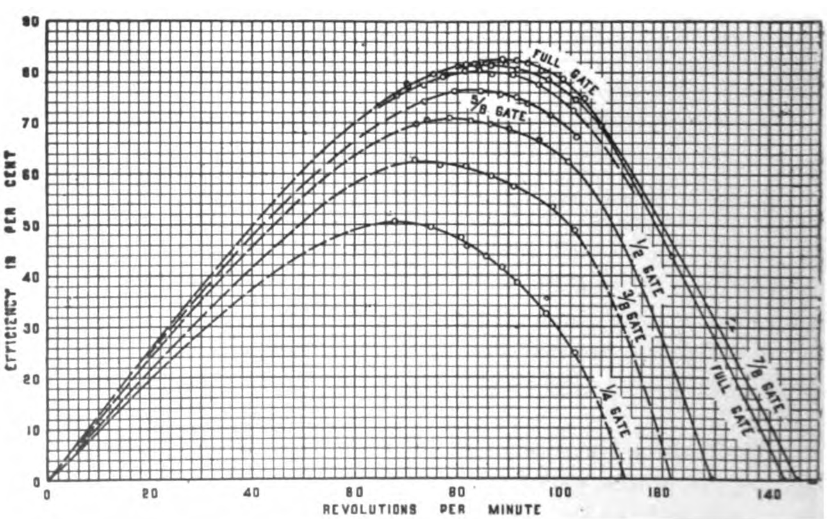


Fig. 211.—Efficiency Speed Curves of a 48" "Victor Turbine."

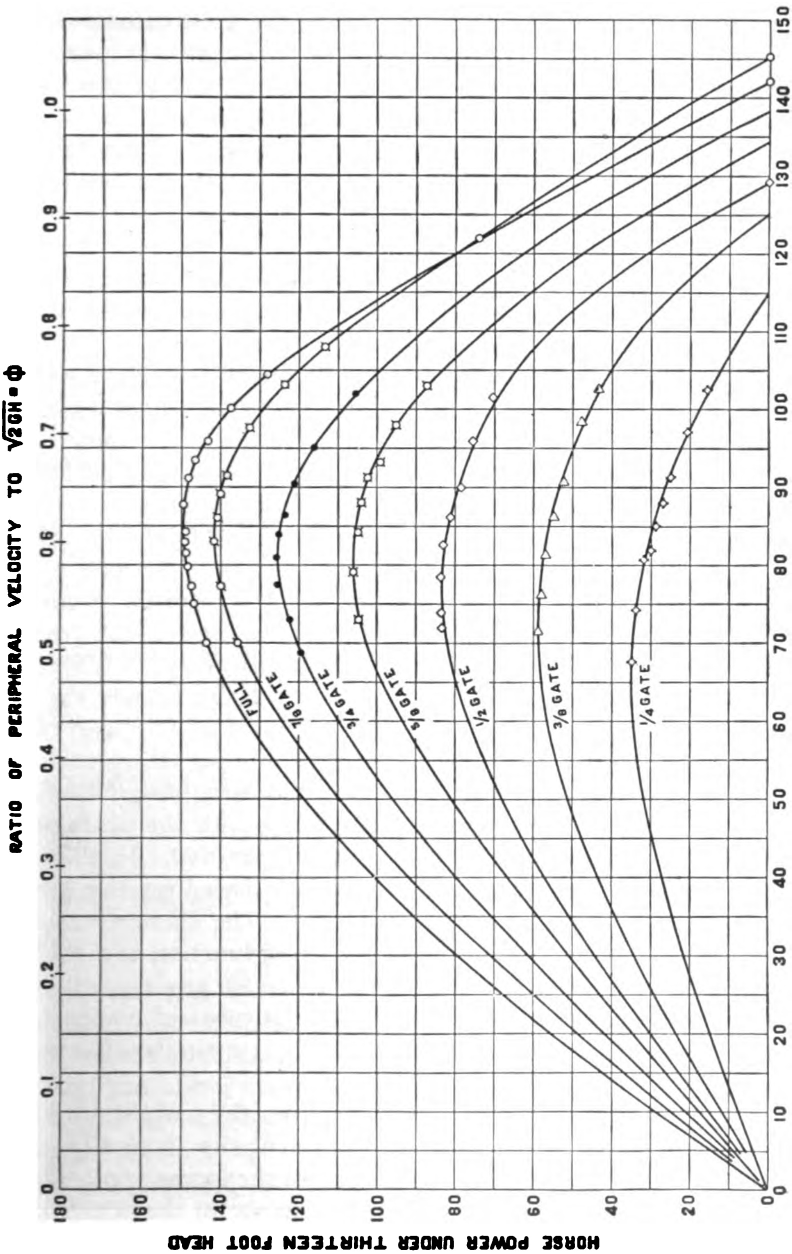


Fig. 212.—Power-Speed Curves of a 48" "Victor Turbine."

mum amount of work, from a given motor, will be obtained. That is to say,—at a certain fixed speed the maximum work and the maximum efficiency of a given wheel will be obtained, and at any speed below or above this speed, the power and efficiency of the wheel will be reduced. These conditions vary considerably according to the type and design of the wheel considered and also according to the gate opening at which the wheel may be operated.

The efficiency curves of a 48″ Victor turbine, under a thirteen foot head and under various conditions of gate, are shown in Fig. 211. Fig. 212 shows the ϕ-power curve of the same wheel under the same conditions of head and gates.

157. **Relation of Turbine Speed to Diameter and Head.**—The velocity of the periphery of the impeller or buckets of a wheel is not necessarily and in fact is not usually the same as the velocity of the point of application of the resultant of the forces applied to the wheel. This point may be at some considerable distance within the wheel and at a point not easily determined. This point of application of the resultant forces may vary in position with the gate opening. The peripheral diameter is fixed and is therefore more convenient for consideration than the point of application of the forces. The peripheral diameter, or the catalogued diameter, is therefore used in the discussion of the general subject. Many wheels vary in diameter at various points on the periphery (see Fig. 174), and there is no uniform practice among manufacturers in designating such diameters so that the diameters used in the following discussion and the functions based thereon are in accordance with the practice of each maker and are therefore not strictly comparative. In this discussion the laws discussed are equally true if based on any actual diameter or any simple function of the same. The diameter chosen simply influences the magnitude of the derived function and not the character. The discussion holds therefore in each case regardless of the method of measurement except for the purpose of comparison between wheels of various makers in which case similar diameters must be used.

In reaction wheels, the buckets extend from the periphery of the wheel to a point quite near the axis of revolution (see Fig. 128, Diagram I). In such wheels the resultant of the forces applied falls a considerable distance within the circumference of the wheel. In such wheels the peripheral velocity may exceed the velocity of the jet acting on the wheel. In impulse wheels (see Fig. 129, Diagram E) the buckets are small in comparison to the wheel diameter and are located at the periphery; hence, in this class of wheels, the resultant of the forces applied lies at or near the periphery, and the peripheral velocity will be less than that of the jet acting on the wheel.

Taking the velocity of the periphery of the wheel as a function of the velocity due to head, the relations may be expressed by the formula:

(19) $$v' = \varphi\sqrt{2gh} \quad \text{from which}$$

(20) $$\varphi = \frac{v'}{\sqrt{2gh}} = \frac{v'}{v}$$

The velocity of the periphery of the impeller may be expressed by the following formula:

(21) $$v' = \frac{D \pi n}{12 \times 60} = \frac{3.1416\ D\,n}{720}$$

Combining equations (20) and (21) it follows that:

(22) $$\varphi = \frac{3.1416\ D\,n}{720 \times 8.025\sqrt{h}} = .000543\,\frac{D\,n}{\sqrt{h}}$$

From this may also be written:

(23) $$n = \frac{\varphi\sqrt{h}}{.000543\ D} = \frac{1841.6\ \varphi\sqrt{h}}{D}$$

As equation (22) is general, it follows that when φ is constant:

(24) $$\frac{D\,n}{\sqrt{h}} = 1841.6\ \varphi = \Delta \text{ is constant.}$$

If $h = 1$, this will reduce to:

(25) $$D\,n_1 = 1841.6\ \varphi = \Delta$$

The catalogue speed, power and discharge of each series of wheels, as given in the catalogues of manufacturers, are usually based on the conditions of maximum efficiency and constant ϕ.

From the above considerations it follows that in any homogeneous series of wheels, that is in any series of wheels constructed on uniform lines and with dimensions proportional, the wheels of the series are designed to run at the same relative velocity, and therefore

(26) $$\frac{D\,n}{\sqrt{h}} = \frac{D_1\,n_1}{\sqrt{h}}$$

That is to say: *In any homogeneous series of turbines the product of the diameter of any wheel* D, *and the number of revolutions* n, *divided by* $\sqrt{h}$ *will be a constant* Δ *provided* ϕ *remains constant.*

In investigating the values of Δ and ϕ for various makes of wheels, as expressed by the data in the manufacturers' catalogues, it is found that these values vary somewhat for different wheels of a series but are usually practically constant. It will be noted, however, from the efficiency speed curve, shown in Fig. 211, and the ϕ power curve, shown in Fig. 212, that the speed, and consequently the values of ϕ and Δ, may vary somewhat without materially affecting the efficiency or power of the wheel.

It should also be noted from Figs. 211 and 212 that if it is desired to secure the greatest efficiency and power at part gate, the values of ϕ and Δ for a given wheel must be reduced. Table XXVI gives the values of Δ and ϕ for various American wheels, calculated from the catalogues of the manufacturers.

TABLE XXVI.

Showing Relation of Diameter and Speed of Various American Turbines working under Catalogue Conditions.

$$\Delta = \frac{D\,n}{\sqrt{h}} \qquad \varphi = \frac{v'}{v} = .000543\frac{D\,n}{\sqrt{h}}$$

Manufacturer.	Name of Wheel.	Δ		φ	
		Min.	Max.	Min.	Max.
Reaction Wheels.					
T. C. Alcott & Son...	Alcott's Standard High Duty	1210	1254	.658	.682
	Alcott's Special High Duty	1211	1253	.658	.682
Alexander, Bradley & Dunning..........	Syracuse Turbine....	1203	1226	.654	.666
American Steel Dredge Works.............	Little Giant	1235	1462	.671	.794
Camden Water Wheel Works.............	United States Turbine	1372	1588	.745	.864
Chase Turbine Mfg. Co.	*Chase-Jonval Turbine (regular).....	1612	1997	.876	1.084
	*Chase-Jonval Turbine (special)......	1840	2237	.999	1.214
Christiana Machine Co.	Balanced Gate Turbine.	1220	1298	.663	.706

*Note.—Wide variation in constants due to the design being special for various sized wheels (series not exactly homogeneous).

TABLE XXVI—Continued

Showing Relation of Diameter and Speed of Various American Turbines working under Catalogue Conditions.

$$\Delta = \frac{D\ n}{\sqrt{h}} \qquad \varphi = \frac{v'}{v} = .000543 \frac{D\ n}{\sqrt{h}}$$

Manufacturer.	Name of Wheel.	Δ Min.	Δ Max.	φ Min.	φ Max.
Reaction Wheel—Con.					
Craig Ridgway & Son Co.	Double Perfection	1186	1250	.644	.679
Craig Ridgway & Son Co.	Standard	1200	1275	.652	.693
Dayton Globe Iron Works Co.	American Turbine	1218	1295	.662	.704
	†New American Turbine (high head type)	1064	1077	.578	.585
	Improved New American	1632	1738	.886	.944
	Special New American	1284	1340	.697	.727
J. L. & S. B. Dix	Improved Jonval Turbine	1474	1617	.800	.880
Dubuque Turbine & Roller Mill Co.	Flenniken Turbine	1511	1533	.821	.833
Dubuque Turbine & Roller Mill Co.					
Holyoke Machine Co.	McCormick's Holyoke Turbine	1196	1296	.650	.704
	Hercules Turbine	1160	1170	.630	.636
Humphrey Machine Co.	‡IXL Turbine	1198	1209	.652	.657
	‡XLCR Turbine	1196	1206	.652	.656
Rodney Hunt Machine Co.	McCormick Holyoke Turbine	1159	1278	.630	.694
	Hunt McCormick Turbine	1158	1272	.629	.691
	New Pattern Hunt Turbine	1163	1415	.632	.768
	Standard Wheel, 1887 Pattern	1200	1291	.651	.701
E. D. Jones & Sons Co.	Crocker Wheel	1208	1292	.657	.702
James Leffel & Co.	Samson Water Wheel	1543	1554	.838	.844
	Improved Samson	1578	1632	.856	.886
	Standard	1330	1339	.722	.727
	Special	1380	1434	.750	.779
Munson Bros. Co.	Phoenix "Little Giant"	1001	1020	.544	.554

†Catalogue recommends a maximum and minimum speed. Constants given are for the average speed.

‡Tables based on full theoretical power of the water. Wheels are said to give from 75 per cent to 90 per cent efficiency, depending on location.

TABLE XXVI.—Continued.

Showing Relation of Diameter and Speed of Various American Turbines working under Catalogue Conditions.

$$\Delta = \frac{D\,n}{\sqrt{h}} \qquad \varphi = \frac{v'}{v} = .000543\,\frac{D\,n}{\sqrt{h}}$$

Manufacturer.	Name of Wheel.	Δ Min.	Δ Max.	φ Min.	φ Max.
Reaction Wheel—Con.					
Norrish, Burnham & Co.	....	1213	1233	.659	.670
Platt Iron Works Co.	Victor Register Gate.	1181	1221	.641	.663
	Victor Standard Cylinder Gate	1380	1410	.749	.765
Poole Engineering & Machine Co.	Poole-Leffel	1341	1380	.728	.749
T. H. Risdon & Co.	Risdon Standard	1213	1420	.659	.772
	Risdon Turbine Type T. C.	1213	1420	.659	.772
	Risdon Turbine Type D. C.	1213	1420	.659	.772
S. Morgan Smith Co.	Smith-McCormick	1180	1344	.641	.730
	Smith	1655	1679	.898	.911
Trump Mfg. Co.	Standard Trump	1320	1380	.716	.749
Wellman, Seaver, Morgan Co.	McCormick	1212	1260	.658	.684
Impulse Wheels.					
DeRemer Water Wheel Co.	DeRemer Water Wheel	962	1001	.522	.545
Abner Doble Co.	Tangential Wheel	841	848	.456	.460
Pelton Water Wheel Co.	Tangential Wheel	912	921	.495	.500
Platt Iron Works Co.	Victor High Pressure	915	919	.497	.499
The Risdon Iron Wks.	Tangential Wheel	917	920	.498	.499

From equation (26) may be derived

$$(27) \qquad n = \frac{D_1\,n_1\,\sqrt{h}}{D\,\sqrt{h_1}}$$

From this equation the economical speed or correct number of revolutions n for any wheel of diameter D, at any head, $\sqrt{h}$, can be obtained if the revolutions n_1 of any other wheel of the series at head h_1 and of diameter D_1 is known.

If in equation (27), $D=D_1$, the equation reduces to

$$n = \frac{n_1\sqrt{h}}{\sqrt{h_1}} \text{ or } \frac{n}{\sqrt{h}} = \frac{n_1}{\sqrt{h_1}} \tag{28}$$

That is to say: *The economical speed of any wheel will be in direct proportion to the square root of the head under which it acts.*

If in the equation (28), $n = 1$, the equation reduces to

$$n = n_1\sqrt{h} \tag{29}$$

From which it follows that the revolutions of a wheel (n) for any head, h, is equal to the evolutions n_1 for one foot head multiplied by $\sqrt{h}$.

158. **Graphical Expression of Speed Relations.**—The relation expressed by equations 18 to 27, inclusive, between the values of v, ϕ, D, n, and h, are graphically shown by Fig. 213. The theoretical relations between v′ and h, and ϕ as expressed by equation (19) when $\phi=1$, are represented by the upper curved line in the diagram referred to ordinates and abcissas. The relation between ϕ, v and h, where ϕ has a fractional value or is less than 100 per cent., as is the case for all wheels working under practical conditions, is shown by reference to the curved lines below; the fractional value of ϕ as represented by each line is given thereon. The relations between v, D and n are shown by the relations of the straight lines originating near the lower right-hand corner of the diagram referred to ordinates and abcissas, and the mutual relations of all lines on the diagrams show the mutual relations between the various factors that are here considered.

159. **Relations of ϕ and Efficiency.**—In any turbine running under different heads but otherwise under the same physical conditions as to gate opening, setting, draft tubes, etc., the efficiency will remain constant provided the ratio of the velocity of rotation to the theoretical spouting velocity of the water under the given head remains the same. This is to say,—the efficiency of a wheel will remain constant under various conditions of head as long as the value of ϕ remains constant. This law is well demonstrated by experiments made on a 12″ Morgan-Smith wheel at the Hydraulic Laboratory of the University of Wisconsin.* These experiments were made under seven different heads varying from about 7.10 feet to about 4.25 feet. The results of all these experiments have been

* "Test of a Twelve-Inch McCormick Turbine," an unpublished thesis by O. W. Middleton and J. C. Whelan.

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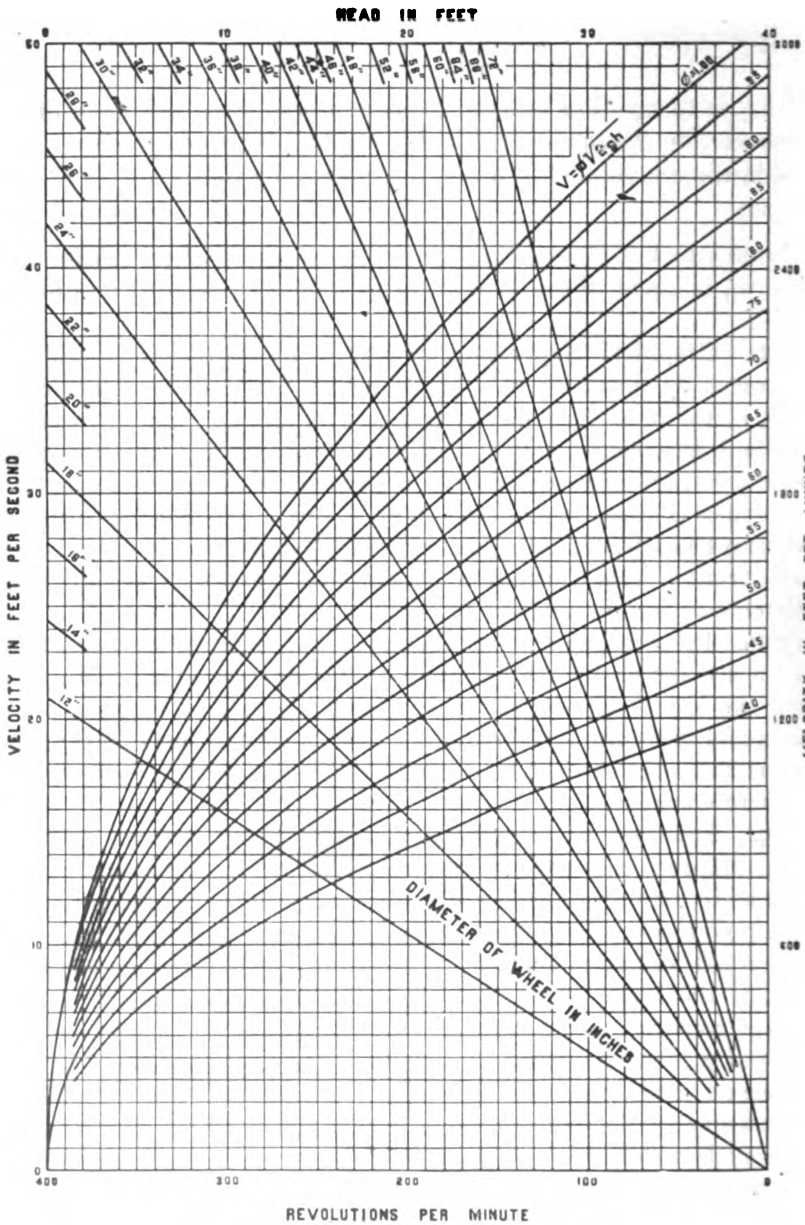

Fig 213.—Speed Relations of the Turbines.

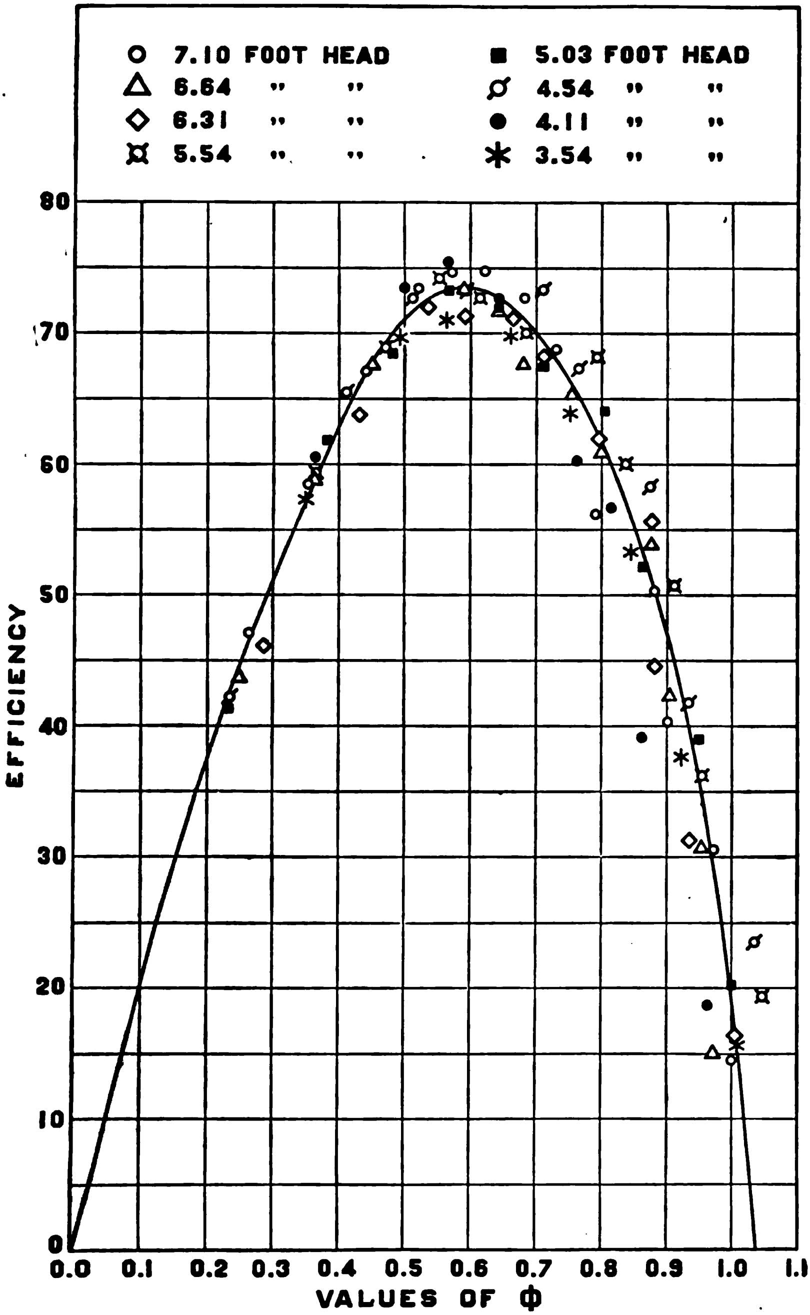

Fig. 214.—Efficiency—ϕ Curve of a 12 "Smith-McCormick Turbine.

platted in a single diagram (see Fig. 214) from which it will be noted that all experiments are fairly close to the mean curve; that the variation therefrom is probably due to experimental errors (principally, it is believed, in the determination of the relative velocities) and that reduction in head shows no uniform decrease in efficiency. The experiments referred to, which are soon to be published in a University bulletin, show that this law is true under all conditions of gate as well as for the full gate conditions, illustrated in Fig. 214. Hence the conclusion may be drawn that the efficiency of a wheel will remain essentially constant if ϕ remains constant at least under moderate changes in head.

160. Discharge of a Turbine at Fixed Gate Opening.—The discharge of a turbine with fixed gate opening, but at various speeds, is not always the same but varies within certain limits and as the speed varies. In some cases the discharge of a wheel increases as the speed increases. (See discharge of Tremont turbine, Fig. 215.) Sometimes the discharge decreases as the speed increases (see discharge of Victor and McCormick turbines, Fig. 215), and sometimes the discharge increases with the speed to a certain point and then decreases with a further increase in the speed (see discharge of Samson and New American wheels, Fig. 215.)

In reaction turbines the discharge takes place first through the guide from which it passes into and through the buckets of the wheel. The relations of these two sets of orifices change as the speed of the wheel changes and affects the total discharge. If during such changes of speed, the ratio, $\phi = \frac{v'}{v}$, remains constant, it is found by experiment that the conditions remain similar to those of any short tube or orifice. The discharge of a turbine may therefore be determined by the formula:

$$q = Ca\sqrt{2gh} \tag{30}$$

And it may be stated: *In a given turbine with fixed gate opening, the discharge will be proportional to the square root of the head, i. e., the discharge divided by $\sqrt{h}$ is constant.*

The values of C and a vary with the opening of the gate or gates, but for any one position are essentially constant.

Let the discharge of a wheel under fixed gate conditions and with a given head, h_1, be given by the formula:

$$q_1 = Ca\sqrt{2gh_1} \tag{31}$$

The discharge of any other head will be proportional to $\sqrt{h}$ and therefore

$$\frac{q_1}{\sqrt{h_1}} = \frac{q}{\sqrt{h}} \text{ hence} \tag{32}$$

$$q = \frac{q_1\sqrt{h}}{\sqrt{h_1}} \text{ or if } h_1 = 1 \tag{33}$$

$$q = q_1\sqrt{h} \tag{34}$$

Therefore, it may be stated: ***In a given turbine with fixed gate opening the discharge at any head h will be equal to the discharge at one foot head multiplied by*** $\sqrt{h}$.

That this law is essentially correct may be demonstrated by experiment. Fig. 216 shows the results from the series of tests on the McCormick turbine, before mentioned, at full gate. Three sets

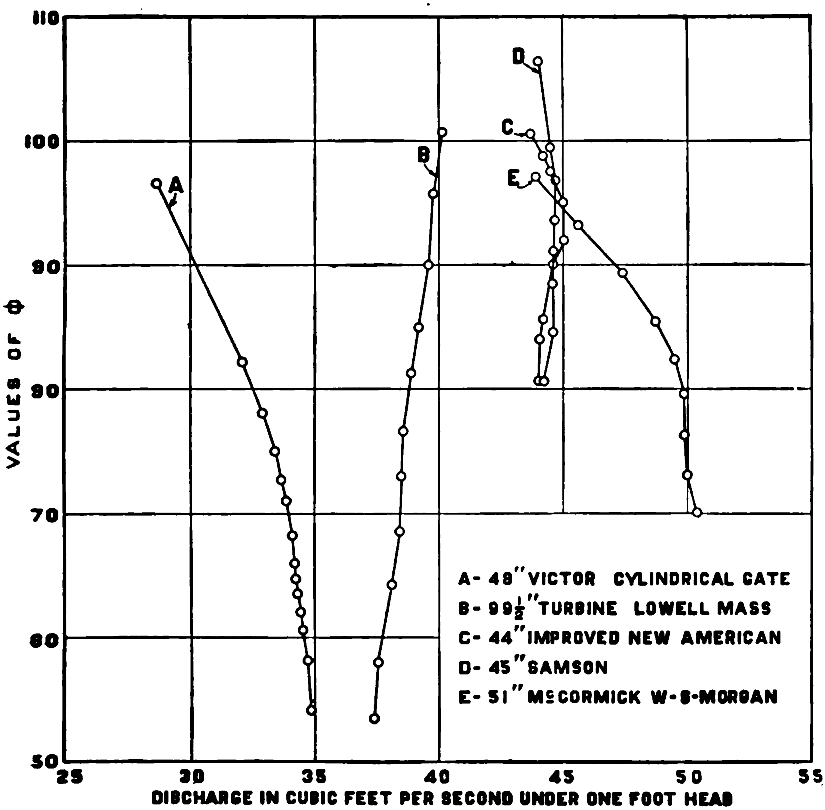

Fig. 215.—Full Gate φ-Discharge Curves of Various Turbines.

of experiments are platted with values of ϕ equal to .35, .65 and .90 and for heads from about 4.25 feet to 7.1 feet. Fig. 217 shows the discharge of this turbine at various gate openings and under seven different heads. For the purpose of this diagram the discharges under each head have been reduced to the theoretical discharge at one foot head by equation 34. It will be noted from both Fig. 216 and Fig. 217 that all experiments where ϕ is the same lie close to the average line, and that the departures from this line are probably due to experimental errors. The results are sufficiently close, however, to demonstrate that the discharge under practical conditions essentially follows the law above expressed.

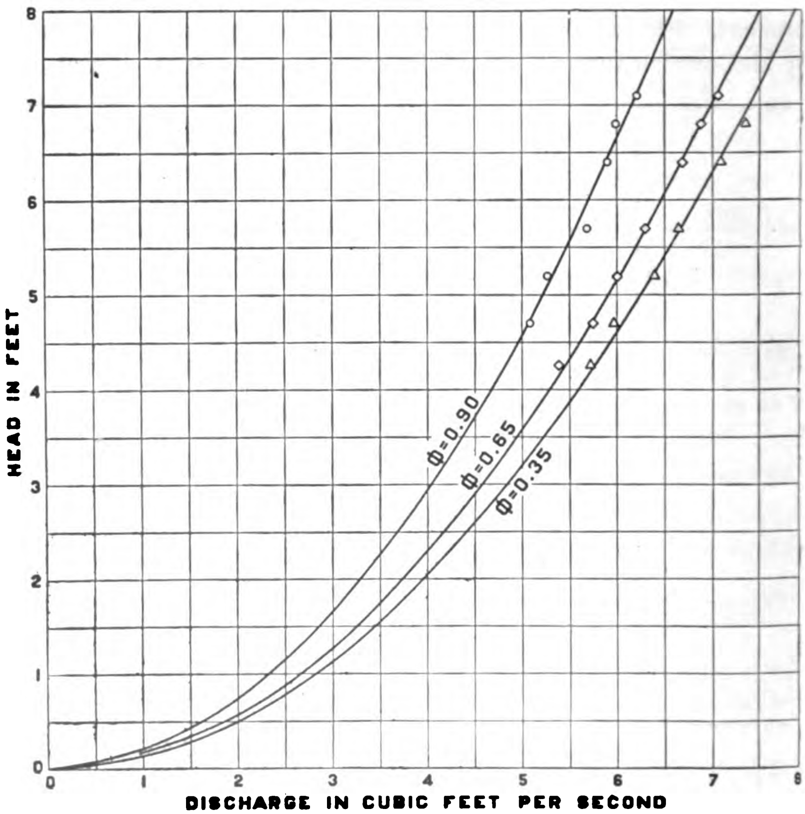

Fig. 216.—The Relations of Head to Discharge of a 12 "Smith-McCormick Turbine.

161. Power of a Turbine.—The power which may be generated by any wheel depends on the head available, the quantity of water which may be discharged through the wheel under the given head, the relative speed at which it may be run, and the efficiency of operation. Hence

$$P = \frac{q\,w\,h\,e}{550} = \frac{q\,h\,e}{8.8} \tag{35}$$

Combining equations (30) and (35) there results

$$P = \frac{C a w \sqrt{2g}\, h^{\frac{3}{2}} e}{550} = \frac{C a \sqrt{2g}\, h^{\frac{3}{2}} e}{8.8} \tag{36}$$

From equation (36) it is apparent that if C, e and a are constant for any given turbine and fixed gate opening, and if the value of ϕ remains constant, the power of the turbine will be in direct proportion to $h^{\frac{3}{2}}$. consequently

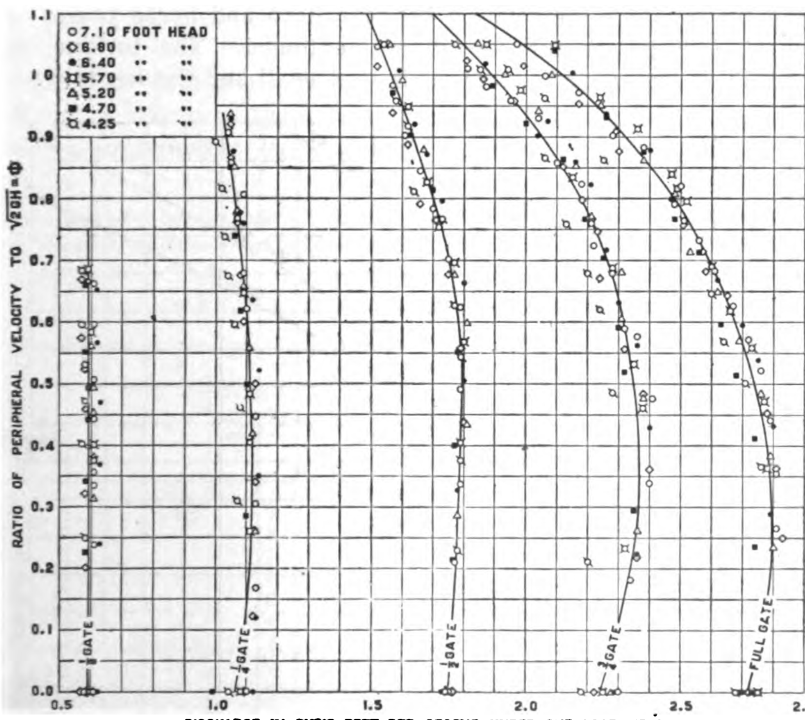

Fig. 217.—Relations of Velocity to Discharge for a 12" "Smith-McCormick" Turbine at Various Gate Openings.

$$\frac{P}{h^{\frac{3}{2}}} = \frac{P_1}{h_1^{\frac{3}{2}}} \tag{37}$$

Equation (37) may be reduced to

$$P = \frac{P_1\, h^{\frac{3}{2}}}{h_1^{\frac{3}{2}}} \tag{38}$$

From which can be determined the power of a wheel at any given head, provided its power at any other head is known.

In equation (38) if $h_1 = 1$, there results

$$P = P_1 h^{\frac{3}{2}} \tag{39}$$

From which it may be stated: *In a given turbine with a fixed gate opening, the power that can be developed at any head will be equal to the power at one foot head multiplied by* $h^{\frac{3}{2}}$.

This law may also be demonstrated experimentally as will be seen by reference to Fig. 218, in which is shown the theoretical curve representing the relation between head and horse power of the 12″ McCormick turbine before mentioned. The turbine on which these experiments were made was small and the heads were

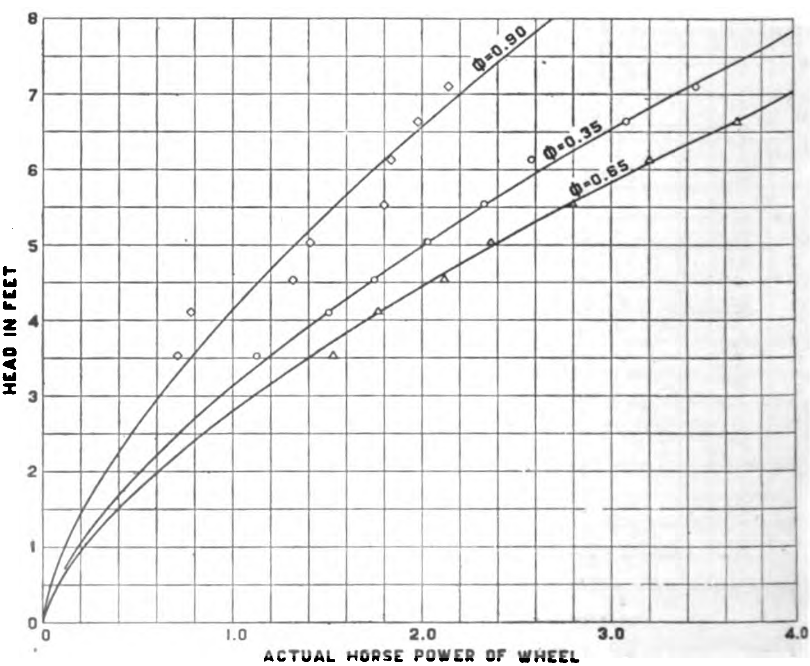

Fig. 218.—Relations of a Power to Head in a 12 "Smith-McCormick Turbine."

limited so that there is some variation from the theoretical curves but the fact expressed by the general law is quite clearly shown.

162. The Relation of Discharge to the Diameter of a Turbine. —In any homogeneous system of water wheels, the diameter, height and corresponding openings and passages are proportional and it follows that in such similar wheels similar areas are proportional to each other and to the squares of any lineal dimension. In such wheels, therefore, the area a of the gate openings is proportional to the square of the diameter of the wheel, and the equation may therefore be written:

$$Ca\sqrt{2g} = K\,D^2 \tag{40}$$

In this equation K is a constant to be determined by experiment. Combining equations (40) and (30) there results

$$q = KD^2\sqrt{h} \tag{41}$$

from which can be obtained, by transposition

$$D = \sqrt{\frac{q}{K\sqrt{h}}} \tag{42}$$

Equation (41) is not only theoretically but is also practically correct, as is shown by the data in Table XXVII, which is also graphically represented in Fig. 219. These data are taken from a paper

TABLE XXVII.

Discharge of thirteen water wheels of the same manufacture but of different diameters, as determined by actual tests, compared with value computed by the formula:

$q = K\,D^2\sqrt{h}$ in which $h = 13$, $K = .0172$

DISCHARGE.

No.	Diameter in inches.	Reduced from actual tests, Cu. ft. per Sec.	Computed (Mean Curve) Cu. ft. per Sec.	Variation from Computed Discharge Cu. ft. per Sec.	Per cent. Variation from Computed Discharge.
1	9	5.17	5.02	+0.15	+2.99
2	12	8.79	8.92	—0.13	—1.46
3	15	13.85	13.93	—0.08	—0.57
4	18	18.85	20.07	—1.22	—6.08
5	12	29.07	27.32	+1.75	+6.41
6	24	35.31	35.68	—0.37	—1.04
7	27	47.81	45.16	+2.65	+5.87
8	30	54.15	55.75	—1.60	—2.87
9	36	77.33	80.28	—2.95	—3.67
10	39	93.51	94.22	—0.71	—0.75
11	42	107.73	109.27	—1.54	—1.41
12	45	128.53	125.44	+3.09	+3.10
13	51	161.07	161.12	—0.05	—0.03

by A. W. Hunking, entitled "Notes on Water Power Equipment," in vol. 13, No. 4, of Jour. Asso. Eng. Soc., April, 1894. In this table are given the discharges of thirteen water wheels of various diameters, the discharges of which were determined from actual tests.

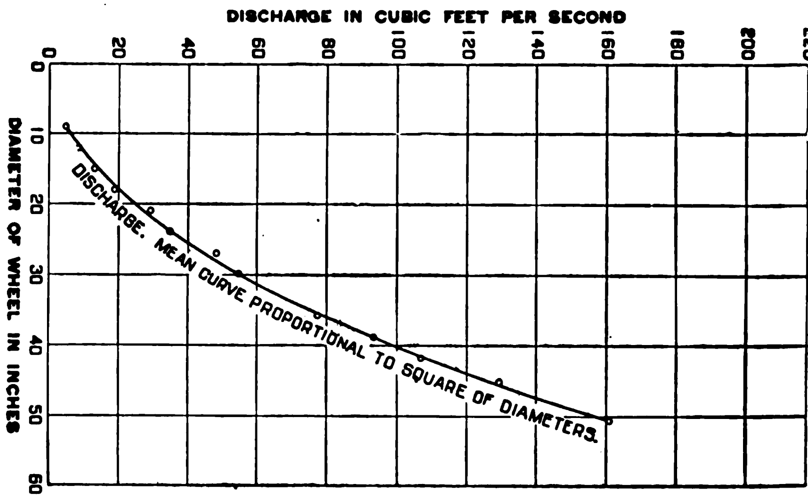

Fig. 219.—Relations of Discharge to Diameter in Reaction Turbine of the same manufacture.

These results have been reduced to the common basis of the discharge at 13 foot head. The computed discharges at 13 foot head on the basis of equation (41) are also given, as well as the percentage of variations of the actual from the theoretical discharges. The wheels were of the same make with inward and downward discharge. The departures or variations from the mean values, as determined by calculation, are probably due both to imperfections in the construction of the wheel and to errors in making the tests. They may be seen, however, to practically conform to the theoretical deductions. The values of the coefficient K, as calculated from the tables contained in the catalogues of various manufacturers of American wheels, are given in Table XXVIII.

163. The Relation of Power to the Diameter of a Turbine.—By substituting the value of q from equation (41) in equation

(35) $$P = \frac{qhe}{8.8}$$

there results

(43) $$P = \frac{D^2 h^{\frac{3}{2}} Ke}{8.8} = \left(\frac{Ke}{8.8}\right) D^2 h^{\frac{3}{2}}$$

TABLE XXVIII.

Showing Relation of Diameter and Discharge of Various American Turbines working under Catalogue Conditions.

$$K = \frac{q}{D^2\sqrt{h}}$$

Manufacturer.	Name of Wheel.	K	
		Min.	Max.
Reaction Wheels.			
T. C. Alcott & Son..........	Alcott's Standard High Duty	.00654	.00860
	Alcott's Special High Duty.	.0157	.0168
Alexander, Bradley & Dunning......................	*Syracuse Turbine.........	.00538	.00622
American Steel Dredge Wks..	*Little Giant..............	.0205	.0340
Camden Water Wheel Works	United States Turbine......	.0214	.0229
Chase Turbine Mfg. Co......	*Chase-Jonval Turbine (regular)....................	.00693	.00913
	*Chase-Jonval Turbine (special).................	.01086	.01346
Christiana Machine Co......	Balanced Gate Turbine.....	.00902	.00952
Craig, Ridgway & Son Co....	Double Perfection..........	.0116	.0142
Craig, Ridgway & Son Co....	Standard..................	.00586	.00659
Dayton Globe Iron Works Co.	*American Turbine........	.00543	.00801
	New American (high head type)....................	.00509	.00644
	Improved New American..	.0233	.0263
	Special New American.....	.0175	.0205
J. L. & S. B. Dix...........	Improved Jonval Turbine..	.00454	.00546
Dubuque Turbine & Roller Mill Co..................	Flenniken Turbine.........	.00652	.0088
Dubuque Turbine & Roller Mill Co..................	McCormick's Holyoke Turbine.....................	.0184	.0191
Holyoke Machine Co........	Hercules Turbine..........	.0162	.0175
Humphrey Machine Co.....	†IXL Turbine.......	.00351	.00536
	†XLCR Turbine...........	.00645	.00953
Rodney Hunt Machine Co..	McCormick's Holyoke Turbine.....................	.01877	.01929
	*Hunt-McCormick Turbine.	.01913	.02867
	New Pattern Hunt Turbine.	.01297	.01543
	Standard Wheel, 1387 pattern.....................	.0123	.0141
E. D. Jones & Sons Co......	Crocker Wheel............	.0175	.0179
James Leffel & Co..........	Samson..................	.0170	.0171
	Improved Samson..........	.022	.022
	Standard..................	.00612	.00640
	Special.................. ...	.00937	.00965
Munson Bros. Co...........	‡Phoenix "Little Giant"...	.00924	.0172
Norrish, Burnham & Co....	....	.00917	.00955
Platt Iron Works Co........	Victor Register Gate.......	.0167	.0186
	Victor Standard Cylinder Gate....................	.0222	.0227

TABLE XXVIII.—Continued.

Showing Relation of Diameter and Discharge of Various American Turbines working under Catalogue Conditions.

$$K = \frac{q}{D^2 \sqrt{h}}$$

Manufacturer.	Name of Wheel.	K Min.	K Max.
Reaction Wheels.—Con.			
Poole Engineering and Machine Co	Poole-Leffel..	.00625	.00827
T. H. Risdon & Co..........	*Risdon Standard Turbine..	.00501	.00698
	*Risdon Type T. C. Turbine	.00753	.00948
	*Risdon Type D. C. Turbine	.0100	.0132
S. Morgan Smith Co........	*Smith-McCormick	.0187	.0238
	Smith......................	.0247	.0256
Trump Mfg. Co.............	Standard Trump...........	.0210	.0263
Wellman, Seaver, Morgan Co.	McCormick................	.0185	.0199
Impulse Wheels.			
DeRemer Water Wheel Co..	*DeRemer Water Wheel....	.000135	.000173
Abner Doble Co............	*Tangential Wheel.........	.000075	.000119
Pelton Water Wheel Co.....	*Tangential Wheel.........	.00010	.000135
Platt Iron Works Co........	Victor High Pressure.......	.0017	.00247
Risdon Iron Works	*Tangential Wheel.........	.000134	.000173

*Wide variation in constants due to the design being special for various sized wheels (series not exactly homogeneous).

†Tables in catalogue based on full theoretical power of the water. Wheels are said to give from 75 per cent to 90 per cent efficiency, depending on location.

‡Munson Bros. Co. make several types of "Litte Giant" turbines causing above wide variation in constants.

As $\left(\frac{Ke}{8.8}\right)$ is constant for a given wheel, as long as ϕ is constant, this expression may be represented by a constant K_2 which may be derived independently for each make of wheel, or may be determined from the equation

(44) $$K_2 = \frac{Ke}{8.8}$$

With this substitution (43) becomes

(45) $$P = K_2 D^2 h^{\frac{3}{2}}$$

That is to say: *With wheels of homogeneous design, the power of any wheel under the given head is in direct proportion to the square of its diameter.* This law is both theoretically and practically correct, as demonstrated by Table XXIX, and Fig. 220, taken from the paper by Mr. Hunking to which reference has previously been

TABLE XXIX.

Horse Power of thirteen water wheels of the same manufacture but of different diameters, as determined by actual tests, compared with values determined by the formula:

$$P = K_2\, D^2\, h^{\frac{3}{2}} \qquad K_2 = .00158 \qquad h = 13$$

HORSE POWER

No.	Diameter in inches.	From Tests.	Computed.	Variation from Computed H. P. in H. P.	Variation from Computed H. P. Percent.
1	9	6.10	6.00	+0.10	+1.67
2	12	10.41	10.67	—0.26	—2.44
3	15	16.49	16.67	—0.18	—1.08
4	18	22.89	24.00	—1.11	—4.62
5	21	33.71	32.67	+1.04	+3.18
6	24	41.53	42.67	—1.14	—2.67
7	27	56.67	54.07	+2.60	+4.81
8	30	63.69	66.68	—2.99	—4.48
9	36	97.45	96.00	+1.45	+1.50
10	39	109.98	112.68	—2.70	—2.40
11	42	133.09	130.69	+2.40	+1.84
12	45	153.82	150.02	+3.80	+2.53
13	51	196.28	192.69	+3.59	+1.86

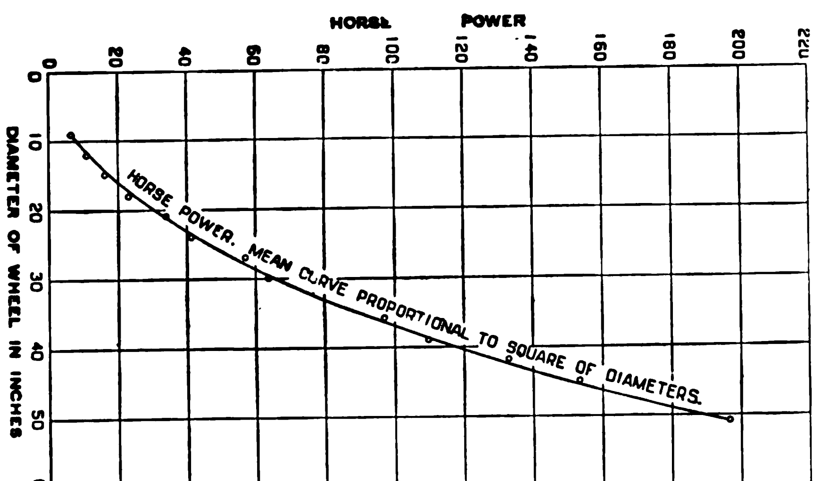

Fig. 220.—Relation of Power to Diameter in Reaction Turbines of the same manufacture.

made. This table and figure illustrate the relation between the theoretical power, as determined by equation (45), and the actual horse power of thirteen wheels of the same manufacture but different diameters, as determined by actual tests.

The values of the constant K_2 for the most efficient relation of power to diameter in various American turbines, as calculated from the tables contained in the catalogues of various American manufacturers of turbines, are given in Table XXX. The values of K_2 and other turbine constants will be found to vary widely in the various types of turbines, not only of different manufacturers but of the same manufacturer. The interpretation of this fact is not that one turbine is, in the abstract and according to the relative value of the constants, more valuable than another, but that each turbine is best fitted for a particular range of conditions for which it was presumably designed.

TABLE XXX.

Showing Relation of Power and Diameter of Various American Turbines Working under Catalogue Conditions.

$$K_2 = \frac{P}{D^2 \; h^{\frac{3}{2}}}$$

Manufacturer.	Name of Wheel.	K_2 Min.	K_2 Max.
Reaction Wheels.			
T. C. Alcott & Son..........	Alcott's Standard High Duty	.000589	.000999
	Alcott's Special High Duty.	.00141	.00155
Alexander, Bradley & Dunning......................	Syracuse Turbine..........	.000483	.000565
American Steel Dredge Wks.	Little Giant................	.00190	.00332
Camden Water Wheel Works	United States Turbine......	.00190	.00207
Chase Turbine Mfg. Co......	*Chase-Jonval Turbine (regular)...................	.000590	.000780
	*Chase-Jonval Turbine (special)................	.000932	.001150
Christiana Machine Co......	Balanced Gate Turbine.....	.000800	.000854
Craig, Ridgway & Son Co....	Double Perfection..........	.00113	.00120
Craig, Ridgway & Son Co....	Standard..................	.000538	.000629
Dayton Globe Iron Works Co.	*American Turbine........	.000434	.000726
	*New American (high head type)..................	.000422	.000588
	Improved New American...	.00212	.00244
	Special New American.....	.00158	.00187
J. L. & S. B. Dix	Improved Jonval Turbine..	.000447	.000532
Dubuque Turbine & Roller Mill Co..................	Flenniken Turbine.........	.000596	.000802
Dubuque Turbine & Roller Mill Co..................	McCormick's Holyoke Turbine.....................	.00167	.00173

TABLE XXX.—Continued.

Showing Relation of Power and Diameter of Various American Turbines Working under Catalogue Conditions.

$$K_2 = \frac{P}{D^2\ h^{\frac{3}{2}}}$$

Manufacturer.	Name of Wheel.	K_2 Min.	K_2 Max.
Reaction Wheel.—Con.			
Holyoke Machine Co........	Hercules Turbine..........	.00147	.00159
Humphrey Machine Co.....	†IXL Turbine.............	.000397	.000620
	†XLCR Turbine...........	.000730	.001310
Rodney Hunt Machine Co...	McCormick Holyoke Turbine....................	.00169	.00173
	*Hunt McCormick Turbine.	.00173	.00260
	New Pattern Hunt Turbine	.00120	.00146
	Standard Wheel, 1887 Pattern....................	.00101	.00122
E. D. Jones & Sons Co......	Crocker Wheel............	.00159	.00163
James Leffel & Co....... ...	Samson....................	.00158	.00159
	Improved Samson..........	.00201	.00202
	Standard	.00056	.00058
	Special....................	.000897	.000920
Munson Bros. Co............	‡Phoenix "Little Giant"...	.000842	.001560
Norrish, Burnham & Co.....	....	.000852	.000885
Platt Iron Works Co.........	Victor Register Gate.......	.00158	.00179
	Victor Standard Cylinder Gate	.00205	.00205
Poole Engineering and Machine Co..................	Poole-Leffel...............	.000625	.000650
T. H. Risdon & Co..........	*Risdon Standard Turbine.	.000485	.000675
	*Risdon Type T. C. Turbine	.000672	.000913
	*Risdon Type D. C. Turbine	.000781	.00135
S. Morgan Smith Co.........	Smith-McCormick	.00169	.00217
	Smith	.00232	.00236
The Trump Mfg. Co..........	Standard Trump...........	.00191	.00241
Wellman, Seaver, Morgan Co.	McCormick................	.00168	.00171
Impulse Wheels.			
DeRemer Water Wheel Co..	*DeRemer Water Wheel...	.000124	.000186
Abner Doble Co.............	*Tangential Wheel.........	.0000055	.0000107
Pelton Water Wheel Co.....	*Tangential Wheel.........	.0000093	.0000130
Platt Iron Works Co.........	Victor High Pressure.......	.000154	.000223
Risdon Iron Works Co......	*Tangential Wheel.........	.0000128	.0000165

*Wide variation in constants due to the design being special for various sized wheels (series not exactly homogeneous).
†Tables based on full theoretical power of the water. Wheels are said to give from 75 per cent to 90 per cent efficiency, depending on location.
‡Munson Bros. Co. make several types of "Little Giant" turbines, causing above wide variation in constants.

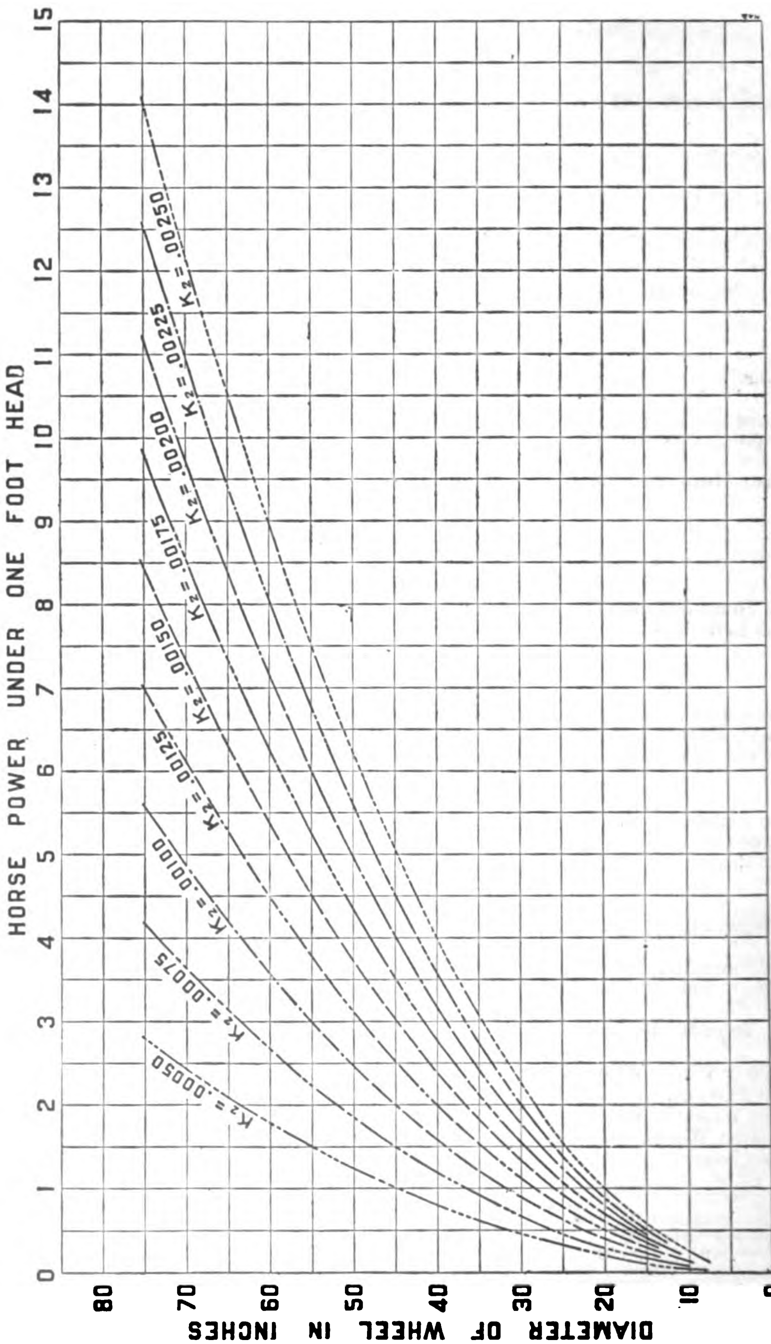

Fig. 221.—Relation of Power to Diameter in Turbines under One Foot Head

As the power of a wheel varies directly with the value of K_2, this constant is a direct measure of comparative power and indicates the relative power that can be developed by various types of wheels of the diameter and under a given head. The range of values for K_2 as found in American practice is shown graphically in Fig. 221 where the power of turbines of various diameter and types under one foot head is given. The power of a wheel varies under different heads as $h^{\frac{3}{2}}$, and therefore the power at any head can be determined directly by multiplying the readings of the graphical table by $h^{\frac{3}{2}}$. For example, from Fig. 221 it will be seen that various types of 40" American wheels, under one foot head, will give from .75 to 4 H. P. and at 16 foot head they will therefore develop 64 times the H. P. at one foot head or from 48 to 256 H. P. within which range a choice must be made.

164. **Relations of Speed to Discharge of Turbines.**—As the speed of all wheels of the same series must be proportional to $\sqrt{h}$, the equation may be written:

$$v' = K_3 \sqrt{h} \qquad (46)$$

from which and from equations (19) and (21)

$$K_3 = \frac{v'}{\sqrt{h}} = \varphi \sqrt{2g} = \frac{\pi D n}{12 \times 60 \sqrt{h}} \qquad (47)$$

From equations (42) and (47) may be derived

$$n = \frac{12 \times 60 K_3 \sqrt{K}}{\pi} \frac{\sqrt{h}}{\sqrt{\frac{q}{\sqrt{h}}}} \qquad (48)$$

As the first term of the last expression is constant, there may be written:

$$K_4 = \frac{12 \times 60 K_3 \sqrt{K}}{\pi} \qquad (49)$$

from which equation (48) may be re-written.

$$n = \frac{K_4 \sqrt[4]{h^3}}{\sqrt{q}} \qquad (50)$$

For a head of one foot, h=1, equation (50) becomes

$$n = \frac{K_4}{\sqrt{q_1}} \qquad (51)$$

Equation (50) may be rearranged to read:

$$K_4 = \frac{n}{\sqrt{h}} \sqrt{\frac{q}{\sqrt{h}}} = n \sqrt{\frac{q}{h^{\frac{3}{2}}}} \qquad (52)$$

TABLE XXXI.

Showing Relation of Speed and Discharge of Various American Turbines Working under Catalogue Conditions.

$$K_4 = n\sqrt{\frac{q}{h^{\frac{3}{2}}}}$$

Manufacturer.	Name of Wheel.	K_4	
		Min.	Max.
Reaction Wheels.			
T. C. Alcott & Son..........	Alcott's Standard High Duty	98.8	114.7
	Alcott's Special High Duty.	154.5	159.2
Alexander, Bradley & Dunning......................	*Syracuse Turbine.........	89.8	108.2
American Steel Dredge Wrks.	*Little Giant	172.0	243.8
Camden Water Wheel Works	United States Turbine......	205.2	239.2
Chase Turbine Mfg. Co......	*Chase-Jonval Turbine (regular)	140.0	174.0
	*Chase-Jonval Turbine (special)................	201.0	255.0
Christiana Machine Co......	Balanced Gate Turbine.....	115.8	126.2
Craig, Ridgway & Son Co....	Double Perfection..........	90.5	97.2
Craig, Ridgway & Son Co....	Standard.................	94.0	101.2
Dayton Globe Iron Works Co.	*American Turbine........	83.0	109.0
	†New American (high head type)	75.4	85.9
	Improved New American...	265.0	268.0
	Special New American.....	170.5	190.0
J. L. & S. B. Dix...........	Improved Jonval Turbine..	84.0	100.0
Dubuque Turbine & Roller Mill Co..................	Flenniken Turbine.........	122.0	156.0
Dubuque Turbine & Roller Mill Co..................	McCormick's Holyoke Turbine.....................	162.0	176.0
Holyoke Machine Co........	Hercules Turbine..........	148.0	154.0
Humphrey Macnine Co.....	‡IXL Turbine.............	71.3	88.3
	‡XLCR Turbine...........	90.5	116.9
Rodney Hunt Machine Co...	McCormick's Holyoke Turbine.....................	159.5	176.0
	*Hunt McCormick Turbine.	161.4	207.5
	*New Pattern Hunt Turbine	132.4	174.8
	Standard Wheel, 1887 Pattern....................	126.0	145.0
E. D. Jones & Sons Co......	Crocker Wheel	161.0	169.8
James Leffel & Co..........	Samson Water Wheel......	201.7	203.0
	Improved Sampson.........	240.0	241.5
	Standard..................	103.7	107.0
	Special...................	134.7	139.8
Munson Bros. & Co..	‡‡Phoenix "Little Giant"	102.0	132.1
Norrish, Burnham & Co.....	....	115.9	120.0
Platt Iron Works Co........	Victor Register Gate.......	153.0	167.0
	Victor Standard Cylinder Gate.................	205.0	212.0

TABLE XXXI.—Continued

Showing Relation of Speed and Discharge of Various American Turbines Working under Catalogue Conditions.

$$K_4 = n\sqrt{\frac{q}{h^{\frac{3}{2}}}}$$

Manufacturer.	Name of Wheel.	K_4	
		Min.	Max.
Reaction Wheels.—Con.			
Poole Engineering and Machine Co.	Poole-Leffel	110.4	121.5
T. H. Risdon & Co	*Risdon Standard Turbine	93.4	117.2
	*Risdon Type T. C. Turbine	100.7	137.3
	*Risdon Type D. C. Turbine	108.0	158.0
S. Morgan Smith Co.	Smith-McCormick	163.7	185.0
	Smith	265.0	266.0
The Trump Mfg. Co.	Standard Trump	194.0	190.0
Wellman, Seaver, Morgan Co.	McCormick	168.5	179.0
Impulse Wheels.			
DeRemer Water Wheel Co.	DeRemer Water Wheel	11.10	13.20
Abner Doble Co.	*Tangential Wheel	6.61	9.20
Pelton Water Wheel Co.	Tangential Wheel	9.21	10.92
Platt Iron Works Co.	Victor High Pressure	37.8	42.2
Risdon Iron Works	Tangential Wheel	10.67	12.10

*Wide variation in constants due to the design being special for various sized wheels (series not exactly homogeneous).

†Catalogue recommends a maximum and minimum speed. Constants given are for the average speed.

‡Tables in catalogue based on full theoretical power of the water. Wheels are said to give from 75 per cent to 90 per cent efficiency, depending on location.

‡‡Munson Bros. Co. make several types of "Little Giant" turbines causing above wide variation in constants.

It is evident that K_4 is constant for all turbines with constant K and K_3; also, for all turbines where q, the discharge, is equal at the same speed, n, and under the same head, h, K_4 must be constant for different heads since n and q are proportional to $\sqrt{h}$. The values of the constant K_4 as calculated from the tables contained in the catalogues of various American manufacturers are given in Table XXXI.

164a. Relation of Speed to Power of Turbines.—From equation (35) may be derived

$$q = \frac{8.8\,P}{eh} \tag{53}$$

From equation (48) may be derived

$$(54) \qquad K_s \sqrt{K} = \frac{\pi n}{12 \times 60 \times \sqrt{h}} \sqrt{\frac{q}{\sqrt{h}}}$$

Combining equations (53) and (54)

$$(55) \qquad K_s \sqrt{K} = \frac{\pi \sqrt{8.8}}{12 \times 60\sqrt{e}}\, n \sqrt{\frac{P}{h^{\frac{5}{2}}}}$$

By transposing

$$(56) \qquad \frac{K_s \sqrt{K}\ 12 \times 60}{\pi} \sqrt{\frac{e}{8.8}} = n \sqrt{\frac{P}{\sqrt{h^5}}}$$

As the first member of the equation is constant for any given wheel, there may be written

$$(57) \qquad K_5 = \left(\frac{K_s \sqrt{K}\ 12 \times 60}{\pi} \sqrt{\frac{e}{8.8}}\right)^2$$

and hence

$$(58) \qquad K_5 = n^2 \frac{P}{\sqrt{h^5}}$$

From equation (58) it will be noted that the value of K_5 under a given head is in direct proportion to the square of the velocity of the wheel and to its power. K_5 is termed the "specific speed" of the wheel. A high value of K_5 is an indication of high speed, and a low value, of low speed.

The values of the constant K_5 as calculated from the tables contained in the catalogues of various manufacturers of American wheels are given in Table XXXII.

Fig. 222 shows graphically the relation of power to speed under one foot head, as expressed by the constant K_5 within the range of practice of American turbine builders.

The use of the diagram may be illustrated as follows:—

At 35 revolutions per minute various types of American wheels will develop from 1 to 5.8 horse power. For the best efficiency, that is for a constant value of ϕ, the number of revolutions of a wheel will vary as $\sqrt{h}$, and the power will vary as $h^{\frac{3}{2}}$. Thus for a 16 foot head these wheels will run four times as fast as for a one foot head or at 140 R. P. M., and will develop 64 times the power that will be developed at a one foot head, or from 64 to 371 H. P., between which limits the wheel must be chosen.

Suppose a wheel is desired to develop 500 H. P. at 150 R. P. M. under 25 foot head. These conditions correspond to 4 H. P. at 30

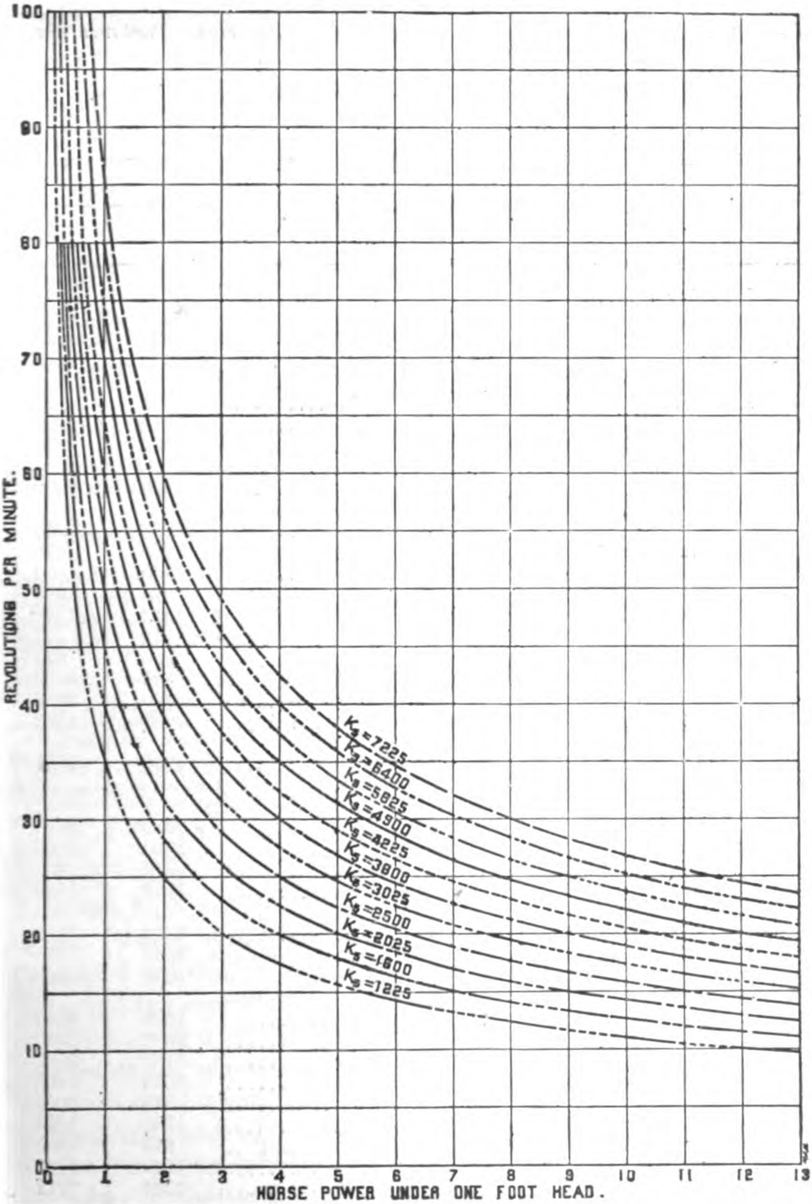

Fig. 222.—Speed Curves of Various Standard American Wheels.

TABLE XXXII.

Showing Relation of Speed and Power of Various American Turbines working under Catalogue Conditions.

$$K_s = n^2 \frac{P}{h^{\frac{5}{2}}}$$

Manufacturer.	Name of Wheel.	K_s Min.	K_s Max.
Reaction Wheels.			
T. C. Alcott & Son..........	Alcott's Standard High Duty	941	1216
	Alcott's Special High Duty.	2152	2300
Alexander, Bradley & Dunning......................	Syracuse Turbine..........	723	830
American Steel Dredge Wrks.	*Little Giant..............	2880	5420
Camden Water Wheel Works	United States Turbine......	3780	4570
Chase Turbine Mfg. Co......	*Chase-Jonval Turbine (regular)....................	1680	2580
	*Chase-Jonval Turbine (special)................	3460	5530
Christiana Machine Co......	Balanced Gate Turbine.....	1220	1475
Craig, Ridgway & Son Co....	Double Perfection..........	840	895
Craig, Ridgway & Son Co....	Standard..................	776	975
Dayton Globe Iron Works Co.	*American Turbine........	623	1140
	†New American (high head type)....................	520	674
	Improved New American ..	6100	6477
	Special New American.....	2490	3293
J. L. & S. B. Dix............	Improved Jonval Turbine..	965	1363
Dubuque Turbine & Roller Mill Co..................	Flenniken Turbine.........	1350	1880
Dubuque Turbine & Roller Mill Co..................	McCormick's Holyoke Turbine......................	2380	2810
Holyoke Machine Co........	Hercules Turbine..........	2030	2155
Humphrey Machine Co.....	‡IXL Turbine.............	572	889
	‡XLCR Turbine...........	1052	1545
Rodney Hunt Machine Co..	McCormick's Holyoke Turbine..........	2310	2810
	*Hunt McCormick Turbine.	2360	3910
	*New Pattern Hunt Turbine	1624	2900
	*Standard Wheel, 1887 Pattern....................	1665	2160
E. D. Jones & Sons Co......	Crocker Wheel....	2360	2630
James Leffel & Co..........	Samson....................	3775	3833
	Improved Samson..........	5013	5400
	Standard.....	948	1063
	Special....................	1730	1858
Munson Bros. & Co.........	‡‡Phoenix "Little Giant"..	843	1600
Norrish, Burnham & Co.....	....	1130	1345
Platt Iron Works Co........	Victor Register Gate.......	2254	2712
	Victor Standard Cylinder Gate	3733	4145
	Victor High Pressure......	129.10	169.50

TABLE XXXII.—Continued.

Showing Relation of Speed and Power of Various American Turbines working under Catalogue Conditions.

$$K_5 = n^2 \frac{P}{h^{\frac{5}{2}}}$$

Manufacturer.	Name of Wheel.	K_5 Min.	K_5 Max.
Reaction Wheels.—Con.			
Poole Engineering and Machine Co.	Poole-Leffel	1170	1239
T. H. Risdon & Co.	*Risdon Standard Turbine	2350	3680
	*Risdon Type T. C. Turbine	3520	5070
	*Risdon Type D. C. Turbine	4690	7370
S. Morgan Smith Co.	Smith McCormick	2640	3013
	Smith	6165	6640
The Trump Mfg. Co.	Standard Trump	3307	4250
Wellman, Seaver, Morgan Co.	McCormick	2380	2862
Impulse Wheels.			
DeRemer Water Wheel Co.	*DeRemer Water Wheel	12.34	18.01
Abner Doble Co.	*Tangential Wheel	4.00	7.62
Pelton Water Wheel Co.	*Tangential Wheel	7.84	11.42
Risdon Iron Works	*Tangential Wheel	8.24	11.22

*Wide variation in constants due to the design being special for various sized wheels (series not exactly homogeneous).

†Catalogue recommends a maximum and minimum speed. Constants given are for the average speed.

‡Tables in catalogue based on full theoretical power of the water. Wheels are said to give from 75 per cent to 90 per cent efficiency, depending on location.

‡‡Munson Bros. Co. make several types of "Little Giant" turbines causing above wide variation in constants.

R. P. M. under one foot head, and would require a wheel having a constant $K_5 = 3600$.

165. Value of Turbine Constants.—The values of the constants discussed in this chapter have been determined from the catalogues of the manufacturers of American turbines and are the values which may be used for determining the manufacturer's standard relations of the wheel for particular and fixed conditions where ϕ is constant, as, for example, the development of a certain power under a fixed head and with a given speed. When the head varies at different times, the value of ϕ also varies and the value of the other coefficients of the turbine, Δ, K, K_2, K_4, and K_5, will also vary. In order to discuss such conditions the laws of the variations of these constants, for any series of wheels, must be known. These laws can be ascertained from a complete test of any one wheel of the series and the laws so determined will hold for the entire series if the series is actually constructed on homogeneous lines. Owing to imperfections in the processes of manufacture, there is actually more or less variation between different wheels of a series. It is therefore desirable, when the approximate size of the wheel needed is known, to secure a test of a wheel of that particular size and hand.

Of the constants discussed, ϕ and Δ express the standard relation recommended by the manufacturer between diameter and speed in the series of wheels he offers. See equations

(23) $$n = \frac{1842\ \varphi\sqrt{h}}{D} \text{ and}$$

(24) $$D = \frac{\Delta\sqrt{h}}{n}$$

The coefficient K is the constant of discharge and shows the standard relation for various types of turbines between the quantity of water discharged and the diameter of the wheel. See equations

(41) $$q = K\,D^2\sqrt{h} \text{ and}$$

(42) $$D = \sqrt{\frac{q}{K\sqrt{h}}}$$

K_2 is the constant of power and shows the standard relation between the diameter of the wheel and the power. See equation

(45) $$P = K_2 D^2 h^{\frac{3}{2}}$$

K_4 is the constant of discharge and shows the standard relation between speed and discharge. See equation

(50) $$n = K_4\sqrt{\frac{h^{\frac{3}{2}}}{q}}$$

K_5 is the constant expressing the standard relation of power and speed for a particular series of wheels. See equation.

(58) $$P = K_5\,\frac{\sqrt{h^5}}{n^2}$$

The catalogue tables of turbines from which the standard values of the constants in the preceding tables have been calculated are presumably based on the actual tests of certain wheels of the series. The actual results of a test of any individual wheel of the series is likely to depart to an extent from the tabular value. Differences will often be found between wheels of different diameters, between wheels of the same diameter but of opposite hand, and even between wheels of the same size and hand which are supposed to be constructed on identical lines.

These differences in results are due to carelessness in construction, or to unusually good construction in the effort to secure special results, where the conditions warrant special effort. Any change in the design of a wheel for the purpose of reducing or increasing the discharge, and hence reducing or increasing its power, will give rise to differences in these coefficients which must be taken into account in any calculations made thereon. A careful study of these coefficients as determined from the actual tests of any wheel, together with a study of the design of the wheel itself, will form the basis of a complete and systematic knowledge of water wheel design.

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CHAPTER XV.

TURBINE TESTING.

166. The Importance of Testing Machinery.—A correct theory based on mathematical analysis forms a valuable foundation for machine design. In the construction of any machine, however, theoretical lines can seldom be followed in all details, and, even if this were possible, the truth of the theory must be demonstrated by actual trial for there are usually many factors involved which cannot be theoretically considered and yet affect practical results. In any machine much depends upon the character of the workmanship, on the class of material used, and on all the details of manufacture, installation and operation as well as on design. All of these matters can hardly be included in a theoretical consideration of the subject, and it therefore becomes necessary to determine the actual results attained by a trial of the machinery under working conditions.

General observations or even a detailed examination of any machine and its operation can rarely be made sufficiently complete to give any accurate knowledge of the quantity or quality of the results which it can and does accomplish. It is only when the actual effect of slight changes in design can be accurately determined by careful experiment that a machine can be improved and practical or approximate perfection attained.

The ease with which such determination can be made is usually a criterion of the rapidity with which the improvements in the design and construction of a particular machine take place. Where such determinations are readily made, rapid advancement results, but where they are costly and require a considerable expenditure of time or money, the resulting delays and expenses usually so limit such determinations that good results are attained but slowly.

The invention of the steam engine indicator and the Prony brake placed in the hands of the engineer instruments by means of which he could readily determine the action of steam within the engine cylinder and the actual power developed therefrom. The knowledge thus gained has been one of the most potent factors in the rapid advancement of steam engineering.

The physical results of radical modifications or changes in design are sometimes quite different from those anticipated by the designer. Improvement in any machine means a departure from the tried field of experience and the adoption of new and untried devices or arrangements. Frequently a line of reasoning, while apparently rational, is found to be in error on account of unforeseen conditions or contingencies and the results anticipated are not borne out in the actual practical results. Unless, therefore, such results are carefully and accurately determined by exact methods the actual value of changes in design may never be known or appreciated and designs may be adopted which, while apparently giving a more desirable form of construction, actually accomplish less than the form from which the design has departed.

167. **The Testing of Water Wheels.**—The value of the testing of water wheels was recognized by Smeaton who tested various models of water wheels about the middle of the Eighteenth Century. Methods of turbine testing were also devised with the first development of the turbine, which have been potent factors in the improvement of the turbine. While the methods of testing have been greatly improved since that time, they have not as yet reached a state that can be considered reasonably satisfactory, and turbine testing has not become so general as to assure the high grade of design and workmanship in their manufacture as in other machinery where testing is more easily and regularly practiced.

The principal causes of the backward condition of turbine testing lie in the difficulties and expense of making an accurate test in place, and the expense and unsatisfactory results of testing turbines in a testing flume where the head and capacity are so limited as to confine satisfactory tests to heads of 17 feet or less and to wheels of a capacity of about 250 cubic feet per second, or less if the full head of 17 feet is to be maintained. There is an urgent demand for accurate and economical methods for the measurement of the water used and of the power developed by water wheels in place, that can be readily and quickly applied without the almost prohibitive expense of the construction of expensive weirs and other apparatus now used for such purposes. Apparently slight variations in turbine construction produce radical changes in practical results. The high results achieved under test by a well-designed and well-constructed wheel is no assurance that wheels of the same make and of the same design, even though they be of the same size and even from the same pattern, will give similar results. This is especially true when the contingencies of competition and the knowledge that a test of the wheel is impossible, or at least highly improbable, offer a premium on careless construction and cheap work.

A brief examination of the work already done in this line, and of the methods now in vogue, may afford suggestions for future improvements and development in this important work.

168. Smeaton's Experiments.—John Smeaton, the most experienced and eminent engineer of his time, made a series of experiments on the power and effect of water used by means of various forms of water wheels for mill purposes. Accounts of these experiments were published in the Transactions of The Royal Society of England in 1759. Until that time the relative values of the different types of water wheels of that day were very poorly understood and appreciated.

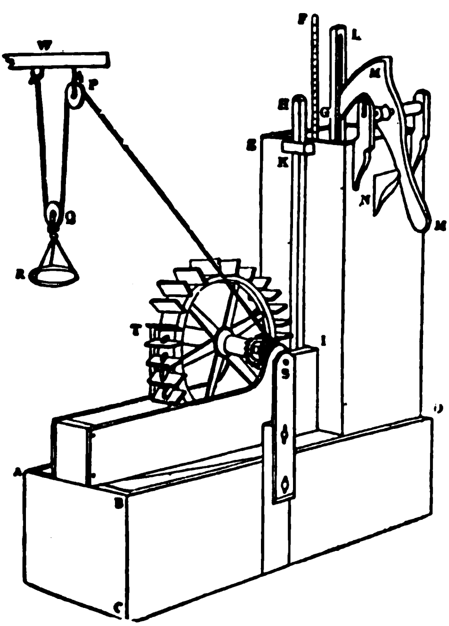

Fig. 223.—Smeaton's Apparatus for Testing Water Wheels.

Smeaton's apparatus for measurement of the power of overshot and undershot wheels is shown by Figs. 223 and 224 taken from "The Encyclopedia of Civil Engineering" by Edward Cressy. Water was pumped by means of the hand pumps from the tail basin, X, to the supply cistern, V, from which it was admitted to the wheel through an adjustable gate. The power developed was measured by the time required to raise a known weight through a known height by means of a cord passing through a system of pulleys and attached to a small winding drum or collar upon the wheel shaft. This drum revolved only when, by slight longitudinal movement, it was made to engage a pin on the shaft.

In these experiments Smeaton found a maximum efficiency of

32 per cent., and a minimum efficiency of 28 per cent. for undershot wheels. He also observed that the most efficient relations between the peripheral velocity of the wheel and velocity of the water were attained when the former was from 50 per cent. to 60 per cent. of the latter, and that the force that could be exerted by a wheel to

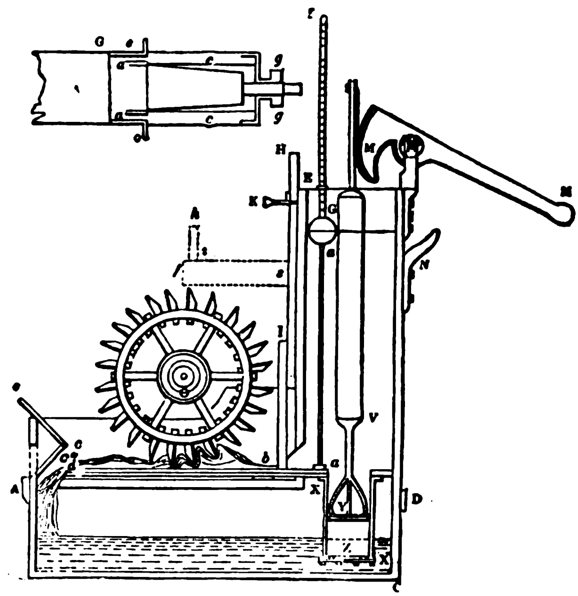

Fig 224.—Section of Smeaton's Apparatus for Testing Water Wheels.

advantage was from 50 per cent. to 70 per cent. of the force required to maintain it in stationary equilibrium.

For overshot wheels Smeaton found that the efficiency varied between 52 and 76 per cent. From his experiments he concluded that the overshot wheel should be as large as possible, allowing, however, a sufficient fall to admit the water onto the wheel with a velocity slightly greater than that of the circumference of the wheel itself, and that the best velocity of the circumference of the wheel was about three and one-half feet per second. This speed he found applied both to the largest as well as to the smallest water wheel.

From these experiments Smeaton concluded that the power of water applied directly through the exertion of its weight by gravity, as with the overshot wheel, was more effective than when its power was applied through its acquired momentum, as in the undershot wheel, although his line of reasoning indicated otherwise. The later development of impulse wheels shows that his reasoning was correct, and that the low efficiency of the impulse wheel was due to the method of applying the momentum of the water rather than to any inherent defect in the impulse principle.

The experiments or tests of Smeaton, while crude and imperfect and performed upon wheels which were merely models, afforded a comparative measurement of the efficiency of the undershot, overshot and breast wheels then in use and had a marked effect on the further selection of such wheels.

169. **The Early Testing of Turbine Water Wheels.**—The testing of turbine wheels began many years ago in France before the turbine became well known in the United States.*

Fourneyron began the study of the early forms of turbines as early as 1823, and, in 1827, he introduced his well-known wheel and also brought into notice a method of systematic testing of the same by means of the Prony brake.

"La Societé d' Encouragement pour l' Industrie Nationale" is credited by Thurston with the introduction of a general system for the comparison of wheels and correct methods of determining the efficiency.** Other engineers immediately accepted this method of comparison of wheels. Morin, in 1838, reported the results of a trial of a Fourneyron wheel as giving an efficiency of 69 per cent. with only slight changes in values for a wide range of speed. With another wheel he obtained 75 per cent. efficiency.†

Combes tested his reaction wheel and found that an efficiency of about 50 per cent. could be obtained.‡

The first systematic test of turbines in the United States was made by Mr. Elwood Morris of Philadelphia in 1843 and reported in the Journal of The Franklin Institute for December of that year. The maximum efficiency reported was 75 per cent. This result was reached when the value of ϕ for the interior circumference of the Fourneyron turbine was .45. In 1844 Mr. James B. Francis determined the power and efficiency of a high breast water wheel

* See "The Systematic Testing of Water Wheels in the United States," by R. H. Thurston, Trans. Am. Soc. Mech. Eng. vol. 8.

** See "Memoire sur les Turbines Hydrauliques," by H. Fourneyron, Brussels, 1840.

† See "Experiences sur les Power Hydrauliques," Paris, 1838.

‡ See "Mechanics of Engineering," Weisbach. Translated by A. J. DuBois. Hydraulics and Hydraulic Motors, vol. II, part I, p. 470.

in the City of Lowell, using a Prony brake fitted with a dash-pot to prevent irregular operation.

In 1845 Mr. Uriah A. Boyden made a trial of a turbine designed by himself, using the Prony brake, and obtained an efficiency of 78 per cent. as the maximum. In 1846 a similar test of one of the Boyden turbines was made at the Appleton Mills in Lowell, and an efficiency of 88 per cent. was reported. He continued the work of the testing of water wheels for several years and tested many wheels of various types.* Mr. Francis introduced the system of testing wheels which were to be used by purchasers of water from the water power company which he represented. The chief purpose of the tests was that the wheels might be used as meters in determining the amount of water used by the various purchasers.

In 1860 the City of Philadelphia undertook a comparative trial of various turbines in order to determine their relative merits for used in the Fairmount Pumping Plant. The results of these tests given in Table XXXIII are somewhat questionable but have a comparative value.

TABLE XXXIII.

Water Wheel Tests at Philadelphia in 1860.

Name of Wheel.	Kind of Wheel.	Per cent of Effect	3 per cent added for frict'n	Where built.
Stevenson's second wheel	Jonval	.8777	.9077	Paterson, N. J.
Geyelin's second wheel	Jonval	.8210	.8510	Philadelphia, Pa.
Andrews & Kalbach's third wheel	Spiral	.8197	.8497	Bernville, Pa.
Collin's second wheel	Jonval	.7672	.7972	Troy, N. Y.
Andrews & Kalbach's second wheel	Spiral	.7591	.7891	Bernville, Pa.
Smith's, Parker's fourth trial	Spiral	.7569	.7869	Reading, Pa.
Smith's, Parker's third trial	Spiral	.7467	.7767	Reading, Pa.
Steven's first wheel	Jonval	.7335	.7635	Paterson. N. J.
Blake	Scroll	.7169	.7469	East Pepperell, Mass.
Tyler	Scroll	.7123	.7423	West Lebanon, N. H.
Geyelin's first wheel	Jonval	.6799	.7099	Philadelphia, Pa.
Smith's, Parker's second wheel	Spiral	.6726	.7026	Reading, Pa.
Merchant's Goodwin	Scroll	.6412	.6712	Guilford, N. Y.
Mason's Smith	Scroll	.6324	.6624	Buffalo, N. Y.
Andrew's first wheel	Spiral	.6205	.6505	Bernville, Pa.
Rich	Scroll	.6132	.6432	Salmon River, N. Y.
Littlepage	Spiral	.5415	.5715	Austin, Texas.
Monroe	Scroll	.5359	.5659	Worcester, Mass.
Collin's first wheel	Jonval	.4734	.5034	Troy, N. Y.

* See "Lowell Hydraulic Experiments."

170. The Testing of Turbines by James Emerson.—One of the men who did much valuable work of this character was Mr. James Emerson who designed a new form of dynamometer of the transmitting kind. At the request of Mr. A. M. Swain, Mr. Emerson designed a Prony brake, embodying this dynamometer for the purpose of testing a Swain turbine in a flume built from designs by Francis. The results obtained by Mr. Emerson from this test were so satisfactory that The Swain Turbine Company decided to open the flume for the purpose of a competitive test of all turbines which might be offered for this purpose. Announcement of this test was dated June 16th, 1869. The pit was fourteen feet wide, thirty feet long, and three feet deep, measured from the crest of the weir. The best results of this competitive test, the accuracy of which has since been questioned by Mr. Emerson, were attained with the Swain and Leffel wheels. The former ranged from 66.8 up to 78.9 per cent. efficiency, and the latter from 61.9 to 79.9 per cent. efficiency. This competitive test was the beginning of a series of such tests as well as of a general system of the public testing of turbines. The testing flume was opened to all builders and users of turbine wheels and such tests have been continued in the United States up to the present time.

The report of the results of this test attracted the attention of Mr. Stewart Chase, then agent of The Holyoke Water Power Company, who, recognizing its very great importance, secured the adoption of a systematic testing of water wheels at Holyoke for the benefit of the Company and wrote to Mr. Emerson as follows:

"The testing of turbines is the only way to perfection, and that is a matter of great importance. Move your work to Holyoke and use all the water that is necessary for the purpose, and welcome, free of charge."

Mr. Emerson, who had been conducting the testing of water wheels as a matter of private business at Lowell, at which place he was obliged to pay for the water used, at once accepted the liberal offer thus tendered him and removed to Holyoke where he continued the testing of water wheels until it was taken in hand by The Holyoke Water Power Company.

The reports of Mr. Emerson's work were published and undoubtedly were the means of bringing a number of wheels up to a state of high efficiency. The reports were found to be full of valuable data, and, although not systematically arranged, formed an extensive and valuable collection of figures.*

In 1879, The Holyoke Water Power Company, for the purpose of determining the standing of wheels offered for use at that place,

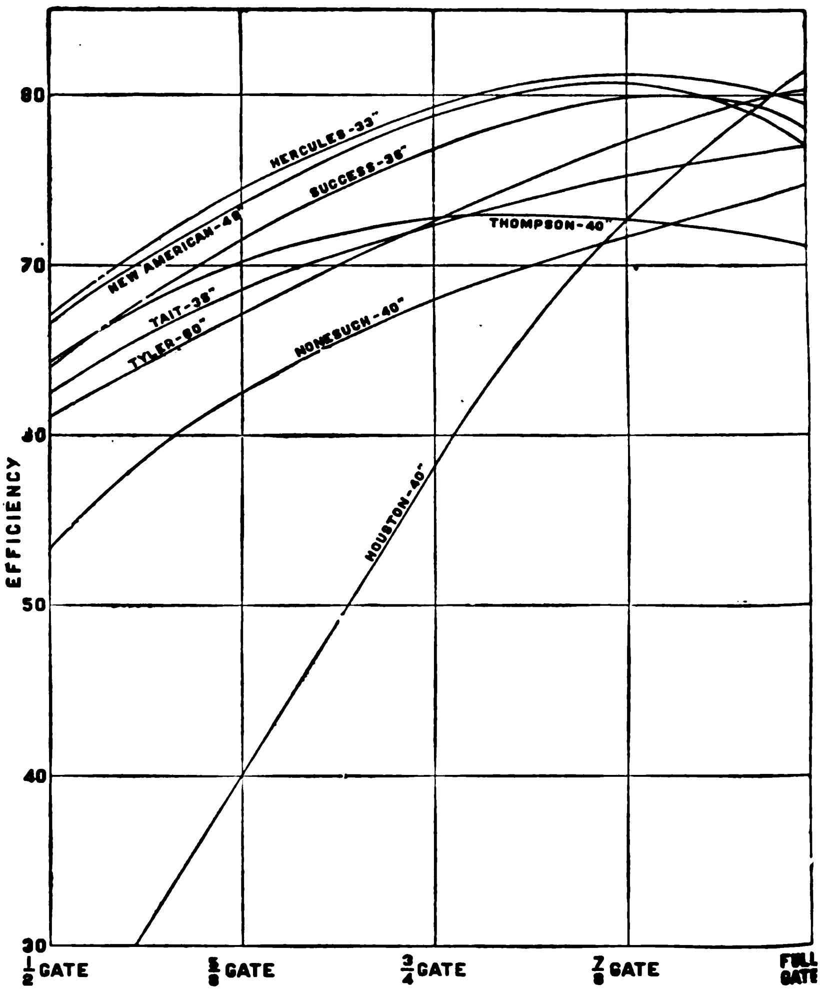

Fig. 225.

arranged for a comparative or competitive turbine test at the flume constructed by Mr. Emerson at Holyoke. The wheels were set under the direction of Mr. Emerson and a part of the tests were

* See James Emerson's "Hydro-Dynamics."

made or witnessed by Mr. Samuel Weber and Mr. T. G. Ellis. Their report was accompanied by a graphical diagram (Fig. 225 and Table XXXIV) on which they commented as follows:

"By examining the diagram and table, the peculiarities of the several wheels will be readily seen. It will be observed that the Houston turbine, which has the highest percentage of effect at full gate, is really the least efficient at from half to three-quarters, and from half to full gate, of all those shown on the diagram, and is only superior to the Nonesuch at from three-quarters to full gate, and that by a very trifling amount; so that the wheel which apparently has the highest percentage is really the least desirable for actual use. The Thompson turbine, which has the lowest percentage of those shown at full gate, rises to the sixth place at from one-half to full gate, and to the fourth place at from one-half to three-quarters gate. The Tyler turbine, which has the second highest percentage at full gate, falls to the sixth place at from one-half to three-quarters gate. The Hercules turbine, which stands third only at full gate, takes the first rank at from half to full gate, or any of its subdivisions. The New American turbine, which stands only fifth in the percentage at full gate, is second only to the Hercules at from one-half to full gate or either of its subdivisions, and, indeed, differs from the Hercules very slightly in its useful effect through the whole range shown.

"Taking the average useful effect of the wheels shown from one-half to full gate as a measure of their efficiency, their relative value is in the order shown in the table."

TABLE XXXIV.

Showing Average Percentage at Part Gate.

Name.	½ to ¾ Per cent.	¾ to Full Per cent.	½ to Full Per cent.
Hercules	.737	.805	.771
New American	.732	.795	.763
Success	.708	.786	.747
Tyler	.665	.766	.715
Tait	.680	.744	.712
Thompson	.696	.721	.709
Nonesuch	.619	.712	.666
Houston	.397	.717	.557

The report of Mr. Emerson covered a much larger number of wheels. The diagram accompanying Mr. Emerson's report* is reproduced in Fig. 226.

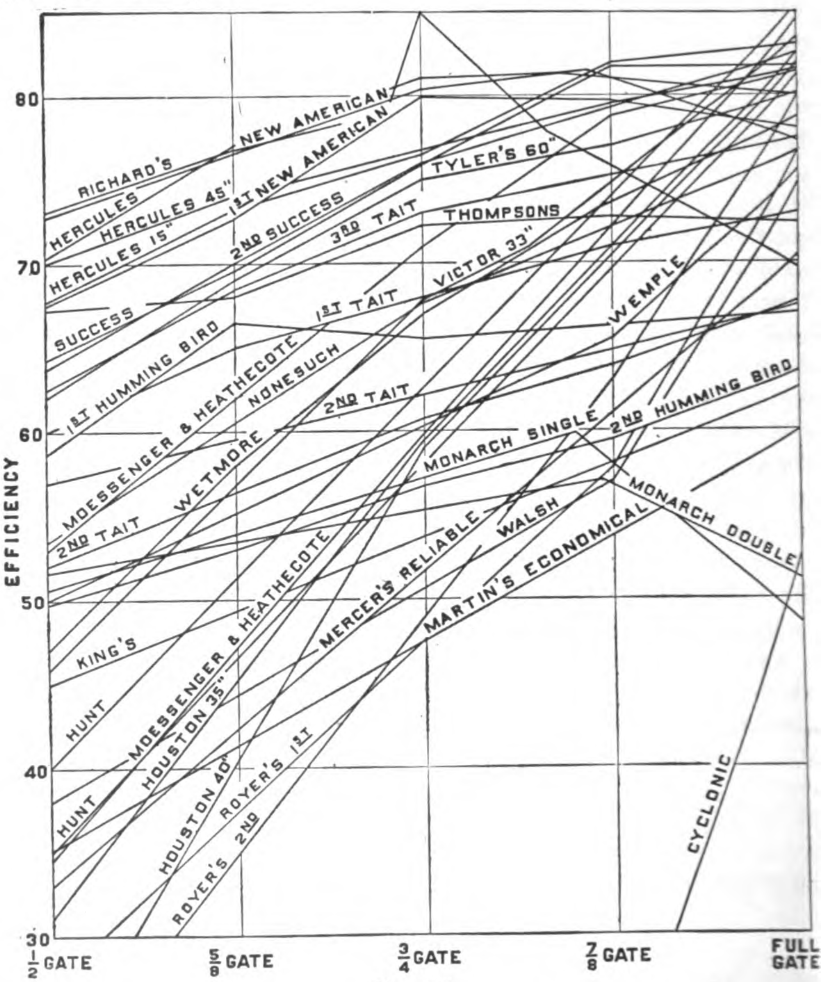

Fig. 226.

171. The Holyoke Testing Flume.—The later work of systematic testing of American turbines has been carried on principally at the Holyoke flume.

† "The object aimed at by the Water-power Companies of Lowell and Holyoke, in the establishment of testing flumes for turbines,

* Emerson's "Hydro-Dynamics," page 300.

† "The Systematic Testing of Water Wheels," by R. H. Thurston.

is the determination of the power and efficiency, the best speed, and the quantity of water flowing at from whole, to, say, half gate, so exactly that the wheel may be used as a meter in the measurement of the water used by it. The quantity of water passing through the wheel, at any given gate-opening, will always be practically the same at the same head, and the wheel having been tested in the pit of the testing flume, and its best speeds and highest efficiency determined, and a record having been made of the quantity of water discharged by it at these best speeds and at all gates, the turbine is set in its place at the mill, speeded correctly for the head there afforded, and a gauge affixed to its gate to indicate the extent of gate opening. The volume of water passing the wheel at various openings of gate having been determined at the testing flume, and tabulated, the engineer of the Water-power Co. has only to take a look at the gauge on the gate, at any time, or at regular times, and to compare its reading with the table of discharges, to ascertain what amount of water the wheel is taking and to determine what is due the company for the operation of that wheel, at that time. The wheel is thus made the best possible meter for the purposes of the vender of water."

The present Holyoke Testing Flume was completed in 1883. The plan of this flume is shown in Figs. 227 and 228.

The testing flume consists of an iron penstock, A, about nine feet in diameter, through which the water flows from the head race into a chamber, B, from which it is admitted through two head gates, G,G, into the chamber, C, and from thence through trash racks into the wheel pit, D. Passing through the wheel to be tested, it flows into the tail-race, E, where it is measured as it flows over a weir, at O. The object of the chamber, B, is to afford opportunity for the use of the two head gates, G,G, to control the admission of water, and consequently the head acting on the wheel. There is also a head gate at the point where the penstock, A, takes in water from the first level canal. A small penstock, F, about 3 feet in diameter, takes water from the chamber, B, independently of the gates and leads to a turbine wheel, H, set in an iron casing, in the chamber, C, in order that this wheel can run when C and the wheel pit, D, are empty. The wheel, H, discharges through the floor at the bottom of C, and through the arch, I, and the supplementary tail-race, K, into the second level canal. This wheel is used to operate the repair shops; also to operate the gates, G. The chamber, C, is bounded on one side by a tier of stop-planks, L, and,

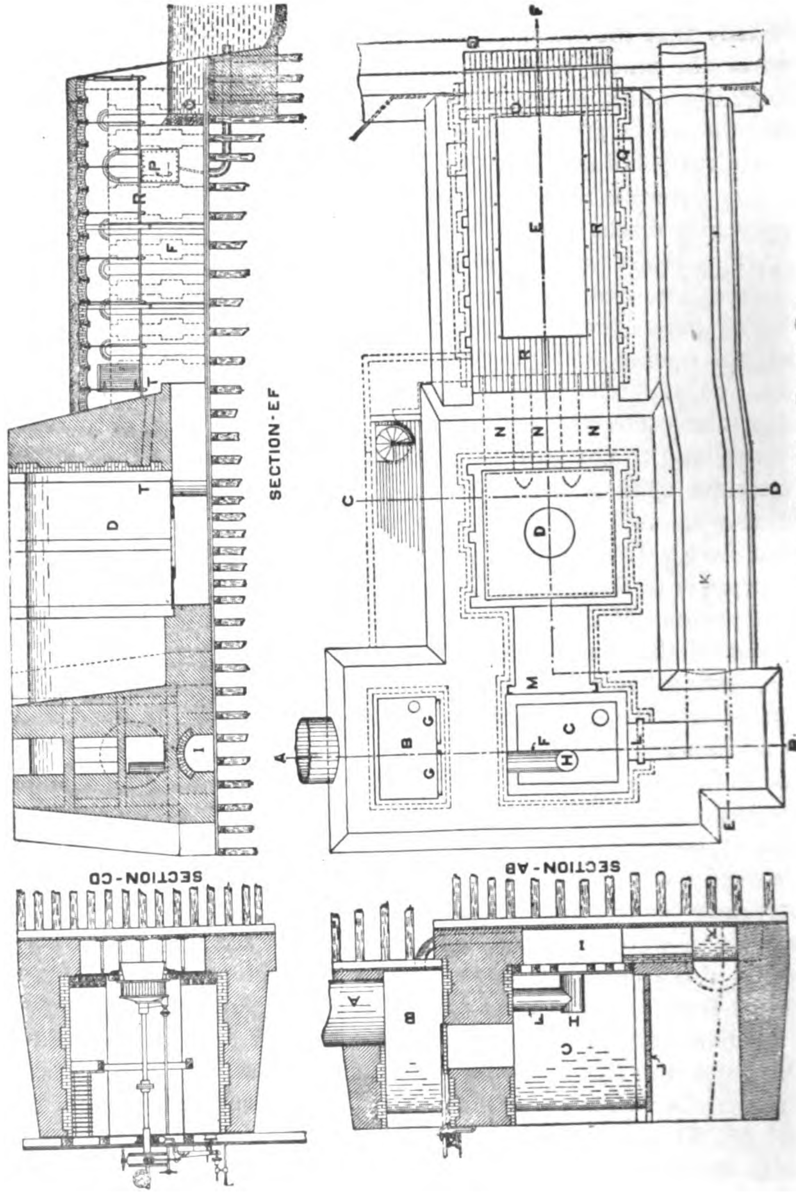

Fig. 227.—Testing Flume of Holyoke Water Power Co.

on another side, by a tier of stop-planks, M. The object of the stop-planks, L, is to afford a waste-way out of the chamber, C. This is of especial use in regulating the height of the water when testing under low heads. The water thus passed over the planks, L, falls directly into the tail-race, K. and passes into the second level. The stop-planks, M, are used when scroll or cased wheels

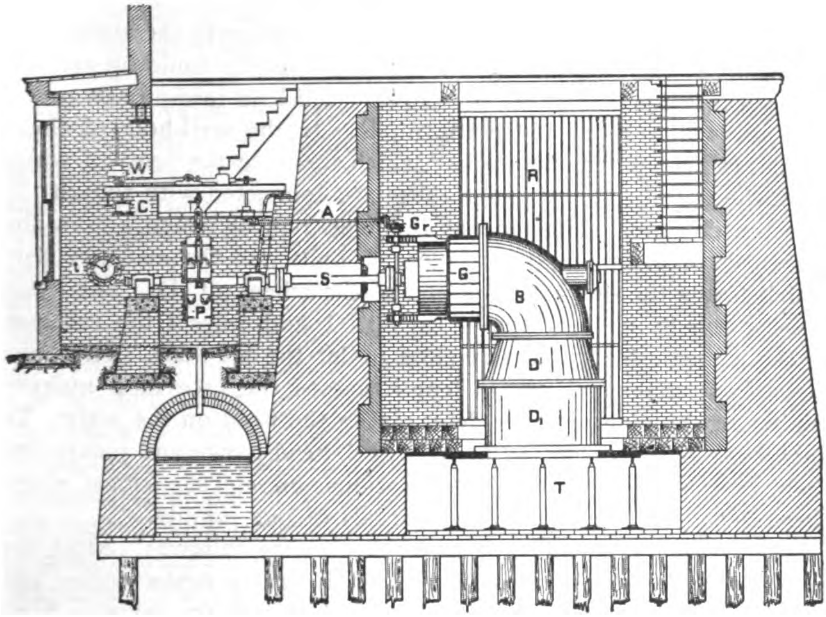

Fig. 228.—Testing Flume of Holyoke Water Power Co. Arranged for Testing Horizontal Turbines.

are tested. In such cases D is empty of water and the wheel case in question is attached by a short pipe or penstock from an opening cut in the planks, M. Flume wheels are set in the center of the floor of D, and D is filled with water. They discharge through the floor of D and out of the three culverts, N,N,N, into the tail-race, E. Horizontal wheels are set in the pit, D, with their shafting projecting through a stuffing-box in the side of the pit (See Fig. 228). At the down-stream end of the tail-race is the measuring weir, O (Fig. 227). The crest of the weir is formed of a strip of planed iron plate twenty feet in length. The depth of water on the weir is measured in a cylinder, P, set in a recess, Q, fashioned in the sides of the tail-race. These recesses are water-tight, and the observer is thus enabled to stand with the water-level at convenient height for accurate observation. The cylinder, P, is connected with a pipe that crosses the tail-race or weir box about ten feet back of the weir crest. The pipe is placed about one foot above the floor and is perforated in the bottom with ⅛ inch holes. A platform, R, surrounds the tail-race, and is suspended from the iron beams that carry the roof. Above the tail-race is the street, over which the wheels to be tested arrive on wagons from which they are lifted by a traveling crane that runs on a frame-work over the street, and by means of which the wheels are carried into the building and are lowered into the wheel pit, D. Spiral stairs lead into a passageway that leads in turn to the platform, R. In the well-hole of these stairs are set up the glass tubes which measure the head of water upon the wheel. These gauge tubes are connected with the pit, D, and the chamber, C, by means of pipes, one of which enters the wheel pit through a cast iron pipe, T, built into the masonry dam which forms the down stream end of the wheel pit, D. The other pipe passes back under the wheel pit, D. and crosses the tail-race at the extreme back line and close under the pit floor. This pipe is perforated throughout its length across the race in a manner similar to the pipe used for determining the head on the weir. To enable the observers at the brake wheel, head gauge and measuring weir to take simultaneous observations, an electric clock rings three bells, simultaneously, at intervals of one minute.

The usual method of testing a wheel is as follows: After the wheel is set in place (See Figs. 227 and 228) a brake pulley and Prony brake are attached to the shaft, the gates are set at a fixed opening and water is admitted. The runaway speed of the wheel is first determined with the brake band loose, after which a weight is applied and the brake tightened until the friction load balances the weight. As soon as this balance is attained, which requires only a few seconds, the revolution counter is read and the heads in the head-race, tail-race and on the weir are observed. Observations are repeated simultaneously each minute at the stroke of the bell and for a period of from three to five minutes. The weight is then changed and the observations repeated for a different load and speed. After observations are made over the range of speeds desired, the gate opening is changed, and a similar series of observations are made for the new gate opening. This is repeated for each desired gate opening, usually from full gate to about one-half gate.

The results are calculated and reported in the form shown in Table LX. It is usually stated in the report whether the test is made with a plain or conical draft tube, whether plain or ball bearings are used, and also the pull necessary, at a given leverage, to start the turbines in the empty pit. No attempt is made in these reports to describe the bearings or finish of the wheels in detail.

The maximum head available is about 17 feet under small discharges and this decreases to about 9 feet under a discharge of 300 cubic feet per second. The capacity of the tail-race and weir is hardly sufficient for the accurate measurement of the latter quantity.

172. **The Value of Tests.**—There can be no question as to the very great value of carefully-made tests of any machine. It must be borne in mind, however, that any test so made represents results under the exact conditions of the test, and, in order to duplicate the results, the conditions under which the test was made must be duplicated. Any changes in the design or finish of the wheels, any alterations in the method of setting, or in the gates, draft tube or other appurtenances connected with the same are bound to affect the power and efficiency to a greater or less extent.

It is unfortunate for the world's progress that the records and conditions of failures are seldom made known. The record of a failure, while of great value from an educational standpoint, may considerably injure the reputation of an engineer or manufacturer, and consequently results of tests and experiments, unless fully satisfactory, are seldom published or known except by those closely interested. For this reason, the published tests of water wheels usually represent the most successful work of the maker and the best practical results he has been able to secure. Tests, unless fairly representative, do not assure that similar turbines of the same make, or even similar turbines of the same make, size and pattern, will give the same efficient results unless all details of their design, construction, and installation are duplicated. There is no doubt that in many cases the published tests of water wheels are the final consummation of a long series of experiments, made in order to secure high results, and do not give assurance that such results can be easily duplicated. The manufacturers have acknowledged this by calculating their standard tables on a basis of power and efficiency below that of the best tests they are able to obtain, and it is only a matter of reasonable precaution for the engineer, who is utilizing the results of any such tests for the purposes of his design, to discount the test values to such an extent as will assure him that his estimates will be fulfilled.

The total losses given above correspond well with current practice. Under the best conditions efficiencies greater than 83 per cent. are often obtained, and, under unfavorable condition,s with poor design and poor construction, efficiencies much less than the minimum of 72 per cent. are common. While these losses can never be entirely obviated they should be reduced to the practical minimum that good design and good workmanship will permit.

175. **Measurement of Discharge.**—The discharge, q, of the wheel is commonly measured in cubic feet per second and should represent only the actual discharge through the wheel itself. This discharge is usually measured, after it has passed the wheel, by the flow over a standard weir. Any leakage around the wheel into the weir box or from the weir box around the weir must be determined and deducted from or added to the amount passing the weir. The actual weir discharge must be known either by a direct calibration of the weir or by the construction of the weir on lines for which the discharge coefficients are well established. Errors in weir measurements often reach values of nearly 5 per cent. due to the erroneous use of coefficients obtained from other weirs not strictly comparative.

The head on the weir must be accurately determined by means of a hook gauge which should usually read to .001 of a foot. An error of .01 foot in reading the head on the weir represents about 1 per cent., and an error of .001 about .1 per cent., in the computed discharge with a 1.5 foot head on the weir and a much greater error at a lower head.

The construction of weirs in the tail-race of power plants, especially where large quantities of water are used under low heads, involves an expense which is often prohibitive. In addition to this, the construction of such weirs in plants working under low heads would often seriously reduce the head and alter the working conditions.

Other methods of accurately determining the flow should be developed. There are two methods which seem to give promise of good results:

First: By the careful determination of the velocities of flow in the cross-section of the head or tail-race at points far enough from the wheel to guarantee steady flow. This may be done by means of a carefully calibrated current meter, a pitot tube, or by floats To secure good results these instruments must be in the hands of one familiar with their use and with the sources of error to which each is liable if carelessly used. (See Chapter XI.) This method involves no loss in head.

Second: By the construction in the head or tail race of submerged orifices of known dimensions and of a character for which the coefficient of discharge has been determined. Some work in this line has been done at the University of Wisconsin (See pages 43 to 45) which will soon be made accessible in detail in a bulletin now in press. This method will involve only small losses of head and by a sufficient range of experiments can perhaps be made nearly as accurate as weir measurements.

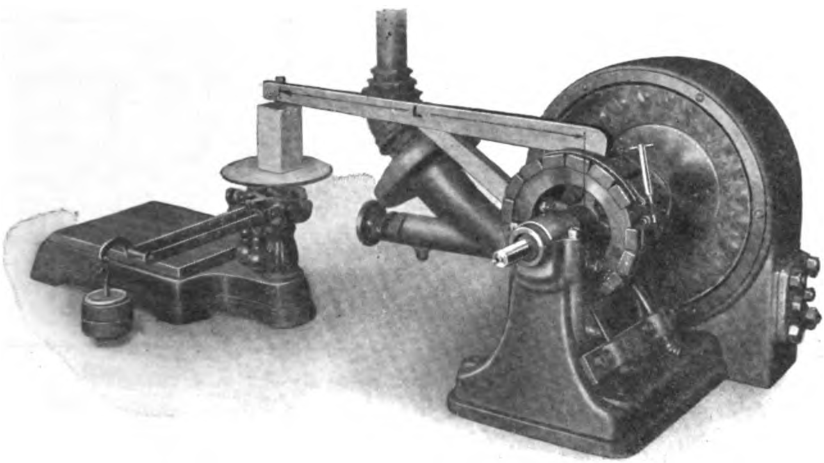

Fig. 229.—Doble Tangential Wheel Arranged for Brake Test.

176. Measurement of Head.—The power of water applied to the wheel depends on both quantity and head. The head is more easily measured than the quantity, but, nevertheless, requires considerable care for its accurate determination.

The head on the wheel must be measured immediatly at the wheel both for the head-water and tail-water. If measured some distance away it is apt to include friction losses, which should not be charged against the wheel in raceways, penstocks and gates. The measurement of head should usually be to about .01 feet, although this depends on the magnitude of the heads involved.

177. Measurement of Speed of Rotation.—The speed of the wheel is usually recorded in revolutions per minute and may be

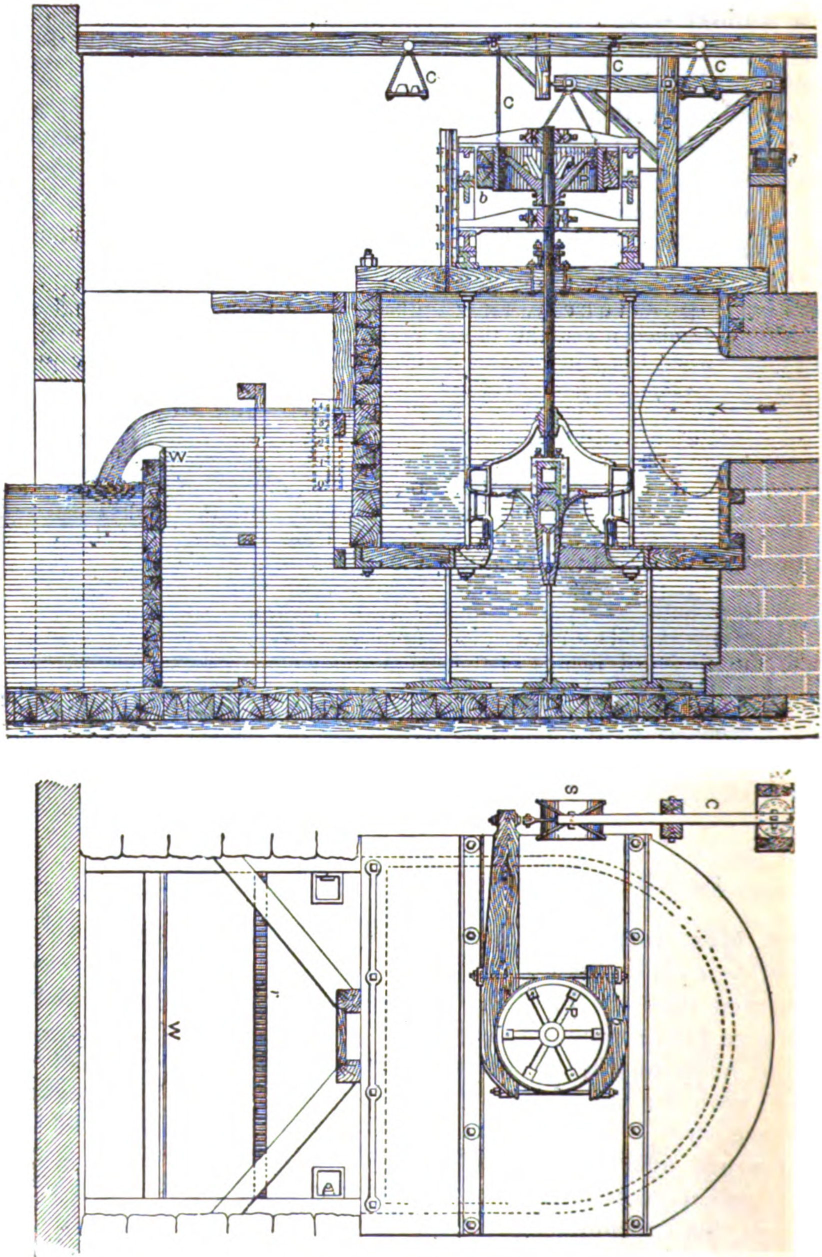

Fig. 230.—Section and Plan of **Apparatus for Testing Swain Turbine (by James B. Francis).**

determined by a revolution-counter which records the number of revolutions made in a given interval of time; or by a "tachometer" which, by means of certain mechanism, indicates at once on a dial the revolutions per minute. The latter method is more convenient if the instrument is correct, but frequent calibration and adjustment are necessary and a correction must usually be applied to values thus observed.

The revolution-counter is more accurate, and, while not so convenient, is to be preferred.

178. **Measurement of Power.**—The power of the wheel may be determined by placing a special brake pulley on the turbine shaft and applying a resistance by means of a Prony brake or some other form of dynamometer. This resistance is then measured by some form of scales (See Figs. 229 and 230). The power thus consumed by the friction of the brake can be calculated by equation (1)

$$(1)\quad P = \frac{2\pi\, l\, n\, w}{33000} \text{ in which}$$

P = Horse power
l = length of lever or brake arm from center of revolution, in ft.
n = revolution per minute.
π = ratio of the circumference to the diameter of a circle = 3.1416.
w = weight on the scale in pounds.

This is the method applied in all laboratory work (see Fig. 229) and is that used at the Holyoke Testing Flume. If properly applied, it is probably subject to minimum error. When wheels are tested in place, it is sometimes more convenient, and often essential, to determine the power output from the current generated by electrical units, which, when measured by aid of the known efficiency of the generator, will give the actual power of the wheel. If these units be direct-connected so that little or no transmission loss is involved, and if the generator is new and its efficiencies have been accurately determined, the errors involved by this method are comparatively small. The transmission of the power before measurement through gearing, through long shafts and bearings or by other means, involves losses, the uncertainties of which must be avoided if accuracy is essential.

179. **Efficiency.**—The efficiency of a machine is the ratio of energy delivered by the machine to that which was supplied to it and it may have various significations.

In an impulse wheel (See Section 152) the theoretical energy of the water in the forebay in foot pounds per second is:

$$(2)\qquad E = qwh$$

The energy just inside the outlet of the pipe is

$$(3) \qquad E_1 = qw\,(h' + h'')$$

The energy of the jet is

$$(4) \qquad E_2 = \frac{qwv^2}{2g}$$

and the theoretical power delivered to the bucket is

$$(5) \qquad E_3 = \frac{qw\,(1 - \varphi)\,v\,(1 - \cos\alpha)\,\varphi\,v}{g}$$

If e represents the actual ft. lbs. of work delivered by the wheel per sec. then

(6) $\frac{e}{E}$ = the efficiency of the entire installation including pipe, jet, wheel, etc.

(7) $\frac{e}{E_1}$ = efficiency of the water wheel, including nozzle and buckets,

(8) $\frac{e}{E_2}$ = efficiency of the runner, and

(9) $\frac{e}{E_3}$ = hydraulic efficiency of the bucket

In the testing of water wheels, the efficiency (7), $\frac{e}{E_1}$, is the ratio ordinarily to be determined since it involves the losses in the noz-

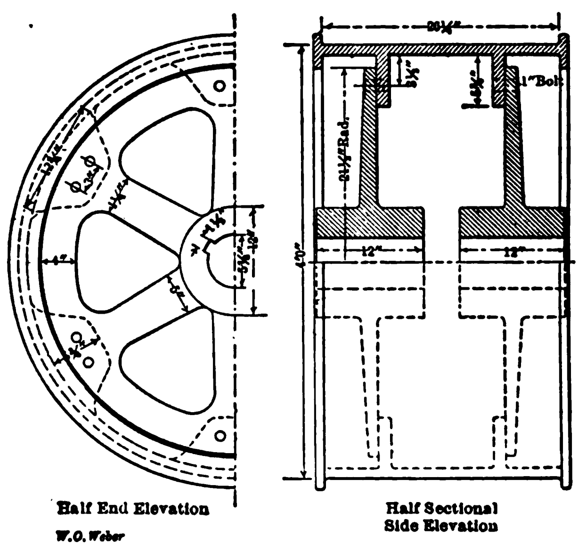

Fig. 231.

zle, jet and buckets as well as from residual energy in the water discharged by the buckets, all of which are properly chargeable to the operation of the wheel.

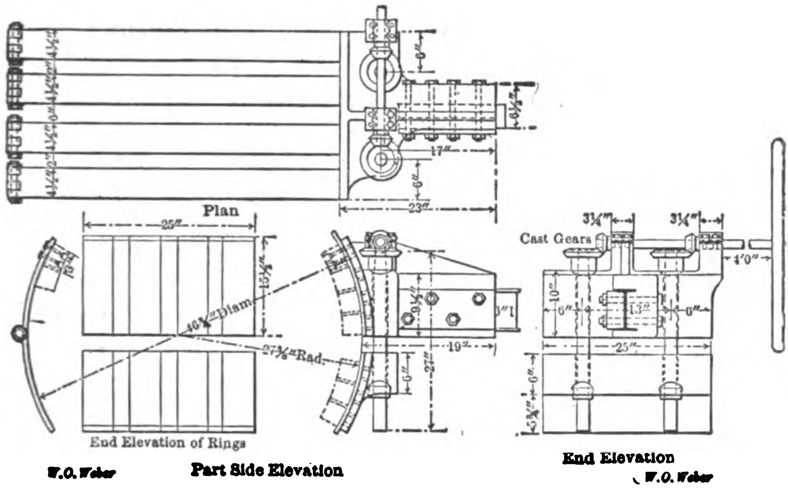

Fig. 232.

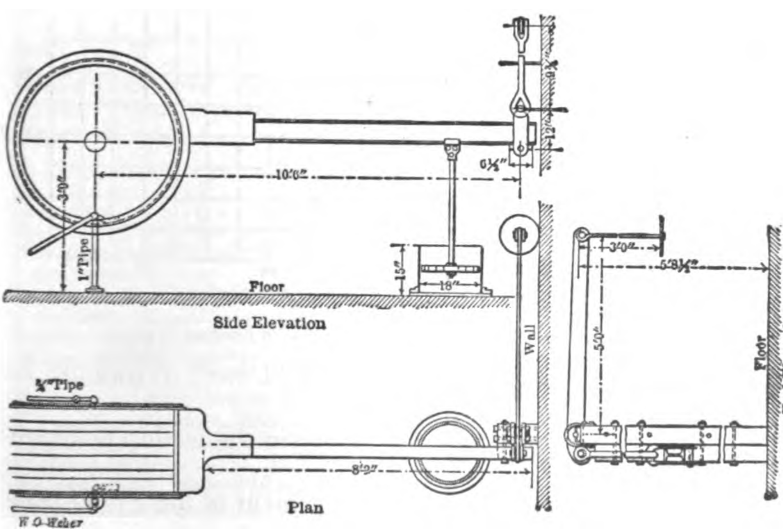

Fig. 233.

The efficiency represented by (9) involves only the effects of losses of energy by the water in passing over the buckets and its theoretical value is 100 per cent. for all values of Φ. It eliminates the effect of uneconomical speed of rotation of the wheel which leaves residual lost energy in the water discharged by the buckets and not properly chargeable to bucket imperfections. It would be determined only in a detailed study or test made for the first purpose above mentioned.

180 Illustration of Methods and Apparatus for Testing Water Wheels:—Fig. 230 shows the apparatus used for testing turbines on a vertical shaft, by Mr. J. B. Francis to test a Swain wheel at the

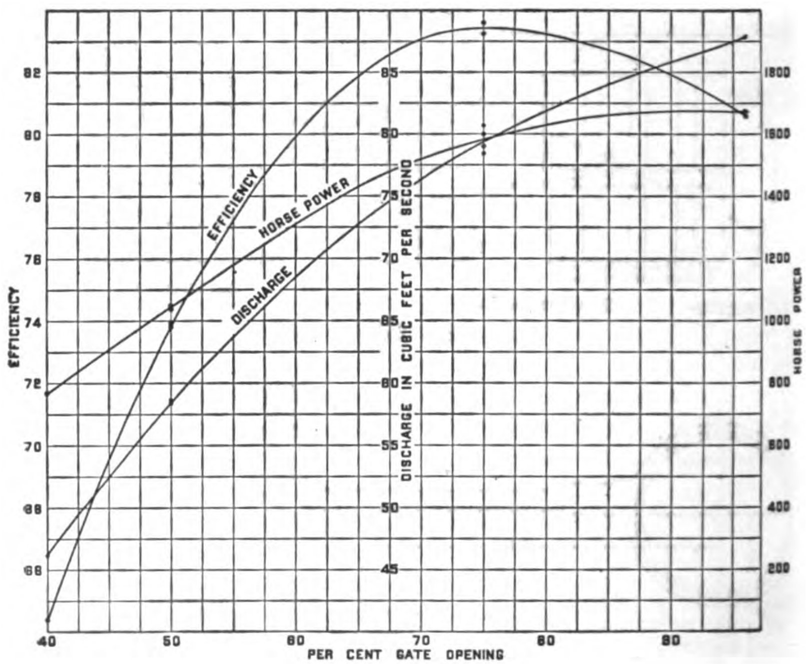

Fig. 234.

Boott Mills, Lowell, Massachusetts (See "Lowell Hydraulic Experiments.")

The section represents a vertical turbine in the testing plant with testing apparatus in place.

The plan of the plant shows the arrangement of the Prony brake. In these drawings P is the friction pulley; b is the brake; c are counter balances to remove the load of the brake from the wheel shaft; L is the bent lever or steel beam for transferring horizontal motion to a vertical lift; S is the scale pan for the weight; d is the dash-pot; w is the weir for measuring the water, and r is the rack for stilling the water after leaving the wheel.

Figs. 231, 232, 233, show the brake wheel and Prony brake details used by Mr. William O. Weber for determining the efficiency of various turbine water wheels as described by him in a paper on "The Efficiency Tests of Turbine Water Wheels," (See vol. 27, No. 4, American Society of Mechanical Engineers). (See also Section 171, Experiments at the Holyoke Testing Flume.)

181. **Tests of Wheels in Place.**—In April, 1903, a Leffel turbine was tested at Logan, Utah, at the station of The Telluride Power Transmission Company, by P. N. Nunn, Chief Engineer. The wheel was directly connected to a General Electric generator the efficiency of which has been determined as follows:

125 per cent load..........................96.7 per cent. efficiency
100 per cent load..........................96.2 per cent. efficiency
75 per cent load..........................95.3 per cent. efficiency
50 per cent load..........................93.5 per cent. efficiency
25 per cent load..........................88.0 per cent. efficiency

The output of this generator was used as a basis for calculating the work done by the water wheel.

The results of the tests and methods of calculation are shown in Table XXXV and graphically illustrated in Fig. 234.

TABLE XXXV.

Test of High Head Leffel Horizontal Turbine at Logan Station of Telluride Power Trans. Company, Logan, Utah. Efficiency of Test at Constant Speed, April 28, 1903.

P. N. Nunn, Chief Engineer.

Gate opening..............	0.75	0.50	0.40	0.50	0.75	0.96
Head on 15 feet weir in feet	1.394	1.132	0.969	1.129	1.368	1.475
Discharge of weir in cubic feet per second...............	81.85	59.76	47.27	59.66	79.55	88.94
Leakage around weir in second feet.................	0.85	0.85	0.85	0.85	0.85	0.85
Exciter water in second feet	1.98	1.98	1.98	1.98	1.98	1.98
Water through turbine in second feet..............	80.72	58.63	46.14	58.53	78.42	87.81
Pressure at shaft center in pounds per square inch..	86.5	87.3	87.5	87.2	86.5	86.2
Effective head above shaft center in feet............	199.3	201.2	201.6	200.9	199.3	198.6
Vacuum head measured in feet.....................	10.4	10.6	10.8	10.6	10.4	10.3
Total working head in feet	209.7	211.8	212.4	211.5	209.7	208.9
Theoretical horse power....	1921	1409	1112	1405	1866	2082
K.W. output at Sw. Bd.....	1152	739	500	737	1123	1210
Generator efficiency	0.965	0.952	0.935	0.952	0.965	0.967
Brake horse power of turbine	1600	1041	717	1038	1560	1677
Efficiency of turbine.......	0.833	0.738	0.644	0.739	0.836	0.806
Gate opening..............	0.75	0.50	0.40	0.50	0.75	0.96

NOTE—Speed, 400 R. P. M. (normal).
Generator efficiency taken from test of machine made by The General Electric Company. (Record of test in office of chief engineer).

A similar test of one of a number of wheels installed by The James Leffel Company in the plant of the Niagara Hydraulic Power and Manufacturing Company was made in December, 1903, by Mr. John L. Harper, engineer of that company. The following table XXXVI is the condensed data of the test of wheel No. 8 which is also illustrated by Fig. 235.

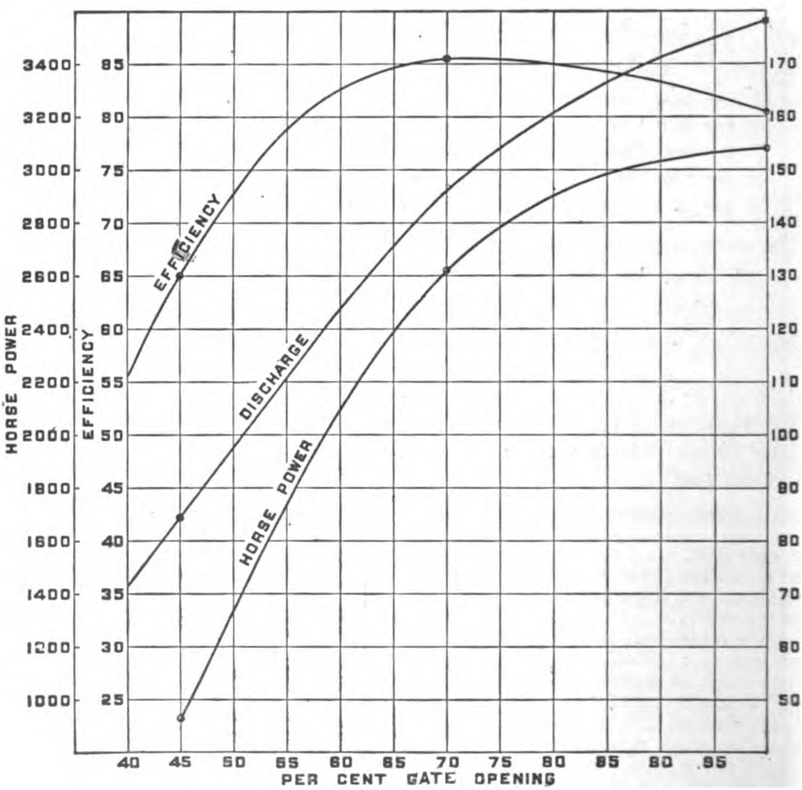

Fig. 235.

The water was measured by a standard contracted weir 16.23 feet long and discharge computed by Francis' formula:

$$q = 3.33\,(L - 0.2h)\,h^{\frac{3}{2}}$$

The load was computed from the voltmeter and ammeter readings of two generators Nos. 5 and 12 which were both driven by this wheel and then corrected for the generator loss by a factor estimated from the shop tests of the generators.

TABLE XXXVI.

Test of a Double Horizontal Leffel Turbine installed in the plant of the Niagara Hydraulic Company, Niagara Falls, N. Y.

	Gate Opening.		
	.45	.7	1.0
	Dec. 5th		Dec. 6th
Time	3:21 p. m.	5:01 p. m.	4:59 p. m.
Hook gauge reading (corrected)	1.365	1.978	2.257
Discharge of wheel by Francis' formula	84.76	146.6	178.3
Head on turbine	213.0	212.4	212.7
Hydraulic horse power	2045	3528	4320
R. P. M	255	259	250
*Generator No. 5**			
Volts	178	178	184
Amperes	5065	5020	5833
Efficiency	.92	.92	.92
Horse power taken from wheel by generator	1314	1302	1563
*Generator No. 12***			
Volts	Friction	12200	13000
Amperes	Load	57.7	60.5
Effciency	Only	.95	.955
Horse power taken from wheel by generator	17	1720	1912
Total horse power output of wheel	1331	3022	3475
Efficiency of wheel	.651	.856	.805

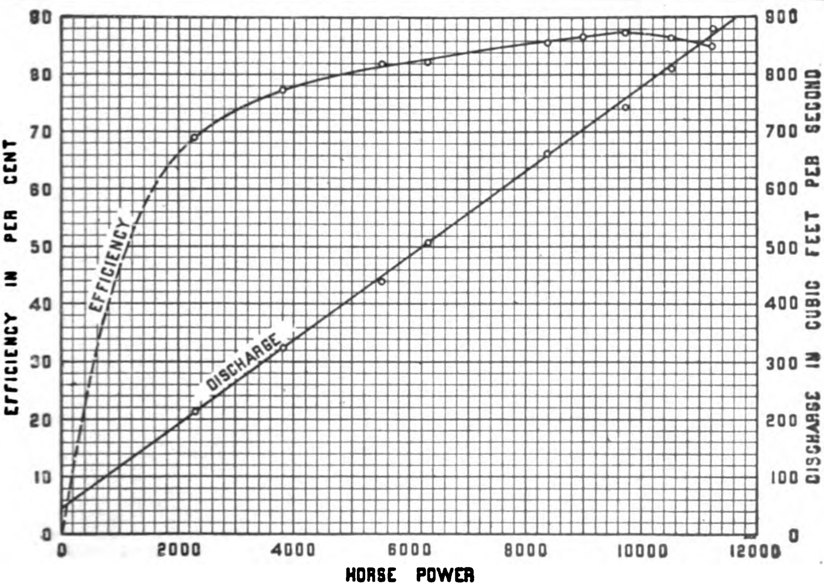

Fig. 236.

* Generator No. 5 is a G. E. 5000 A. 175 V., D. C. machine.

** Generator No. 12 is a Bullock 1000 K. W., 3 phase A. C. generator.

The 10,500 h.p. turbine manufactured by the I. P. Morris Company for the Shawinigan Power Company was also tested in a similar manner. A brief outline of this test is given on page 416 The graphical result of the same is shown by Fig. 236. Fig. 237 illustrates the test of a 25″ Victor High Pressure Turbine, manufactured by the Platt Iron Works Co., at the Houck Falls Power Station at Ellensville, New York.

The results of various tests at the Holyoke Testing Flume, collected from divers sources, are given in the appendix. Most of the later tests have been furnished by manufacturers and represent the best results of modern turbine manufacture.

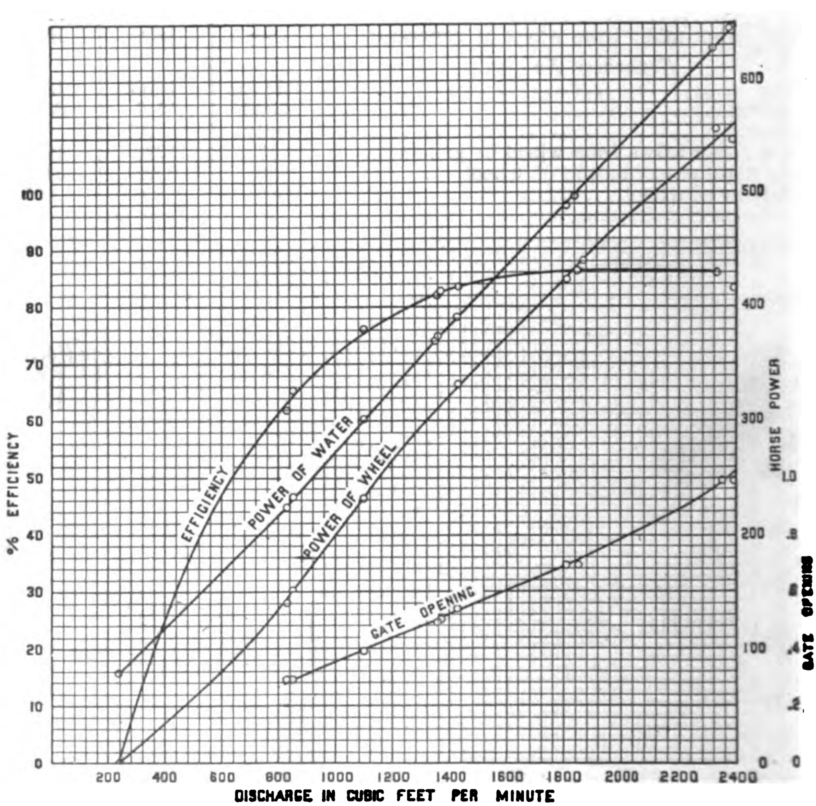

Fig. 237.

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CHAPTER XVI.

THE SELECTION OF THE TURBINE.

182. Effect of Conditions of Operation.—For high and moderate falls the variations in head under different conditions of flow are of small importance and water wheels can commonly be placed high enough above tail-water to be practically free from its influences. In such cases variations in head are comparatively so slight as to have little effect on the operation of the wheels which can therefore be selected for a single head. Such conditions are the most favorable for all types of wheels.

When low falls are developed the rise in the tail-water is often comparatively great, and, as the head water cannot commonly be permitted to rise to a similar extent on account of overflow and damage from back water, the heads at such time are considerably reduced. As is pointed out in Chapter V, under such conditions and for continuous power purposes wheels must be selected, if possible, that will operate satisfactorily under the entire range of head variations that the conditions may demand, or at least under as great a range of such variations as practicable.

In some cases the change in head is so great that no wheel can be selected which will work satisfactorily under the entire range of conditions. In other cases, the head becomes so small that the power which can be developed is insufficient without a large and unwarranted first cost. In many such cases the use of the water power plant must be discontinued, and, if the delivery of power must be continuous, it must be temporarily supplemented or replaced by some form of auxiliary power.

In Chapter XVII it is shown that, almost without exception, great variations take place in every power load and that a plant must therefore be designed to work satisfactorily under considerable changes in load. Most plants are called upon to furnish power for a considerable portion of the time under much less than their maximum load, but must occasionally furnish a maximum load for a short period.

If power is valuable, and the quantity of water is limited, it is desirable to select a wheel that will give the maximum efficiency for the condition of load under which it must operate for the greater portion of the time and that will also give, if possible, high efficiency under the head available at the lowest stages of the water. High efficiency is not essential to economy during high water, for there is plenty of water to spare at such times; neither is high efficiency as important during unusual load conditions, which obtain for only brief intervals, as it is during the average conditions under which the plant operates.

183. Basis for the Selection of the Turbine.—In Chapter XV the testing of water wheels has been discussed and a number of tabulated results of such tests are given. (See appendix D). The standard water wheel tables are calculated from the results of these tests but the values of power and efficiency, as given therein, are usually reduced somewhat for safety from the results determined experimentally. Such tests also give data for a much broader consideration of the question, and for the determination of the results that can be obtained under the actual conditions of installation and operation, even when such conditions are subject to wide variations.

In Chapter XIV the hydraulics of the turbine are discussed, various turbine constants are considered, and the constants are calculated for a number of standard American turbines in accordance with the conditions of operation as recommended in the catalogues of their makers. It will be seen from a study of the tables that the turbines designed and built by various manufacturers sometimes have widely different constants, indicating that each is best adapted to certain conditions of which the values of these constants are an index.

It is also shown that the various constants for a homogeneous series of wheels may be calculated from experimental data for any desired condition of gate opening and fixed value of ϕ, and that from these constants the operating results, that is, the discharge, power, speed, and efficiency for any wheel of the series, with the given gate opening and value of ϕ and for any desired head, can be calculated. For most purposes, where the head is constant or where the range in heads and other conditions to be considered are not extreme, the necessary calculations can be readily made from a satisfactory test, by applying some of the formulas developed and discussed in Chapter XIV. The formulas of greatest value for this purpose are as follows:

1 $$\varphi = \frac{D\,n}{1842\sqrt{h}}$$

2 $$n_1 = \frac{1842\,\varphi}{D}, \text{ when } h = 1$$

3 $$\Delta = \frac{D\,n}{\sqrt{h}} = \frac{D_1\,n_1}{\sqrt{h_1}} \quad \text{when } \varphi \text{ is constant.}$$

4 $$\frac{n}{\sqrt{h}} = \frac{n_1}{\sqrt{h_1}} \quad \text{when } \varphi \text{ and D are constant.}$$

5 $$n = n_1\sqrt{h} \quad \text{when } \varphi \text{ and D are constant.}$$

6 $$K = \frac{q}{D^2\sqrt{h}} = \frac{q_1}{D_1^2\sqrt{h_1}} \quad \text{when } \varphi \text{ is constant.}$$

7 $$\frac{q}{\sqrt{h}} = \frac{q_1}{\sqrt{h_1}} \quad \text{when } \varphi \text{ and D are constant.}$$

8 $$q = q_1\sqrt{h} \quad \text{when } \varphi \text{ and D are constant.}$$

9 $$K_2 = \frac{P}{D^2\,h^{\frac{3}{2}}} = \frac{P_1}{D_1^2 h_1^{\frac{3}{2}}} \quad \text{when } \varphi \text{ is constant.}$$

10 $$\frac{P}{h^{\frac{3}{2}}} = \frac{P_1}{h_1^{\frac{3}{2}}} \quad \text{when } \varphi \text{ is constant.}$$

11 $$P = P_1 h^{\frac{3}{2}} \quad \text{when } \varphi \text{ and D are constant and } h_1 = 1.$$

12 $$K_s = \frac{n^2 P}{\sqrt{h^5}}$$

In using these formulas it must be remembered that each is essentially correct only when the condition specified after each equation obtains; also that as long as ϕ remains constant the efficiency obtained by the test will remain practically constant for the same wheel, under all conditions of head. It should also be noted that, with a fixed diameter of wheel and a fixed head, ϕ and n are in direct proportion, and most calculations can be made by a direct consideration of the values of n without a determination of the value of ϕ.

When the operating results are calculated for a wheel of a given series but of a diameter differing from that on which the experiments were made, the results are liable to differ from the true results on account of variations in manufacture, and allowance must be made for such differences, at least until the art of manufacturing turbines has further advanced.

184. Selection of the Turbine for Uniform Head and Power.—The conditions of operation, as catalogued, are usually based upon tests of a few turbines of the series, and represent the best conditions of operation for that series of wheels as determined by such tests. Where the conditions of installation and operation are fixed, and are not subject to radical changes in head or to great variations in the demand for power, the selection of a wheel may be made by inspection directly from the catalogues. This method of selection is based on the assumption that the catalogue data is correct, which assumption should be verified by the records of an actual test of the series of wheels and, if possible, of the size and hand which are actually to be used.

The examination of the many catalogues of turbine manufacturers, in order to determine the wheel best suited to the conditions, is a tedious method of procedure and can be greatly shortened by brief calculations which are described in the following sections:

185. The Selection of a Turbine for a Given Speed and Power to Work under a Given Fixed Head.—It is frequently necessary to select a turbine which must have a given speed and power in order to successfully operate machinery for which such requirements obtain. It is desirable to select a wheel which will furnish essentially the amount of power required as all machinery will work more efficiently and more satisfactorily at or near full load conditions. It is also desirable to use a single turbine rather than two turbines, and if more than one turbine is required, the least number found practicable should usually be selected because the multiplication of units involves an increase in the number of bearings which must be maintained and kept in alignment.

To determine the best installation of turbines necessary to fulfill the given conditions, the value of K_5 as given by equation (12) should be determined. Having determined the value of K_5, a turbine should be selected having a constant K_5 not less than the amount determined, and if it is desired to operate the turbine at its maximum efficiency, the value of K_5 for the turbine selected should not greatly exceed the value found by computation. If the value of K_5 as computed greatly exceeds the value of K_5 for the various makes of turbines, then the power must be divided between two or more units in order that the conditions may be satisfied. As K_5 is in direct proportion to P, one-half, one-third or any other fraction of K_5 will give the value of K_5 for a wheel having a similar fractional value of the power, P, and will therefore show the type of wheel which must be selected in order that two, three, or more will do the work in question. The great variations in the value of K_s for different types of wheels and the influence of this variation on the relation of speed and power will be seen by reference to Fig. 222 which shows the curves of relation between revolution and power of various wheels for one foot head. This may be used for any other head by considering the revolutions in proportion to the square root of the head and the power in proportion to the three-halves power of the head. A brief study of this diagram will show its use more plainly. For example: under a one foot head, and for 30 revolutions per minute, turbines may be selected that will deliver from 1.3 to 6.6 horse power.

Suppose we desire to determine the power that will be available under a 16′ head at 100 revolutions per minute. 100 revolutions per minute at 16′ head would correspond to 25 revolutions per minute at 1′ head.

For since

$$\frac{n_1}{n} = \frac{\sqrt{h_1}}{\sqrt{h_1}},$$

$$\text{therefore } n^1 = \frac{\sqrt{1}}{\sqrt{16}} 100 = .25 \times 100 = 25.$$

At 25 r. p. m. the diagram shows that turbines are obtainable that will give 1.8 to 10 horse power at one foot head.

The power at 16 foot head will be to the power at one foot head as the three-halves power of the head. The three-halves power of 16 is 64; hence the power at 16 feet will be 64 times the power at one foot head, and, hence, wheels under a 16 foot head operated at 100 revolutions per minute, will furnish from 122 to 657 horse power and the most satisfactory wheel within these limits for the problem at hand can be selected.

The diagram, however, is a convenience, not a necessity, and a problem can often be more readily solved by the direct application of equation 12. If, for example, it is desired to operate a turbine at 100 revolutions per minute under 16 foot head to develop 400 h. p., the corresponding value of K_s will be

$$K_s = \frac{n^2 P}{\sqrt{h^5}} = \frac{100 \times 100 \times 400}{\sqrt{16^5}} = 3906$$

By examination of Table XXXII it will be found that the Victor Standard Cylinder Gate or the United States Turbine wheels have practically this value of K_5 and will therefore fulfill the conditions. Having determined from the calculated value of K_5 the makes and types of the several wheels which will satisfy the requirements, the size of the wheel may immdiately be determined by determining the value of K_2 for the same series of wheels from Table XXX, Chap. XIV, and calculating the size of the wheel by the use of formula 9.

Thus for the Victor Standard Cylinder Gate wheel the value of K_2 is 0.00205. Therefore from equation (9)

$$D = \sqrt{\frac{P}{K_2 \; h^{\frac{3}{2}}}} = \sqrt{\frac{400}{.00205 \times 64}} = 55.2''$$

which is the size of this series of wheels needed to fulfill the assumed conditions.

Having thus selected several possible wheels, tenders for these wheels may be invited from their makers. These tenders should be accompanied by an official report of a Holyoke test for the wheel in question, or, if this is not available at the time, for the next larger and the next smaller wheels of the series which have been tested. From these tests the catalogue values of K_2 and K_5 which were used in their selection can be checked. In addition to this the several prospective wheels may be compared as to their operation at part gate, which comparison is equally important for the final choice to be made.

As the wheels are seldom or never tested for the head under which they are to work, and as tests are not always available for the size of wheel to be used, it is necessary to predict from the test data furnished by the wheel makers the efficiency, power and water-consumption curves which can be anticipated under the given head. This can be done as illustrated in the next two articles.

186. To Estimate the Operating Results of a Turbine under one Head from Test Results secured at another Head.—For the purpose of illustrating the methods of calculation, Table LXXIII. may be considered. This table gives the results of certain tests of a 33" special, left-hand turbine wheel, with conical draft tube and balance gate, manufactured by the S. Morgan Smith Company. While the heads in the different experiments of this test vary slightly, they are so nearly uniform that the table may be considered as developed under a uniform head of 17.15 feet. If greater accuracy is desired, however, the square root of the actual head can be considered each time.

Let it be assumed that the wheel is to be operated under a 20 foot head and with a speed of 200 r. p. m. with the average load at about .765 gate. The maximum efficiency at .765 gate is represented by experiment No. 43 of this table. In order that the wheel shall work under the new head with this efficiency, equation (4) must be satisfied. In all of these equations the primed characters are used to represent the experimental conditions. The most efficient revolutions under the new head will therefore be determined as follows:

$$n = \frac{172.75 \times 4.46}{4.14} = 186 \text{ r. p. m.}$$

The wheel to be chosen must, however, in this case operate at 200 revolutions per minute. At 200 r. p. m. the wheel will not run at its maximum efficiency. The actual efficiency at this speed may be determined by finding what speed at the experimental head corresponds with the speed to be used, and noting the efficiency corresponding to the same. This is done on the assumption that the efficiency remains constant as long as ϕ remains constant which is shown to be essentially true by Fig. 214, Chap. XIV.

The revolutions under 17.15 ft. head corresponding to 200 r. p. m. under 20 feet will be determined as before:

$$n' = \frac{200 \times 4.14}{4.46} = 187 \text{ r. p. m.}$$

The result, 187 r. p. m., lies between the conditions of experiments 41 and 40. By proportion, the efficiency corresponding to 187 r. p. m. will be found to be about 83.25 at .765 gate.

If the efficiency corresponding to 187 r. p. m. in the table is now determined from each gate opening, it will be found that at full gate the efficiency will be slightly below that shown in experiment 15. and can be determined by interpolation, or graphically, to be about 81%. At gate .948 the efficiency can be determined in the same way to be about 82.75%. At gate .883 the results will fall between experiments 69 and 70 and the efficiency will be found to be about 86%. At gate .851 the result falls below experiment 54. and, by calculation from a graphical diagram or by interpolation, the results are found to be about .86. At gate .702 the revolutions correspond closely with experiment 56, and the efficiency from the table is found to be 81.35%. At gate .636 the revolutions fall between experiments 25 and 26 and, by proportion, the efficiency is found to be 80.62%. At gate .556, the efficiency is found, by proportion, to be 77%. To determine the power of the wheel under the new conditions, and for each condition of gate, the power of the wheel as found by the test must be determined for the same value of ϕ. The power of the new head can then be calculated by use of formula (11).

In the same manner the discharge of the turbine can be determined by finding the value of q corresponding to the value of ϕ for the experimental head, and from this value so determined the value of q under the 20 foot head can be calculated by formula (7). The results of these calculations, together with the efficiency as determined for 20 foot head and for 200 revolutions per minute, are given in Table XXXVII.

Having computed a similar table for each of the several prospective wheels the one best suited to the given conditions can be chosen.

TABLE XXXVII.

Showing Horse Power, Discharge and Efficiency of 33-inch Special Left Hand S. Morgan Smith Turbine, with 20-foot head and 200 R. P. M.

Calculated from test of 33-inch wheel under a head of 17.15 feet.

Proportional Gate Opening.	Horse Power	Discharge, cubic feet per second.	Efficiency.
1.000	222.1	120.4	**81.6**
.948	220.1	117.0	**83.2**
.883	217.7	111.3	**85.6**
.851	212.7	109.2	**86.3**
.765	188.1	97.5	**83.2**
.702	165.1	87.8	**81.3**
.636	154.7	81.5	**80.7**
.556	136.7	75.0	**79.0**

187. To Estimate the Operating Results of a Turbine of one Diameter from Test Results of Another Diameter of the Same Series.—It is always desirable for the purpose of calculations to use the results of a test made on a wheel of the same size and hand as that which is to be used in the installation for which the wheel is being considered. It is seldom, however, that all of the various sizes of wheels in a series of wheels have been tested, and the manufacturers therefore frequently base their estimates and guarantees of wheels of an untested size on the test of some other wheel of the series which may be larger or smaller than the wheel offered. Sometimes tests of wheels both larger and smaller than the wheel to be used are available, in which case both sets of tests should be used as a basis of calculation.

Let it be assumed that a 40″ wheel is to be installed of the same series as the 33″ wheel just considered, and that no tests of such a wheel are obtainable. The tests of the 33″ wheel may therefore be used as the best information available. Let it be assumed that the 40″ wheel is to be operated under a 9 foot head. For these calculations formula (3) must be satisfied.

Let it be assumed that the wheel is to operate at nearly full load and the best efficiency is desired at about .85 gate. From the tests it will be found that at .85 gate, and with a 17.15 foot head and 191 revolutions, the wheel gave 85.97% efficiency and 170.08 horse power. Substituting these values in equation (3) there results:

$$\frac{33 \times 191}{4.14} = \frac{40 \times n}{3}, \text{ from which } n = 114 \text{ r. p. m.}$$

One hundred and fourteen revolutions per minute is therefore the speed under which the wheel must operate in order to give this maximum efficiency at this gate.

Let it be assumed, however, that the wheel must be run at 120 r. p. m., on account of the class of machinery to be operated By substituting the value $n = 120$, in equation (3), it is found that $n' = 202$. The experimental efficiency at 202 r. p. m. under the 17.15 foot head and with the 33″ wheel, will therefore, correspond to 120 revolutions under a 9 foot head with a 40″ wheel, and will indicate the efficiency under which the wheel will operate under these conditions. This is found to be about 81.5 at .85 gate.

In order to determine the horse power of the wheel under the new conditions, the horse power of the wheel under the test conditions must first be determined for that gate; the resulting horse power can then be determined by equation (9).

For 202 r. p. m. at 17.15 foot head for this 33″ wheel P=158 which, substituted in equation (9), gives

$$\frac{158}{33 \times 33 \times 71} = \frac{P}{40 \times 40 \times 27}, \text{ from which } P = 88.$$

In the same manner, the discharge of the larger wheel under the lower head can be determined by equation (6), and q is found to equal 104 cu. ft. per second.

In this way the discharge, efficiency and power of the larger wheel under the chosen r. p. m. can be determined for each condition of gate, as shown in Table XXXVIII.

TABLE XXXVIII.

Showing Horse Power, Discharge and Efficiency of a 40-inch Special Left Hand S. Morgan Smith Turbine, with a 9-foot head and 120 R. P. M.

Calculated from test of 33-inch wheel under a head of 17.15 feet.

Proportional Gate Opening.	Horse Power	Discharge cubic feet per second.	Efficiency.
1.000	100.	119.	82.1
.948	100.	112.	84.2
.883	92.	108.	82.5
.851	88.	104.	81.5
.765	76.	91.	78.1
.702	68.	83.	77.8
.636	64.	76.	78.8
.556	58.	73.	75.1

188. To Estimate the Operating Results of a Turbine under Variable Heads from a Test made under a Fixed Head.—Where the variations in the head under which a wheel is to operate are considerable, the variation in ϕ, and consequently in n, are sometimes found to be beyond the limits of the test. Where the test conditions are not greatly exceeded, the experiments may be extended graphically without any serious error.

Let it be assumed that the 33″ wheel above considered is to be operated under a maximum head of 25 feet, and that the head will decrease to 16 feet at times of high water; also, that the wheel is to be operated for the major portion of the time under about .75 gate. The best condition for operation is shown by test 43, which shows an efficiency of 86.3% at n′ = 172.75 r. p. m.

n may be calculated from equation (4) for the 25 foot head as follows:

$$n = \frac{172.75 \times 5}{4.14} = 208 \text{ r. p. m.}$$

That is: the best number of revolutions for a 25 ft. working head would be 208 r. p. m. The best number of revolutions for a sixteen foot head would be determined as follows:

$$n = \frac{172.75 \times 4}{4.14} = 166 \text{ r. p. m.}$$

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The wheel, for the best efficiency, should be run at a different speed for each head, but under practical conditions of service must be run at a constant speed.

Let it be assumed that, on account of the machinery operated, it is desirable to adopt for the plant a speed of 200 r. p. m. Let the 25 foot head be first considered. For considering the 25 foot head the equivalent value of n under the test conditions is found as follows:

$$n' = \frac{200 \times 4.14}{5} = 167 \text{ r. p. m.}$$

It will be noted from experiment 44 that at 169.25 r. p. m. the efficiency is 85.55. At 167 revolutions per minute the efficiency would therefore be about 85%. Under a sixteen foot head n must also equal 200 r. p. m., hence, for this case, the equivalent value of n′ for the test conditions is

$$n' = \frac{200 \times 4.14}{4} = 208 \text{ revolutions.}$$

Test 39 shows that, with 206.25 revolutions, the efficiency is 76.66. At 208 revolutions the efficiency is therefore less than this amount and the probable efficiency under these conditions can be estimated by platting the relation between revolutions and efficiency as shown in Fig. 238. By prolonging the line from the actual experiments, the efficiency indicated for 208 revolutions, under the experimental conditions, is found to be about 76%. As far as efficiency is concerned, therefore, the arrangement is very satisfactory, for a sufficiently high efficiency will be obtained under conditions of high water, and when the quantity of water used is immaterial.

The relations of efficiency to speed, under the experimental conditions and at various gate openings, are shown by the points platted on Fig. 238. Through these points mean curves are drawn, which are extended where necessary to intersect the abscissa of 167 revolutions, which corresponds to the condition of efficiency for 25 foot head, and to the abscissa of 208 revolutions, which corresponds to the condition of efficiency for a 16 foot head. From these results the relations of efficiency at various gates and at the two heads named are platted in Fig. 239.

The relations of power to speed are shown by Fig. 240, which has been platted in the same manner as Fig. 238. From Fig. 240,

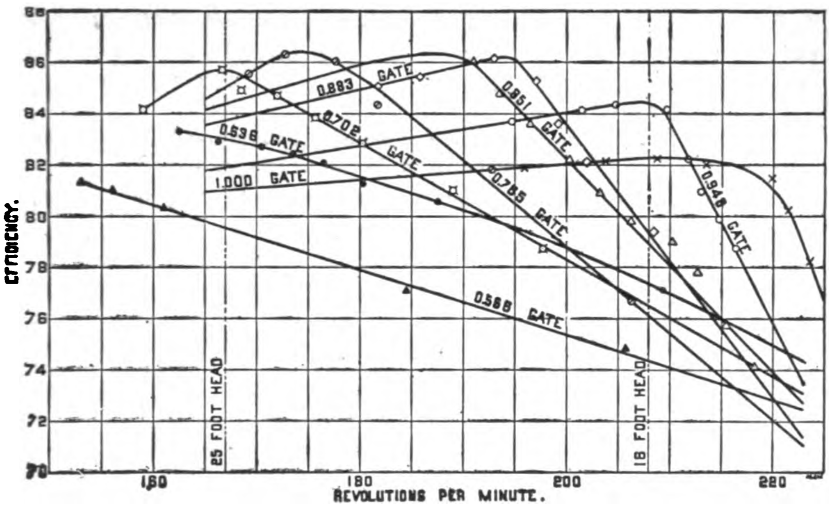


Fig. 238.—Curves Showing the Efficiency Obtained at Various Speeds under a Test Head of about 17.15 Feet from a 33-Inch Special Left-Hand Wheel with Balance Gate, Manufactured by the S. Morgan Smith Co.

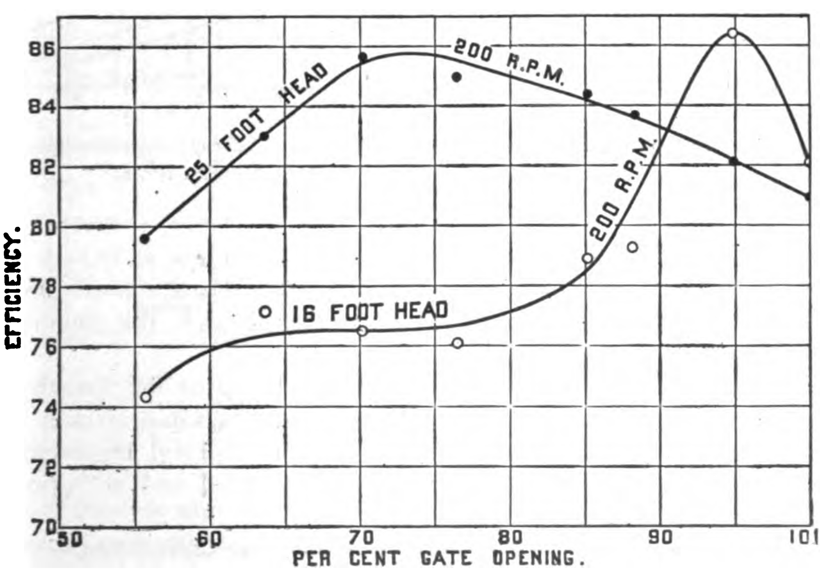


Fig. 239.—Curves Showing Estimated Efficiency at Various Gate Openings and at Two Heads for 33-Inch S. Morgan Smith Wheel. (Taken from Fig. 238.)

the power of the wheel at 25 and 16 feet can be determined by equation (10).

The power at 25 feet will be

$$\frac{h^{\frac{3}{2}}}{h'^{\frac{3}{2}}} = \frac{125}{70.6} = 1.77 \text{ times the power determined by the experiment at 17.15 feet and 167 r. p. m.}$$

The power at 16 feet will be

$$\frac{h^{\frac{3}{2}}}{h'^{\frac{3}{2}}} = \frac{64}{70.6} = .91 \text{ times the power, as determined by the experiment at 17.15 feet, and at 216 r. p. m.}$$

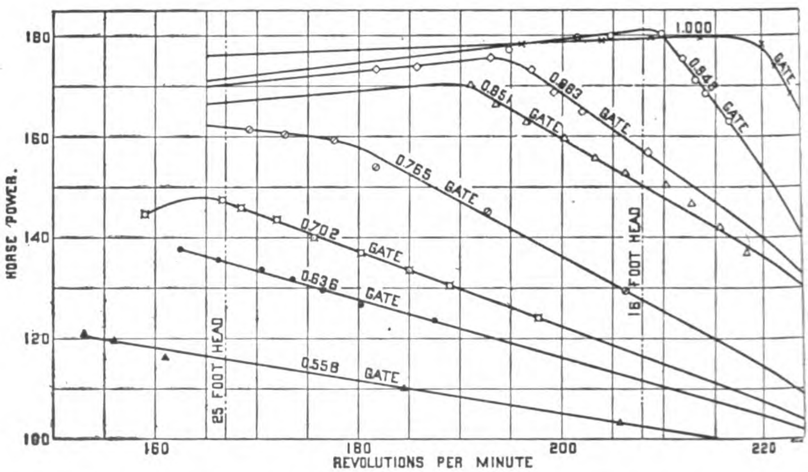


Fig. 240.—Curves Showing the Power Obtained at Different Speeds under a Test Head of about 17.15 Feet from the S. Morgan Smith 33-Inch Wheel.

.91 times the power, as determined by the experiment at 17.15 feet, and at 216 r. p. m. Curves of the power of this wheel under 25 and 16 foot heads, and at various gates, as determined in this manner. are shown by Fig. 241.

The experimental relations of speed and discharge for the wheel are shown in Fig. 242 which was platted in the same manner as the diagrams for efficiency and power. A graphical representation of the discharge under 25 and 16 foot head and at various gates is shown in Fig. 243.

189. A More Exact Graphical Method for Calculation.—The method outlined in section 188 is subject to some error as the results are platted regardless of head. The graphical method is therefore applicable without correction only when the experimental head remains nearly constant. For a more complete, accurate and satisfactory analysis the discharge, power and revolutions should be reduced to their equivalents i. e. at one foot head

$$q_1 = \frac{q}{\sqrt{h}}, \qquad P_1 = \frac{P}{h^{\frac{3}{2}}}, \qquad n_1 = \frac{n}{\sqrt{h}}$$

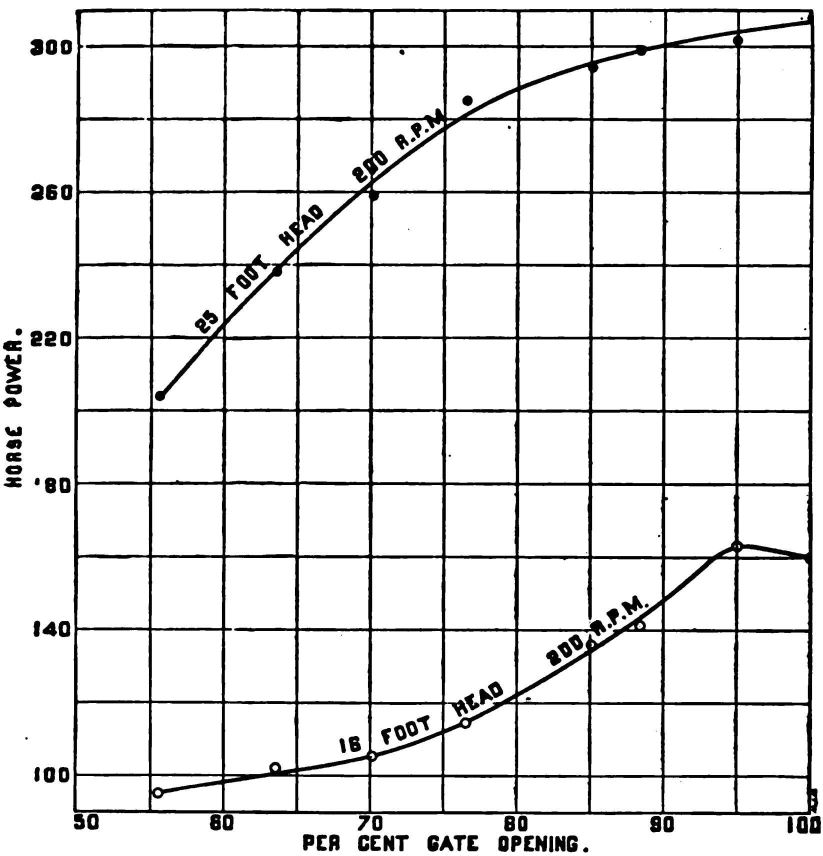


Fig. 241.—Curves Showing Estimated Power Obtained at Various Gate Openings and at Two Heads for 33-Inch S. Morgan Smith Wheel. (Taken from Fig. 240.)

and platted as shown in Fig. 244 where the r. p. m. under one foot head is used as abscissas, and the power, discharge and efficiencies are used as ordinates. The condition at any given number of revolutions under a given head can be calculated by dividing the given number of revolutions by the square root of the head. The

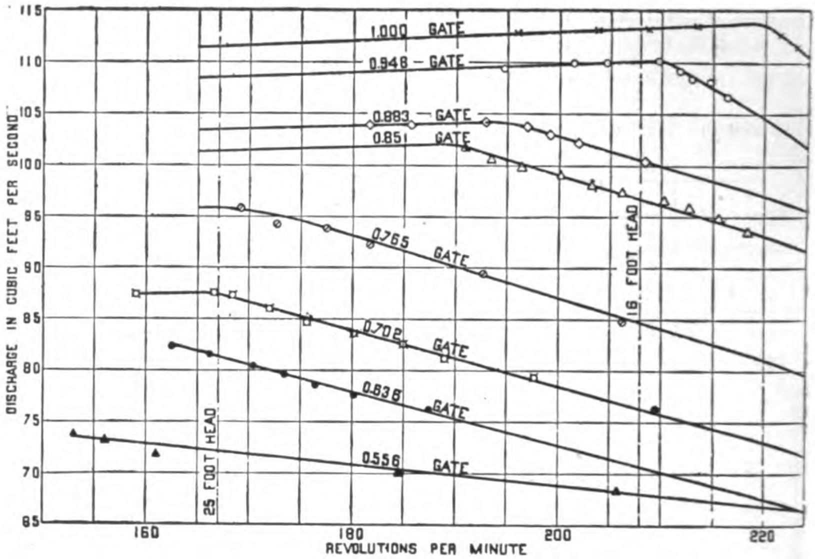


Fig. 242.—Curves Showing the Discharge at Various Speeds under the Test Head of about 17.15 Feet of the 33-Inch S. Morgan Smith Wheel.

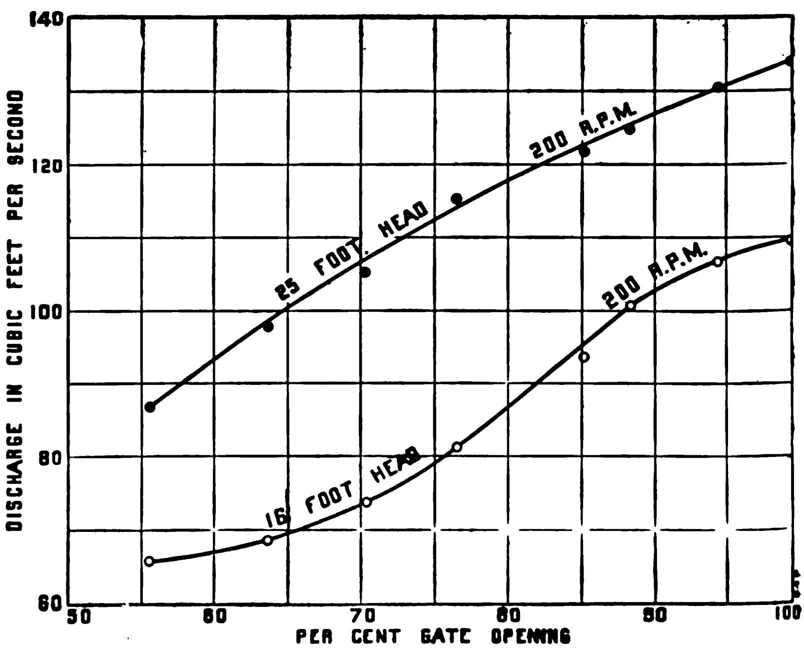


Fig. 24 —Curves Showing the Estimated Discharge at Various Gate Openings and at Two Heads for the 33-Inch S. Morgan Smith Wheel. Taken from Fig. 242.)

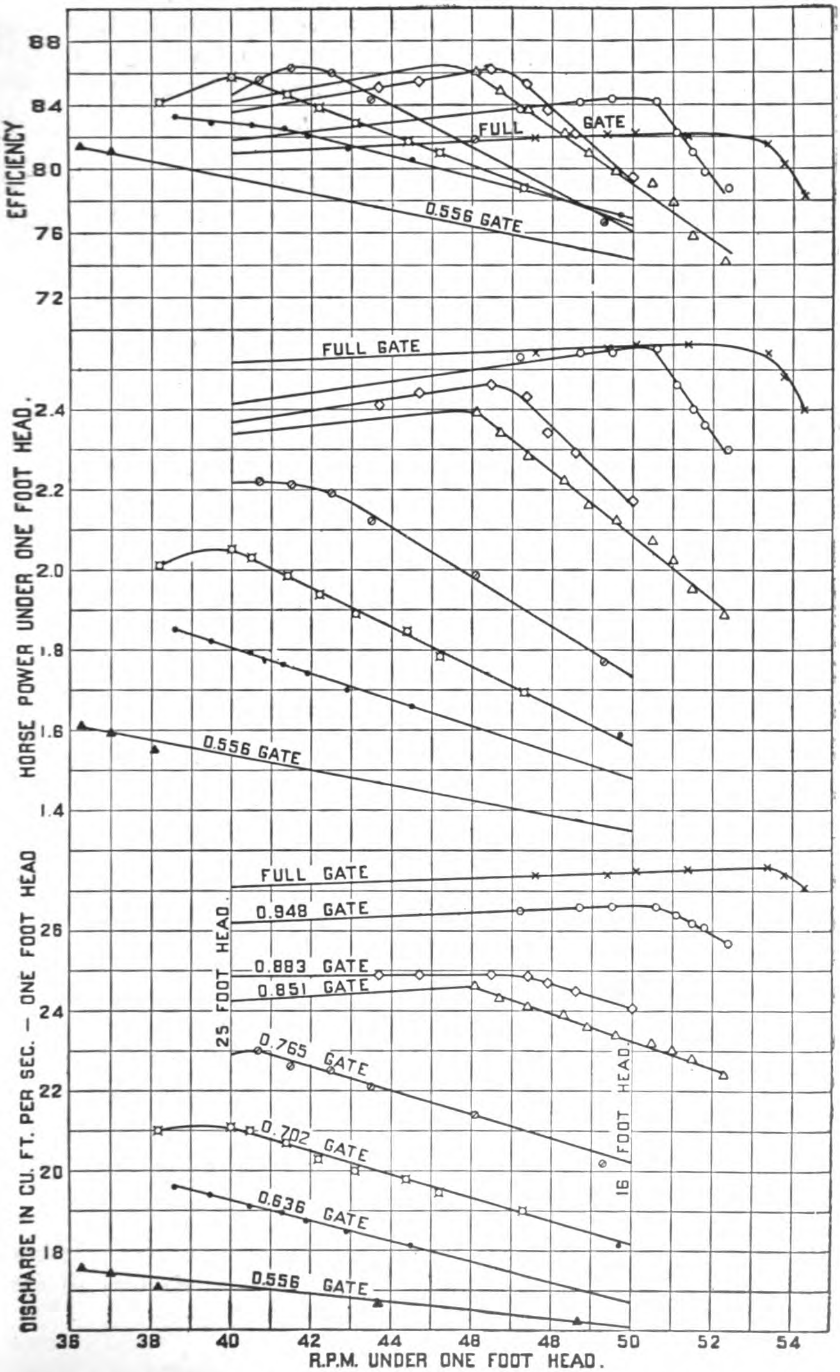


Fig. 244.—Curves of the 33-Inch S. Morgan Smith Wheel for One Foot Head.

result is the comparative revolutions under one foot head, and a line drawn vertically at the point so located on the diagram will give the basis of calculations for power and discharge by multiplying by $h^{\frac{3}{2}}$ and $h^{\frac{1}{2}}$, respectively, for each gate opening and by reading the efficiency direct.

For the wheel under 200 revolutions at 25 and 16 foot heads the equivalent speeds on the diagram are 40 and 50, respectively, Lines drawn vertically at these points will intersect the curves of efficiency, power and discharge and if reduced by a similar method will give curves essentially the same as those shown in Figs. 239, 241 and 243. This is probably the best method for common use in studying, from test data, the operation of a wheel under a variable head.

190. The Construction of the Characteristic Curves of a Turbine.—It is frequently desirable to make a more thorough analysis, based on the available test, of the conditions under which a wheel can operate. For this purpose, the writer finds the use of what he has termed "the characteristic curve" of a turbine to be the most comprehensive method for such an analysis.

For this purpose, prepare a diagram on which the ordinates represent the values of ϕ and the r. p. m. under one foot head, and the abscissas the discharge of the wheels in cubic feet per second under one foot head. It is also found desirable to show on the upper margin of the diagram the horse power under one foot head with 100% efficiency, corresponding to the discharge shown below. For each experimental result the values of ϕ and of the discharge under one foot head are determined by formulas (1) and (7). The point representing these values is then platted on the diagram, and the efficiency, as determined by the test for that experiment, is written closely adjoining the platted point. This is done for each experiment at each condition of gate. After all the experimental points are platted, and the resulting efficiency at each given point is expressed, lines of equal efficiency are interpolated on the drawing, and will indicate the general law of the variation of efficiency as represented by the test.

It is, of course, possible to reduce the horse power determined for each experiment to the theoretical horse power under one foot head, and record it at the corresponding point, and then interpolate horse power curves, as in the case of the efficiency curves. It has been found by the writer, however, to be more satisfactory to use the horse power scale at the top of the diagram, together with the efficiency lines already drawn, for the calculation and platting of the horse power curves. The horse power at any point will, of course, equal the theoretical horse power expressed at the upper margin, multiplied by the efficiency at the given points.

In determining the horse power curve, it is best to assume the horse power of the desired curve, and then determine its location in regard to the theoretical horse power from the equation.

$$\text{A. H. P} = \text{T. H. P.} \times \text{Efficiency.}$$

For example, on Fig. 245, if it is desired to plat the curve representing 2 A. H. P. it may be done as follows:—The line representing two actual horse power will intersect the 70% efficiency line at two points whose abscissae are determined from the T. H. P. scale by the equation

$$\text{T. H. P.} = \frac{\text{A. H. P.}}{\text{Eff.}} = \frac{2}{.70} = 2.86$$

If, therefore, the two points of intersection of the abscissa 2.86, as indicated on the upper T. H. P. scale, with the 70% efficiency line, are marked, two points will be established on the 2 A. H. P. line. As many of the lines of equal efficiency and equal horse power can be drawn on the diagram as may be desired, but if the lines of the drawing or diagram are too numerous, confusion will result rather than clearness.

One of the most complete sets of experiments with, or tests of, a turbine water wheel which the writer has been able to obtain is the set of experiments made for the Tremont and Suffolk Mills at the Holyoke Testing Flume, December 3-5,1890, on a 48 inch Victor turbine, with cylinder gate (See "Notes on Water Power Equipment," by A. H. Hunking), which is given in full in Table LX.*

From this table, and in the manner above described, a characteristic curve of this wheel has been prepared, and is shown by Fig. 245. In this Figure the efficiency curves are shown in black, the horse power curves are shown in red, and the lines showing the relations of discharge and ϕ at various gate openings are shown by the dotted lines connecting the experimental points.

191. **The Consideration of the Turbine from its Characteristic Curve**:—From this characteristic curve the action of the wheel under all conditions of operation within the experimental limits of ϕ can be readily determined. The use of the characteristic

* See Appendix—D.

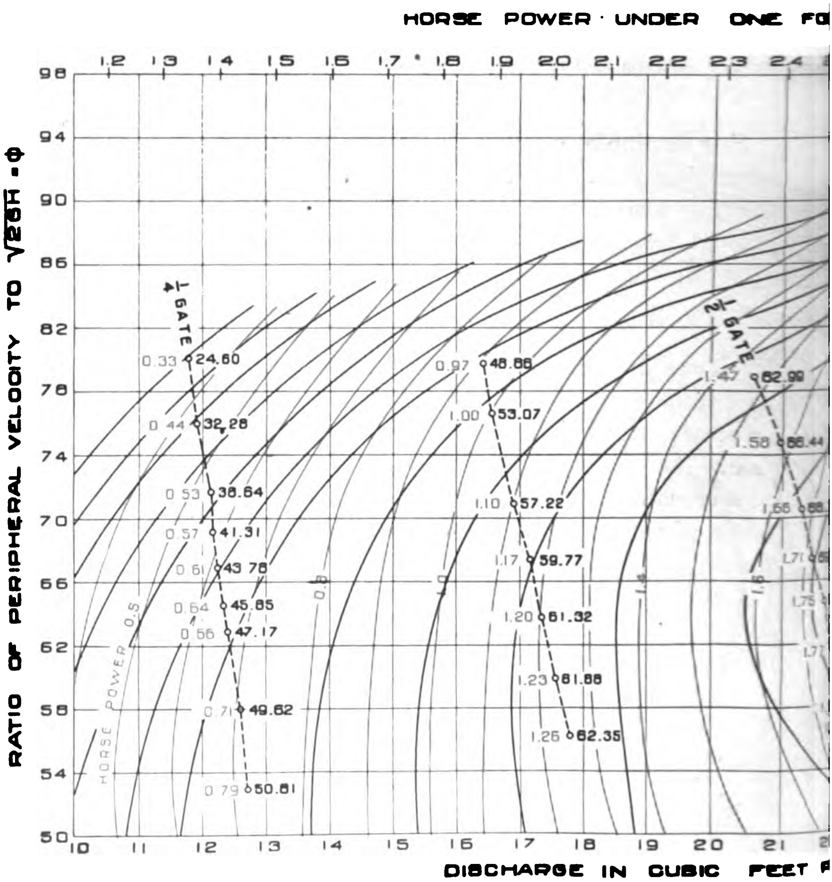
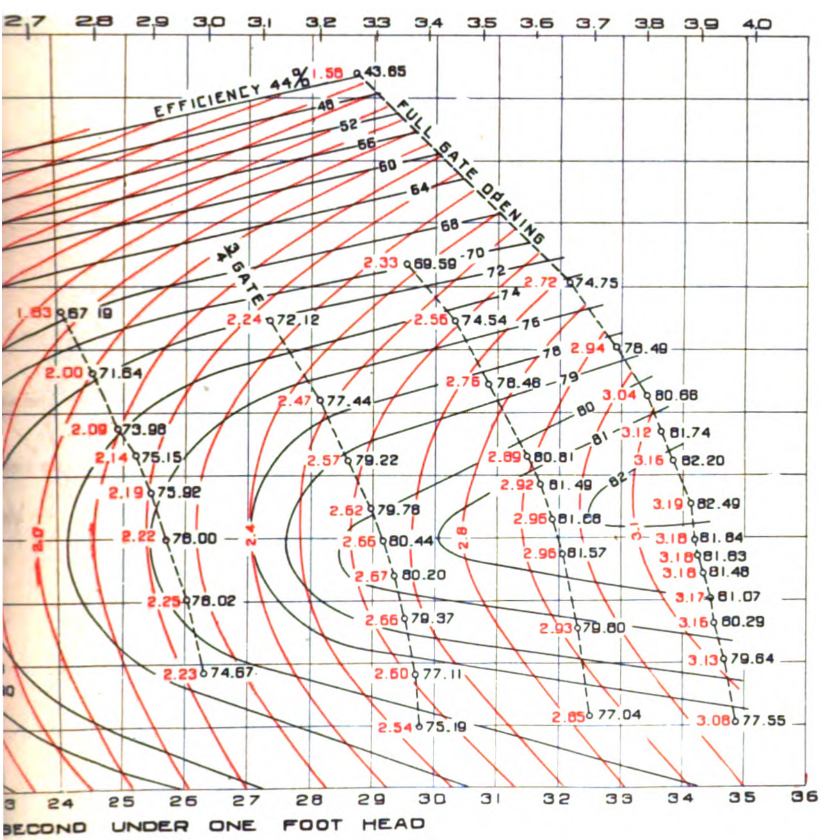


Fig. 245.—"Characteristic Curve" of a

ꟾD WITH 100 PERCENT EFFICIENCY



ꟾh Victor Turbine, with Cylinder Gate.

curve is based upon the assumption that the efficiency will remain constant for a variable head as long as ϕ remains constant.

The efficiency and horse power lines as interpolated, are subject to errors of interpolation, the extent of which can be readily judged from the diagram made. The conditions of the test are approximately checked by this diagram, for any marked irregularities in these curves must be due to errors in testing, or to poor workmanship.

By inspection it is possible to decide immodiately the value of ϕ that must be maintained in order to maintain the maximum efficiency at any particular condition of gate. For example: if the maximum efficiency at full load is desired, ϕ with this wheel should equal about .69. If the maximum efficiency at .75 gate is desired, the value of ϕ should be about .65, and for maximum efficiency at .50 gate, ϕ should be reduced to about .64.

Knowing the head under which the wheel is to operate, the necessary number of revolutions at any head can be calculated by formula (1) or by multiplying the r. p. m. at one foot head by the $\sqrt{h}$ and the conditions of operation, in regard to both power and efficiency at all gates, will be determined by the intersection of a horizontal line through the chosen value of ϕ with the efficiency and horse power lines. If, for example, it is decided that ϕ shall be .66, a horizontal line running directly through the diagram at $\phi = .66$ will, by means of the various points of intersection with the gate opening, efficiency and horse power lines, give all information desired and from it can be calculated the efficiency, speed, discharge and horse power of the wheel for the head under which it is to operate. The intersection of this .66 ϕ line with the various efficiency curves will give the relation of efficiency to discharge with one foot head. The discharge under the required head can be calculated by equation (8), i. e. by multiplying the discharge shown at the bottom of the diagram (one foot head) by $\sqrt{h}$. The efficiencies at each gate position will remain unchanged by this change in head since ϕ is fixed at .66. If a 16 foot head be considered, the discharge at any point will be four times the discharge read from the diagram.

The relation of horse power to discharge is also shown by the intersection of the ϕ line with the horse power curves. The actual horse power under any head can be determined by equation (11) i. e. by multiplying the horse power, as read from the diagram (one foot head) by $h^{\frac{3}{2}}$. The horse power at 16 foot head will therefore be 64 times that given by the diagram.

If it is desired to utilize the characteristic curve for the consideration of a wheel of another size but of the same series, the power and discharge must be multiplied by the ratio $\frac{D_1^2}{D^2}$

All of the various types of curves showing the results of opera-

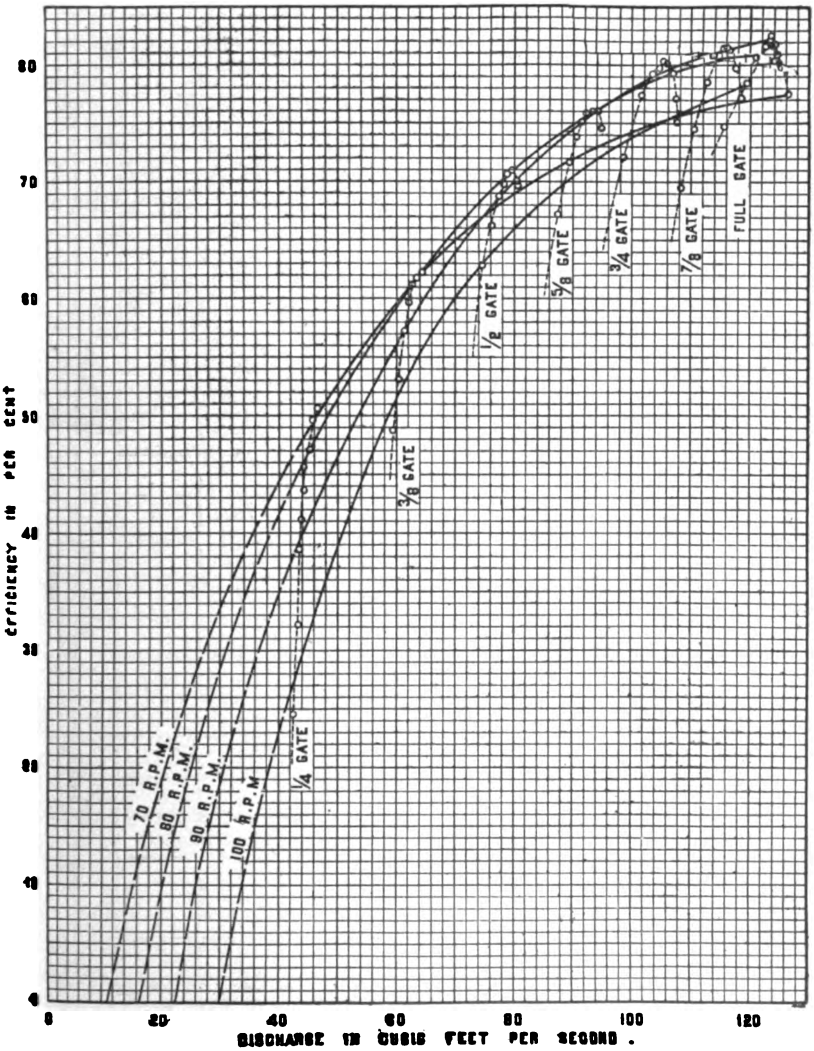


Fig. 246.

tion of the wheel as hitherto described are shown by, or can be calculated from, the characteristic curve.

Fig. 246, showing the relation of the number of revolutions to the efficiency and discharge of the wheel, is one example of such use.

192. Other Characteristic Curves.—Fig. 247 is the characteristic curve of a 44 inch "Improved New American" turbine showing the

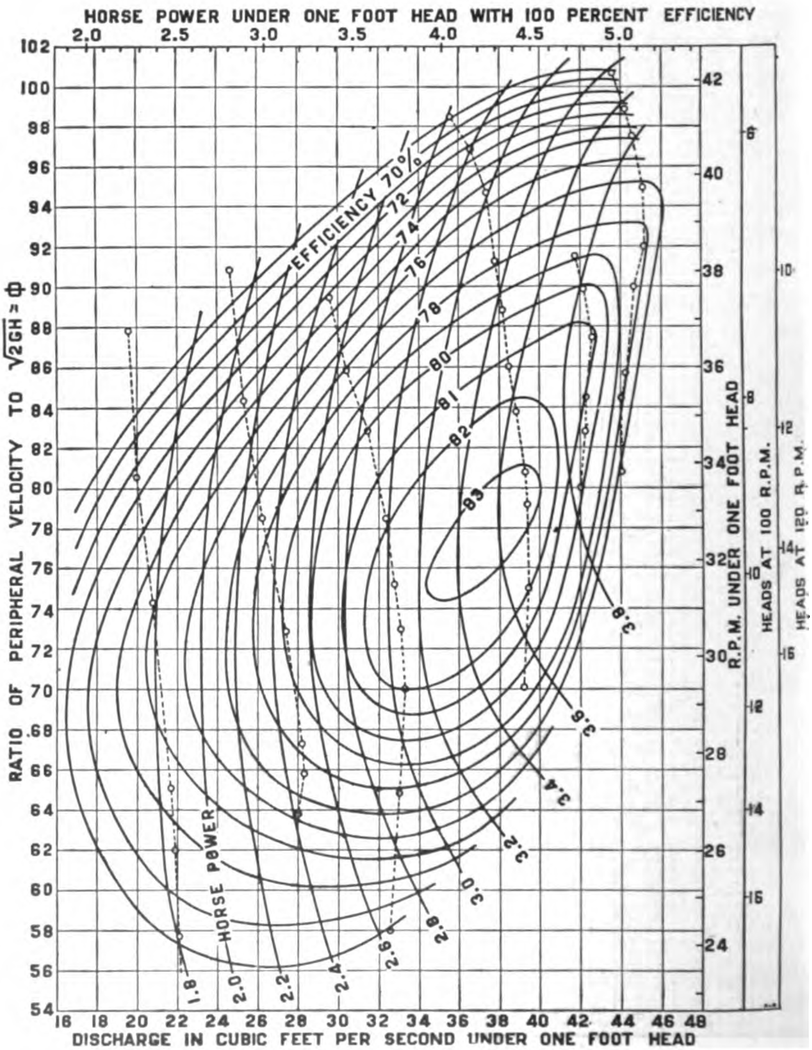


Fig. 247.—Characteristic Curve of a 44-Inch "Improved New American" Turbine.

operation of the wheel through a considerable range of heads. The outer line entitled "Head at 120 r. p. m., shows the values of ϕ and n_1 at which the wheel would have to operate to maintain 120 r. p. m. at the indicated heads. The location of these points may be determined in two ways: First.—By calculating the values of

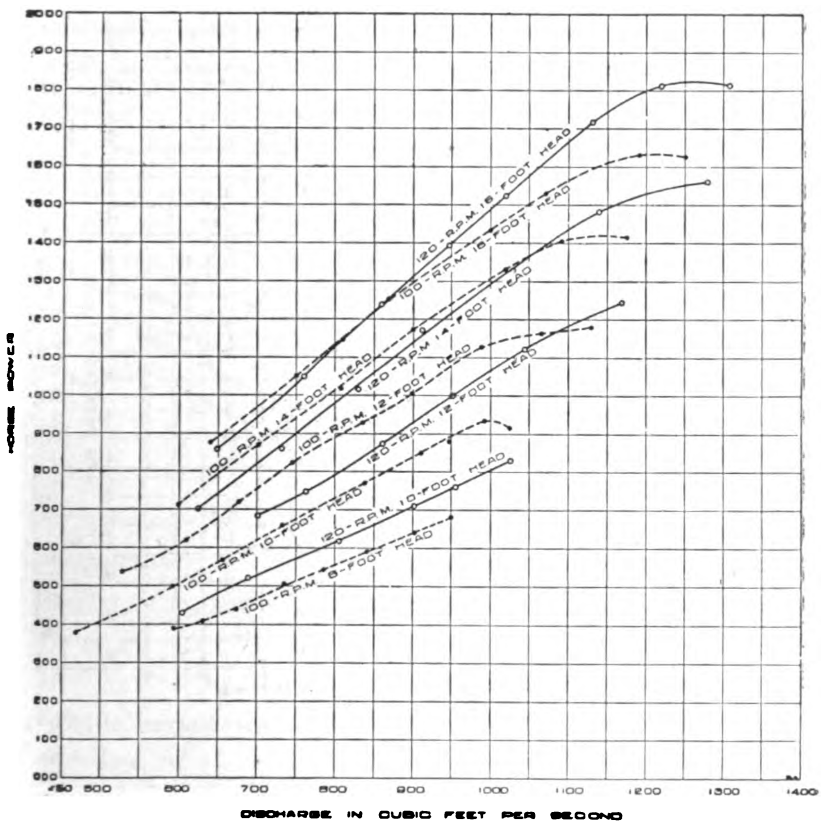


Fig. 248.—Curves Constructed from Fig. 247 Showing the Power at Two Speeds of Six "Improved New American" Wheels.

ϕ for a given head and number of revolutions, and locating the corresponding point from the scale on the left of the diagram: Second.—By dividing the number of revolutions by the square root of the head and fixing the point by the corresponding revolutions under one foot head, as shown on the scale of r. p. m. at the right of the diagram.

At 14 foot head the wheel will operate at about the maximum efficiency. If the head be decreased to 12′, the relative efficiencies will still remain fairly satisfactory, but will decrease rapidly at 10′ as shown by a horizontal line drawn through the corresponding point. It is also evident that at 8′ the efficiency becomes very low, and below this head the wheel would probably be unable to maintain 120 r. p. m.

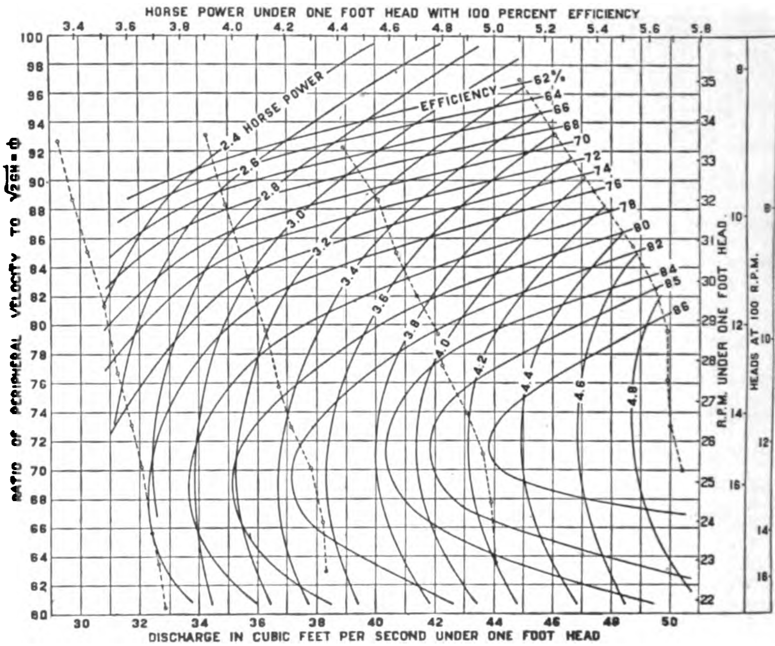


Fig. 249.—Characteristic Curves of a Wellman-Seaver-Morgan 51-Inch McCormick Wheel.

The second line at the right shows the value of ϕ and n_1 at various heads when operating at 100 revolutions per minute. At this speed the wheel will operate satisfactorily under heads from 14′ to as low as 7′, or even less. The efficiency at 14 foot head in this case will be less than at 120 r. p. m., and the efficiency of operation will increase as the head diminishes to the 9 and 10 foot point, where the best efficiencies are obtained at 100 r. p. m. Below this point the efficiency of operation will gradually decrease. Provided the revolutions per minute are satisfactorily selected, it will be seen that the wheel will meet successfully a wide variation in the operating conditions.

Fig. 248 is a diagram constructed from this characteristic curve and shows the power of six turbines of this series but of 49″ diameter connected tandem to a horizontal shaft and operated at the various heads and revolutions above discussed. The curves show the condition both at full and at part gates. The gradual change

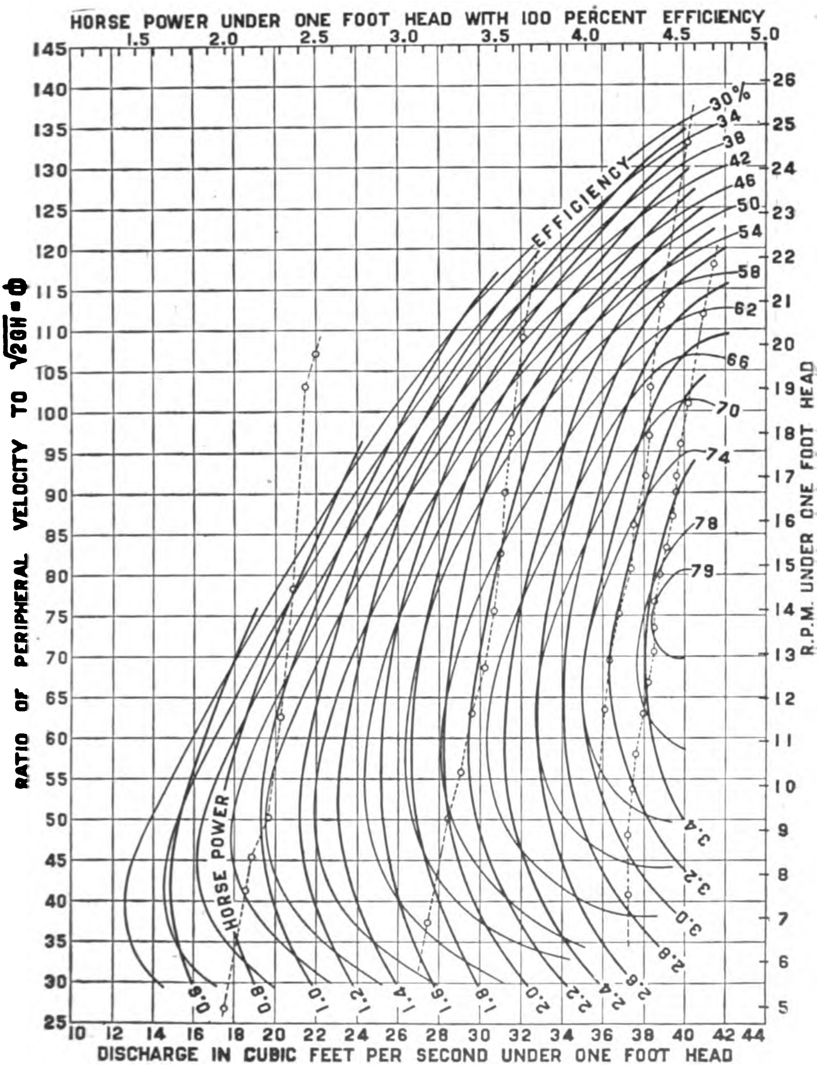


Fig. 250.—Characteristic Curves of the 99½-Inch Tremont Fourneyron Wheel.

in the relative position of the 100 and the 120 r. p. m. curves, as the head changes, should be noted.

Fig. 249 shows the characteristic curve of a 51" McCormick turbine, as manufactured by Jolly Brothers for the Wellman-Seaver-Morgan Company. At the right of the diagram are shown the relative values of ϕ and at the left the values of n for heads from

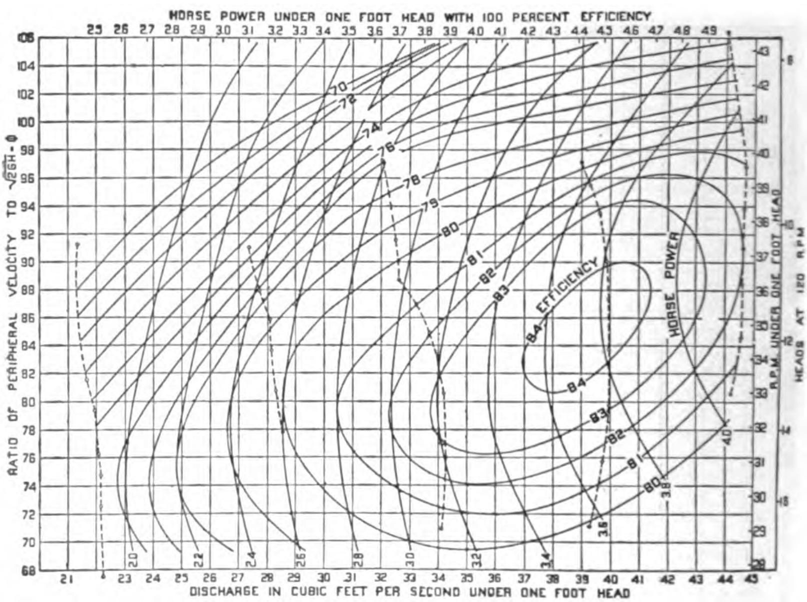


Fig. 251.—Characteristic Curves of a 45-Inch "Samson" Wheel. (James Leffel & Co.)

16 to 8 feet, at 90 and 100 r. p. m. This curve shows that this wheel will work satisfactorily under a wide range of conditions, if a suitable speed is chosen.

Fig. 250 is the characteristic curve of the Tremont turbine tested by James B. Francis, and described in the "Lowell Hydraulic Experiments." This wheel was a Fourneyron turbine of about 700 horse power at 13' head.

Fig. 251 is the characteristic curve of a 45" Leffel turbine, which has been selected for the Morris Plant of the Economy Light and Power Company, now under construction on the Des Plaines River, about twelve miles south of Joliet, Illinois. It is to be operated at 120 revolutions per minute and under variations in head from 16 to 8 feet. Eight units, each consisting of eight of these wheels, connected tandem, are to be installed to operate eight 1,000 K. W. alternating generators. This diagram was prepared from the test sheet accompanying the bid of the James Leffel & Company. In the construction of the wheels for the plant, an attempt was made to so alter them as to maintain a high efficiency for a

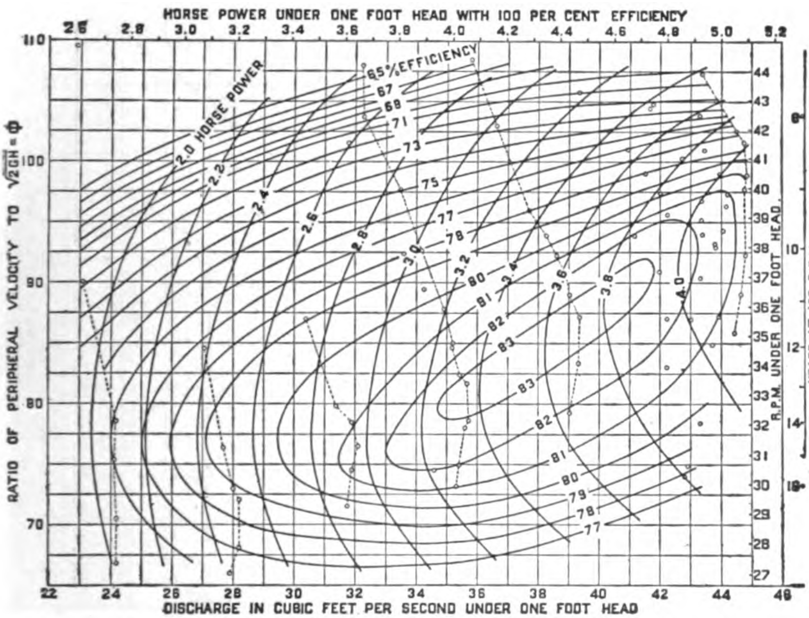


Fig. 252.—Characteristic Curves of a 45-Inch "Samson" Wheel. (James Leffel & Co.)

greater range of gate conditions than ordinarily obtained. Fig. 252 shows a characteristic curve of one of the new wheels as constructed for this plant. The analysis was made for the purpose of estimating the results which would probably be secured under service.

In Fig. 253 are shown the discharges, powers, and efficiencies of one unit of eight wheels under all heads from 8 to 16 feet at full and seven-eighths gate. Allowances would have to be made in order to take into account the difference between the operation of the eight wheels in the horizontal position connected in tandem, and in the position in which they were tested; but the diagram shown gives an analysis from which fairly satisfactory conclusions can be drawn.

193. Graphical Analysis as Developed by H. B. Taylor under supervision of W. M. White.—A valuable method of graphical

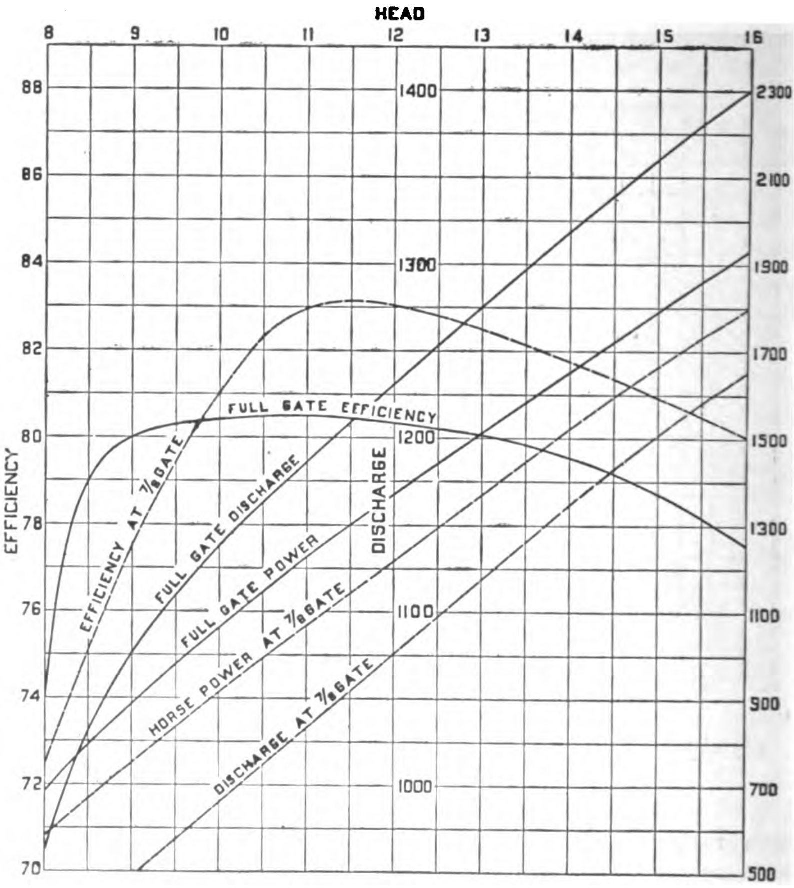


Fig. 253.—Curves Showing the Efficiency and the Maximum and Ordinary Power and Discharge of One Unit of 8 45-Inch Samson Wheels.

analysis is shown in Bulletin No. 2 of the I. P. Morris Company, in which is discussed the variations in power and efficiency of a turbine wheel capable of giving 13,500 horse power under a head of 65 feet, and at a speed of 107 revolutions per minute. This wheel was designed by this Company for the McCall-Ferry Power Company, and was to work under heads varying from 50 to 70 feet.

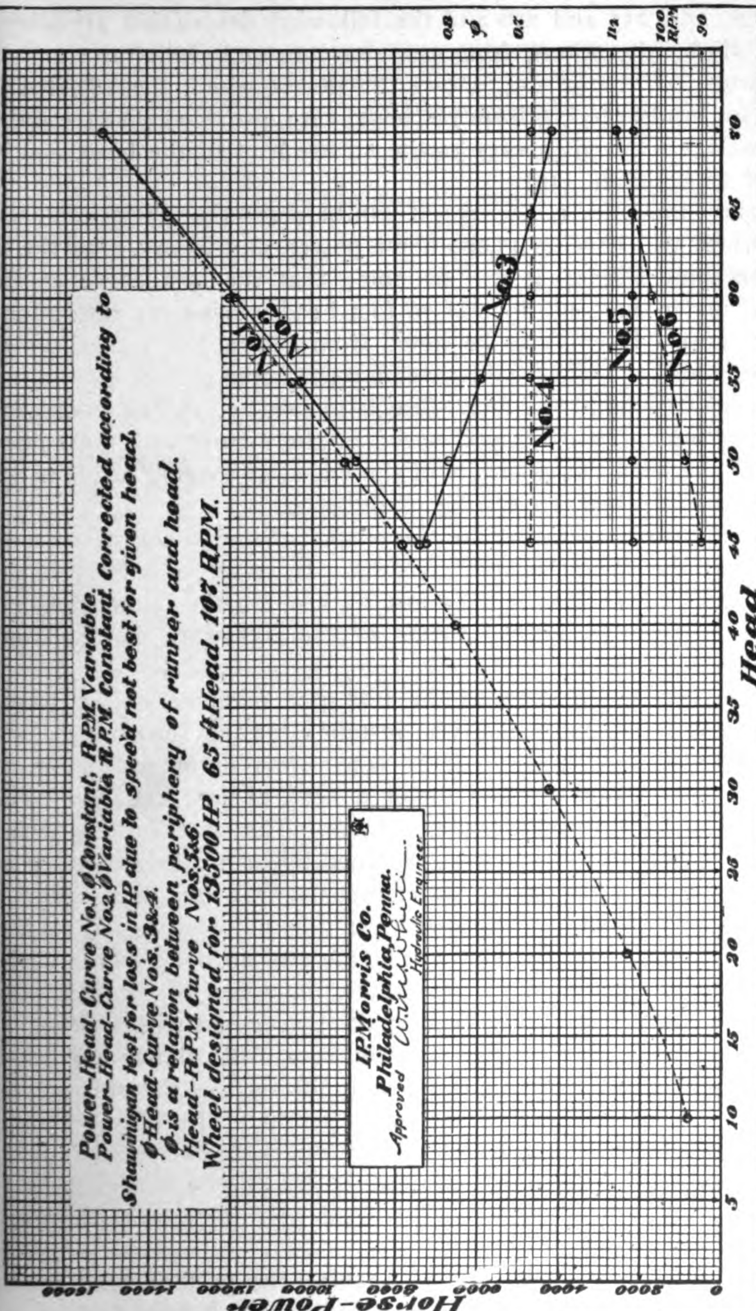


Fig. 254—Relation of Power and Head. 13500 H. P. Turbine designed for McCall Ferry Power Co. (Reproduced from Bull. No. 2 of I. P. Morris Co.)

Figs. 254, 255 and 256 and the following description are taken, with slight alterations, from the above named Bulletin.

Curve No. 1. Fig. 254, shows the power which the wheel will give for heads varying from 70 feet to zero, provided that the revolutions are allowed to vary as the square root of the head, and is based on equation (10).

From Curve No. 1, Fig. 254, it will be noted that at 70 foot head the wheel will develop 15,000 horse power, and from Curve No. 6, of the same Figure, it will be noted that the best speed of the wheel under the conditions of 70 foot head will be 111 revolutions per minute. It will also be noted from Curve No. 1 that, under 50 foot head, the wheel will develop 9,150 horse power, if it be run at 94 revolutions per minute. That is to say, by keeping a constant ratio between the peripheral speed of the runner and the square root of the head the efficiency of the wheel at varying heads is not changed for any given setting of the gate.

In order to properly utilize the output of the wheel, it is necessary that the speed be kept constant. In order to determine the amount of power that will be lost by keeping the speed constant while the head varies, the curves of Fig. 255 were platted from actual observations.

Curve No. 1, Fig. 255, is the full gate readings of the 10,500 horse power turbine, which was installed for the Shawinigan Water and Power Company. This wheel was designed for 10,500 horse power when working under a head of 135 feet, and when running at 180 revolutions per minute. The observations which are platted on this curve were obtained by using the generator as a brake for the wheel, and a water rheostat was used as a means of loading the generator. The speed was then adjusted to 180 revolutions per minute at the wide open gate and an observation made. By varying the field of the generator, the speed of the unit was varied without materially affecting the power and without moving the gate of the wheel. Observations were made above and below the normal speed through as wide limits as the rheostat in the field circuit of the generator would permit. The power output was determined by means of accurately calibrated electrical instruments. The speed was determined by an accurately calibrated tachometer. The curves on this sheet give the relation between ϕ and horse power.

Referring back to Fig. 254, and taking the 50 foot head conditions, it should be noted that for a constant speed of 107 revolu-

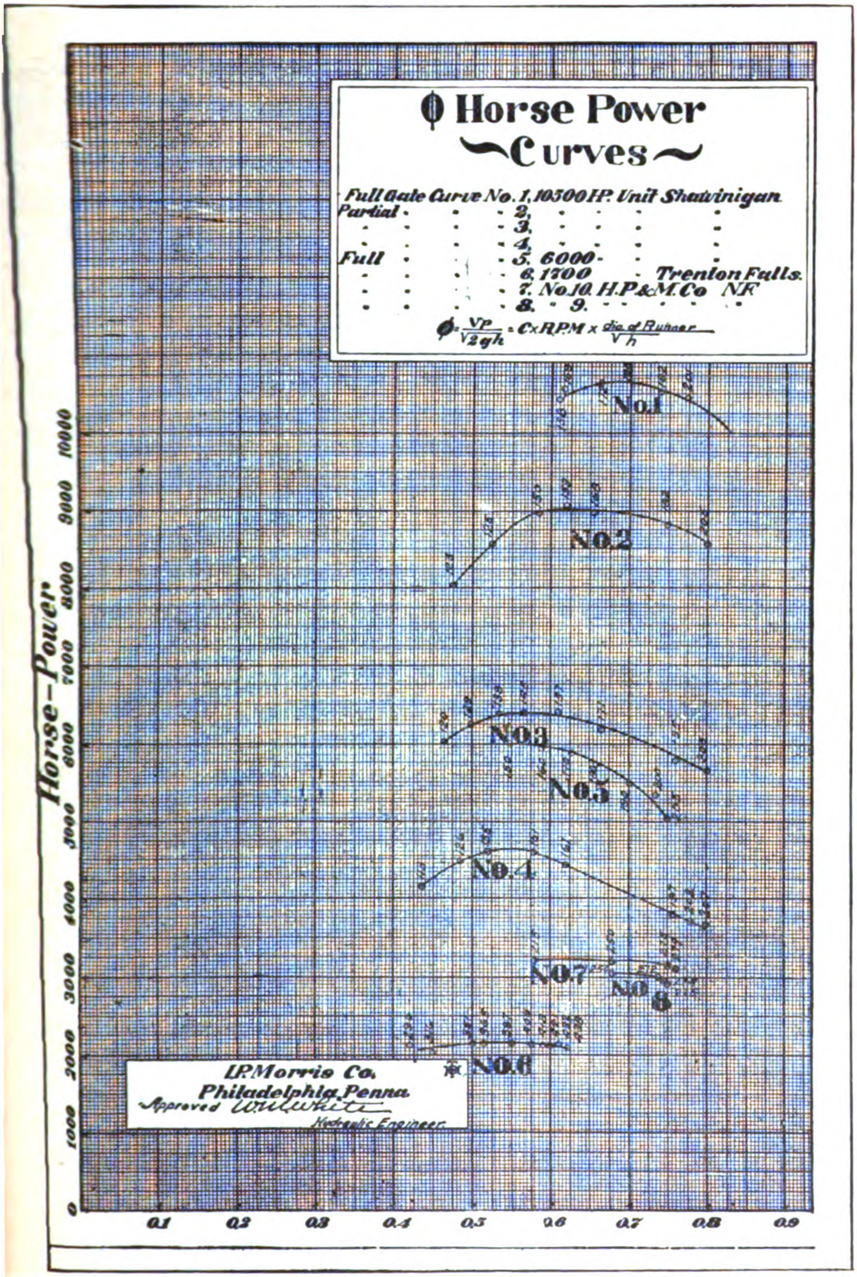


Fig. 255.—Curves of φ and Power of Several I. P. Morris Wheels. (Reproduced from Bull. No. 2 of I. P. Morris Co.)

tions per minute ϕ would have to increase from the normal value of about .68 to .80. By referring again to Fig. 255, it will be noted that when ϕ was 0.8, with full gate opening, the power dropped from 10,650 horse power to 10,250 horse power, or about 3.3 per cent. From this fact the normal power as shown by Fig. 1 may be corrected for the new speed of rotation and a point on Curve No. 2, Fig. 254 obtained, giving the actual power which would be developed by the wheel under the 50 foot head, and running at the constant speed of 107 revolutions per minute. Curve No. 2 is platted in this manner from Curve No. 1.

As a check to Curve No. 1, Fig. 255, Curves Nos. 5, 6, 7, and 8 have been platted, all of which were made from actual observations, in the same manner as Curve No. 1. All of these wheels are of the Francis inflow type, and were designed for $\phi = .7$, except Curve No. 6, which is an outward flow Fourneyron wheel, and was designed for $\phi = .5$. Curve No. 5 is for a 6,000 horse power wheel with gates in the draft tubes. The shape of the curve shows that the gate was probably not entirely open when the observations were made.

In Fig. 256 has been platted efficiency curves, which the designed wheel would give under varying heads, and running at a constant number of revolutions. Curve No. 1 is an exact duplicate of the efficiency curve which was obtained on a 3,500 horse power wheel working under 210 foot head, and making 250 revolutions per minute. The wheel is of the Francis inflow type, with double runners, fitted with movable guide vanes, similar to those which are proposed to be used in the wheels for the McCall-Ferry Power Company.

It will be noted that the efficiency of the wheel reaches 82.3 per cent. at about seven-eighths power, the efficiency dropping to 81½ per cent. at full gate. It will be noted that the efficiency is very high at part load. This was accomplished in the design of the wheel by sacrificing a higher efficiency at full load. This curve has been taken as typical of the efficiency which would be obtained by the wheel proposed for the McCall-Ferry Power Company, when working under a 65 foot head. The efficiency curve of the 10,500 horse power wheel which was supplied by the I. P. Morris Company to the Shawinigan Water and Power Company (See Fig. 236), gives higher results than the curve selected, but it was thought that Curve No. 1 is the best for a typical curve.

Curve No. 1, Fig. 256 was platted by assuming that, at full gate, 3,500 horse power corresponded to 13,500 horse power in the wheel to be designed. The part gate points of the curve were obtained by proportion. Curve No. 3 represents the efficiency and power of the wheel when working under 50 foot head, and at 94 r. p. m.

Point X on this curve was obtained in the following manner: First, read on Curve No. 1, Fig. 254 the power which the wheel would give under the 50 foot head, and revolutions best suited. This is found to be 9,150 horse power. On Scale B, Fig. 256 a line is drawn from 9,150 horse power to zero, forming Curve No. 10. To find what the efficiency would be at 8,000 horse power under the 50 foot head, take the point at 8,000 horse power on Scale B, projected horizontally until it intersects Curve No. 10, and 11,800 horse power will be read from Scale A. From the efficiency curve directly over 8,000 horse power on Scale A, the point, X, will be found on Curve No. 3, which gives the efficiency of the wheel when developing 8,000 horse power under the 50 foot head, and running at the revolutions best suited, namely 94.

This wheel is to run, however, at 107 revolutions per minute, under all conditions of head, and it is necessary to correct Curve No. 3 for the drop in power and efficiency due to the increase in speed.

Referring to Curve No. 1, Fig. 255, it will be noted that the power varies when the speed varies, and in the calculations of efficiency in Fig. 256, it has been assumed that the efficiency varies directly as the power. In other words, it has been assumed that the quantity of water does not vary when the revolutions are changed with the constant setting of the gate. This is not strictly true but for the observations as platted on Curve No. 1, Fig. 255 the quantity of water would probably vary only one-half of one per cent., increasing as the revolutions increase from 158 to 201.

Referring to Fig. 254, and the 50 foot head, it will be noted that when the speed is increased from the best speed of 94 revolutions to the desired speed of 107 revolutions, the power falls 3.3 per cent. and the power and efficiency of the full gate point on Curve No. 3, Fig. 256 can be decreased 3.3 per cent, resulting in the full gate point on Curve No. 2.

Referring to Fig. 255, Curves Nos. 1, 2, 3, and 4, it will be noted that the slope of these curves between $\phi = 0.7$ and $\phi = 0.8$ is about the same, and, therefore, the power and efficiency of all the points

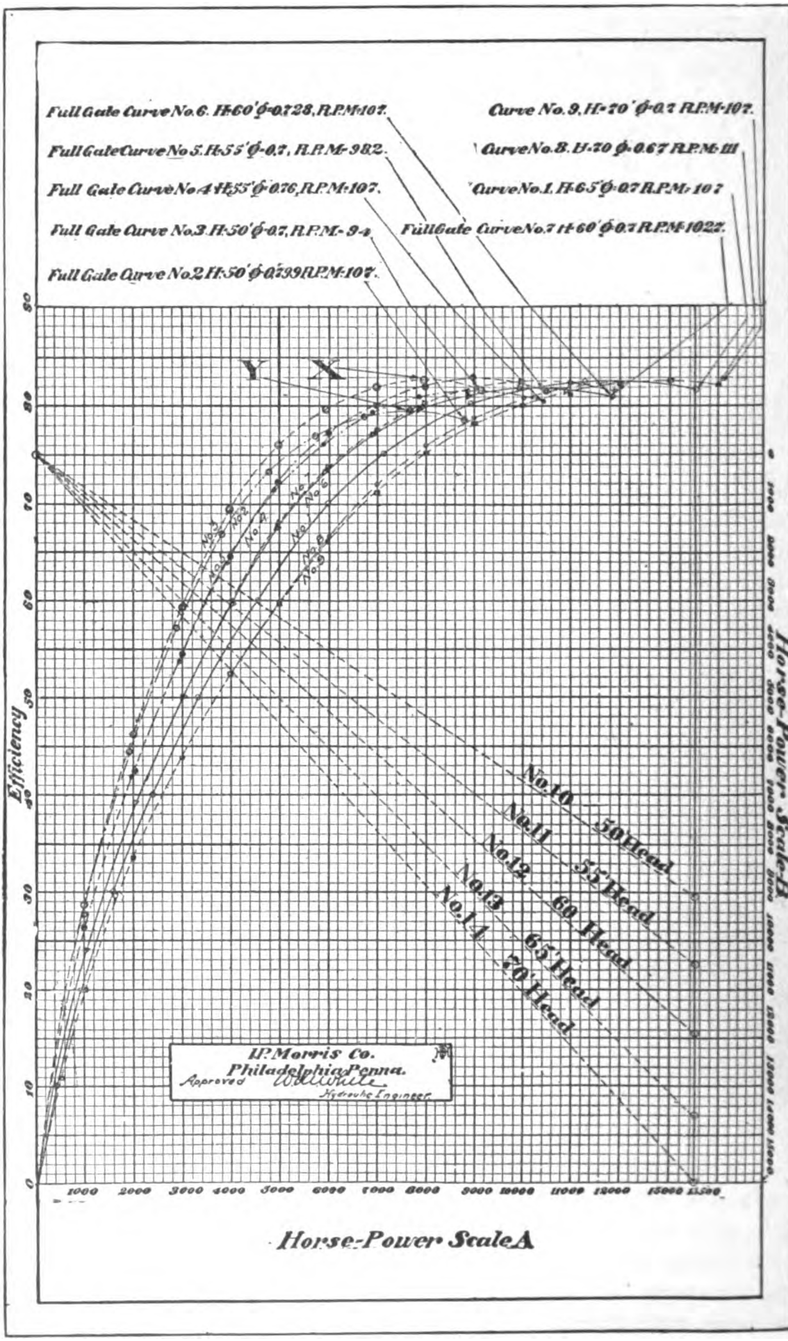


Fig. 256.—Estimated Efficiency—Power Curves of the Proposed McCall-Ferry Wheel. (Reproduced from Bull. No. 2 of I. P. Morris Co.)

on Curve No. 3, Fig. 256, can be reduced by the same percentage, namely, 3.3 per cent. In this manner Curve No. 2, Fig. 256. is obtained, which gives the power and efficiency of the wheel when working under the 50 foot head, and running at the speed of 107 revolutions per minute. In the same manner Curves Nos. 5 and 7 are platted, Curves Nos. 4 and 6 being deduced therefrom, respectively. In the same manner Curve No. 9 is platted, and Curve No. 8 deduced therefrom. It will be noted that Curve No. 8 lies on the opposite side of the parent curve to that of the other curves. Curve No. 8 crosses Curve No. 9 at 13,500 horse power on Scale A, and beyond this point would drop below Curve No. 9. The reason Curve No. 8 falls to the left of Curve No. 9, and shows greater efficiency at part gate for the 70 foot head, is because when ϕ changes from 0.7 to 0.65, Fig. 255, the partial gate Curves Nos. 2, 3. and 4, Fig. 255, show the increase in power and efficiency. These points, however, cannot be very definitely determined, but it does show that the assumption is correct that the designed wheel, working under the head of 70 feet, and running at 107 revolutions, will show higher percentage of efficiency at part gate than when running at the 65 foot head and the same powers.

The curves on Fig. 256 show that the efficiency is not seriously affected by keeping the speed of the wheel constant under the varying conditions of head. They do show, however, that the power is seriously affected by keeping the speed of the wheel constant under the varying conditions of head. The endings of the various curves show the maximum power, as read on Scale A, which the wheels will give under that head.

These curves, therefore, give the performance of the wheel when running at a constant number of revolutions, and working under varying heads from 50 to 70 feet. The curves, of course, are not absolutely correct. They show, however, fairly accurately, the amount of variation in efficiency and power which may be expected from the actual conditions obtained with the proposed wheel under the head for which it was designed.

CHAPTER XVII

THE LOAD CURVE AND LOAD FACTOR, AND THEIR INFLUENCE ON THE DESIGN OF THE POWER PLANT.

194. Variation in Load.—All power plants are subjected to more or less change in load, and this continually changing load has an important bearing on the economy of the plant, and should be carefully considered in its design and construction.

If the power output of any plant be ascertained, minute by minute or hour by hour, either by means of recording devices or by reading the various forms of power indicators usually provided for such purposes, and a graphical record of such readings be made, a curve varying in height, in proportion as the power varies from time to time, will result. This curve is termed the daily load curve. The load curve itself will vary from day to day as the various demands for power vary, but it usually possesses certain characteristic features which depend on the load tributary to each plant and which vary somewhat as the seasons or other conditions cause the load to vary.

The characteristics of the load curve, due to certain demands, can be quite safely predicted. A power plant in a large city, for example, will carry a comparatively small continuous night load. This, in dark weather and in winter, will be increased by the early risers who are obliged to go early to shop and factory. These demands usually begin to affect the load curve about 5 A. M. and may cease wholly, or in part, by 7 A. M., depending on the season and latitude. From 7 to 8 A. M. the motor load begins to be felt. This may reach a maximum from 10 to 12, and usually decreases from 12 to 2 during the lunch hours. The maximum load usually comes in the afternoon when business reaches a maximum, and when the largest amount of power and also light (in the late afternoon) are used. The load begins to decrease after the evening meal, as the demand for light lessens, and may again increase somewhat as the theatres and halls open for evenings' amusements. The character of the load curves, due to various loads, is best understood by a study of the actual curves themselves.

195. Load Curves of Light and Power Plants.—The curves shown in Fig. 257 are from the plants of the Hartford Electric Light Co., of Hartford, Conn., and will illustrate variation of the load curve at different seasons of the year. These curves were taken from an article in "The Electrical World and Engineer" of March 8th, 1902. This plant is a combined water and steam power plant, and is provided with a storage battery to assist in equalizing the load. These curves are described as follows:

"On a week day in March, 1901, the maximum load was 1720 k. w. and the total energy output was 30249 k. w. hours. The average hourly load was then 1260 k. w. or 46 per cent. of the maximum load. On this same day the battery discharged at the rate of 260 k. w. at the peak of the load. In the early morning hours of this day the load on the system, apart from battery charging, reached its minimum at 612 k. w., or only 22.5 per cent. of the maximum load. In June, 1901, the maximum load on a certain week day was 1390 k. w., and the minimum 250 k. w., or 18 per cent. of the former. The total output on this day was 2505 k. w. hours, so that the average load during the 24 hours was 1046 k. w. or 75 per cent. of the maximum. In January, the maximum load came on between 4 and 5 P. M., when lighting was the predominant factor, but in July the greatest demand came on the system in the latter part of the forenoon, and must have been made up in large part by requirements for electric power. By December 1901, the maximum load reached 2838 k. w. and the minimum 612 k. w. The approximate capacity of all connected lamps and motors in that month was 8530 k. w. The maximum load for the December day of 2838 k.w. is only 33 per cent. of the connected capacity. On this day the total output was 3219 k. w. hours, so that the average load during the 24 hours was 1342 k. w. This average is 15 per cent. of the total capacity."

Fig. 258 is a combined annual load curve for several years, and not only shows the increase in the electrical output of this system for the years from 1898 to 1905, but also the annual monthly change in load from a maximum in December or January to a minimum in June or July. This variation fortunately accompanied similar variation in the flow of the Farmington River on which most of the power was developed.

Up to the middle of 1898 the entire load of this Company was carried by a single water power plant. The natural increase in demand for power necessitated the construction of a second plant

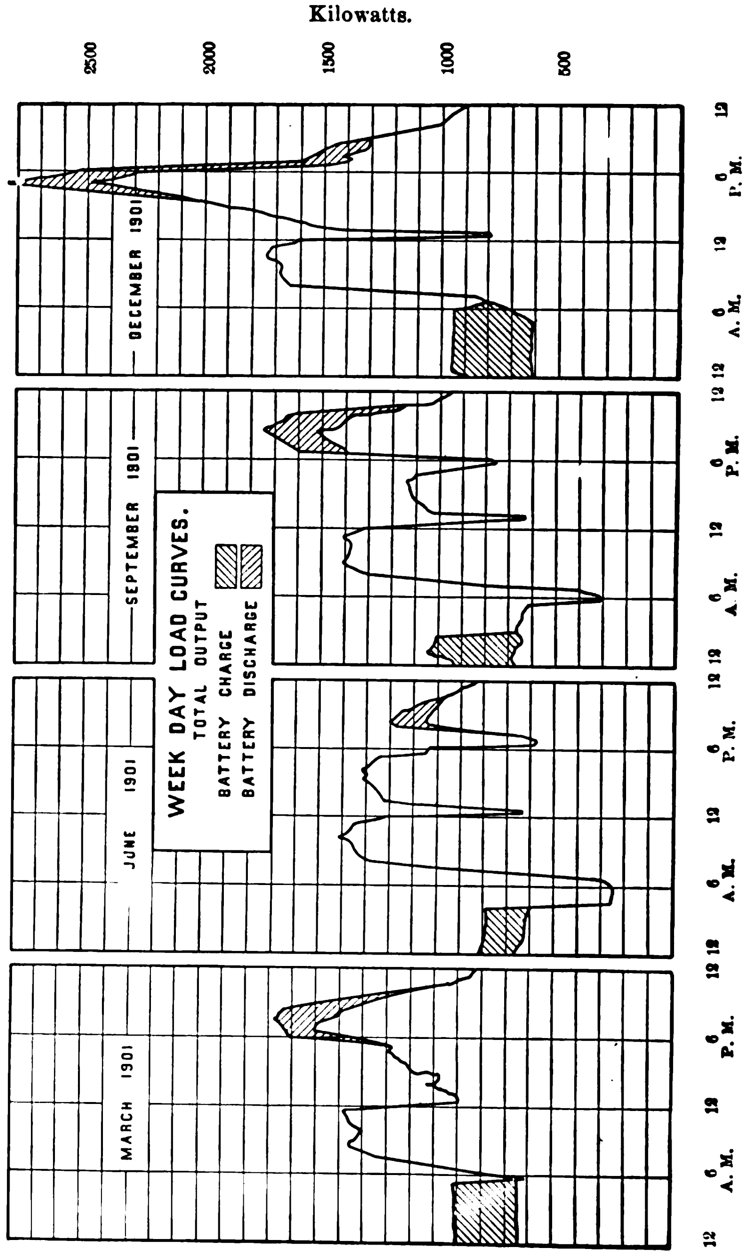


Fig. 257.—Hartford Electric Light Co., Load Curves. (Electrical World and Engineer, Mar. 8, 1902.)

on the same river, and up to January 1905, the two water power plants were able to carry most of the load, steam auxiliaries, however, being occasionally used, as indicated by the dotted line.

Fig. 259 shows daily load curves from the Christiania Power Stations, of Christiania, Norway. In this figure are shown the maximum, the minimum, and a mean curve for the entire year. The

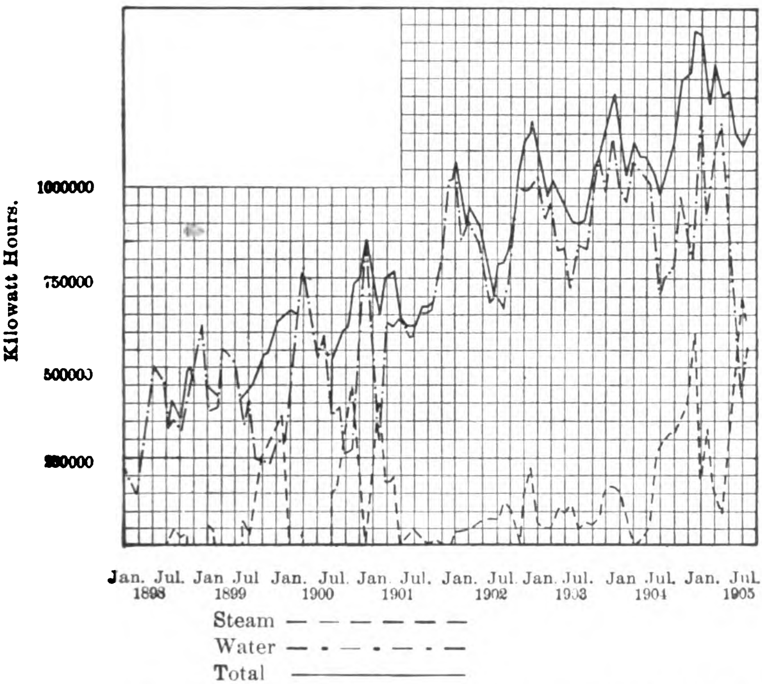


Fig. 258.—Energy Output of Hartford Electric Light Co. (From Electrical World and Engineer.)

difference between the maximum and minimum curves is here very marked. This is readily ascribed to the high latitude of Christiania as the long twilights of summer render lighting at that season almost unnecessary, while the very short and dark days of winter create not only a high maximum but a high continual demand during the entire day. No data as to kind of load is available.

Fig. 260 is a power curve from the New York Edison Company. On August 1st, 1905, there were connected up to the system of the New York Edison Company an equivalent of 1,651,917 incandescent lamps, 22,093 arc lamps, 2,539 k. w. in storage batteries and 99,258 H. P. in motors. The lighting load forms 52.2 per cent of the connected load.

The effect of extraordinary conditions on the load curve and the necessity of some kind of storage to provide for the same, is well illustrated by Fig. 261 which shows the effect on the load curve

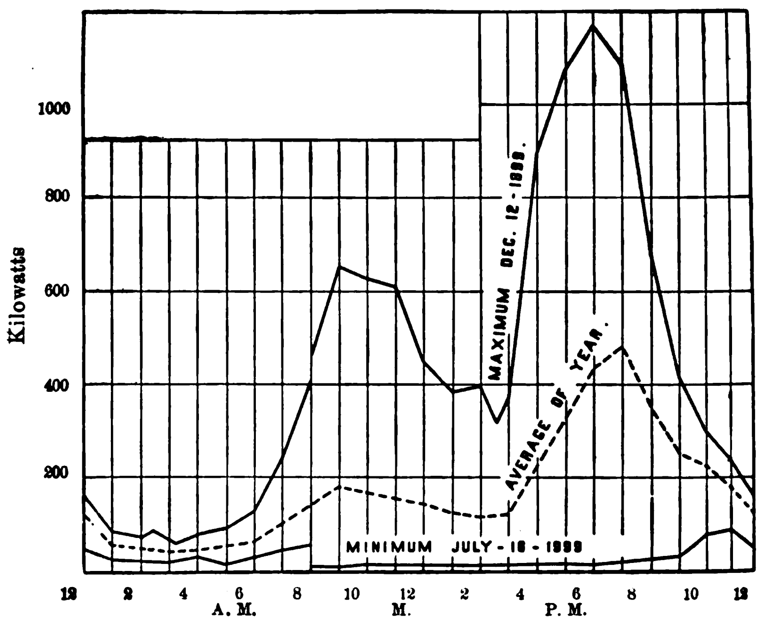


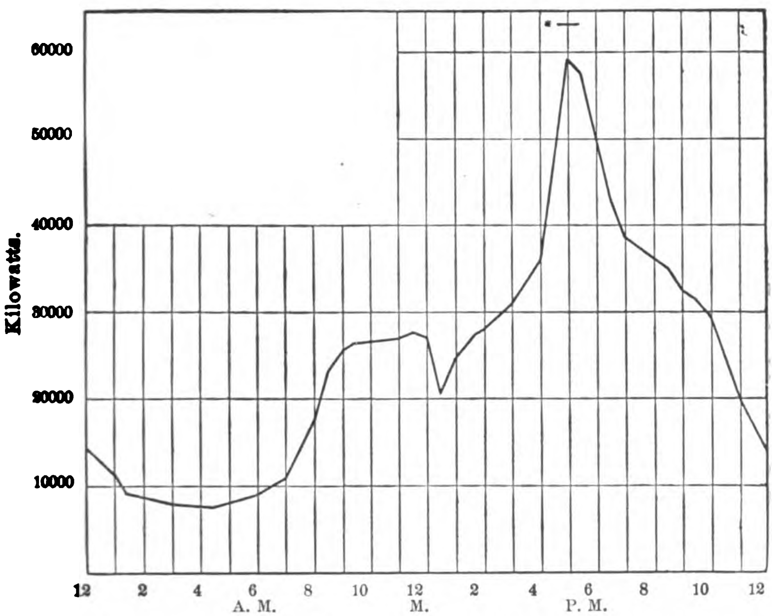
Fig. 259.—Typical Electric Lighting Load Curves. Christiana, Norway, Power Stations.

of a lighting plant of a sudden thunderstorm. When such a storm occurs in the late afternoon the light load from schools, offices, stores, etc., may be suddenly thrown on, and the result may be an extraordinary load which the plant must meet.

196. **Factory Load Curves.**—Shop and factory loads are supposed to be the most uniform in character, yet they are subject to great variation, due to the sudden turning on or off of the machines. Fig. 262 shows the load curve of the Pennsylvania Railroad Shops at Altoona, Pennsylvania.

The shops of the Pennsylvania Railroad are located in and around Altoona, Pennsylvania, in groups, each group being supplied by its own power station. No data as to the number and power of motors connected up is available, but the following shows to some extent how the load is divided. The Machine Shop power plant embraces

3—300 k. w. generators, 1 Brush arc generator (power unknown), and a 40 H. P. Thompson-Houston arc generator for lighting shop and grounds. At the Car Shops 4–250 k. w. and 1–625 k. w. generators are used. Current is supplied to 75 arc lights in shops and yards. At the Junita shops 3–300 k. w. generators are used for power purposes only. At South Altoona the generating station



New York Edison Co., Load Curve, day of Max. load, Dec. 22, 1904.

*Including 3900 K. W. delivered directly at 6600 Volts A. C.

Fig. 260.—Typical Electric Lighting Load Curve.

embraces 1–50 k. w., and 2–500 k. w., and 2–300 k. w. generators. The loads are quite variable, as would be expected in a railroad shop, there being some very heavy machines in intermittent operation, one planer running as high as 80 H. P., while 20 H. P. motors are numerous. The normal load is less than the maximum, but the latter is frequently reached.

A, B and C, Fig. 263, are three typical factory load curves which represent types of load curves from three different electric power stations, A in an Eastern, B in a Central, and C in a far Western state. These curves are taken from an article on "The Economics of Electric Power" in Cassier's Magazine for March, 1894. The circuits from these stations are exclusively motor circuits, the number of motors connected being given in the following tables;

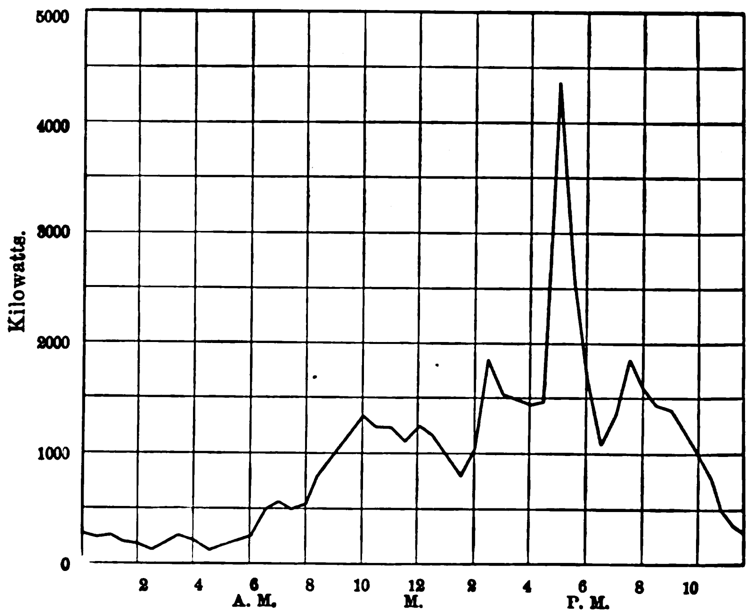


Fig. 261.—Sharp Thunder Storm Peak, Dickenson St. Station, Manchester, Eng.

A			B			C		
Size of Motor (H. P.)	No. in Use.	Combined H. P.	Size of Motor (H. P.)	No. in Use.	Combined H. P.	Size of Motor (H. P.)	No. in Use.	Combined H. P.
½	3	1½	⅛	3	⅜	¼	4	1
1	31	31	¼	2	½	½	1	½
2	10	20	½	1	½	1	5	5
3	19	57	1	15	15	2	3	6
5	10	50	2	14	28	3	4	12
7½	3	22½	3	5	15	5	3	15
10	12	120	5	12	60	6	5	30
13	5	75	7½	12	90	8½	3	25½
20	2	40	10	15	150	10	6	60
25	4	100	15	9	135	14	6	84
50	1	50	20	5	100	15	1	15
......	....	....	25	3	75	17½	1	17½
....	....	....	30	3	90	25	1	25
....	....	....	40	1	40	30	3	90
....	....	....	....	....	....	40	1	40
....	....	....	....	....	....	60	2	120
....	....	....	....	....	....	70	1	70
Total...	100	567	....	100	799⅜	....	50	616½

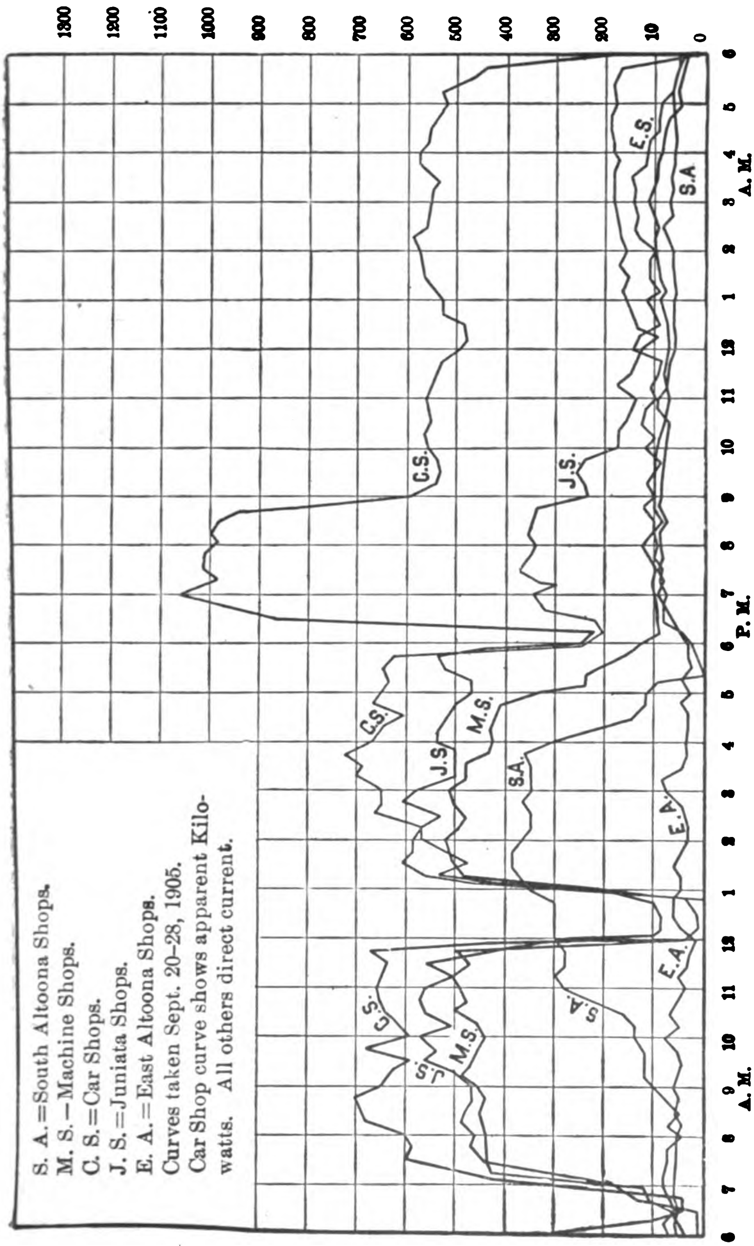


Fig. 282.—Load Curves Pennsylvania R. R. Shops, Altoona, Pa. (From Electrical World and Engineer, Aug. 18, 1906.)

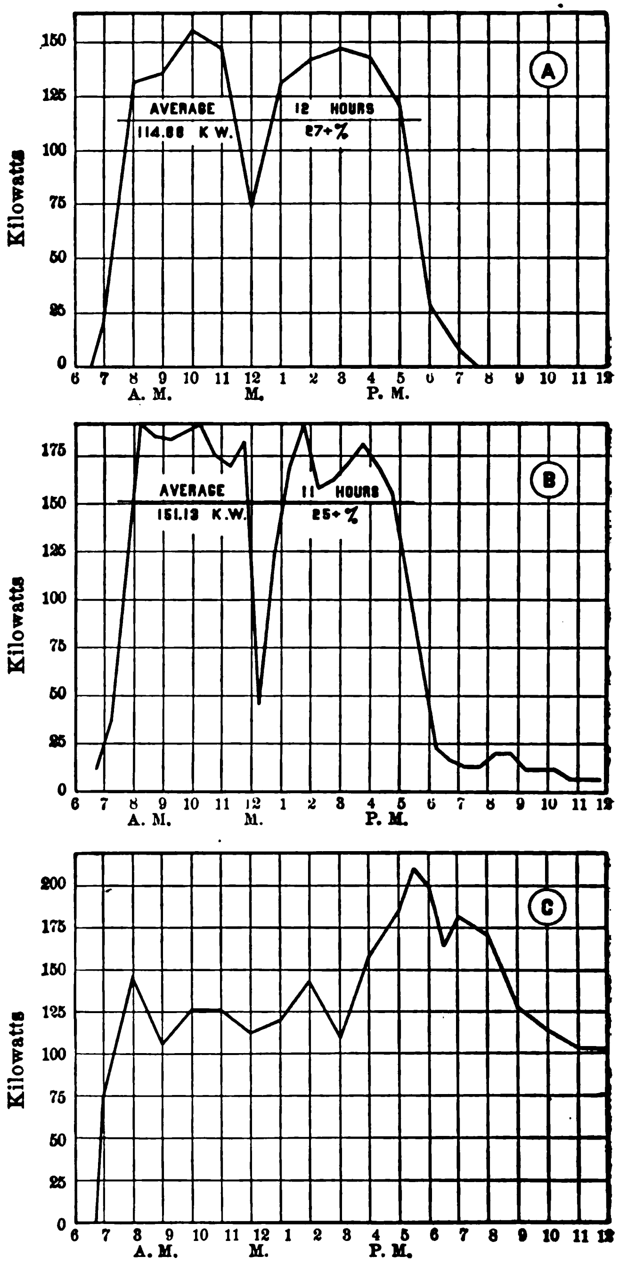


Fig. 263.—Typical Factory Load Curves. (Cassier's Magazine.)

On the circuits covered by the diagram B some of the motors are five miles and more distant from the power stations.

One deduction which may be made from a study of these curves is that in an electrical power system where a considerable number of motors are employed the initial dynamo plant need not be equal to the total motor load. In the case in hand the curves show that the generator need be but from 25 per cent. to 40 per cent. of the

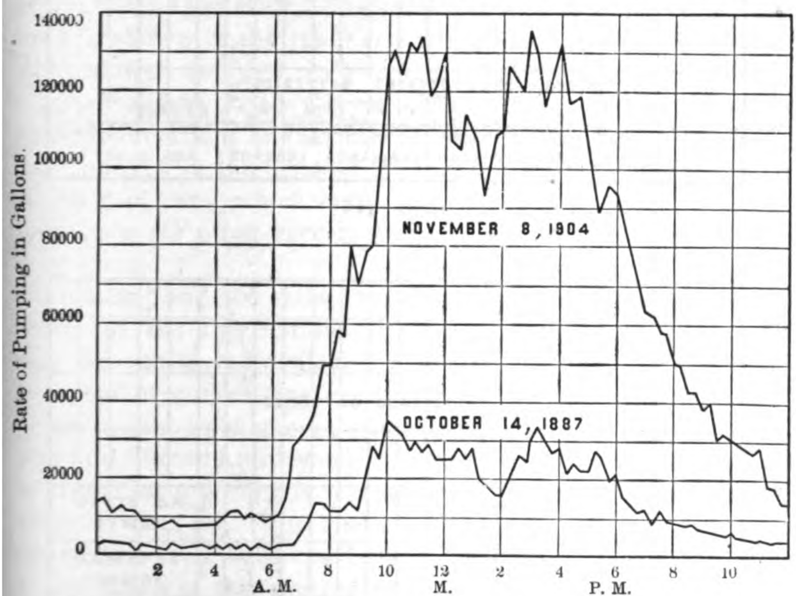


Fig. 264.—Maximum Days of Pumping.—London Hydraulic Supply. (Cassier's Magazine.)

rated capacity of the motors connected. In order to check off this phenomenal condition actual meter readings were taken monthly from fifty-three different shops covering a period of from four to six months, current to these shops being sold on the meter basis. The results showed that only 25½ per cent. of the nominal capacity of the motors was employed, thus practically checking the conditions indicated by the diagrams of the central power stations.

197. Load Curve of London Hydraulic Supply Company.—Fig. 264 is a load curve of The London Hydraulic Supply Company, which is rather exceptional in that the power is used almost entirely for running elevators and is therefore almost exclusively a

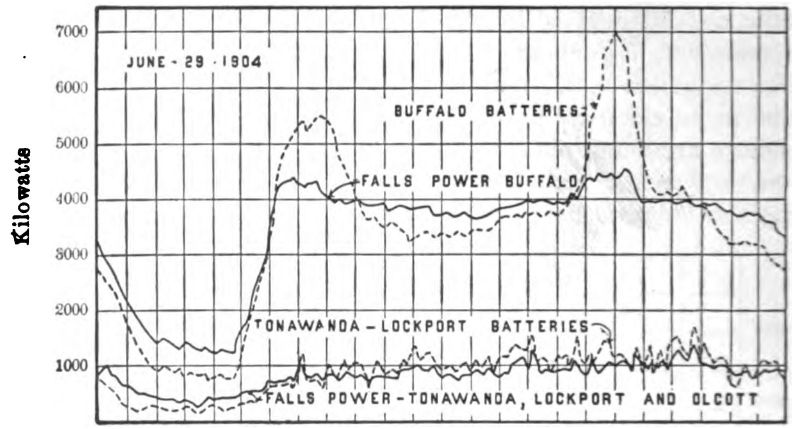


Fig. 265.

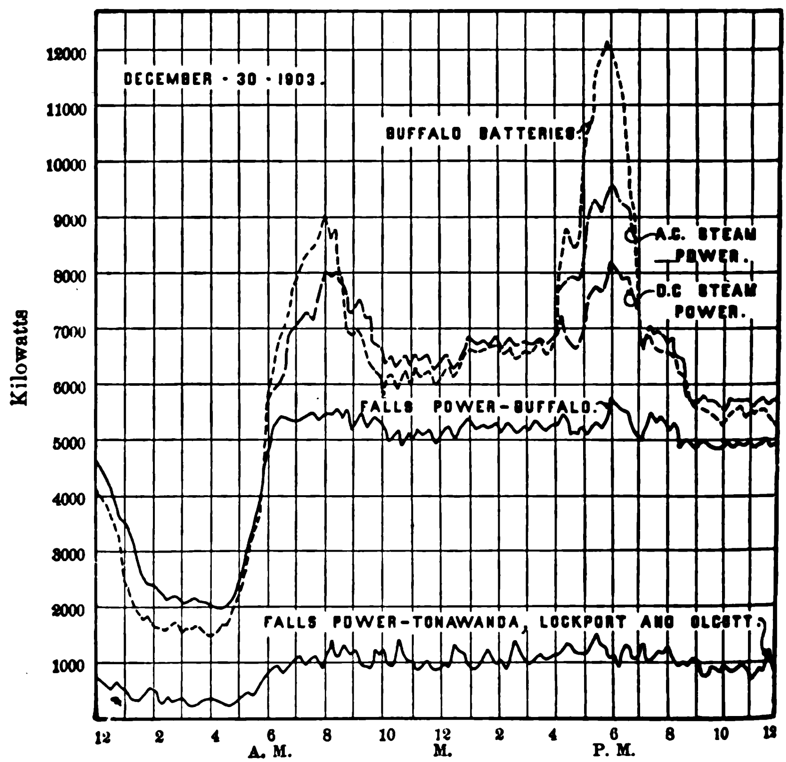


Fig. 266.—Typical Railway Load Curves, International Ry. Co. (From Electrical World and Engineer.)

day load. The London Hydraulic Supply Company furnishes water under a pressure of 750 pounds per square inch through a system of mains 86 miles long. In 1894, 2915 machines were connected to this system, of which 650 were passenger elevators, 2000 freight elevators and cranes, 90 presses of various kinds, 95 motors, and 80 fire hydrants. Each 1000 gallons of water pumped represents 8.738 H. P. hours, therefore, the maximum on the diagram represents about 1200 H. P. The preponderant influence of the elevator load is shown in the rapid rise from 6 to 10 A. M. and the somewhat slower decline from 4 to 12 P. M.

198. **Railway Load Curves.**—The power load most subject to violent fluctations is that utilized for railway purposes. The sudden changes in the demand for power occasioned by stopping and starting of cars, which may, under some conditions, occur simultaneously are often very rapid and the resulting load fluctuations very great.

Figs. 265 and 266 show two sets of curves taken from the power charts of the International Railway Company of Buffalo, which may be considered typical for electric railways. Each chart has two sets of curves, one for the city lines, on which the traffic is purely urban in character, and the other for the Tonawanda, Lockport and Olcott Line, which is an interurban line. In either set the total load at any time is represented by the ordinate to the highest curve in that set. The amount of load carried by any portion of the system is represented by the difference between the ordinates to the curve of that portion and to the curve next below. On the urban lines two peaks will be observed, one at 8 A. M. and one at 6 P. M., for both winter and summer, the afternoon peak of the former being nearly 75 per cent greater than the latter, however. The load curve of the interurban line appears to be nearly uniform throughout the year.

The data, on page 432, concerning these curves are taken from "The Electrical World and Engineer" of December 10, 1904.

199. **Load Conditions for Maximum Returns.**—It is manifest that no plant will receive its maximum returns without operating at full load all of the time; that if it operates at less than full load its income will be reduced unless more is charged for power so delivered; and that if the load carried for a large portion of the time is comparatively small and the returns for such power are not proportionately large the plant may be found to be an unprofitable

investment. On every plant the fixed charges, which include interest on first cost, depreciation charges and taxes, continue at a uniform rate every hour of the day and every day of the year. The operating expenses increase somewhat with the total amount of power furnished but not in proportion. An increase in the total

Data from Curves of Figure 265.

	Purchased Power.			Storage Batteries.			Grand Total.
	Tonawanda.			Tonawanda.			
	Buffalo.	Lock-port. Olcott	Total.	Buffalo.	Lock-port.	Total.	
Maximum H. P.	6,114	1,985	8,099	3,752	635	4,387	12,486
Minimum H. P.	1,667	319	1,985	79	40	119	2,104
Average H. P.	4,636	1,221	5,857	1,262	274	1,536	7,393
H. P., hours.	111,272	29,302	140,574	8,406	3,480	11,886	152,460
K. W., hours.	83,009	21,859	104,868	6,271	2,596	8,867	113,735

Maximum number of cars in service in Buffalo, 408.
Average volts at D. C. busbars, 592.
State of weather: 8 a. m., cloudy; 6 p. m., fair.
Temperature: 8 a. m., 66 degrees F.; 6 p. m., 74 degrees F.

Data from Curves of Figure 266.

	Purchased Power.			Steam Power.				Grand Total.
	Tonawanda.							
	Buffalo.	Lock-port. Olcott	Total.	Niagara St.	Virginia St.	Total.	Buffalo.	
Maximum H. P.	7,622	2,025	9,647	3,414	2,064	5,478	3,970	19,095
Minimum H. P.	2,303	199	2,502	953	715	1,668	79	4,249
Average H. P.	6,002	1,149	7,151	2,115	1,641	3,756	1,224	12,131
H. P., hours.	144,046	27,584	171,630	38,442	4,367	42,809	7,344	221,783
K. W., hours.	107,458	20,578	128,036	28,678	3,238	31,936	5,479	165,451

Average volts at D. C. busbars, 592.
State of weather: 8 a. m., cloudy; 6 p. m., cloudy.
Temperature: 8 a. m., 20 degrees F.; 6 p. m. 26 degrees F.

output of a given plant, therefore, means a direct increase in the net earnings of the plant and unless the power plant is constantly operating at its maximum capacity, its earning efficiency is not at the highest point.

It will be noted at once that if a machine can be operated at its full capacity for the entire time, that the work done will be done under the most economical conditions as far as each unit of output (Horse Power Hour or Kilo-Watt Hour) is concerned. The interest on the first cost and other fixed charges will be distributed among the maximum number of power units. The cost of wear, and the repairs, while they increase with the amount of power furnished, are not in direct proportion thereto, and decrease per unit as the average load carried reaches nearer the maximum of the machinery used. The same is true of the cost of attendance and most other operating expenses.

200. The Load Curve in Relation to Machine Selection.—A comparison between the average load carried and the maximum load will show the relation between the machinery which it is necessary to install and the active work which it has to do, and furnishes a basis for the study of the possible earnings of the plant.

The ratio of the average to the maximum load is called the "load factor." Some engineers use the term "load factor" as representing the ratio between the average load actually carried and the maximum capacity of the machinery operated. The writer however, prefers the term "machine factor" to represent this ratio. The same term is also sometimes applied to the ratio of the average load to the machinery in hourly operation, but to this the term "hourly machine factor" seems more applicable. The ratio of the average load to the total capacity of the station would seem best represented by the expression "capacity factor."

In order to have a plant work at the maximum advantage, it must be designed to fit the contingencies of the load. The operation of a machine at partial load is not only expensive on the basis of fixed charges, but is still more so on account of the decreased efficiency under such conditions.

With a varying load, efficient operation usually involves the installation of two or more generators of such capacity that a single unit will furnish the power required during the hours of minimum demand and at the same time operate at a fairly efficient rate. As the daily demand for power increases, additional units are started and operated, still under economical conditions, and at the peak of the load one or more additional units may be cut in and operated for the limited time during which the maximum demands prevail. Such an arrangement assures reasonable economy of operation at all times, even when great changes of load are of daily occurrence.

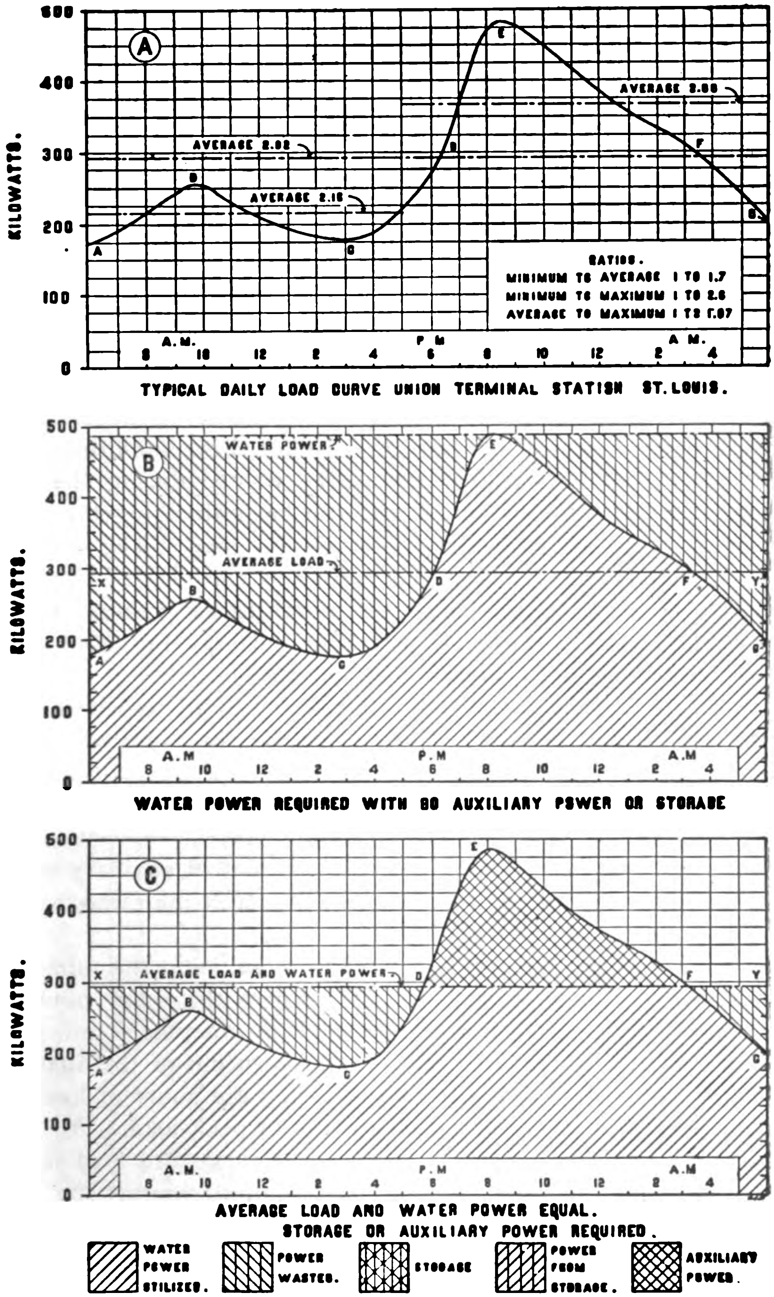
201. Influence of Management on Load Curve.—The relations of the "load curve," the "load factor," the "machine factor" and the "capacity factor" are, or may be, to an extent controlled by the business management of any plant, and by the selection and the character of the load to be carried, where such selection is possible. Each consumer of power will develop a particular curve due to the character of the work done, and it is frequently possible, by a judicious selection of customers, and especially by a proper grading of rates, to raise the load factor and thereby decrease the cost of operation and increase the net profits from the plant. A study of the probable plant factors is necessary for the judicious selection of machinery in order to attain the most efficient operation and, in a hydraulic plant, in order to properly design it and conserve the maximum energy of the stream that is being developed.

202. Relation of Load Curve to Stream Flow and Auxiliary Power.—Some of the relations between the load factor and the conditions under which a hydraulic plant may have to be operated are shown by Figs. 267, 268 and 269.

In Fig. 267, diagram A shows a typical daily load curve from the terminal station at St. Louis, a curve quite similar in general character to those previously shown.

Diagram B shows the power that must be developed by a stream in order to take care of the load represented by this load curve, under conditions where no auxiliary power or storage are available. In this case, it will be noted that the available water power must be equivalent to or greater than the maximum peak load, and that all power represented by the area above the load line, amounting in the case illustrated to about 40 per cent. of the total available power, will be wasted.

Diagram C illustrates a condition where the average load and water power are equal. In this case, pondage or storage, represented by the cross-hatched area below the average line, may be utilized to furnish the peak power represented by the cross-hatched area above the average line. Without pondage, the cross-hatched area below the average load line will represent the energy wasted, and the cross-hatched area above the average load line will represent the energy which must be supplied by auxiliary power. Without pondage the power of the stream must be utilized as it passes, and in the diagram B, of Fig. 267, the power represented above the load line under such conditions must be wasted.



RELATION OF POWER SUPPLY AND DEMAND

Fig. 267.

These same conditions are shown both by diagram C, Fig. 268, and diagram A, Fig. 269. In the latter, with water power above the average load of the plant, the peak load must be supplied by auxiliary power, although more water power than would be sufficient to handle it is daily wasted.

Diagram B, Fig. 268, shows a condition with low water power no storage available, and the power less than the average load. In this case the water power wasted is comparatively small, and the amount, and especially the capacity, of the auxiliary power becomes large.

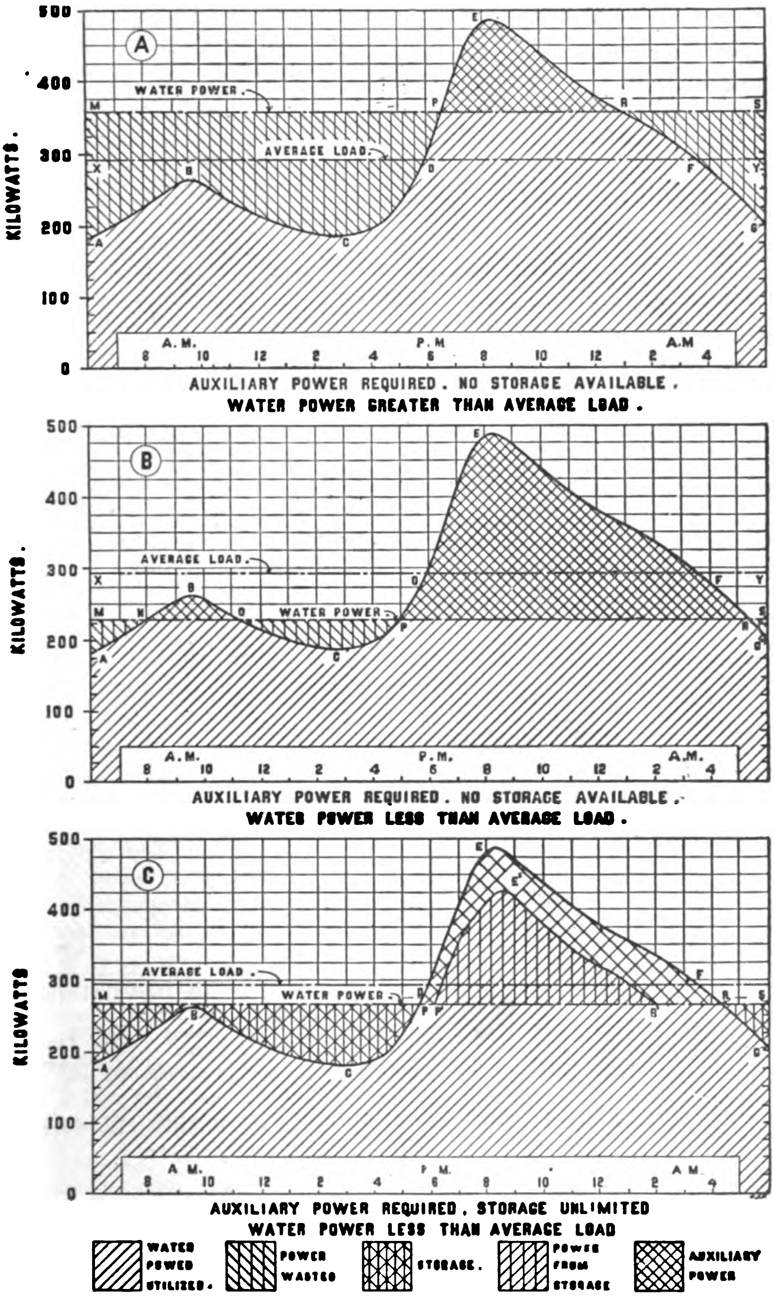
Diagram C, Fig. 268, represents a water power condition, where the power available is less than the average load, where storage is practically unlimited, and some auxiliary power is necessary in order to carry the peak of the load. Under these conditions, the water power, which would otherwise be wasted during the time of minimum load, is impounded, and can be utilized together with the auxiliary power at times of maximum load. The diagram shows a method of utilizing the minimum capacity of auxiliary power by utilizing the stored water power to its greatest advantage, and utilizing auxiliary power uniformly throughout the period where auxiliary power is demanded.

Diagram A, Fig. 269, represents the same conditions where storage is limited, and auxiliary power is necessarily required to help out the peak load conditions. In this case only a certain amount of the spare water can be stored, the balance being wasted at times where it cannot be continuously utilized.

The conditions for reducing the total amount of auxiliary power by utilizing the storage to advantage is shown in the same manner as in diagram C, Fig. 268.

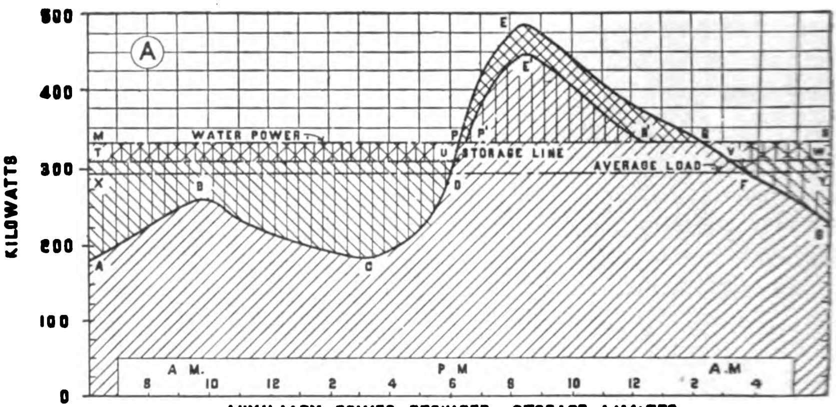
Diagram B, Fig. 269, shows a method of utilizing the minimum capacity of auxiliary power in a plant where the water power is below the average load and the pondage is practically unlimited. This is accomplished by the continuous operation of the auxiliary plant and the storage of water power during the hours of low consumption, for utilization during the hours of peak load.

A careful and detailed study of the load curve and load factor; the method of increasing the latter and of designing the most economical plant to take care of the condition to be met; and the adjustment of rates to attain equitable returns to the investor at reasonable price to the consumer, are matters of plant design worthy of the best efforts of the engineer.

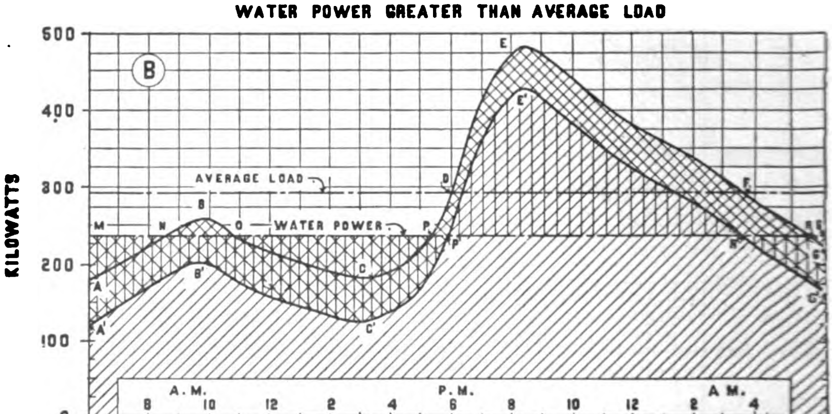


RELATION OF POWER SUPPLY AND DEMAND.

Fig. 268.



AUXILIARY POWER REQUIRED STORAGE LIMITED
WATER POWER GREATER THAN AVERAGE LOAD



AUXILIARY POWER [MINIMUM REQUIRED] IN CONTINUOUS SERVICE
STORAGE UNLIMITED

RELATION OF POWER SUPPLY AND DEMAND

Fig. 269.

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CHAPTER XVIII.

THE SPEED REGULATION OF TURBINE WATER WHEELS.

203. **The Relation of Resistance and Speed.**—The power delivered by any water wheel may be expressed, in terms of resistance overcome by the wheel through a known distance and in a known time by the formula (See equation 1, Section 177, Chap. XVI).

$$P = \frac{2\pi \, l \, w \, n}{33000} \qquad (1)$$

The second term of this equation may be divided into two factors: first,

$$\frac{2\pi \, l \, w}{33000}$$

which may be called the resistance factor and which is the resistance overcome or power produced by the wheel per revolution per

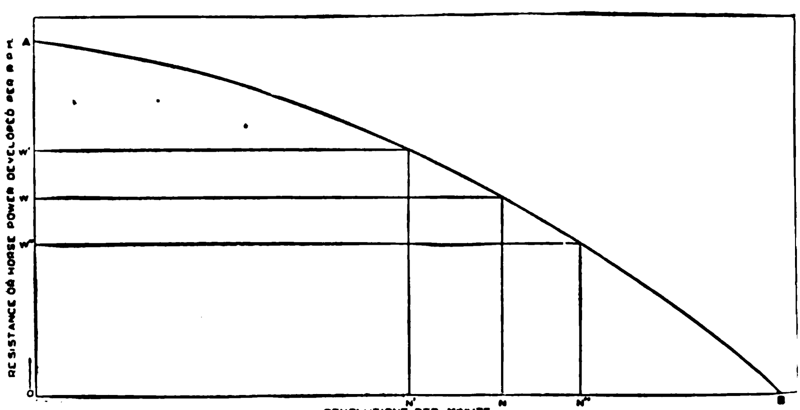

Fig. 270.

minute; and n, the number of revolutions per minute. The product is the horse power of the wheel.

In any wheel operating with a fixed gate opening and under a fixed head the speed, n, will always increase as the resistance, w, decreases, and will decrease as the resistance increases.

In Fig. 270 the line AB shows the relation of speed to resistance in a turbine operated with a single fixed gate opening and for the full range of load conditions (as determined by experiment) from A, at which the resistance, w, was so great as to hold the motor stationary, to B where the resistance was completely removed and the entire energy of the applied water was expended in overcoming the friction of the wheel, or rejected as velocity en-

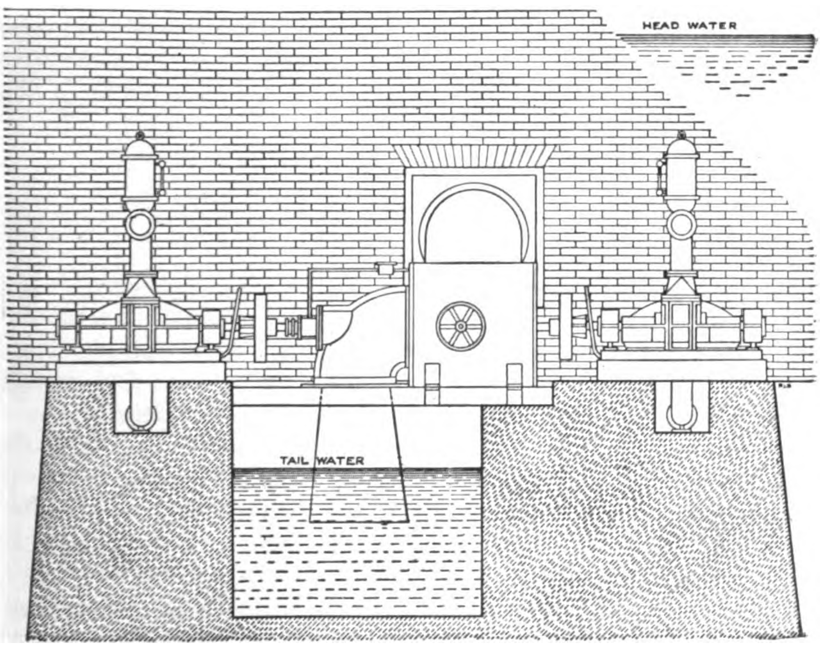

Fig. 271.

ergy in the water discharged therefrom. From this figure it is evident that if, at any fixed gate opening, a wheel is revolving at a given speed, n, and the resistance, w, is decreased to w″ the speed will increase to n″, while if the resistance increases to w′ the speed will decrease to n′.

204. Self-Regulation in a Plant with Variable Speed and Resistance.—At Connorsville, Indiana, is a pumping plant (Fig. 271) in which a horizontal shaft turbine is directly connected through friction clutches to two rotary pumps. For operation the turbine gates are opened until the pump, or pumps, speeding up to a suitable r. p. m., produces the desired pressure in the distributing system. The work of the pump under these conditions in pumping water at the speed of operation against the desired pressure equals the work done by the quantity of water q passing through the turbine, less friction and other losses. If the pressure falls, the loads become unbalanced: i. e., the resistance is reduced and the turbine and pump increase in speed until the balance is restored. If the pressure rises the machine slows down until there is again a restoration of balance between the power of the turbine, the pump load and friction losses.

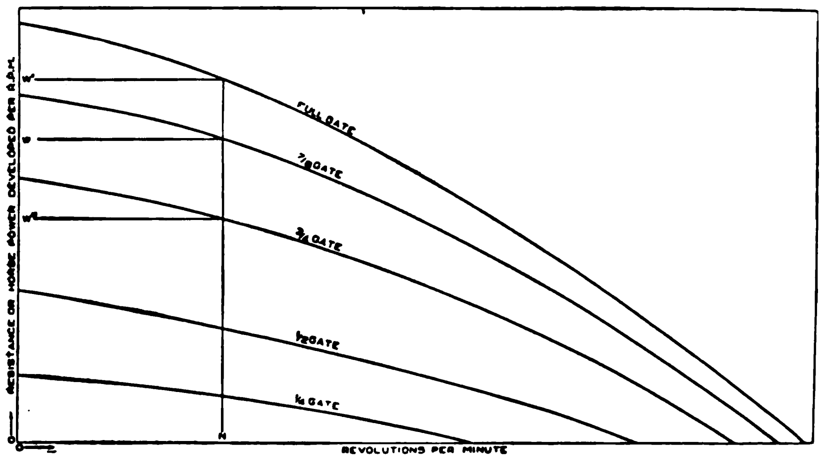

Fig. 272.

To pump water against an increased pressure, it is necessary to increase the gate opening of the turbine. In its regular daily work the varying demand for water is thus supplied by the self-regulation of the two machines used and no governor is needed. The conditions of operation are similar to those illustrated in Fig. 270.

205. The Relations Necessary for Constant Speed.—Fig. 272 is a diagram drawn from experimental or test observations and similar to Fig. 270 except that the relations between speed and desistance are shown for various gate openings.

It is evident that if the wheel must operate at a fixed speed, n, and the resistance, w, increases to w′ or decreases to w″, it will be necessary to increase the gate opening from 7/8 gate to full gate in the first case and to decrease it to 3/4 gate in the second case in order to maintain the speed uniform.

An examination of the load curves described in Chapter XVII shows that changes in load are constantly in progress. For the satisfactory operation of water wheels, under these constant and irregular changes in load, automatic regulation of the turbine gates becomes necessary. This is accomplished through the water wheel governor which regulates the gates through the various classes of gate mechanisms described in Chap. XIII.

206. The Ideal Governor.—The power output of a water turbine in terms of energy applied to the wheel is expressed by the formula.

(2) $$P = \frac{qH'E}{8.8} \quad \text{where}$$

q = cu. ft. per second of water used by the wheel.
H' = net available head.
E = efficiency of the wheel.
P = horse power developed.

Any sudden increase or decrease of load, w, will produce a corresponding decrease or increase, respectively, in the speed, n, of the machine as shown by Fig. 270 unless the energy applied to the turbine is immediately changed to correspond. The ideal turbine governor would effect a change in output by varying only q, thus obtaining perfect water economy by conserving unneeded water for future use. This is not possible in practice as head, water, and therefore efficiency are usually wasted when operating a wheel under other than its normal load and during the change in load.

207. Present Status.—The success of the comparatively recent application of hydraulic power to the operation of alternators in parallel and to the generation of current for electric lighting street railway and synchronous motor loads has been largely dependent upon the possibility of obtaining close speed regulation of the generating units accompanied with good water economy and without undue shock upon machinery and penstocks while working under extremely variable loads.

The degree of success thus far obtained in the development (necessitated by the above conditions) of automatic turbine governors, although achieved from the experimental standpoint almost exclusively, has been remarkable. Instances are now by no means uncommon where hydro-electric units working upon variable loads are controlled as satisfactorily as modern steam driven units. To accomplish this result the conditions must be especially favorable.

Success in this feature of hydro-electric design is by no means uniform, however, and the frequent failure to realize satisfactory results can often be ascribed to the lack of proper consideration of the arrangement of the mechanical, hydraulic, and electrical elements of the plant, wheels, and generators, rather than to any inherent defects in the governor itself. The power plant, the turbines, the generators, and the governors are commonly designed by four different parties without proper correlation of study and design. At present neither experimental data nor theoretical formula are available by which the hydro-electric engineer can design his plant for an assumed speed regulation, or can predetermine the speed regulation which is possible with a given installation or the

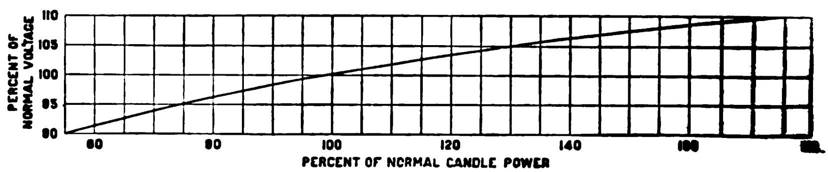

Fig. 273.

time required for the return to normal speed,—and yet the governor builder is commonly required by the engineer to guarantee these operating results. The predetermination of speed variations during portions of the steam cycle and at load changes has received careful study in the design of reciprocating steam engines and the desirable per cent of speed regulation is freely guaranteed and readily obtained through careful study and analysis by the designer. The same amount of study is warranted but seldom or never given to the problem of speed regulation in water power work.

208. Value of Uniform Speed.—Uniform, or nearly uniform, speed is of great economic value in the operation of a plant but adds to the first cost and may also result in a waste of water. The correct solution of any given problem of speed regulation involves a compromise between first cost, water economy and speed regulation.

A pecuniary value cannot well be placed upon good speed regulation. It differs from poor speed regulation chiefly in procuring a more satisfactory operation of motor driven machinery and in producing a more constant incandescent light. Fluctuations in the brightness of a light are annoying, and tend to create dissatisfaction among consumers. Fig. 273 shows the general way in which the candle power of an incandescent light varies with the impressed voltage.* A pressure variation of 5 per cent., and hence also a speed variation of a similar amount, is shown to produce a much larger variation in candle power of the light,—in this case about 25 to 30 per cent.

209. The Problem.—Where (as in Fig. 271) a turbine is operating under balanced conditions and the resistance changes in magnitude, the turbine does not at once assume the new speed relations corresponding to the change in resistance. The inertia of the moving parts of the wheel and of the column of water in the penstock,

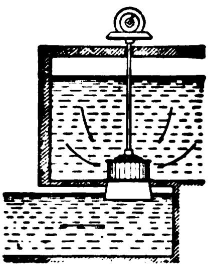

Fig. 274.

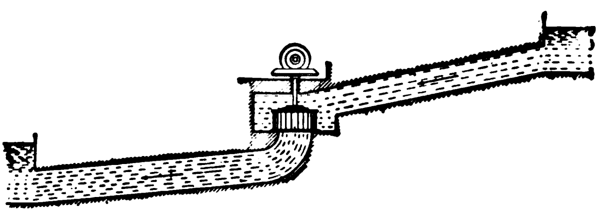

Fig. 275.

turbine and draft tube, tends to maintain uniformity of speed, and the wheel gradually changes in speed to that corresponding to the new conditions. In such cases the speed of operation is not essential and the delay in reaching the speed corresponding to the resistance or work the turbine must perform is usually unimportant.

When, as in Fig. 272, the wheel is designated to operate at a fixed speed, the uniformity of speed becomes a matter of greater or less importance depending on the character of the work the wheel is to perform. In this case the inertia of the wheel and of all rotating parts of other machinery connected thereto tends to maintain a constant speed. On the other hand, the flow of water in penstock, turbine, and draft tube must be changed in quantity, (Eq. 2), hence in velocity, and its inertia therefore tends to produce a change in head and to produce effects opposite to those desired for efficient regulation.

The conditions of installation have a marked effect on the difficulties of turbine governing. If (as in Fig. 274) the turbine is installed in an open pit and has only a short draft tube, and the water

*See American Electrician, Vol. XIII, No. 7. July, 1901, by F. W. Wilcox.

flows to the gates from every direction, the velocity of flow from all directions is very low. The quantity of water which moves at a high velocity is confined to that in the wheel and draft tube and the change in the velocity and momentum, due to a change in the gates, produces no serious effects. If, however, water be conducted to and from the wheel through a long penstock and draft tube (as illustrated by Fig. 275) the conditions become quite different. In this case a large amount of energy is stored in the moving column of water and a change in its velocity involves a change in its kinetic energy which may, if an attempt is made at too rapid regulation, leave the wheel deficient in energy when increased power is desired, or, when the power is decreased, may produce such shocks as will seriously affect regulation or perhaps result in serious injury to the penstock and wheel.

210. Energy Required to Change the Penstock Velocity.—An increase or decrease of load requires an ultimate increase or decrease in velocity of the water in the penstock. Work has to be done upon the water to accelerate it and must be absorbed in order to retard it. The total avaliable power which can be expended for all purposes at any instant during the acceleration is (since vH is proportional to qH) proportional to the product of the instantaneous velocity and the supply head. This total power is thus definitely limited and, hence, *the work required to accelerate the water must be obtained at the expense of the work done upon the wheel.*

Thus, when an increase of load occurs the gate is opened by the governor, and the immediate result is a decrease in the power output of the wheel, even below its original value, and is diametrically opposed to the result desired. This counter effect may last for several seconds, and, unless sufficient reserve energy in some form is available to partially supply this deficiency, the speed of the wheel may fall considerably before readjustment to normal power can take place.

In the same way an excess of energy must be absorbed to decrease the velocity at time of decreasing load. This may be expended upon the wheel thus increasing the speed above normal, or it may be dissipated in one of several ways to be discussed later.

The water in the draft tube must be accelerated and retarded at each change of gate opening and its kinetic energy changed at the expense of the power output in exactly the same manner as that in the penstock. For this reason it should be included in all calculations as a part of the penstock. One additional precaution must be taken: if the draft head is large a quick closure of the turbine gate may cause the water in the draft tube to run away from the wheel (actually creating a vacuum in the draft tube) and then return again causing a destructive blow against the wheel.

211. Hunting or Racing.—The regulation of both steam engines and hydraulic turbines as now accomplished is one of degree only since a departure from normal speed is necessary before the governor can act. Since the immediate effect of the gate motion is opposite to that intended, the speed will depart still further from the normal. This tends to cause the governor to move the gate too far with the result that the speed will not only return to normal as soon as the inertia of the water and of the rotating parts is overcome, but may rush far beyond normal in the opposite direction. The obvious tendency is thus to cause the speed to oscillate above and below normal to the almost complete destruction of speed regulation.

A successful governor must therefore "anticipate" the effect of any gate movement. It must move the gate to, or only slightly beyond, the position which will give normal speed when readjustment to uniform flow in the penstock has taken place. A governor with this property or quality is commonly said to be "dead-beat." In Chap. XIX several expedients are shown for the automatic elimination of excessive racing.

212. Nomenclature.—The following symbols will be used in the mathematical discussions which follow:

A = cross-sectional area of penstock in sq. ft.

$B = \dfrac{V + v_e}{V - v_e}$

C = friction coefficient for flow in pipe lines $= \dfrac{1}{2g}\left(1 + f\dfrac{1}{d} + \text{etc.}\right)$

D_a = maximum rise of water in standpipe above the forebay when full load ($v = v_f$) is rejected by the wheels.

D' = drop of water in standpipe below original friction gradient all influences considered.

D = ditto, friction in penstock neglected.

D_b = drop of level in standpipe below forebay.

d = diameter of penstock (closed circular) in feet.

e = 2.71828 = base of natural system of logarithms.

F = cross-sectional area of the standpipe in square feet.

f = "friction factor" in penstock.

g = acceleration due to gravity in feet per second per second.

H = total available power head in feet.

H' = effective head at the wheel $= H - h_F$ for any given uniform velocity, V, in the penstock.

h = instantaneous effective head at the wheel during changes of velocity in the penstock.

h_a = head which is effective at any instant in accelerating the water in the penstock and draft tube.

h_F = friction loss in penstock for normal flow with a given head and gate opening.

h_f = variable head lost by friction entrance, etc., in penstock when the velocity is v.

I = moment of inertia or fly wheel effect of revolving parts in pounds at one ft. radius = ft.^{2} lbs.

K = energy delivered to the wheel.

ΔK = excess or deficient energy delivered to wheel during change of load.

ΔK_1 = excess or deficient energy delivered to wheel due to excess or deficiency in quantity of water during load change.

ΔK_2 = ditto,—due to energy required to accelerate or retard the water in the penstock.

ΔK_3 = ditto,—due to sluggishness of gate movement.

K' = kinetic energy in foot pounds of revolving parts at speed S.

$\Delta K'$ = increment (+ or —) in K' due to load change.

$$k = \frac{2gH}{lV}$$

$$k' = \frac{2gH}{2.31V}$$

l = length of penstock in feet.

M = slope of the v-t curve when $v = \frac{v_0 + v_1}{2}$ (equation 19).

p_0 = initial horse power output from the water wheel.

p_1 = the horse power output from the water wheel corresponding to the new load.

Q = discharge of the wheel under normal effective head H' for any given load.

q = instantaneous discharge of wheel in cubic feet per second during load change.

R = ratio of actual deficient or excess work done on wheel to that computed.

S = normal r. p. m. of the wheel and other rotating parts.

$\Delta S = S - S_1$ = temporary change in speed.

S_1 = speed in revolutions per minute after load change.

T' = approximate time required for acceleration or retarding of water from velocity v_0 to v_1.

T'' = the time required for the governor to adjust the gate after a change of load.

t = variable time after gate movement.

V = normal (and hence maximum possible) velocity in the penstock with given head and gate opening.

v = instantaneous variable velocity in the penstock while adjusting to a new value.

v_0 = velocity in penstock at the instant of gate change.

v_1 = velocity in the penstock required for new load.

w = weight of a cubic unit of water in lbs.
Y = maximum departure of head, h, from normal with use of standpipe,—discharge of wheel assumed constant at the abnormal head (see D_a and D_b).
y = variation of water level in the standpipe from forebay level = H — h.
δ = speed regulation or per cent variation of speed from normal.

213. Shock or Water Hammer Due to Sudden Changes in Velocity.—The acceleration or retardation of a moving body requires an unbalanced force. Since acceleration and retardation are identical, except as to sign, the required accelerating force may in all cases be expressed as follows:

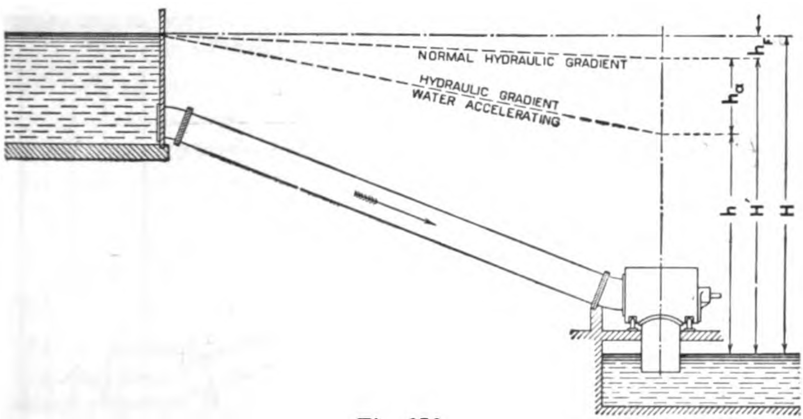

Fig. 276.

$$\text{Force} = \text{mass} \times \text{acceleration.}$$

Acceleration, or the rate at which the velocity increment increases per increment of time, is expressed by the formula:

$$\text{Acceleration} = \frac{dv}{dt} \tag{3}$$

The mass of water to be accelerated is

$$\text{Mass} = \frac{A\,l\,w}{g} \tag{4}$$

Figs. 276 and 277 show the conditions existing during an increase and decrease of velocity respectively. If the draft tube were closed at the lower end and no water leaving, there would be a total force, equal to the hydraulic pressure over the area of the penstock, or wAH, tending to move the water.

If the water is flowing with a velocity v the turbine offers a resistance to flow represented by the effective head, h, at the wheel, and the penstock offers a resisting head h_f composed of friction, entrance, and other losses. If the velocity remains uniform, h=H', and the forces are balanced thus:

$$H = H' + h_f \tag{5}$$

If the opening of the turbine gate is now suddenly increased, the head H' at the wheel, will fall to the value, h, (shown in Fig. 276) which is required to force the given amount of water, Av, through

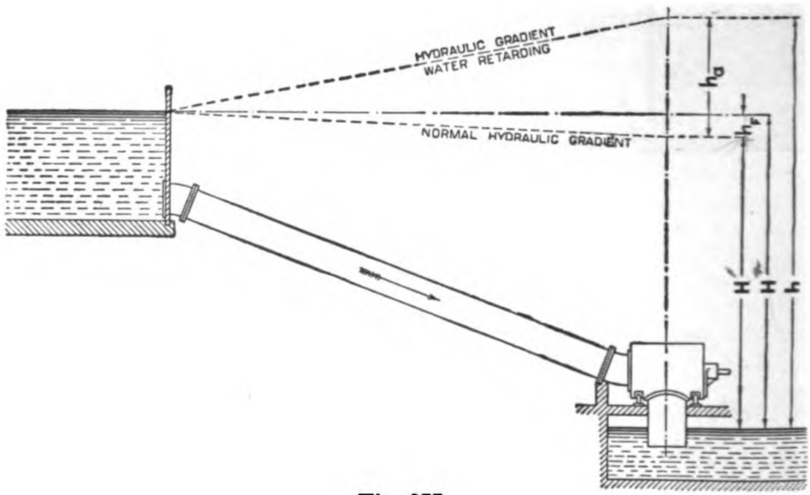

Fig. 277.

the wheel. On the other hand, if the gate opening is decreased the pressure head must rise above H' (as shown in Fig. 277) in order to discharge the water through the wheel. This change h_a in the head H' disturbs the equilibrium of forces shown by equation (5) making

$$h_a = H - h - h_f \tag{6}$$

Only the head h_a is effective in accelerating or retarding the water and the force resulting from this head is wAh_a. Substituting this value and those of equations (3) and (4) in equation (2) we obtain:

$$wAh_a = \frac{Alw}{g} \cdot \frac{dv}{dt} \quad \text{or}$$

$$h_a = \frac{l}{g} \cdot \frac{dv}{dt} = \frac{l}{g} \times (\text{rate of velocity change}) \tag{7}$$

The value of h_a given by formula (7) is a general expression for the change in pressure-head due to a change of velocity or for the head which must be impressed to produce a desired change in velocity. When in excess of the static pressure as shown in Fig. 277, it is commonly called "water hammer." (See Appendix —.)

If the closure of the gates is rapid the value of h_a is large and the column of water is set into vibration or oscillation. If the partial closure of gate is sufficiently slow to allow a distribution of each increment of pressure along the pipe, this oscillatory wave is avoided and the pressure produced at any instant during closure (given by equation (7) is that which is necessary to retard the moving column of water at the rate at which its velocity actually decreases at that instant and can be reduced below any assumed maximum allowable value by a sufficiently slow gate movement.

When a penstock is long, these oscillatory waves become a source of great danger to the turbines and also to the penstock, especially at bends. The extinction of a velocity of 4 feet per second at a uniform rate in one second in a pipe 1,600 feet in length would create a pressure-head of about 200 feet, or a total longitudinal thrust on the pipe line at each bend, and upon the wheel gate, if 24" in diameter, of about 20 tons.

These dangers are further augmented by the fact that several waves, if succeeding each other by an interval which is approximately a multiple of the vibration period of the pipe, may pile up, so to speak, crest upon crest and cause a pressure which no possible strength of parts could withstand.

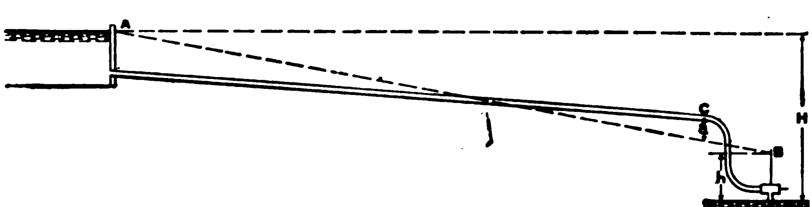

Fig. 278.

214. Permissible Rate of Gate Movement.—Gate movements must be sufficiently slow to avoid oscillatory waves of dangerous amplitude. No general quantitative rule can be given for the required rate of movement. It can be more rapid the shorter the penstock and the smaller the velocity in the same. The danger is much smaller during opening than during closure of a gate and the rate of gate movement could well be made much more rapid in the former than in the latter case.

The rapidity with which a gate should be opened is limited for feeder pipes with an initial flat slope as shown in Fig. 278.

Let h′ be the lowest head obtained in opening the gate at an assumed rate and AB, the resulting hydraulic gradient. In case the gate opens so rapidly as to cause the distance, a, at any point along the pipe to exceed suction limit, the water column in the penstock will separate (the portion of the column above A not being able to accelerate as rapidly as that below) and will again reunite with a severe hammer blow. Failure to observe this precaution probably caused the destruction of the feeder pipe of the Fresno, California, power plant. The rate to be used can be chosen after a determination, by the method discussed in Appendix —, of the pressures resulting from several assumed rates of movement. The method is tedious but justifiable in many cases.

215. Regulation of Impulse Wheels.—It is impracticable, if not impossible, to build a pipe line strong enough and well enough anchored at all points to withstand the enormous pressures and longitudinal thrusts which would result from rapid gate closures in a long closed penstock such as commonly used for impulse wheels. The adjustment of quantity, q, for changes in load of short duration is hence impossible in such closed penstocks and the expedient usually adopted is to "deflect" the jet from the wheel by changing the direction of discharge of a pivoted nozzle. This requires that the "needle valve" (See Fig. 195) or gate maintain a jet sufficient to carry peak loads; hence causing a waste of water at all other times. This condition is commonly improved somewhat by adjusting the valve about once each hour by means of a slow motion hand wheel for the maximum peak load liable to occur during that hour.

An automatic governor has recently been invented which moves the needle valve or gate slowly, thus adjusting for changes of load of long duration while it still retains the deflector to provide for abrupt changes in the load curve. (See Fig. ^ .)

Another device proposed for use in this connection is a by-pass nozzle arranged to open as the needle valve rapidly closes, and then automatically close again at a rate sufficiently slow to reduce the excess pressure to safe limits. One advantage in favor of this arrangement is that the jet would then always strike the center of the buckets which is found to considerably reduce their wear.

An automatic relief valve of hydraulic or spring type is nearly always used but serves more as an emergency valve to reduce water hammer pressures than as a by-pass to divert water from the wheel for the purpose of governing? For this latter use the spring type of valve has proven unsatisfactory.

In some cases the water discharged from high head plants is used below for irrigation and must be kept constant, thus doing away with the necessity of varying the velocity in the feeder pipe for a varying load.

Mr. Raymond D. Johnson proposes for these high head plants, the use of large air chambers or "Surge Tanks," placed near the wheels, of a sufficient size so that the governor can control the needle valve directly, thus dispensing with the deflector and by-pass and doing away completely with the waste of water occasioned by their use. He has derived formulas by which he claims to accurately proportion these tanks for an assumed maximum allowable range of head fluctuation or surge.*

216. Influences Opposing Speed Regulation.—Abrupt changes in the demand for power of a considerable proportion of the total capacity of a plant, take place at times in modern power plants. Three causes tend to make the change in output of a wheel lag behind the change in demand placed upon it; viz.: (1) the fact that the governor, however sensitive, does not act until an appreciable change of speed occurs, and then not instantly; (2) the fact that some time is required for the readjustment of penstock velocity, even after the gate movement is complete; (3) the necessity of changing the velocity, and hence of overcoming the inertia of the water in the penstock and draft tube at each change of load.

Each of these influences is directly opposed to speed regulation, as will appear in the succeeding articles, since each causes the power supplied to a wheel, at time of increasing load, to fall short of the demand, the deficiency being supplied at the expense of the speed from the kinetic energy stored in the rotating parts. The expression for the total deficient work, i. e. foot,pounds, is:

$$\Delta K = \Delta K_1 + \Delta K_2 + \Delta K_3 \qquad (8)$$

for which see equations 22 and 23 and Section 221.

217. Change of Penstock Velocity.—Assuming the gate movement to take place instantly, we will have the condition illustrated

* See "The Surge Tank in Water Power Plants," by R. D. Johnson. Trans. Am. Soc. M. E., 1908.

in Figs. 276 or 277, for which equation 7 was derived (See Section 213). Solving equation (7) for $\frac{dv}{dt}$ we have:

$$(9) \qquad \text{Acceleration} = \frac{dv}{dt} = \frac{g}{l} \times (\text{accelerating head}) = \frac{g}{l} h_a$$

The accelerating head as shown in equation 6 is $H - h - h_f$. It is the general principles of hydraulics that the head lost in flow through any opening, pipe, orifice, etc., varies as the square of velocity.

It was shown in Section , Chapter XVI, that the quantity flowing through a turbine varies as the square root of the head. Remembering that the quantity is proportional to the penstock velocity, we have:

$$(10) \qquad \frac{q}{Q} = \frac{v}{V} = \frac{\sqrt{h}}{\sqrt{H'}}, \quad \text{from which}$$

$$(11) \qquad h = \frac{v^2}{V^2} H' \quad \text{Now}$$

$$(12) \qquad h_f = (1 + f\frac{l}{d} + \text{etc.}) \frac{v^2}{2g} \; * \quad \text{Hence,}$$

$$(13) \qquad \frac{h_f}{h_F} = \frac{v^2}{V^2} \quad \text{Or}$$

$$(14) \qquad h_f = \frac{v^2}{V^2} h_F$$

From equation (6)

$$h_a = H - h - h_f = H - H' \frac{v^2}{V^2} - h_F \frac{v^2}{V^2} \quad \text{or}$$

$$(15) \qquad h_a = H - (H' + h_F) \frac{v^2}{V^2}$$

And from equation (5)

$$(16) \qquad h_a = H - H \frac{v^2}{V^2} = H (1 - \frac{v^2}{V^2})$$

Hence from equation (9)

$$(17) \qquad \frac{dv}{dt} = \frac{gH}{l} (1 - \frac{v^2}{V^2})$$

The integration of this equation as given in Appendix — gives the following equation for the curve of velocity change in the penstock following a sudden change of gate opening:

$$(18) \qquad v = V \frac{B \text{ anti log } k't - 1}{B \text{ anti log } k't + 1}$$

As shown in Appendix B this value of v approaches but never equals the value of V. The form of the curve for an increasing velocity is shown in Fig. 279.

* See Merriman's Treatise on Hydraulics, p. ——, equation. ——.

218. Effect of Slow Acceleration on Water Supplied to Wheel.— Since velocity in the penstock, discharge of wheel, and load are approximately proportional to each other, the ordinates of Fig. 279 may be taken to represent loads. The load demand remains at a constant value v_0 from A to B, where it suddenly increases to v_1, following the line A B C D T. The supply, however, assuming an instantaneous gate movement, follows the line A B D F. Now, the total quantity of water supplied to, and hence

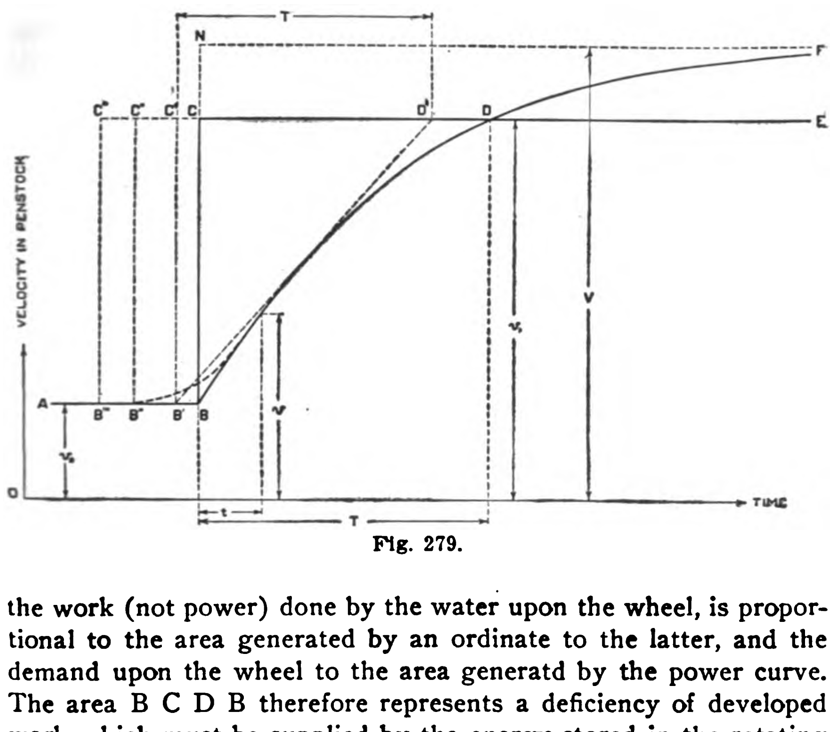

Fig. 279.

the work (not power) done by the water upon the wheel, is proportional to the area generated by an ordinate to the latter, and the demand upon the wheel to the area generatd by the power curve. The area B C D B therefore represents a deficiency of developed work which must be supplied by the energy stored in the rotating parts.

For practical purposes this area may be assumed equal to the area L of the triangle B′ C′ D′, where the line B′ D′ is tangent to the curve B M D at the point of mean velocity $\frac{v_0 + v_1}{2}$

The slope of the line B′ D′ for this mean velocity is readily obtained from equation 17. Call it M, then

$$(19) \qquad M = \frac{B'\,C'}{C'\,D'} = \frac{v_1 - v_0}{T'} = \frac{gH}{l}\left[1 - \frac{(v_0 + v_1)^2}{4V^2}\right] \text{ and}$$

$$(20) \qquad T' = \frac{v_1 - v_0}{M}$$

$$(21) \qquad \text{Area } B'C'D' = L = \frac{(v_1 - v_0)T'}{2} = \frac{(v_1 - v_0)^2}{2M}$$

This value of L is expressed in feet and represents the deficiency of lineal distance moved by the water column in the penstock. The deficiency of supplied water in cu. feet is, hence, A L and the deficiency of undeveloped work is

$$(22) \qquad \Delta K_1 = ALwH = \frac{AwH}{2M}(v_1 - v_0)^2$$

219. Value of Racing or Gate Over-Run.—At D, Fig. 279, the supply line B D F crosses the load line C D E, and the speed which was lost from B to D begins to pick up again.

The necessity also for an overrun of the governor is shown by Fig. 279. If the demand line were A B N F and the gate opened to the same place as before, giving the supply line B D F, the supply of power would approach, but theoretically never equal, the demand and the speed would hence never pick up to normal. The

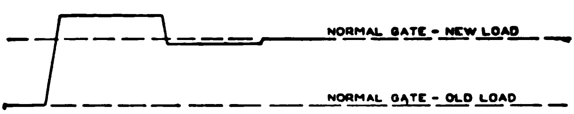

Fig. 280.

gate movement should therefore be similar to that shown in Fig. 280 in order to give the gate the small overrun which is necessary to bring the speed back to normal.

220. Energy Required to Change the Penstock Velocity.—The energy involved in the change of velocity above described results in an excess or deficiency of energy delivered to the wheel (See Section 210). The amount of this excess or deficient energy is readily determinable. The kinetic energy in foot pounds stored in the moving column of water is $K_2 = \frac{Wv^2}{2g}$ or

$$K_2 = \frac{62.5Alv^2}{64.3} = .972\ Alv^2$$

The amount which must be diverted from the wheel or dissipated when the velocity changes is therefore

$$(23) \qquad \Delta K_2 = 0.972\ Al\,(v_1^2 - v_0^2)$$

In this case l should be taken as the combined length of penstock and draft tube.

This deficient energy must be supplied, or the excess absorbed, by means of a flywheel or the installation of a stand-pipe connected with the penstock closely adjoining the wheel.

221. Effect of Sensitiveness and Rapidity of Governor.—Referring again to Fig. 279, suppose the increase of load to take place at B‴ giving the load line AB‴ C‴ E. After an interval from B‴ to B″, the speed has dropped an amount depending upon the sensitiveness of the governor. The gate will then begin to open; the velocity in the penstock accelerating meanwhile along the dotted line B‴Y. The lack of sensitiveness of the governor has therefore added a deficient work area of B‴ B″ C″ C‴, and the sluggishness of its motion an additional area C″B″ B C, approximately. This deficiency ΔK_3 can be only roughly approximated without the detailed analysis given in Appendix —.

222. The Fly-Wheel.—A fly-wheel is valuable for the storage of energy. Work must be done upon it to increase its speed of rotation, and it will again give out this energy in being retarded. From the laws of mechanics the number of foot pounds of kinetic energy stored in a body by virtue of its rotation is given by the formula:

$$K' = \frac{2I\pi^2S^2}{g\,60^2} = \frac{2 \times 3.1416^2}{32.15 \times 60^2}\, I\, S^2 \quad \text{or}$$

$$K' = .00017\, I\, S^2 \tag{24}$$

The amount of energy which must be given to or absorbed from the fly-wheel in order to change the speed is

$$\Delta K' = 00017\, I\, (S_0^2 - S_1^2) \tag{25}$$

Thus a fly-wheel can store energy only by means of a change in speed. By means of a sufficiently large moment of inertia the speed change of a fly-wheel, for any given energy storage, $\Delta K'$, can be reduced to any desirable limit.

The n ed of a fly-wheel effect to carry the load of a hydro-electric unit during changes of gate, and while the water is accelerating in the penstock at an increase of load has led to the development of a type of revolving field generator, whose rotor has a high moment of inertia and is therefore especially adapted for speed regulation usually making the use of a fly-wheel unnecessary.

Warren* has simplified the expression for $\Delta K'$ (See equation 25), substantially as follows:

* See "Speed Regulation of High Head Water Wheels," by H. E. Warren, in Technology Quarterly, Vol. XX. No. 2.

From equation (24):

$$(26)\qquad \frac{K_1'}{K_2'} = \frac{.00017\,I\,S_1^2}{.00017\,I\,S_2^2} = \frac{S_1^2}{S_2^2} \quad \text{Hence,}$$

$$(27)\qquad \frac{K_1' - K_2'}{K_2'} = \frac{S_1^2 - S_2^2}{S_2^2} = \frac{(S_1 + S_2)(S_1 - S_2)}{S_2^2}$$

Put $S_1 - S_2 = \Delta S$

and $K_1' - K_2' = \Delta K'$

For small differences between S_1 and S_2 equation (27) becomes approximately:

$$(28)\qquad \frac{\Delta K'}{K'} = \frac{2S \times \Delta S}{S_2} = \frac{2 \times \Delta S}{S} \quad \text{or}$$

$$(29)\qquad \Delta K' = \frac{2K' \times \Delta S}{S}$$

Or the percentage change in speed is

$$(30)\qquad \delta = \frac{100 \times \Delta S}{S} = \frac{50 \times \Delta K'}{K'}$$

223. The Stand-Pipe.—The function of the stand-pipe is two-fold: (1) to act as a relief valve in case of excess pressures in the penstock; (2) to furnish a supply of energy to take care of sudden increases of load while the water is accelerating, and to dissipate the excess kinetic energy in the moving water column at time of sudden drop in load. For these purposes it should be of ample diameter and placed as close to the wheel as possible.

The analytical determination of the effect of a given stand-pipe upon speed regulation is very difficult if not quite impossible. Furthermore, it is not necessary, since the drop in effective head at an increase of load may (except in the case of maximum possible load) be compensated for by an increase of gate opening, hence maintaining a constant power and speed or at least a satisfactory degree of speed regulation. Thus the action of a stand-pipe in storing energy differs radically from that of the fly-wheel as the latter can store or give out energy only by means of a change of speed in the generating unit.

The determination of the range of fluctuation of water level in an assumed stand-pipe, and the time required for return to normal level for various changes of load on the wheel, will assist greatly in the design of the stand-pipe.

Fig. 281 shows the condition when a stand-pipe is used. Assume that the wheel is operating under part load. The water normally stands a height h_F below the supply level. If the load suddenly increases, the gates open, and the water level begins to fall, thus causing an accelerating head $h_a = H - h - h_f$. Equation 9 then applies as before, where h_a becomes $(h - cv^2)$.

If the governor keeps step with the change in head by increasing the gate opening to maintain a constant power then

$$q\,h = q_1\,h_1$$

$$q\,(H - y) = Av_1\,(H - h_F) = Av_1\,(H - cv_1^2) \qquad \text{or}$$

$$(31) \qquad q = \frac{Av_1\,(H - cv_1^2)}{H - y}$$

The rate of water consumption by the wheel at any instant is q; the rate at which the water is supplied by the penstock is Av; and the rate of rise or fall of the water surface in stand-pipe is therefore:

$$(32) \qquad \frac{dy}{dt} = \frac{dh}{dt} = \frac{Av - q}{F} = \frac{A}{F}\left[v - \frac{v_1\,(H - cv_1^2)}{H - y}\right]$$

The solutions of equations 9 and 32, which are necessary for determining the curves of variation of head and velocity, is imprac-

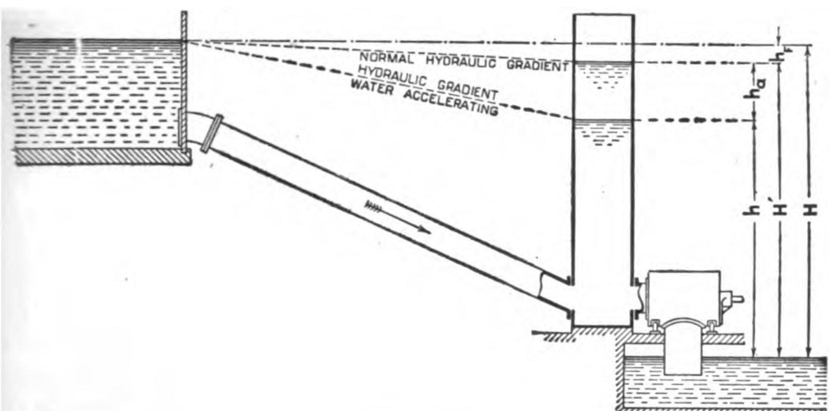

Fig. 281.

ticable, if not impossible, hence a different treatment is proposed and considered in Appendix.

If q be assumed constant ($=Av_1$) during the adustment of penstock velocity and the friction loss, cv^2, in the penstock be neglected, then equations 9 and 32 simplify and become integrable. The resulting equations, showing the variations of v and y, are true harmonics or sine curves. The effect of friction and governor action is to produce a damped or somewhat distorted harmonic as discussed in Appendix —. Any change of load thus starts a series of wave like fluctuations of penstock velocity and stand-pipe level which continue until this wave energy has been entirely expended in friction.

Analogous to all other wave motions these waves may pile up, (if two or more gate movements succeed each other by short intervals which are approximately multiples of the cycle, 2T) causing a very great flucuation in head and velocity. In fact by assuming a proper combination and succession of circumstances no limit can be assigned to the range of fluctuation or "surge" which may occur. The probable combination of circumstances which will occur in any plant depends largely upon the character of the load. Overflows from stand-pipes due to these surges have been known to do considerable damage and it is desirable to either provide for this overflow either at the top or by relief valves at the bottom, or build the stand-pipe high enough to prevent it and thus gain the additional advantage of conserving the water which would otherwise waste.

If the change of load is assumed to occur when the water is at its normal level then the analysis given in Appendix— furnishes the following formulas:

$$(33) \qquad T = \pi \sqrt{\frac{Fl}{Ag}}$$

$$(34) \qquad Y = \sqrt{\frac{Al}{Fg}}\,(v_1 - v_0)$$

$$(35) \qquad D^2 - 2HD = -\frac{2A}{F}\left[\frac{1}{2g}(v_1^2 - v_0^2) + \frac{HT}{\pi}(v_1 - v_0)\right]$$

$$(36) \qquad D_b = D + c\,v_0^2$$

$$(37) \qquad D_a^2 = \frac{A}{F}\,v_0^2\left(\frac{1}{g} - \frac{cT}{3}v_0\right)$$

The value of T from equation (33) is one-half a wave cycle or the time required for return to normal head after a change of load. It is obtained by neglecting both friction and the compensating effect of the governor. These influences increase T in very nearly the ratio that D exceeds Y.

Y from equation (34) is the maximum head fluctuation, or maximum value of y, also obtained by neglecting friction and governor action.

D from equation (35) is the maximum drop in standpipe level corresponding to Y except that governor action is included. If this value of D is added as shown in equation (36) to the initial friction loss, cv_0^2, the result agrees very closely with the value of the maximum drop D where friction is included and is much more simple than the more exact equation given in Appendix —.

A reasonable assumption for determining the probable maximum height to which the water will rise in the stand-pipe is that full load is instantly thrown off the unit when the normal full load velocity v_f exists in the penstock. This assumption leads to equation (37).

The verification of these formulas and some additional ones is given in Appendix —, and an example of their application in section 230.

224. The Air Chamber.—There is a practical limit to the height to which a stand-pipe can be built. A high stand-pipe is also less effective due to the inertia of the water in the stand-pipe itself which must be overcome at each change of load, thus introducing to a lesser degree the same problem as in a penstock without stand-pipe. For some such cases the top of the tank can be closed and furnished with air by a compressor. The design of air chambers has been investigated by Raymond D. Johnson.* An air chamber is less effective in equalizing the pressure than a standpipe of the same diameter.

225. Predetermination of Speed Regulation for Wheels Set in Open Penstocks.—The influences which oppose speed regulation have been partly discussed. At an increase or decrease of load there is a deficiency or excess of developed power due to (1) the inability of the governor to move the gate upon the instant that the load changes; (2) the necessity of accelerating or retarding the water in the penstock and draft tube as previously discussed. If no stand-pipe is used, reliance must be placed upon the fly-wheel effect of turbine, generator and additional fly wheel, if necessary, to absorb or give out the excess or deficiency of input over output of the plant at this time.

The first influence opposed to speed regulation, that of slow gate movement, is of chief importance (a) where the plant is provided with large open penstocks and short draft tubes; (b) where an ample stand-pipe, placed close to the wheel, and a short draft tube are used; (c) in the regulation of an impulse wheel where no attempt is made to change the velocity of water in the feeder pipe.

Mr. H. E. Warren† has analyzed this case essentially as follows:

"As long as the output from the wheel is equal to the load, the speed S and kinetic energy K′ of the revolving parts will remain constant. The governor is designed to adjust the output of the wheel to correspond with the load, but it cannot do this instanta-

* See Trans. of Am. Soc. M. E., 1908.

† See article by H. E. Warren on "Speed Regulation of High Head Water Wheels," previously referred to in Section 222

neously. Consequently, during the time T required to make the adjustment of the control mechanism after a load change there will be a production of energy by the water wheel greater or less than the load. The entire excess or deficiency will be added to or subtracted from the kinetic energy of the revolving parts, and will become manifest by a corresponding change in speed.

Neglecting friction losses, and assuming that the power of the water wheel is proportional to the percentage of the governor stroke and that the movement of the governor after a load change is at a uniform rate, the excess or deficient energy which goes to or comes from the revolving parts after an instantaneous change of load from L_0 to L_1 is measured by the average difference between the power of the wheel and the new load during the time T″, while the governor is moving, multiplied by T″ or expressed in foot pounds:

$$\Delta K' = \frac{p_0 - p_1}{2} \times T'' \times 550 \qquad (38)$$

From equation 24 the kinetic energy of the rotating parts is:

$$K' = .00017\ IS^2$$

From equations 24, 30 and 38

$$\delta = \frac{50 \times (p_0 - p_1) T'' \times 550}{2 \times .00017\ I\ S^2} \quad \text{or}$$

$$\delta = 81,000,000 \frac{T''}{I\ S^2} (p_0 - p_1) \qquad (39)$$

226. Predetermination of Speed Regulation, Plant with Closed Penstock.—In this case the rotating parts must absorb or deliver up an amount of energy $\Delta K'$ (equation 29), equivalent to that given for ΔK in formula

$$\Delta K = \Delta K_1 + \Delta K_2 + \Delta K_3 \qquad (8)$$

where, from equation 22,

$$\Delta K_1 = \frac{AwH}{2\ M} (v_1 - v_0)^2 \qquad (22)$$

M being obtained from equation

$$M = \frac{gH}{l} \left[1 - \frac{(v_0 + v_1)^2}{4V^2} \right] \qquad (19)$$

The value of ΔK_2 is obtained by equation

$$\Delta K_2 = 0.972\ Al\ (v_1^2 - v_0^2) \qquad (23)$$

There is no simple way, as discussed in section **221**, of determining K_3. It must be estimated or analyzed graphically as in Appendix C.

From equation

$$K' = .00017\ I\ S^2 \qquad (24)$$

If R is the proportion of this theoretical energy which is given to the rotating parts at a decrease in load, or which the rotating parts must give out during an increase of velocity and load then

$$(40) \qquad \Delta K' = R \times \Delta K$$

and we have from equation

$$(30) \qquad \Delta K' = \frac{50 \times R \times \Delta K}{K'}$$

$$= \frac{50 \times R \times \Delta K}{.00017\ I\ S^2} \quad \text{or}$$

$$(41) \qquad \delta = 294,000 \frac{R \times \Delta K}{I\ S^2}$$

Solving for I we find the moment of inertia of the rotating parts, which is necessary to obtain any desired percentage of regulation to be

$$(42) \qquad I = 294,000 \frac{R \times \Delta K}{\delta\ S^2}$$

Although there can be no doubt as to the accuracy of the form of equations 41 and 42 yet their value for other than comparative purposes depends upon the accuracy with which we can estimate R. With perfect efficiency of the wheel under all conditions, R would be unity, but in actual cases R must be determined by experiment or by the graphical method given in Appendix —. It will be less for decreasing than for increasing loads since the inefficient operation of the wheel assists speed regulation in the former case, and hinders it in the latter. In addition to this fact, the excess energy at a decrease of load can be partially dissipated through a relief valve, or a by-pass, etc. For practical cases it is therefore necessary to investigate only the case of increasing load.

A detailed analysis of a particular problem can be made, as in Appendix —, by which the velocity in the penstock, effective head, power of wheel, speed, etc., can be determined for each instant during the period of adjustment. From this also the time of return to normal speed can be determined. The method is somewhat tedious, but justifiable nevertheless.

227. Predetermination of Speed, Regulation, Plant with Stand-pipe.—If the stand-pipe is of suitable diameter and close to the wheel the speed regulation will approach that obtainable in open penstock and as investigated by Warren in Section 225. Otherwise the problem becomes that of a plant with a closed penstock, of a length equal to that of the draft tube, plus the penstock from stand-pipe to wheel.

228. Application of Method, Closed Penstock.—An example of the analysis of a problem in speed regulation is as follows:

Assume the 48" Victor cylinder gate turbine, whose characteristic curve is shown in Fig. 245, page —. Suppose it is supplied with water through a penstock whose diameter is 8 feet, and whose length combined with that of the draft tube is 500 feet. The head is 50 feet which for ϕ=.664 gives 180 R. P. M. = S.

Neglecting all losses of head except that in the turbine, we find from the characteristic curve for various loads as follows:

	Full load.	.8 Load	½ Load.	¼ Load.
Brake Horse Power	1120.00	900	560.00	280.00
Quantity of water per sec. (cu. ft.	240.00	210	145.00	97.80
Velocity in Penstock, V	4.77	4.18	2.88	1.94
Efficiency of wheel	.82	.754	.68	.505

The above values will be considered as applying to the entire plant since the loss in the penstock is small in this case.

Assume the load to increase suddenly from one quarter load to 0.8 load, while the gate at the same time opens to full load position. The number of foot pounds of work which must be done to accelerate the water from a velocity of 1.94 feet per second to 4.18 feet per second is found from equation 23 to be

$$\begin{aligned}\Delta K_2 &= 0.972\ Al\ (v_1^2 - v_0^2)\\ &= 0.972 \times 50.3 \times 500\ (4.18^2 - 1.94^2)\\ &= 0.972 \times 50.3 \times 500 \times 13.73\\ &= 335{,}000 \text{ foot pounds.}\end{aligned}$$

Referring to section 226, p. —, to find the amount of deficient work due to insufficient supply of water we have

$$\frac{v_0 + v_1}{2} = 3.06,$$

From equation 19, section 226

$$\begin{aligned}M &= \frac{32.15 \times 50}{500}\left(1 - \frac{3.06^2}{4 \times 4.77^2}\right)\\ &= \frac{32.15 \times 50}{500} \times .897\\ &= 2.88\end{aligned}$$

From equation 22,

$$\begin{aligned}\Delta K_1 &= \frac{50.3 \times 62.5 \times 50}{2 \times 2.88}(4.18 - 1.94)^2\\ &= 187{,}000 \text{ foot pounds.}\end{aligned}$$

The total deficiency for which formulas have been derived is hence,

$$\begin{aligned}\Delta K &= \Delta K_1 + \Delta K^2 + (\Delta K_3 \text{ undeterminable})\\ &= 335{,}000 + 137{,}000\\ &= 472{,}000 + \text{ft. lbs.}\end{aligned}$$

By means of the detailed graphical analysis given in Appendix —— this deficiency is found to be 600,000 foot pounds for gate movement in one-half second showing that the estimated value should have been increased in this case by 12.7 per cent. $(R = 1.127)$ to compensate for neglecting the effect of slow (½ second) gate movement, or K_3. It must be remembered that this quantity, ΔK, is the deficiency of theoretical hydraulic work done upon the wheel. For reasons discussed in Appendix —, it will, however, be found to differ but slightly from the deficiency of wheel output, in this case 586,000 ft. pounds.

To determine the speed regulation which can be obtained, assume a generating unit whose rotor has a fly-wheel effect, or moment of inertia, I, of 1,000,000 lbs. at one ft. radius. The normal speed $S = 180$, $\Delta K = 472{,}000$ ft. lbs., and R (in general to be estimated. but in this case obtained by the graphical method given in Appendix —, is 1,127. Therefore from equation (43)

$$\delta = 294{,}000 \frac{1.127 \times 472{,}000}{1{,}000{,}000 \times 180^2} = 5.42\%$$

If a fly-wheel is to be designed for a given regulation say 4 per cent., then the required moment of inertia of same is, from equation (42).

$$\begin{aligned}I &= 294{,}000 \frac{R \Delta K}{S^2}\\ &= 294{,}000 \frac{1.157 \times 472{,}000}{4 \times 180^2} \quad \text{or}\\ I &= 1{,}355{,}000 \text{ ft.}^2 \text{ lbs.}\end{aligned}$$

229. Application of Method, Open Penstock.—As the penstock and draft tube are shortened, the excess or deficient energy area, ΔK_3, obtained during the gate movement becomes an increasing proportion of the whole until for a large open penstock and short draft tube the developed power ceases to lag and follows practically the same law of change as the gate opening. The estimation of excess or deficient energy, and consequently of speed, is then very simple by means of Mr. Warrens equation (39). For illustration: assume the same wheel as in the preceeding section, obtaining the outputs of 280 H. P.$=P_0$ at one-fourth load and 1120 H. P.$= P_1$ at full load, as in the other installation. Assume the same moment of inertia 1,000,000 and that the gate movement takes place in ½ second as before. Then $T'' = ½$; $S = 180$.

This gives

$$\delta = 81,000,000 \frac{0.5}{1,000,000 \times 180^2}(1120 - 280) = 1.05\%$$

This is a much closer regulation than obtained with the long penstock.

230. Application of Method, Plant with Stand-pipe.—Assume a plant where the wheels develop 39,000 H. P. under 375 head, thereby requiring about 1100 cu. ft. of water per second (assuming 83 per cent. efficiency of the wheels). Assume this water is supplied through four 7′ pipes about 4800 feet long, requiring a velocity in the feeder pipes at full load of about 7.15 feet. Suppose four pipes all connected at the lower end to a stand-pipe 30 feet in diameter. If a sudden load change, of about one third of the total is to be provided for this would require an ultimate change of velocity in the penstock from about 4.76 feet per sec. at two-thirds load to 7.15 feet at full load, or $v_0 = 4.76$, and $v_1 = 7.15$. Now,

$$A = 4 \times \pi \frac{7^2}{4} = 154 \text{ sq. ft.}$$

$$F = \pi \frac{30^2}{4} = 707$$

From equation 33 the time required for return to normal head, or the half period of oscillation, is

$$T = \pi \sqrt{\frac{707 \times 4800}{154 \times 32.15}} = 82 \text{ seconds}$$

This would perhaps be increased to nearly 100 seconds, due to the use of additional water during this period of low head, as discussed in Appendix — , but the value 82 should be used in equation 35.

Equation 34 gives for the drop in water level in the stand-pipe,

$$Y = \sqrt{\frac{154 \times 4800}{707 \times 32.15}}\ (7.15 - 4.76)$$

$$= \sqrt{32.5} \times 2.39 = 13.6 \text{ feet.}$$

The more exact equations, 35 and 36, give for D and D_b

$$D^2 - 2 \times 375\, D = -\frac{2 \times 154}{707}\left[\frac{4800}{64.3}(7.15^2 - 4.76^2) + \frac{375 \times 82}{3.14}(7.15 - 4.76)\right]$$

or

$$D^2 - 750\, D + 11,120 = 0$$

Solving this quadratic equation gives

$$D = \frac{750 - \sqrt{750^2 - 4 \times 11,120}}{2} \quad \text{or}$$

$$D = \frac{750 - 719}{2} = 15.5 \text{ feet}$$

$$C = \left(1 + .015\frac{4800}{7}\right)\frac{1}{64.3} = .176$$

$$D_b = 15.5 + c \times 4.76^2 = 15.5 + .176 \times 4.76^2 = 19.5 \text{ feet}$$

No attempt will be made to estimate the greatest drop in level which might occur, due to an addition of waves.

231. **Governor Specifications.**—The present practice of requiring the governor builder to guarantee the speed regulation of a plant, in the design of which he has had no voice, without even giving him the necessary information regarding the hydraulic elements which are considered in this chapter is wrong. It is partly the outgrowth of the modern tendency to specialize, but perhaps more largely due to a lack of understanding on the part of the engineer of the nature of the problem, and a resulting desire to shift the responsibility for results upon some one else who is better informed upon the subject and thus protect results financially as well as save his own reputation in case of failure.

Governor specifications should call for a guarantee of the

(a) *Sensitiveness* or per cent load change which will actuate the governor;

(b) *Power* which the governor can develop, and *force* which it can exert to move the gates;

(c) *Rapidity* with which it will move the gates;

(d) *Anti-racing qualities,* such as number of gate movements required to adjust for a given load change (See figure 280), or percent. over-run of the gate, etc.

(e) *General requirements* of material, strength, durability, etc.

Beyond this point the governor designor has no control. The engineer can, however, by choosing a generator whose rotor has a high moment of inertia (which quantity should be stated in tenders for supplying the generators), by the addition of a fly-wheel, if necessary; by the construction of a stand-pipe; by means of a relief valve, and very largely, also, by the general design of the penstocks, draft tubes, etc., greatly improve the governing qualities, and, in fact, reduce the speed variation to any desirable limit which the nature of load to be carried, magnitude of load changes anticipated, and economy of first cost will warrant.

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CHAPTER XIX.

THE WATER WHEEL GOVERNOR.

232. **Types of Water Wheel Governors.**—In all reaction turbines and in all impulse turbines, with the exception of tangential wheels, the governor affects regulation, i. e. controls the output, and hence the speed of the wheel, by opening or closing the regulating gates, thus varying the amount of water supplied to the wheel. In tangential wheels, under high head, this method of control, for obvious reasons (See section 215), becomes difficult and in extreme cases impossible and in such cases the governor must be arranged to affect regulation by the deflection of the jet from the bucket. (See Fig. 282).

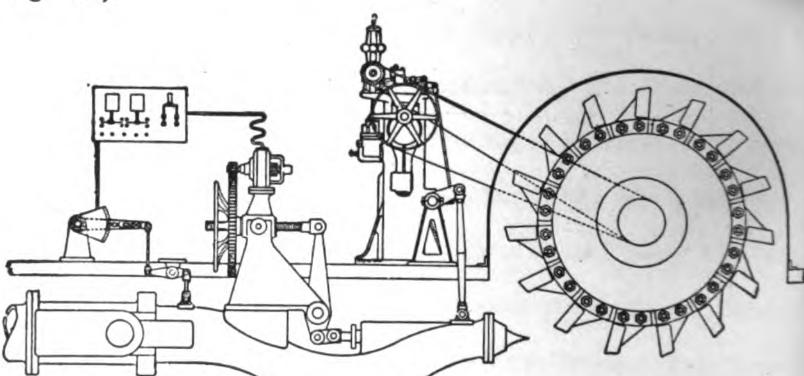

Fig. 282.—Governing Impulse wheel with Automatic Needle and Deflecting Nozzle (after Warren).

The force required to move the turbine gates is large (sometimes 50,000 lbs. or more) and it is therefore evident that they cannot be moved by the direct action of the centrifugal ball governors, as with steam engines, but must be moved by a "relay."

The relay, as its name implies, is a device for transmitting energy from a source of energy independent,— as to quantity—of the centrifugal governor balls but controlled by them in its application.

If the relay is of *"mechanical type"* the power required to operate it and the gates is transmitted, when needed, from the wheel by means of shafts, gears, friction-clutches, belts and pulleys or other mechanical devices. In mechanical governors the flyballs may actuate pawls, friction gears or other mechanical devices which will bring the relay into action.

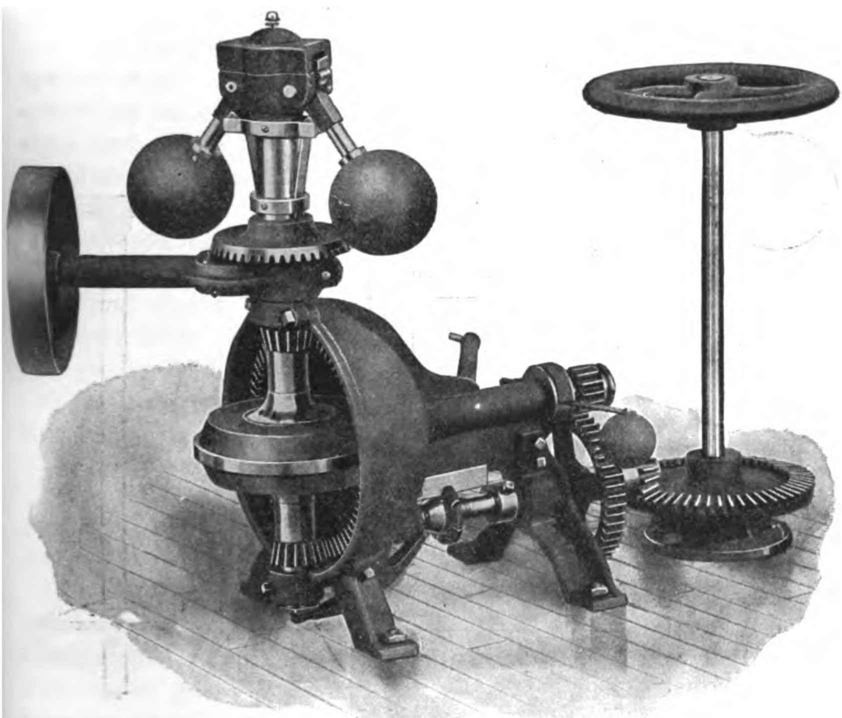

Fig. 283.—Woodward Standard Governor.

If the relay is of the *hydraulic type*, it usually consists of a piston connected by some mechanical device to the gate rigging and moved by means of the hydraulic pressure of water taken from the penstock, or other source, or by oil supplied under high pressure from a reservoir. The pressure of the oil in the reservoir is maintained by compressed air supplied by power taken from the wheel itself. The oil thus used in moving the piston is exhausted into a receiver from which it is pumped back into the supply reservoir. The hydraulic relay is commonly controlled by the ball governor through the medium of a small valve which by its motion either admits the actuating water (or oil) directly to the cylinder or to a secondary piston controlling a larger admission valve.

Electrical methods of actuating the relays controlled by means of governor balls have been used to some extent but are not nearly so common as mechanical or hydraulic devices.

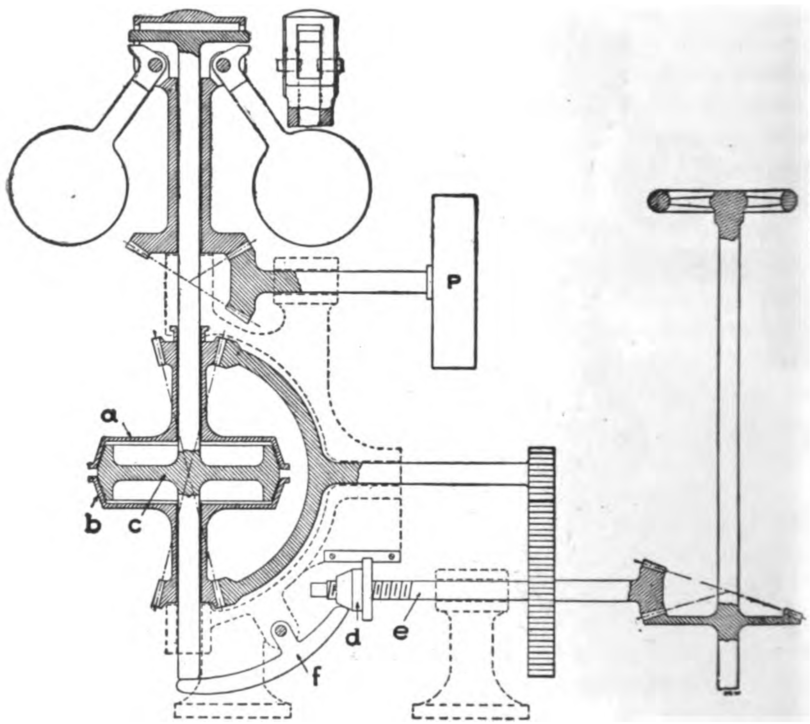

Fig. 284.—Diagramatic Section of Woodward Simple Mechanical Governor.

233. **Simple Mechanical Governors.**—Fig. 283 is a view and Fig. 284 a diagramatic section of a simple mechanical governor of the Woodward* Standard type. On the upright shaft are two friction pans (a and b). (See also Fig. 287). These pans are loose on the shaft, the upper one being supported in position by a groove in the hub and the lower one by an adjustable step-bearing. Between these pans, and beveled to fit them, is a double-faced, friction wheel (c) which is keyed to the shaft. This shaft and friction wheel run

*Woodward Governor Co., Rockford, Ill.

continuously and have a slight endwise movement. They are supported by lugs on the ball arm and therefore rise and fall as the position of the balls varies with the speed.

When the speed is normal, the inner or friction wheel revolves freely between the two outer wheels or pans which remain stationary. When a change of speed occurs, the friction wheel is brought against the upper or lower pan as the speed is either slow or fast. This causes the latter to revolve and, by means of the bevel gearings, turn the gates in the proper direction until the speed is again normal. As the gate opens, the nut (d) travels along the screw (e) which is driven through gearing by the main governor shaft and as the gate reacts, the nut (d) coming in contact with the lever (f) throws the vertical shaft upward and the governor out of commission.

This type of governor may be used to advantage where the water wheels operate a number of machines, connected to a main shaft and where, in consequence, the friction or constant load is a considerable percentage of the total load. In such cases the changes in load may not be a large percentage of the total load and the temporary variations in speed, which occur at times of changes of load, may not be of sufficient importance to necessitate the installation of a quick acting governor.

When the water wheel is direct connected to a single machine, and the friction load is comparatively small, the relative change in load, and the consequent possible changes in speed, is much larger.

In such cases the type of governor above shown will result in a serious hunting or racing (See Section 211) of the wheel during considerable changes of load, and in unsatisfactory regulation. In such cases governors with compensating or anti-racing devices must be used for satisfactory regulation.

234. Anti-Racing Mechanical Governors.—The Woodward Compensating Governor.—Fig. 285 is a view and Fig. 286 is a diagramatic section of a Woodward vertical mechanical governor of the compensating type.

In the simple Woodward governor (See Figs. 283 and 284) the power necessary to actuate both the centrifugal governor balls and the relay is transmitted through a belt to a single pulley, P. In the Woodward compensating type of governor the relay is operated in a similar manner by a single pulley, P, while the centrifugal governor balls are actuated by an independent pulley, q, having an independent belt connected to the wheel shaft or to some other revolving part connected therewith. From the driving pulley, q, power is transmitted to the governor balls through a shaft and gearing. The shaft supporting the centrifugal governor balls is hollow, and on the ball-arms are two lugs which connect with a

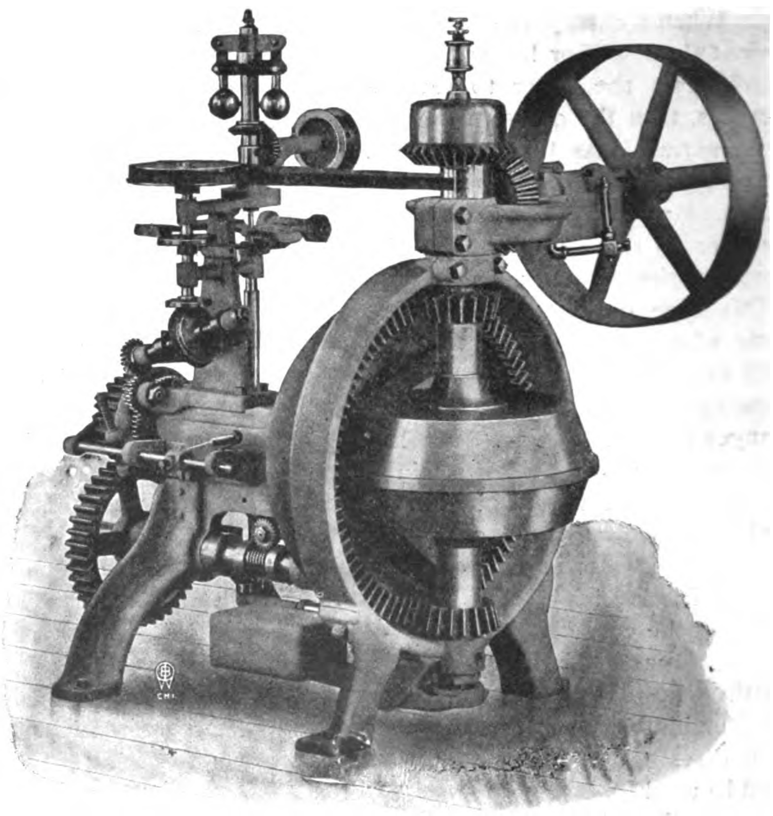

Fig. 285.—Woodward Compensating Governor.

spindle (f), which therefore rises and falls as the positions of the governor balls vary with the speed.

The movement of the centrifugal governor balls causing the spindle, f, to rise and fall changes the position of the tappet arm, g, to which it is connected, and causes one or the other of the two tappets, tt', to engage a double-faced cam, h. This cam is continuously rotated by means of the pulley above it, driven by a belt connected with the main vertical shaft of the relay. The tappets are connected to a common suspension arm to which the vertical spindle, f, is attached. The suspension arm is hinged to the lever arm, j. The lever arm is connected to the shaft, K, which can be rotated on its bearings and which is connected with a tension rod, l, by an eccentric at the bottom. The tension rod, l, is in turn connected by

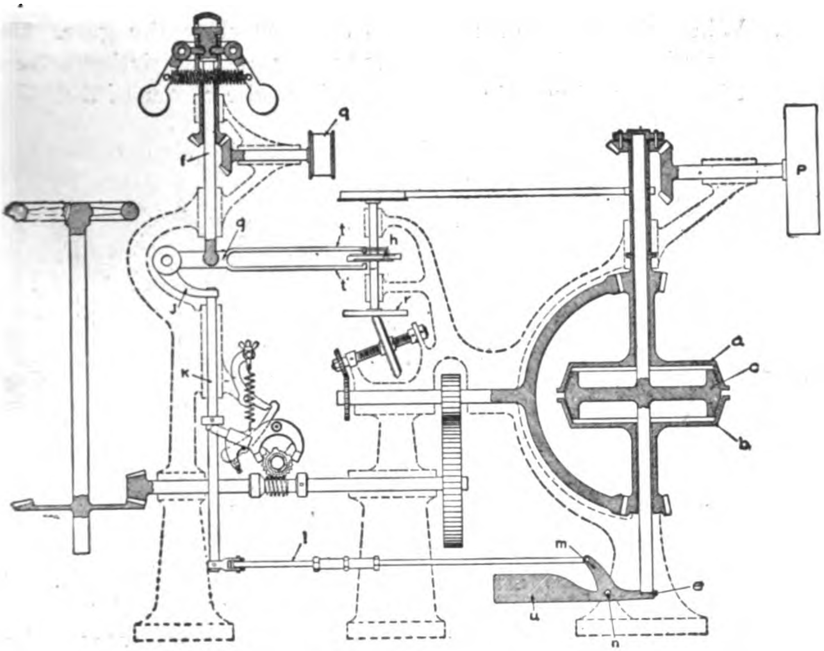

Fig. 286.—Diagramatic Section of Woodward Vertical Compensating Mechanical Governor,

a lever, m, with the vertical bearing, e, on which the main shaft of the friction cone rests. This bearing is movable around the fulcrum, n, and is counterbalanced by an arm and weight, u.

When either of the tappets engages the rotating cam, the resulting movement turns the rocker shaft, K, and, through its connection, raises or lowers the vertical bearing, e, which causes the friction wheel, c, to engage either the upper or the lower of the friction pans, a and b, as in the case of the simple governor.

The compensating or anti-racing mechanism is just below the rotating cam. It is essentially alike in all of the Woodward compensating types of governors and is described in the governor catalogue as follows:

"On the lower end of the cam shaft is a friction disc, r, (Fig. 286) which rests on a rawhide friction wheel on a diagonal shaft. The hub of the friction wheel is threaded and fits loosely on the diagonal shaft which is normally at rest. The effect of the continually rotating friction disc upon the rawhide wheel is evidently to cause it to travel along the threaded diagonal shaft to the center of the disc. When the governor moves to open or close the gate, the diagonal shaft, which is geared to it, is turned and the friction wheel is caused to travel along the shaft away from the center of the disc

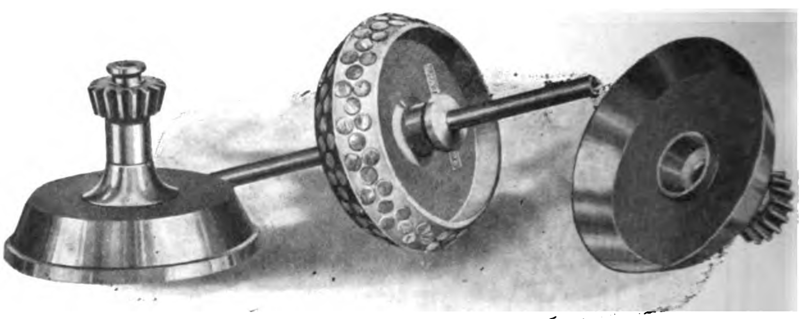

Fig. 287.—Friction Cone and Pans of Woodward Governor.

and thus raise or lower the cam shaft so as to separate the cam from the tappet which is in action, before the gate has moved too far, thus preventing racing. As soon as the gate movement ceases the disc causes the friction wheel to return to the center of the disc along the threaded shaft."

To prevent the governor from straining when the gate is fully open or closed, suitable cams are mounted on the stop shaft. "When the gates are completely opened, the cam engages the speed lever and holds it down so that it cannot raise the lower tappet sufficiently to engage the revolving cam; this does not, however, interfere with the upper tappet, to prevent the closing of the gates, should the conditions demand. The closed gate stop acts in a similar manner on the upper tappet but does not interfere with the lower tappet being engaged, should the conditions demand that the gate be opened. In addition to these stops, the governor is provided with a safety stop whose function is to immediately close the gates should the speed governor stop through breakage of the belt or any other cause."

235. **Details and Applications of Woodward Governors.**—Fig. 287 shows the construction of the friction gearing of the Woodward Mechanical Governor. In the inner friction driving cone, corks are inserted in holes drilled in the rim and these are ground off true so that they project about one-sixteenth inch. This seems to give a very reliable friction surface not readily affected by either water or

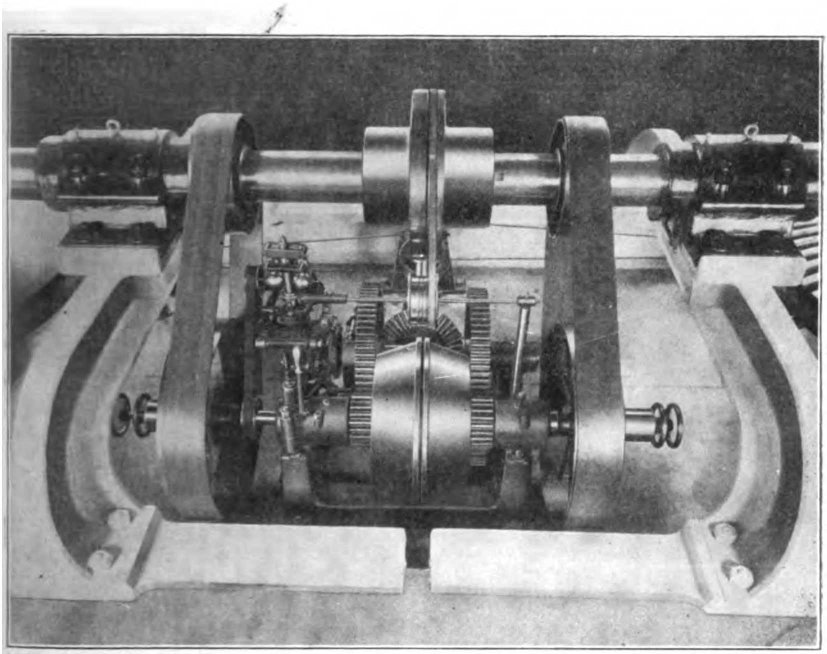

Fig. 288.—Woodward Horizontal Compensating Mechanical Governor at Hydro-Electric Plant of U. S. Arsenal, Rock Island, Ill.

oil, and it is claimed to be superior to either leather or paper for this purpose. In order to cause the friction wheel to engage smoothly and noiselessly, a plunger attached to the shaft, just below the inner friction wheel, fits rather closely into a dash-pot formed in the lower pan.

Fig. 288 shows a horizontal compensating type of Woodward governor as installed to control the gates of the turbines in the Hydraulic Power Plant of the U. S. Arsenal at Rock Island, Illinois. The cables shown at the back of the cut operate the gates of the turbine. On the gate shafts of the latter are sheave wheels to which the cables are attached. These sheave wheels are fitted with clutches so that any gate may be disconnected from the governor. Each gate is provided with an indicator showing its position This provides means of coupling properly, after being disconnected, without closing the gates of the other wheels. Each governor is arranged to control six turbines, belonging to two different units. Two belts are provided so as to drive from either unit. The gover-

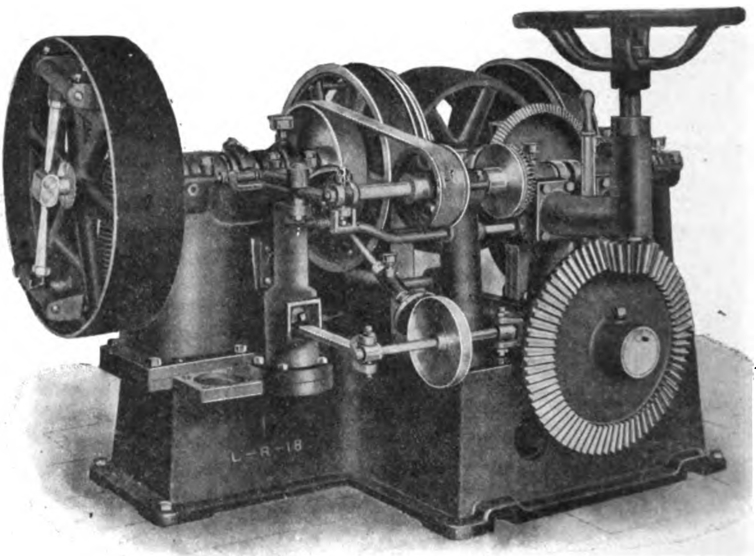

Fig. 289.—Lombard-Replogal Mechanical Governor.

nor can thus be used to control three wheels on either side or all six when the two units are running in multiple.

236. The Lombard-Replogle Mechanical Governor.*—Fig. 289 shows a Lombard-Replogle mechanical governor. The principles of operation of this governor are better illustrated in the diagram, Fig. 290.

In the diagram A is a spherical pulley with its shaft turned down and treaded as at X. B and B are revolving concave discs lined with leather which are continuously revolving in opposite directions. C and C are lignum vitae pins flush with the leather. D and D are compression springs for controlling the pressure between the disks and the sphere. When the spherical pulley A is shifted from its central position in the line of its axis, the springs are

*The Lombard-Replogle Governor Co., Akron, Ohio.

tightened automatically, causing increased traction as the smaller diameters of the sphere engage the larger diameters of the disc. E and E are the centrifugal governor balls so poised as fo require the weight of the pulley A to balance them at normal speed. F is a loose collar to allow independent revolution of the balls EE. G is the point of connection between A and the gates or valve rigging of the wheel to be governed. X is the compensating devise, and is

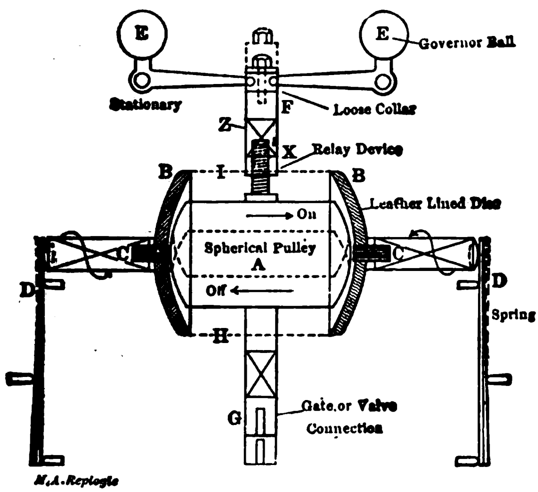

Fig. 290.—Diagram of Lombard-Replogal Mechanical Governor.

for the purpose of reducing and controlling racing. Z is a stationary spindle or connecting link between the collar F and the threaded shaft or pulley A. Z is only stationary in reference to revolution, as it rises or falls with the variations of the governor balls.

The spherical pulley A is normally at rest while the discs BB are continually revolving. A movement of the governor balls raises or lowers the shaft so that the spherical discs rotate the pulley.

The greater the displacement of the shaft the more rapid the revolution since the circle of contact on the disc is increased. The rotation of the spherical pulley A either shortens or lengthens the distance to collar F by means of thread X. "This shortening causes A to be pulled back to the disc centers, thereby cutting the governor out of action" and preventing the gates from moving too far or racing.

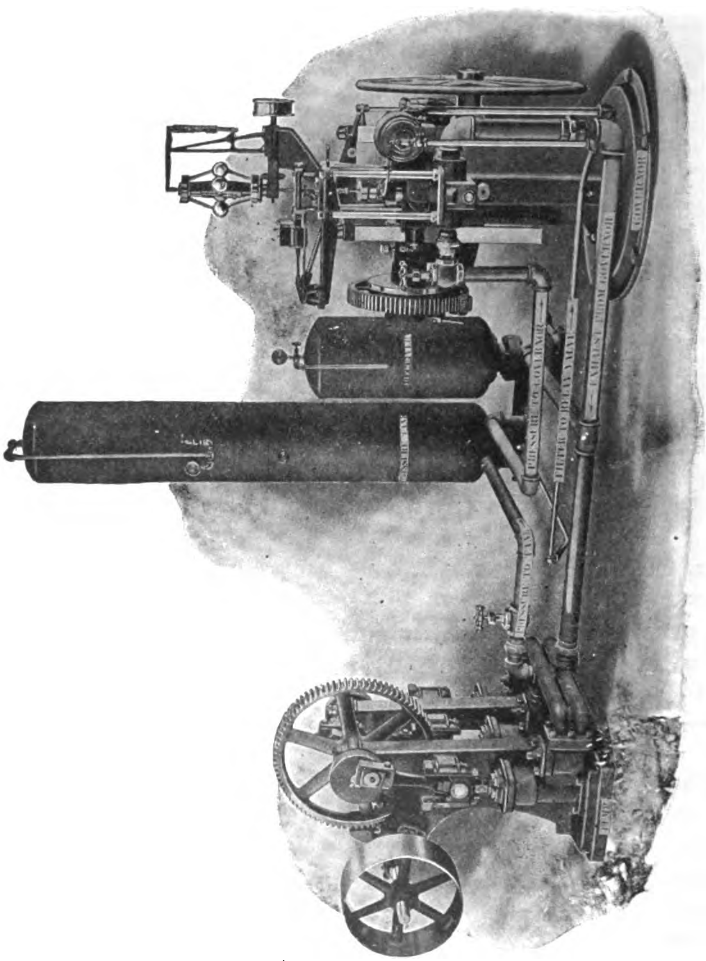

Fig. 291.—Lombard Type N Hydraulic Governor.

237. Essential Features of an Hydraulic Governor.—The essential features of an hydraulic water wheel governor are:

1. A tank for storing oil under air pressure.
2. A receiver tank for the collection of oil used by the governor.
3. A power pump driven from the water wheel shaft.
4. A hydraulic power cylinder for operating the gates.
5. A sensitive contrifugal ball system for controlling a valve which either admits oil directly to the power cylinder or to an intermediate relay cylinder the piston of which operates the admission valve to the power cylinder.
6. An anti-racing or compensating mechanism.

The power pump is continually using power from the wheel to pump the oil from the receiver back to the pressure tank thus gradually storing the energy which is used intermittently to operate the gates.

Fig. 291 illustrates the Lombard Type "N" Governor and shows clearly the relations of the various parts of an hydraulic governor. The centrifugal governor balls are connected by belt to the wheel shaft. These balls control a small primary or pilot valve of the cylinder type which admits oil from the large pressure tank under about 200 pounds pressure into one side of a cylinder where its pressure is exerted against one of two plungers. These plungers control a large valve, also of the cylinder type, which admits oil from the pressure tank to one or the other side of the power piston. The rectilinear motion of the piston is converted, by rack and pinion, into rotary motion for transmission to the wheel gates. The oil used for operating the power pistons and the plungers of the relay is exhausted into the vacuum tank from which it is pumped back into the pressure tank by means of the power pump shown at the left which is driven by belt from the wheel shaft. The speed variation necessary to actuate the governor depends upon the lap of the pilot valve and is adjustable.

238. Details of Lombard Hydraulic Governor.—The details of the Lombard Type N Governor are best shown by the enlarged view of the upper portion of the governor (Fig. 292) and by the section of the relay valve (Fig. 293). The following description of the operation of this governor is taken from the Directions for Erecting and Adjusting Governors.*

"The oil from the pressure-tank is supplied to the working cylinder 62 through the large relay-valve 106, arranged to discharge

*Published by The Lombard Governor Co., Ashland, Mass.

or exhaust oil directly and rapidly into or from either end of the cylinder. The relay-valve 106, through the hydraulic system connected therewith, is under the simultaneous control of the regulating-valve 14 and the displacement-cylinder 107. This is

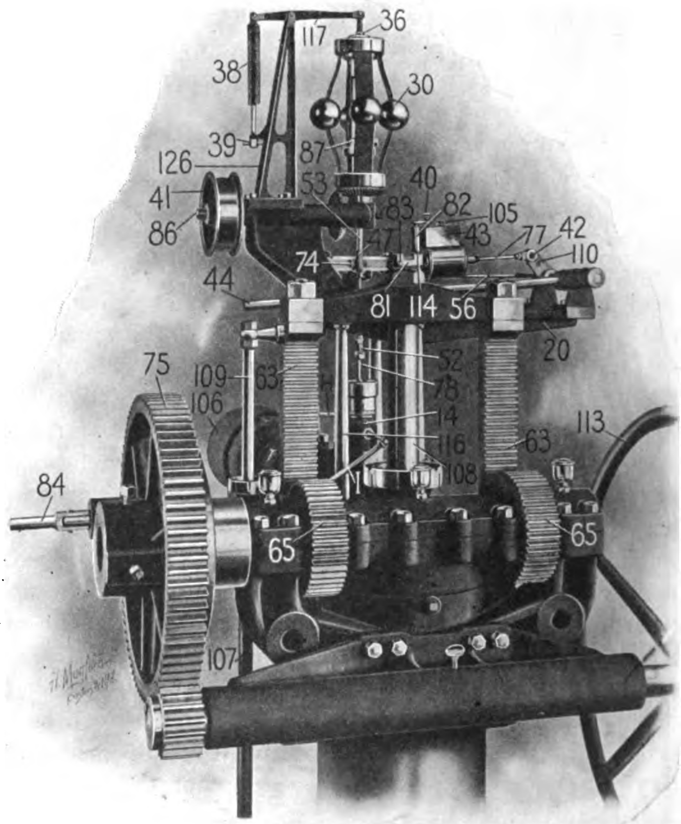

Fig. 292.—Upper Portion of Lombard Type N Governor.

brought about in the following manner. The relay-valve A (See Fig. 293) is moved hydraulically by plungers B and C contained within cylinders D and E forming parts of the relay-valve heads F and G. Plunger B has about one-half the area of plunger C, consequently plunger C can over-power plunger B, if the pressure in cylinders E and D is nearly equal. The cylinder D is permanently in communication with the main pressure supply through the pipe H which also furnishes liquid to the regulating-valve 14. Therefore the tendency of plunger B is always to move valve A towards the relay-valve head G. Cylin-

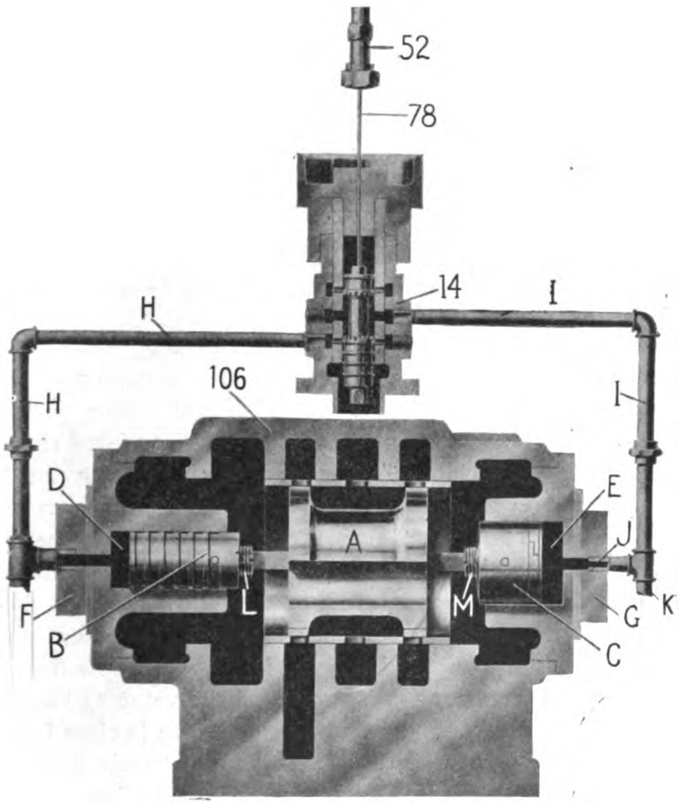

Fig. 293.—Section Lombard Relay Valve.

der E is in communication through pipes I and J with the adjusting-valve 14, and also through the pipes J and K with the displacement-cylinder 107. The regulating valve 14 is capable, when moved in one direction, of admitting liquid under full pressure into the pipe, I, and, when moved in the other direction, of exhausting liquid through the pipe I. In the former case the action is to increase the pressure back of the piston C until it overpowers the piston B, thereby moving valve A towards the relay-valve head F, simultaneously opening the upper cylinder-port to the main exhaust and

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As the displacement-plunger begins to
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... I continues on through K into the c
... relay-valve A then ceases to move any
... main governor-piston, however, conti
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... the moving displacement-plunge
... brought to rest.

... regulating-valve 14 is moved in the
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... C: this allows the plunger B
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Fig. 294.—The Lombard Type R Governor.

...ower Transmission Companies plant and shows the regulation of the Lombard Type B Governor regulating S. Morgan Smith turbines on an electric railroad load. The cars are large and the change in load rapid and large.

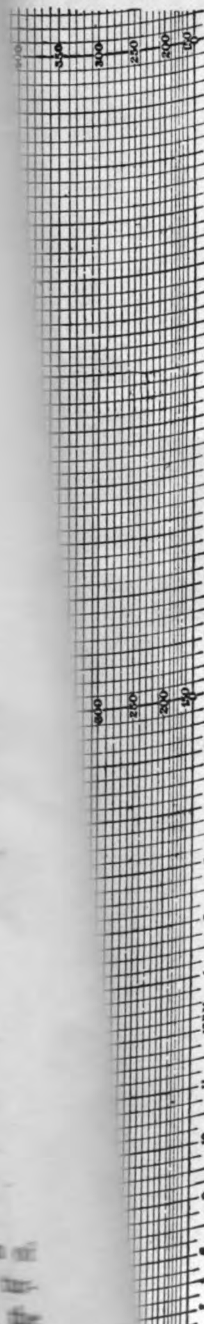


Fig. 296.—Comparative Turbine Regulation.

the lower cylinder-port to the main pressure supply. Instantly the main piston of the governor and with it the displacement-plunger 109 are set in motion.

"As the displacement-plunger begins to move, a space is created back of it, into which a portion of the liquid flowing through the pipe I is diverted. As the motion of the displacement-plunger becomes more rapid, a condition is reached when all the liquid flowing through I continues on through K into the displacement-chamber. The relay-valve A then ceases to move any further. The motion of the main governor-piston, however, continues as long as the regulating-valve 14 is open. When this valve 14 closes, the relay-valve A is immediately thereafter closed, because the liquid in the cylinder E instantly escapes through the pipes J and K into the space beneath the moving displacement-plunger; thus the whole governor is brought to rest.

"When the regulating-valve 14 is moved in the opposite direction by the centrifugal balls so as to allow liquid to escape through the pipe I, there results an immediate loss of liquid in the cylinder E, back of the plunger C; this allows the plunger B to force the relay-valve A towards the relay-valve head G, thus opening the lower cylinder port to the exhaust, and the upper cylinder-port to the pressure supply. The main governor-piston instantly begins to move down, carrying with it the displacement-plunger, thus forcing liquid through the pipes K and I, reducing the flow outward through J, until finally the downward velocity of the displacement-plunger becomes rapid enough to entirely check the outward flow through J. Relay-valve A then remains stationary until the valve 14 has moved to a new position. As soon as regulating-valve 14 is closed, the liquid which has been flowing out through I immediately flows into J and, acting upon the plunger C, restores valve A to its closed position, stopping further movement of the governor. It will be seen that the governor when moving has a constant tendency to close the relay-valve which keeps it in motion, and this relay-valve can be maintained open only so long as the regulating-valve 14 is adding or subtracting oil to or from the system consisting of the pipes I, J, K, and parts connected therewith."

Fig. 294 shows the Lomard Governor, Type R, the smallest of the various governors made by that company. This is a vertical, self-contained oil pressure machine. The oil is stored in a tank formed by the main frame. The governor is designed to exert 2500 pounds pressure and will make an extreme stroke of eight inches in one second.

239. Operating Results with Lombard Governor. — Fig. 295 is a cut from a spead recorder strip taken from the Hudson River

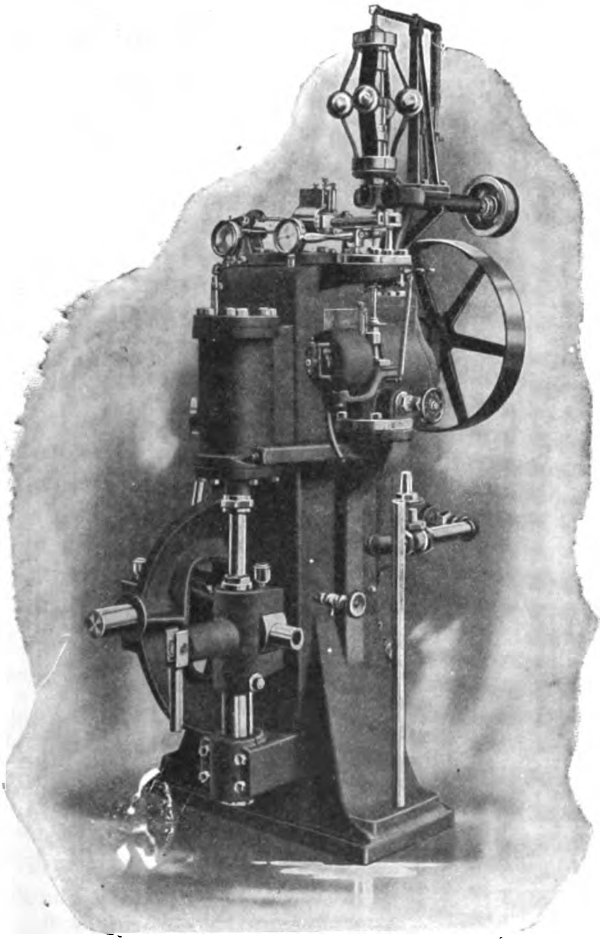


Fig. 294.—The Lombard Type R Governor.

Power Transmission Companies plant and shows the regulation of the Lombard Type B Governor regulating S. Morgan Smith turbines on an electric railroad load. The cars are large and the change in load rapid and large.

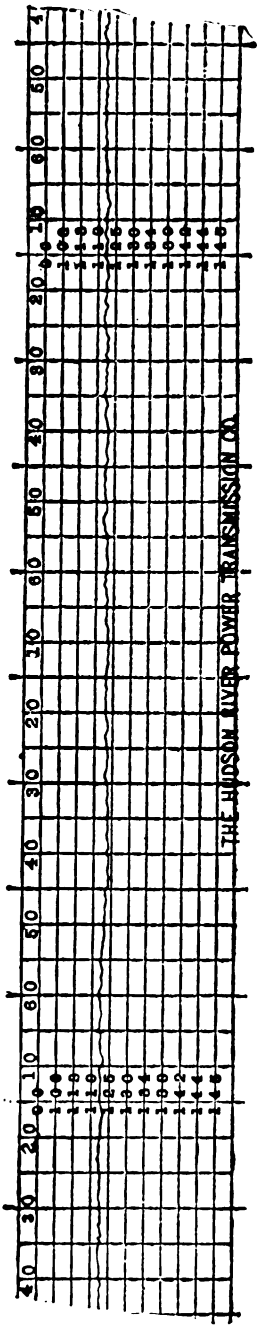


Fig. 295.—Speed Record from Plant of Hudson River Power Transmission Co.

Fig. 296 shows the comparative regulation of two generators in the same plant (See Bulletin No. 107 Lombard Governor Company.) The load was quite variable on account of beaters which had to be driven from the same shaft as the paper making machinery. The original governor used, the work of which is shown in the upper cut, was replaced by a Lombard Type D Governor. The work of the latter is shown in the lower tachometer chart, and the improvement in the uniformity of operation is readily seen by a comparison of the two charts.

240. The Sturgess Hydraulic Governor.*—The Sturgess Type "M" Hydraulic Governor, with the omission of the pump and storage tank, is shown in Fig. 297 and in section in Fig. 298. This governor consists of a shaft-type centrifugal governor G attached to the top of the machine and operated by a belt and pulley P from the turbine shaft. The governor balls BB in this machine control directly by means of a long vertical lever D a small primary or pilot valve S of cylinder type which admits oil to a cylinder controlling the main admission valve S. The main valves, attached to the side of the cylinder, admit pressure directly into the cylinder S and on either side of the piston S which, by its motion, rotates the gate shaft by means of the concealed steel rack R and pinion N, shown in the sectional view, Fig. 298.

The valves for the admission of oil or water, as the case may be, in the cylinder are of the poppet type which avoid "lap" and therefore increase the sensitiveness of the governor. The anti-racing mechanism

*Sturgess Engineering Dept. of The Ludlow Valve Mfg. Co., Troy, N. Y.

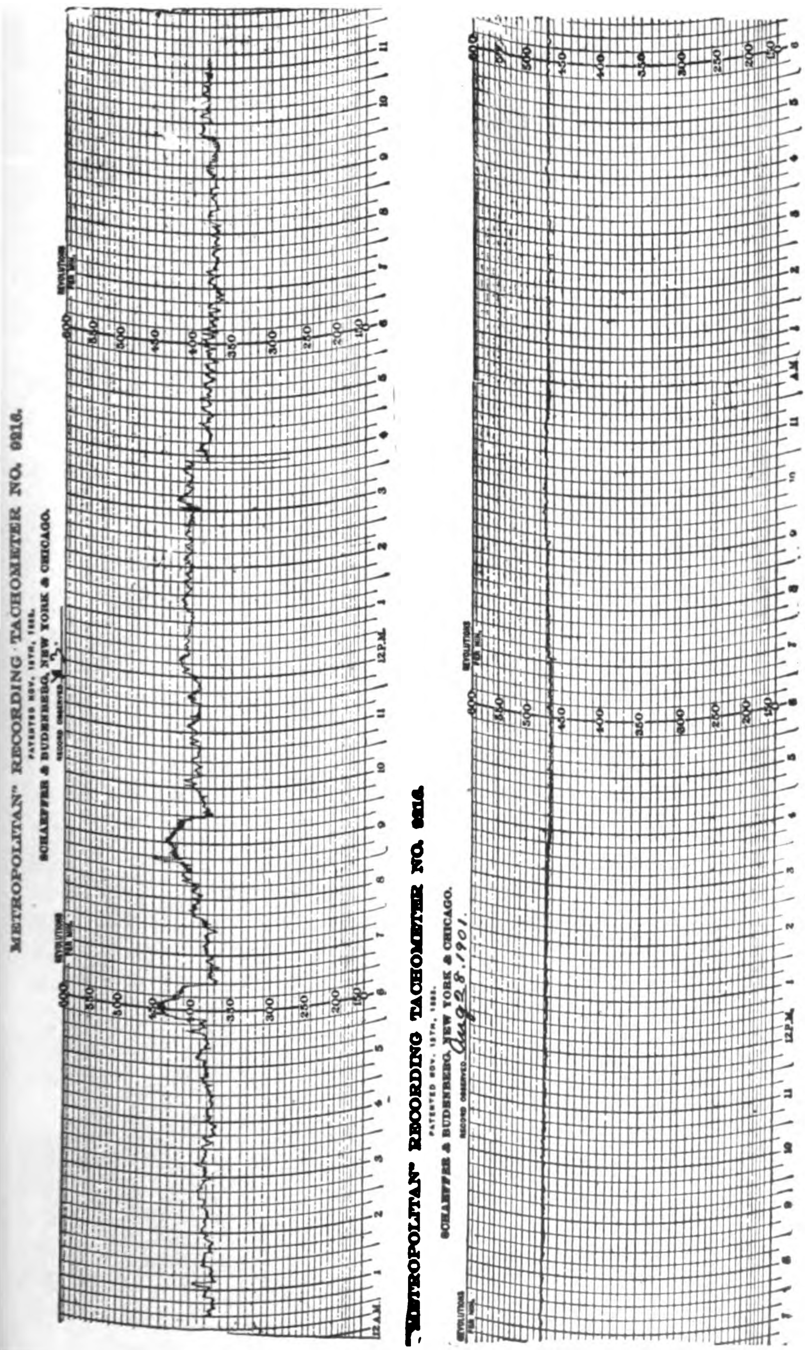


Fig. 296.—Comparative Turbine Regulation.

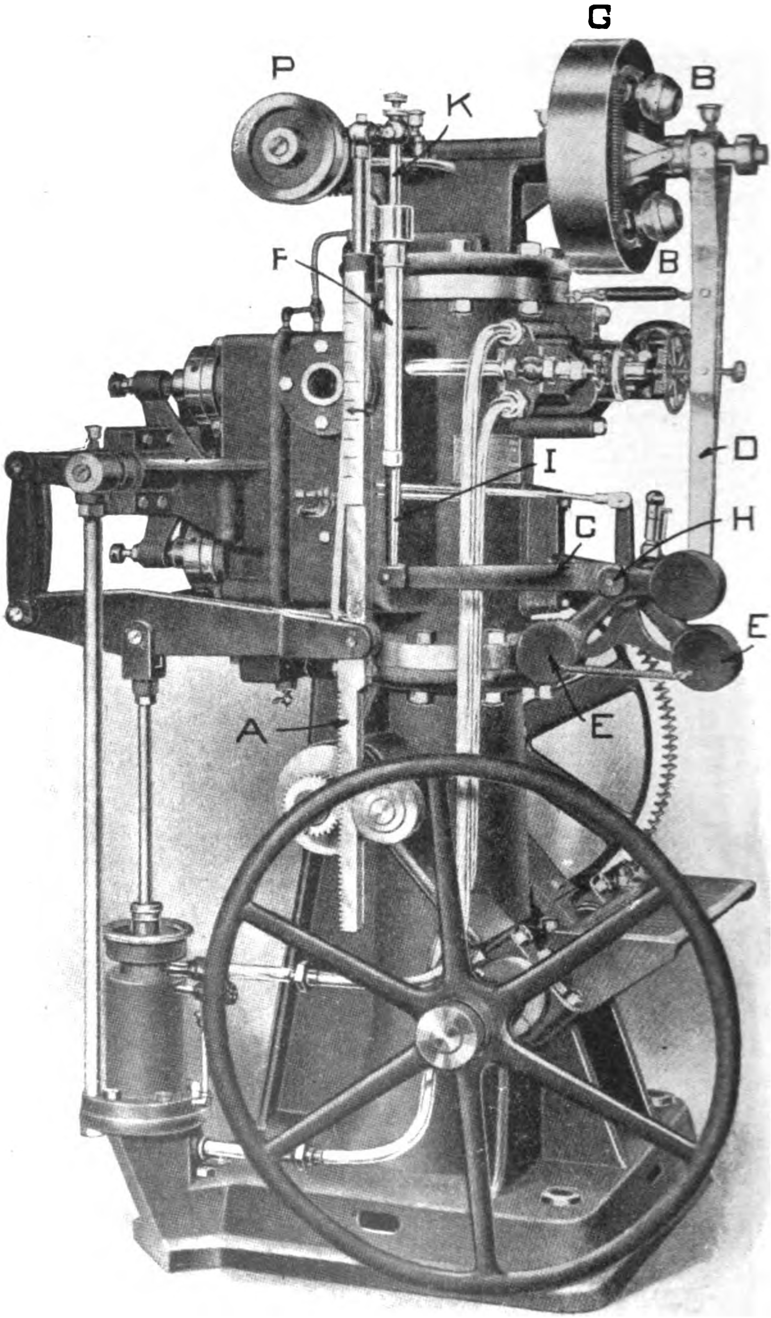


Fig.—297.—Sturgess Type M Hydraulic Governor.

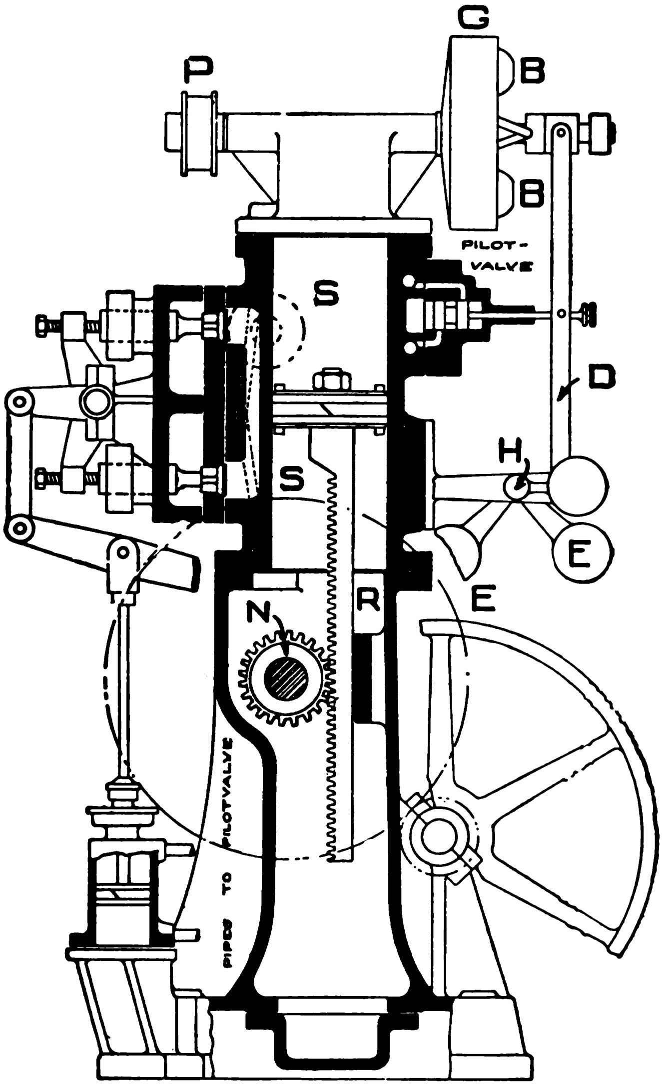


Fig. 298.—Section Sturgess Type M Governor.

consists of a rod A which is attached to the cross head of the governor. At the top of this rod is a projection to which is attached an adjustable piston rod reaching down into the open top dash pot F. The piston rod has a piston attached at its lower end fitting freely into the bore of the dash pot the top of which is formed into a cup which receives the excess oil. The bottom of the dash pot is closed and is attached to a tail piece connected to the counter weighted locker lever, C.

The piston rod and piston are hollow and near the bottom of the piston is a small by-pass which can be regulated by an adjusting screw which controls the rate of flow of the oil in the dash pot. The lever, C, is fixed on the rocker shaft the opposite end of which carries the short arm from which a link is carried to the bottom of the valve lever D which is free to move. Two weights, EE, are

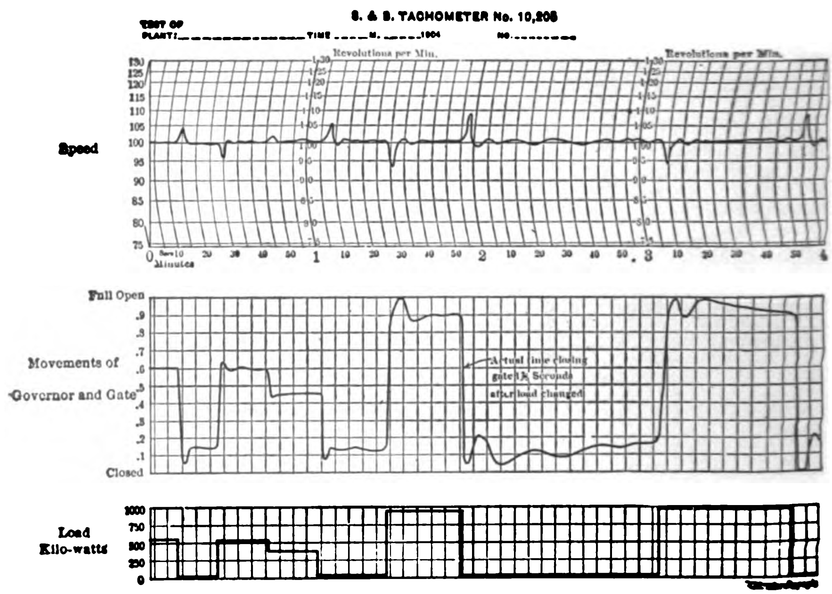


Fig. 299.—Test Results with Sturgess Governor.

hung loosely on the rocker shaft but a pin on the shaft engages with either one or the other of the weights and raises them whenever the rocker shaft moves. The function of the weights therefore is to keep the rocker shaft, and consequently the bottom of the valve lever, in normal position. When the main piston moves, it is obvious that it will tend to raise or lower the dash pot, F, through its connection to the rod I and this movement will swing the lever C and rocker shaft H thus deflecting the bottom of the valve level D so as to compensate in the correct manner. The same movement raises one of the weights E, but as the dash pot permits a slow movement the weights will finally restore all parts to the middle or normal position. In the smaller sizes the pilot valve is omitted and the centrifugal governor balls actuate directly through the lever the main valves of the system.

241. Test Results with Sturgess Governor.—The action of any governor in maintaining a uniform speed may be shown graphically by attaching a recording tachometer to the turbine shaft. In order to fully understand and appreciate the action of the governor, the tachometer chart should be considered together with the load curve and a diagram showing the movement of the governor during the same period.

Fig. 299 shows a governor test made by Mr. John Sturgess on an 1100 K. W. unit. "The curves were traced by a special Schafer & Budenberg tachometer, the readings being sufficiently magnified to bring out the characteristics of the governor. * * * The load changes and governor movements are platted below. Note that when the whole load was thrown off (at 1:55), the speed accelerated about 8 per cent. in an incredibly short time (under 1 sec.), and the governor had the gate shut in 1.4 secs. after the load went off. * * * It is to be noted that after the first quick result at 2:00 mins. the governor slowly oscillated for about another minute, but with gradually increasing gate opening, the speed and load being practically constant. This was due to the water rising in the forebay, and gradually subsiding in a succession of waves, the governor taking care of these fluctuations, in effective head, in a very intelligent manner."*

"The plant in which these tests were made was by no means a good one from the regulation standpoint, for it will be noticed that when the whole load was instantly thrown off the momentary rise of speed was about 8 per cent, although the governor shut the gate from full open position in the extremely quick time of 1.4 secs. There were five wicket gates, having a total of 96 leaves, and a heavy counter-weight to be moved a considerable distance in this interval. **

242. General Consideration.—Mechanical governors are cheaper than hydraulic, but, assuming the same gate movement, they are less effective at increasing loads since the power to move the gates must be taken as needed from the wheel itself instead of being taken

* See American Society M. E., Vol. 27, No. 4, p. 8.

** Catalogue of Water Wheel Governors, Sturgess Engineering Department of the Ludlow Valve Co., p. 23.

from a storage tank as with hydaulic governors. This is a factor of more or less importance in accordance with the degree of regulation required. The difference is manifest principally at low loads when the energy taken by the governor relay from the water wheel is a considerable percentage of the total energy being generated. As the power exerted by the relay is usually comparatively small, the difference in action from this cause between the two types of governors is often unimportant.

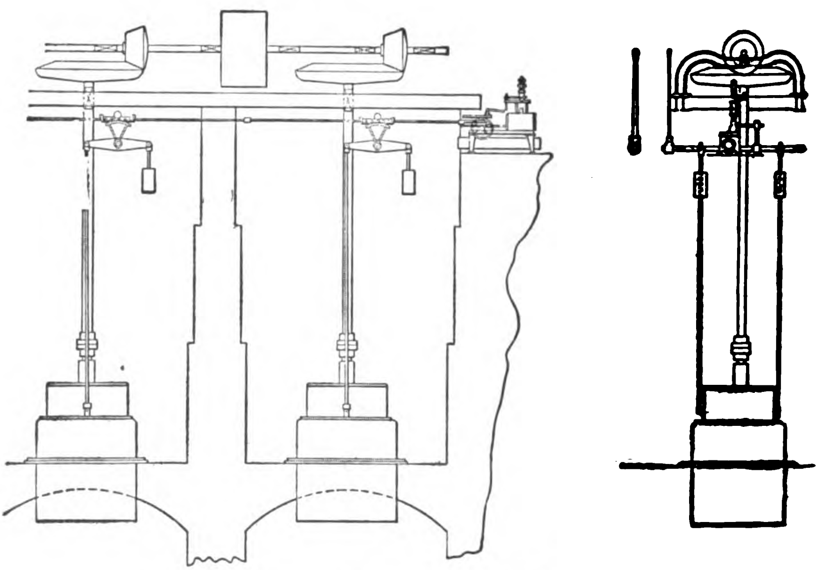


Fig. 300.—Governor Connection by Draw Rods.

The hydraulic governor possesses an additional advantage in its ability to start a stationary wheel into action by means of its stored energy. The mechanical governor depending as it does on the power of the wheel itself is only effective after the wheel has been started by other means.

243. **Control From the Switchboard.**—Electrical devices can now be purchased by which the normal speed of the wheels can be controlled from the switchboard in case the governor is so designed that it can be adjusted while in motion, which is true of most high class machines. It is also possible to start and stop the wheels electrically from the switchboard or from a distant station.

244. Connecton of Governors to Gates.—The following discussion of this subject and the accompanying figures are taken with slight changes, from a paper by Mr. A. V. Garratt.*

" * * * The most successful method of connecting the cylinder gates of several turbines to the same governor is shown in Fig. 300. In this case each pair of drawrods is connected to a pair of walking beams which carry counterweights on their opposite ends. Each walking beam carries a gear sector which engages a rack on a long, horizontal reciprocating member terminating at the governor. The racks on the reciprocating member are "sleeved" on it, and held in place by pins, which may be removed if it is desired to disconnect any turbine from the governor.

"By this method any one, or any combination of turbines, may be handled by the governor or any turbines by hand, at will, by means of a lever shown in the end projection.

"Fig. 301 shows a good method of connecting a governor to a pair of horizontal wicket-gate turbines. It will be noted that the shaft connecting the two gear sectors on the gate stems goes directly to the governor, and is connected to it through a pin clutch which may be opened, and a hand-wheel on the governor may then be used to move the gates by hand. The only improvement on this design which can be suggested would be to eliminate the counter-shaft between the governor pulleys and the turbine shaft by placing the governor beyond the draught-tube quarter-turn, so that the governor pulleys might belt directly to the turbine shaft. The limitations of available space prevented the location of the governor in this manner on the drawing which shows the design used for three units in a modern power plant.

"Frequently the only possible location of the governor prevents anything like direct connection between it and the turbines. In such cases experience has shown that it is wisest to avoid the use of several pairs of bevel gears and long shafts, and in their place use a steel rope drive. This method has great flexibility, and permits of governor locations which would otherwise be impossible. Fig. 302 shows a design of this kind. The governor is located in the only available space, and yet its connection to the turbines is perfectly adequate. The steel rope used is small in size, made of very small wire, especially laid up, and its ends are fixed to the grooved sheaves, which are provided with internal take-ups, so

* See "Speed Regulation of Water Power Plants," by Allan V. Garratt. Cassier's Magazine, May, 1901.

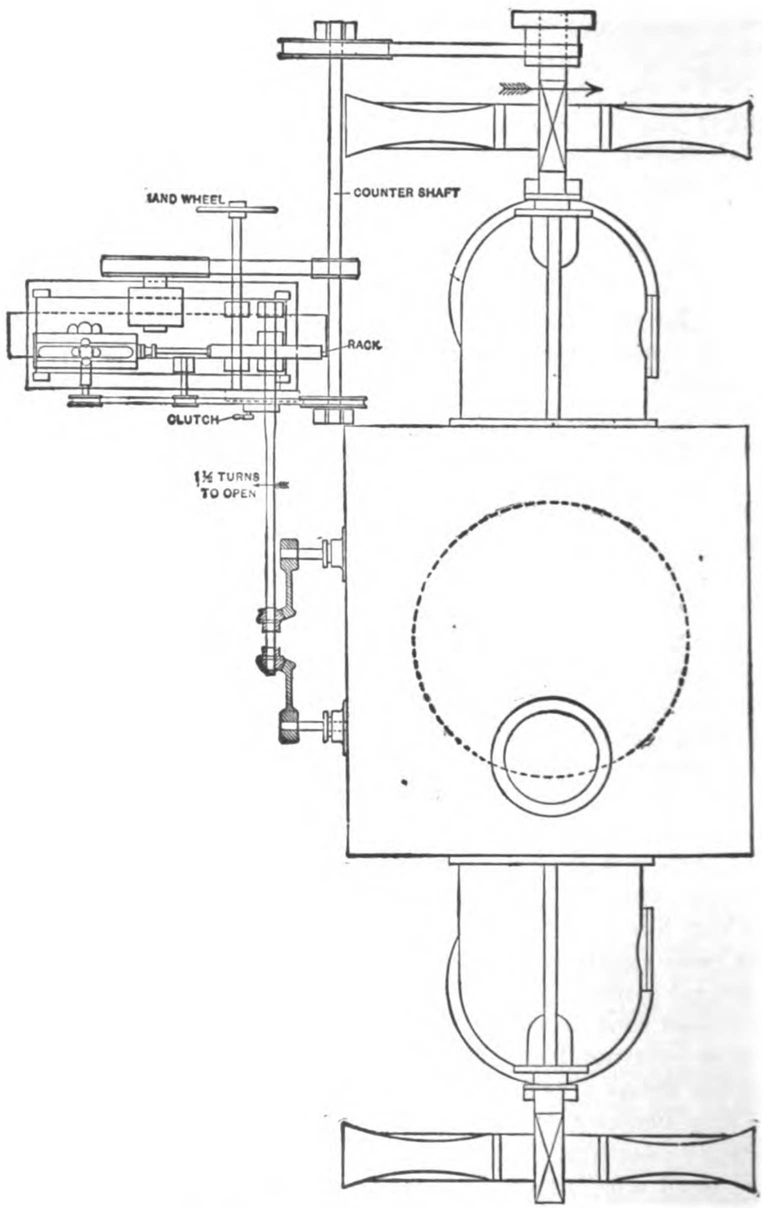


Fig 301—Governor Connection by Shaft and Sectors.

that the rope may be kept tight as a fiddle string. This general method of connection is in successful use in many plants where the requirements for speed regulation are most exacting.

"In the above examples the two ends which have governed the design are simplicity and directness. These two factors should never be lost sight of, and the more completely they are embodied in the design, the better will be the speed regulation. To these two may be added another, and that is freedom from lost motion. These

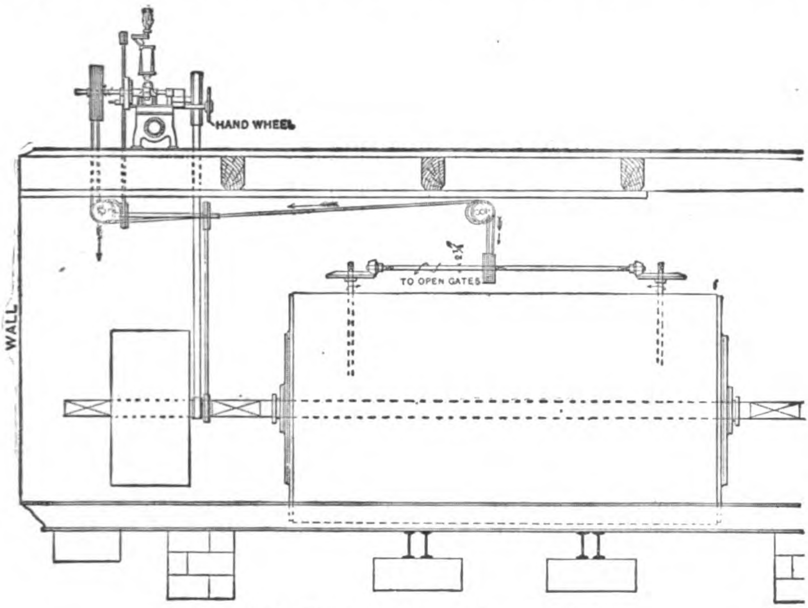


Fig. 302.—Governor Connection by Cable.

three factors are absolutely necessary if successful results are to be expected. The slightest motion of the governor must be transmitted in the simplest and most direct manner, and in the shortest possible interval of time, to the turbine gates."

245. Relief Valves.—Relief valves are very necessary on long feeder pipes and penstocks to avoid excess pressures of an accidental nature as well as those produced by closing of the turbine gates. A group of such valves installed on the end of one of the penstocks of the Niagara Falls Hydraulic Power and Manufacturing Co. is shown in Fig. 303. Relief valves should be arranged to open with a slight excess of the penstock pressure but should close very slowly in order to avoid oscillatory waves. Spring balanced relief valves have proven objectionable for this purpose. If set to open at a small excess pressure they are apt not to close on account of the impact of the discharging water against the valve. In order that they may close, the balancing spring must be so strong that a considerable excess is required to open the valve which does not therefore serve the desired purpose. All types of valves are also hindered by the fact that corrosion is apt to seal the valve so that a considerable excess is required to open it.

246. Lombard Hydraulic Relief Valve.—The Lombard Gov-

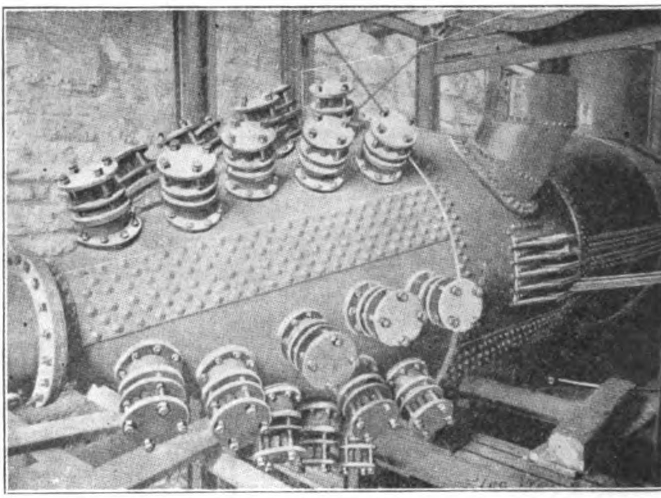


Fig. 303.—Relief Valve on end of Penstock. Niagara Falls Hydraulic Power Manufacturing Co. (Electrical World, Jan. 14, 1899.)

ernor Company have designed a valve in which they claim to have eliminated the difficulties of the spring valve. This valve is shown in Fig. 304* and is described as follows:

"The valve consists of the following parts, viz:—A valve disc, c, capable of motion to or from its seat, b, rigidly connected by means of a rod, i, with the piston, f, in the cylinder, e. The whole valve is bolted to a flange upon the supply pipe, d, wherein the pressure is to be controlled. The area of piston, f, is somewhat greater than that of the valve disc, c, so that when water at the same pressure is behind the piston and in front of the valve there is a positive and strong tendency to hold the valve closed. For the purpose of al-

* Lombard Bulletin No. 101.

lowing the valve disc, c, to open at proper times to relieve excess pressure in the supply pipe, d, there is provided a regulating waste valve, C. This valve is opened or closed by a piston, n, opposed by a very oblong and strong spiral spring, p. Piston, n, is a loose fit in its cylinder, o, so that it moves upward freely in response

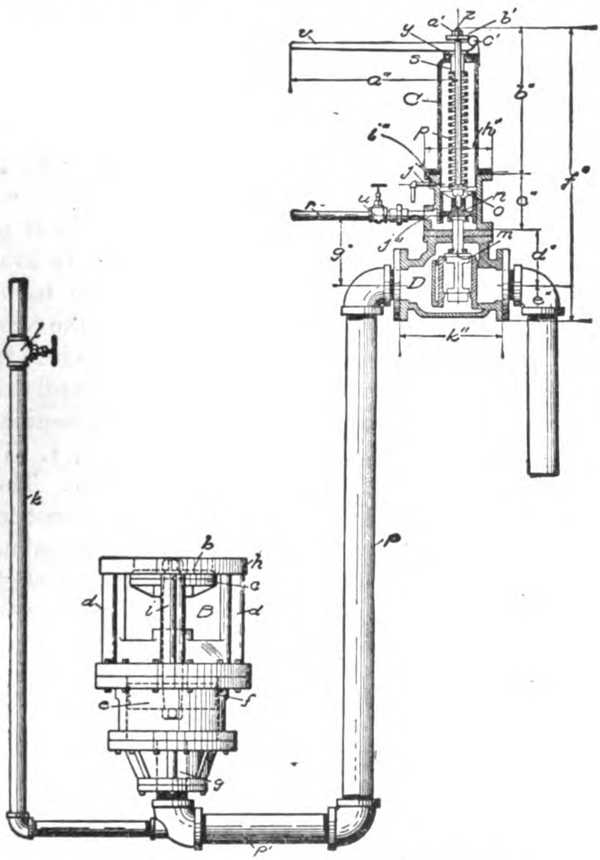


Fig. 304.—Lombard Hydraulic Relief Valve.

to the least excess in pressure upward due to the water in the cylinder, o, opposed to the downward pressure of the spring, p. * * * The piston, n, is connected by means of the stem, m, with a double-seated balanced valve, d, which of course, opens simultaneously with any upward movement of the piston. Water under existing pressure is admitted into the cylinder, e, through the pipe, k, and throttle valve, 1.

"The spring, p, is adjusted by means of the screw, s, and lock-nut, y, so that the effective normal pressure of the water in the chamber is just insufficient to overcome the downward pressure of the spring. The valve, D, will therefore remain closed normally; consequently the main valve disc, c, will also remain closed normally, because water flowing in through the pipe, k, and throttle valve, 1, will produce an excess closing pressure upon the piston, f. When thus adjusted any increase in pressure above the normal will immediately force the piston, n, upward, and will thereby open the balanced valve, D. This instantly relieves the pressure back of the piston, f, which of course then gives way to the superior pressure back of the piston, f, which of course then gives way to the superior pressure in front of valve, c. In this manner practically the whole pressure in front of the valve disc, c, is available for opening it. * * * Valve disc, c, will continue to open until the limit of its travel has been reached, or the pressure in the supply pipe, d, has been reduced to a point where the piston, n, will close the balanced valve, D. Immediately on the closing of balanced valve, D, water begins to accumulate behind the piston, f, flowing in through the throttle valve, 1. This water gradually and surely forces the valve disc, c, to close. The speed of closing is adjustable by the opening through the throttle valve, 1. and may be made as slow as several seconds or even minutes. The closing motion is * * uniform and there is not the slightest tendency to set up vibrations in the water column, a very serious objection to the ordinary types of spring balanced valves which open and close suddenly and are liable in the latter operation to set up water hammer effects even more dangerous than those which they are designed to relieve."

247. **Sturgess Relief Valves.**—The Sturgess Engineering Department of the Ludlow Valve Manufacturing Company makes two forms or relief valves, the "Automatic" and the "Mechanical." The Automatic Relief Valve is shown in Fig. 305 and is described as follows:

"The essential element in the Automatic Relief Valves is a large, very sensitive diaphragm of special construction. This is under the influence of the water pressure in the pipe-line and its movements are communicated to a small pilot valve controlling a hydraulic cylinder, which in turn operates the relieving valve on the relief valve proper. After the pressure in the pipe-line is restored to normal, the relief valve gradually closes automatically.

"The action of this valve is almost instantaneous, and it will fully open on a very small rise of pressure.

"These valves can either be made in self-contained form, or the sensitive parts (diaphragm, pilot valve, and hydraulic cylin-

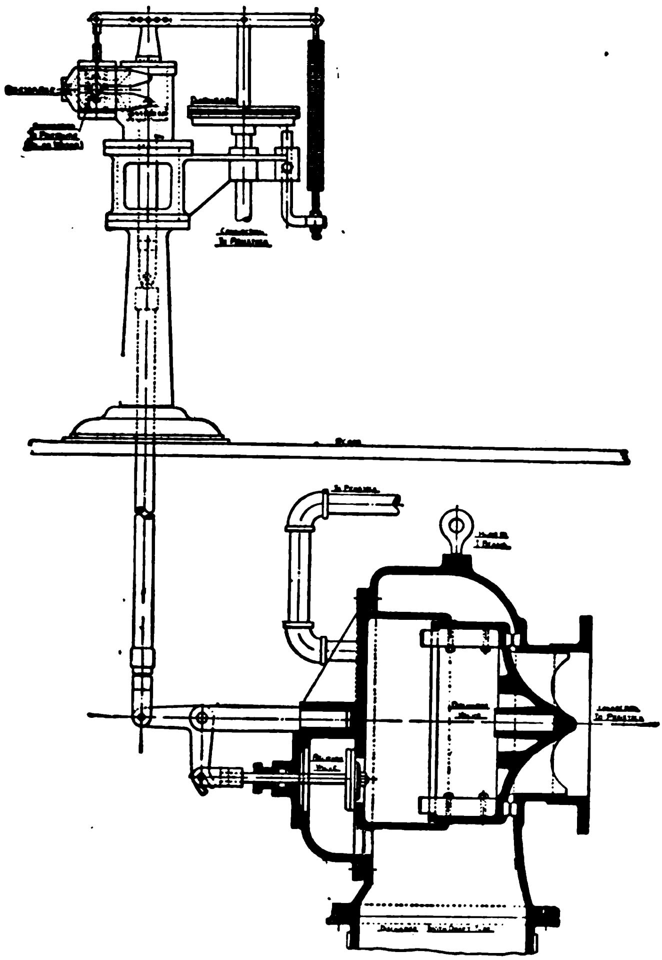


Fig. 305.—Sturgess Relief Valve.

der) may be mounted on a pedestal placed in the power house, and the relief valve proper attached to the penstock or wheel casing, a rod or link being provided to connect the two (as in Fig. 305).

CHAPTER XX.

ARRANGEMENT OF THE REACTION WHEEL.

248. General Conditions.—The reaction turbine may be set or arranged for service in a water power plant in a variety of ways, and the best way may differ more or less with each installation. The arrangement of wheels should always be made with due regard to machinery to be operated, the local conditions that prevail, and especial consideration should be given to securing the greatest economy in the first cost of installation, maximum efficiency and facility in operation, and minimum cost of operation and maintenance.

Impulse water wheels of the tangential type have always been set with their shafts horizontal. An installation with vertical shaft was proposed for one of the first Niagara plants but was not considered on account of the lack of actual experience with such a form of installation. Impulse wheels of the Girard type have been used with both vertical and horizontal shafts. In general, however, because of the high heads under which impulse wheels usually operate, the horizontal shaft arrangement is readily adapted. When an impulse wheel is installed it must be set above the level of maximum tail water, if it is to be operated at all stages of water. The wheel arrangement is therefore dependent principally on the arrangement of the machinery to be operated. By far the greater proportion of such machinery is built with horizontal shafts and hence in most cases where machinery is not special, horizontal shaft arrangements are desirable.

Reaction wheels are often used on streams where the relative variation in position of the tail-water is considerable, and it is both desirable to utilize the full head and to have the wheel set at an elevation at least above the lowest elevation of the tail-water in order that they may be accessible for examination and repairs. By the use of the draft tube this can often be done without the sacrifice of head.. If the wheel must be set below tail-water, gates must be provided for the tail-race with pumps for the removal of the water when access to the wheels is necessary.

The arrangement of reaction water wheels is susceptible only of general classification, which, however, may assist in the understanding of the subject and the selection of the best methods to be adopted under any set of local conditions. Wheels may be set vertically or horizontally, as the conditions of operation demand, without materially affecting their efficiency, provided that in each instance the turbine case, draft tubes, etc., are suitably arranged. The improper design of the setting may materially affect the efficiency of operation in either case.

249. Necessary Submergence of Reaction Wheels.—In order to prevent the formation of a vortex or whirlpool, which will draw air into the wheel and often seriously affect its power and efficiency, it is necessary that the gate openings of the wheel be placed from one to one and one-quarter wheel diameters below the water surface. The head under which the wheel is to operate, however, greatly affects the formation of the vortex. High velocities of flow will facilitate their formation; therefore greater heads will require a greater water covering or other means for the prevention of vortex formation.

As the wheel usually has a greater diameter than the height of the gate it can be set vertically with less danger of air interference than when set horizontally. For this reason the vertical wheels are more readily adapted to low heads and have in the past been more widely used for developments under low and moderate heads.

With both horizontal and vertical wheels the wheel may be protected from the formation of the vortex by a solid wooden float, or may be partially encased or covered with an umbrella-shaped cover the edges of which can be brought below the level of the upper gates of the turbine thus allowing the wheel to be set near the head water surface without the serious interference above mentioned. In all such cases the float or cover must be so arranged as to admit the water to the wheel gates without undue velocity in order to prevent the loss of head. If this is done the efficiency and power of the wheel will not be affected (see Appendix —). Arrangements of this sort were designed by the writer, in the fall of 1906, for the water power plants at Kilbourn and at Dresden Heights.

250. Arrangements of Vertical Shaft Turbines.—Figs. 306 and 307 show twelve typical arrangements of reaction turbines. Figs. A, B, C and D of Fig. 306 show typical arrangements of vertical

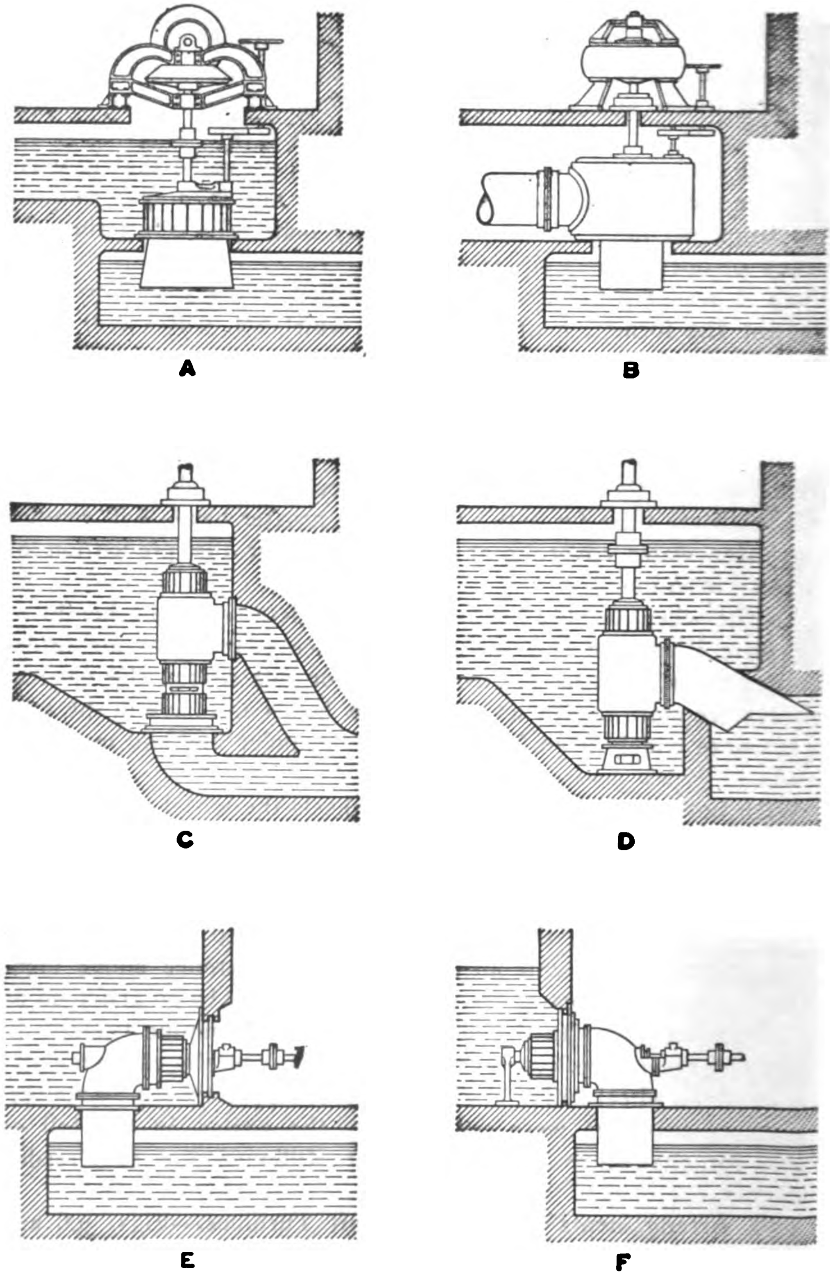


Fig. 306.

wheels. Diagram A is the most common arrangement of the reaction turbine in an open penstock for low head. In this case the wheel is set in a chamber called the wheel pit, the flume, or sometimes the penstock, and is connected with the head race from which it should be separated by gates. The wheel pits in the smaller plants have commonly been constructed of timber; but in the larger plant, they are usually built of a more substantial character,—often of iron or concrete, usually reinforced. Sometimes two or more wheels are set in a single pit; but in the better class of construction, a pit is supplied for each individual wheel or each unit combination of wheels so that each unit can be cut off from its fellows, disconnected from the transmission mechanism to which it is attached, and examined or repaired without interference with the remainder of the plant. Open pits are commonly used for heads up to 18 or 20 feet and may be used for considerably higher heads under favorable conditions.

For higher heads, the arrangement shown in diagram B, or some other form similar thereto, is often found more desirable. In this case closed flumes of steel or reinforced concrete are used, and are connected with the head race by metal, wood, or reinforced concrete pipes to which the term "penstock" is commonly applied. This form of construction permits of the use of vertical wheels with almost any head. In Diagram B the turbine is shown as direct connected to an electrical generator of special design with vertical shaft.

In Diagram A the shaft of the turbine is shown as directly attached to a crown gear which in turn is connected by a spur gear with a horizontal shaft. This horizontal shaft may be direct-connected to a generator as shown in Fig. 325, or may be attached by belting, ropes, cable or other mechanical means with one or more machines which it is designed to operate.

Diagrams C and D show two vertical types of settings of tandem or multiple wheels. Such arrangements are introduced when it is necessary to reduce the diameter of the wheels on account of increased speed, and at the same time maintain the power of installation by increasing the number of wheels for the purpose of direct connection to some machine to be operated .

In all cases where two wheels discharge into a common draft tube sufficient space is necessary between the wheels to prevent interference and consequent loss in efficiency. The arrangement of wheels in this manner therefore requires a considerable amount of vertical space and, under low or moderate head, involves the construction of a wheel pit of considerable depth in order to secure proper submergence of the upper wheel. This arrangement results in the lower wheel being often considerably below the tail-water and necessitates the use of tail gates and a pumping plant to remove the water in order to make the lower wheels accessible. With this design the plant is made comparatively narrow but the greater depth of construction means an additional expense in the foundation work. Vertical wheels of all types involve a design of satisfactory vertical bearings which are usually less accessible than in the case of horizontal bearings which can be placed at an elevation above the power house floor, and are consequently more readily accessible. The stop bearings for single vertical wheels have been long in use and are reasonably satisfactory. The suspension bearing, which is involved in the use of large vertical installations, is not universally satisfactory and, in fact, considerable difficulties have been encountered in so designing a bearing that it will operate without undue expense for maintenance.

251. **Arrangement of Horizontal Turbines.**—Single horizontal wheels of the common type are shown in Diagrams E and F of Fig. 306 and in Diagrams A, B, C, and D of Fig. 307. In each case the gates of the turbine must be readily accessible to the entering water without undue velocity, and the wheel pit, or penstock, must be designed with this requirement in view.

Diagrams E and F, Fig. 306, and A, Fig. 307, show horizontal types of wheels set in an open wheel pit or penstock.

In Diagram E the wheel has the quarter turn set entirely in the pit, and the main shaft passes through a bulkhead in the wall of the station with a packing gland to prevent the passage of water. In this case the water must flow by the quarter-bend and hence, in order to secure sufficiently slow velocity, the wheel pit must be wider or deeper than in the case shown in Diagram F of Fig. 1. Here the gates of the turbine are placed toward the entering water and the flow is interfered with only by the pedestal bearings which, being placed in the center of the crown or cover plate of the wheel, occupy but little space and offer practically no obstruction to flow.

Diagram A of Fig. 307 is essentially the same in arrangement as Diagram F in Fig. 306, except that in this case instead of a metallic quarter-turn and draft-tube, the quarter-turn and draft-tube are constructed in the masonry of the power station and the bulk-

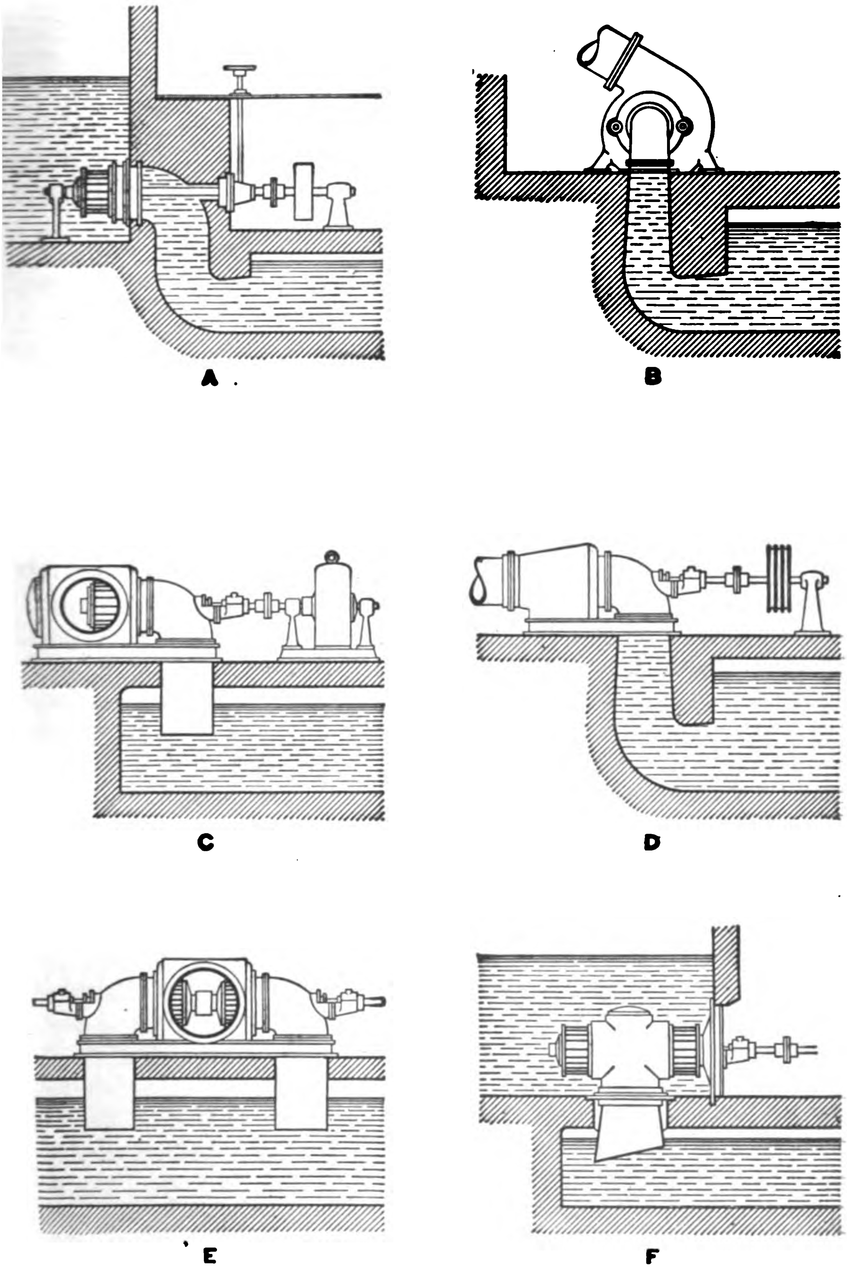


Fig. 307.

head is reduced to simply a packing gland through which the shaft enters the power station.

Diagrams B, C, and D, Fig. 307, illustrate three methods of enclosing a turbine in a closed flume which is connected with the head water by a closed penstock.

In Diagram B the turbine case is spiral, the water enters tangent to the wheel and at right angles to the shaft and is discharged through a metal quarter-bend into a concrete draft-tube.

In Diagram C the water enters the metallic flume in which the wheel is placed at right angles to the shaft, and is discharged through a metal quarter-bend and draft-tube.

In Diagram D the water enters the wheel case parallel to the shaft of the wheel and is discharged through a metal quarter-bend into a concrete draft-tube.

Figs. E and F of Fig. 307 show methods of setting horizontal shaft wheels in tandem. Diagram F is for setting in an open flume or penstock. The two wheels discharge into a common shaft chest and use a common draft-tube. In Diagram E the wheels have a common closed case or flume connected by a penstock with the head waters and each discharges through an independent quarter-turn and an independent draft-tube into the tail-waters beneath. With the closed flume removed, this arrangement can also be used in an open penstock. These diagrams are simply typical of various possible arrangements of wheels that can be adapted with various modifications of detail to meet the local requirements of the engineer for any hydraulic plant which he may be called upon to design.

252. **Classification of Wheels.**—The classification of the arrangement of wheels as shown in Figs. 306 and 307 may be reviewed briefly as follows:

In this review reference is given to various figures in the preceding and following text in which the type of wheel described is illustrated with more or less modifications.

1st. Vertical single wheel, open wheel pit. (See Diagram A, Fig. 306, also Figs. 329, 331, 333 and 334.)

2nd. Vertical single or tandem wheels in metal casing connected by cylindrical penstock with supply. (See Diagram B, Fig. 306, also Figs. 132, 181, 310, 311.)

3rd. Vertical tandem wheels,—two or more wheels in open pit. (See Diagrams C and D, Fig. 306, also Figs. 134, 138, 173, 339.)

4th. Horizontal turbine, open wheel pit, quarter-bend and draft-tube within wheel pit,—quarter bend of metal. (See Diagram E, Fig. 306.)

5th. Horizontal turbine, open wheel pit, quarter-bend, and draft-tube exterior to pit,—quarter-bend may be of metal or concrete construction. (See Diagram F, Fig. 306, also Diagram A, Fig. 307 and Figs. 314, 322.)

6th. Horizontal turbine in spiral case at end of penstock, single or double draft-tube. (See Diagram B, Fig. 307, also Figs. 159, 162, 338.)

7th. Horizontal turbine in cylindrical or conical case at end of penstock. (See Diagrams C and D, Fig. 307, also Fig. 335.)

8th. Tandem horizontal turbines in open wheel pit, single discharge through common or independent draft tubes. (See Diagram F. Fig. 307, also Figs. 315, 319 to 324 inclusive.)

9th. Tandem horizontal turbine in enclosed cylindrical case with common penstock and common or independent draft-tubes. (See Diagram E, Fig. 307, also Figs. 13, 140, 152, 317.)

253. **Vertical Wheels and Their Connection.**—The vertical setting of single wheels is usually the cheapest in first cost, which fact is an important factor that has been largely instrumental in the adoption of this arrangement in most of the older plants. Vertical wheels are most commonly set in open wheel pits. They may, however, be set in a cast iron or steel casing which is then connected to the headrace or dam by a proper penstock. Single vertical wheels can be connected to the machine they are to drive by various means. Belting, transmission ropes, cables, and shaftings, are in common use for such connections. The shaft is usually placed horizontally and is connected by a crown beveled gear and pinion to the wheel. Frequently belts, ropes, and cables are connected by pulleys or sheaves to a short horizontal shaft driven in the same manner. When the power of a single vertical wheel is insufficient, two or more may be harnessed by gearing to a line shaft which may be directly connected to the machine or machines to be operated, or otherwise connected as convenience and conditions may require.

254. **Some Installations of Vertical Water Wheels.**—Figs. 329 to 332 inclusive, show the plans, elevations, sections, and details of a small plant of vertical water wheels designed by the writer for the Sterling Gas, Light and Power Company of Sterling, Illinois. The details of this plant are clearly shown by the illustrations and will be discussed at some length later. This plant is located on the

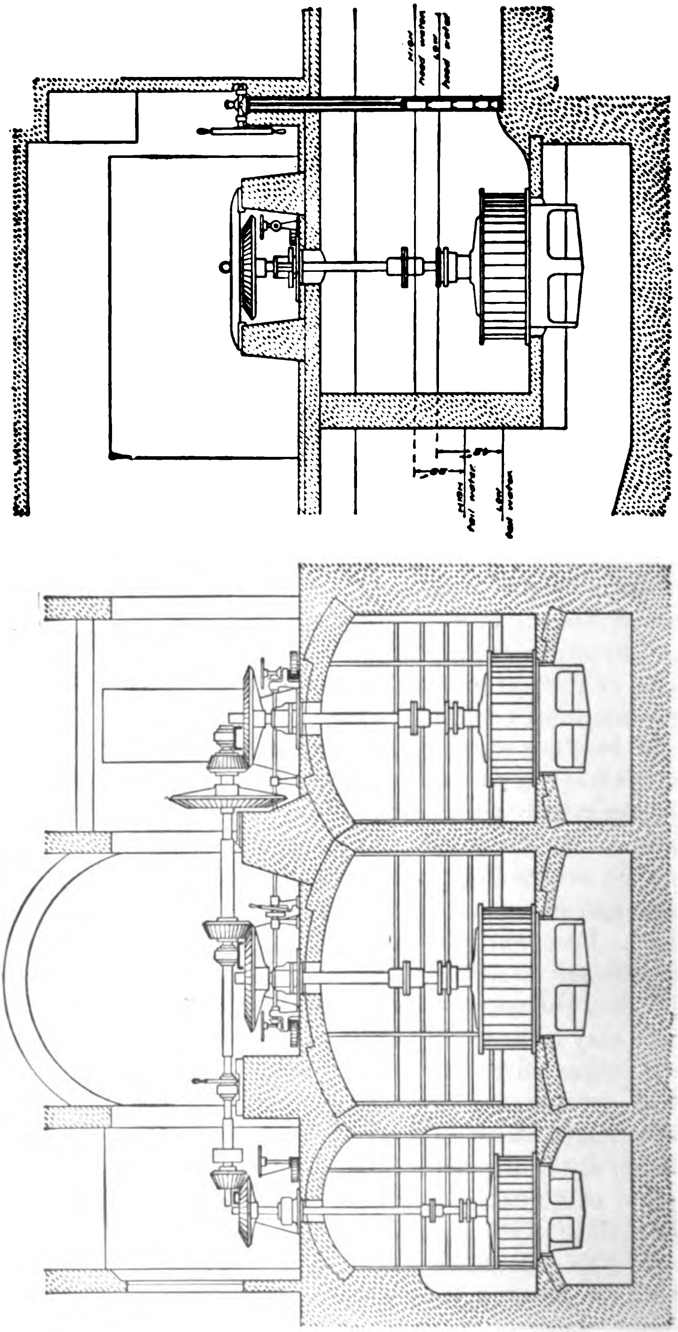


Fig. 308.

Sterling side of the Rock River (See Fig. 345) and is next to the last plant on the Sterling Race. The head developed is about eight feet and the power of each wheel is about 115 h. p. under this head. Each wheel of the installation is set in an independent pit or penstock which can be closed by means of a flume gate. The wheels are connected to a common shaft extending into the power house and connected with pulleys and belts to the generator.

The plan of the South Bend Electric Company at Buchanan, Michigan, is of similar type and is shown on page 544, Fig. 334. The main shaft is here connected with ten turbines and is in turn directly connected to an electric alternator.

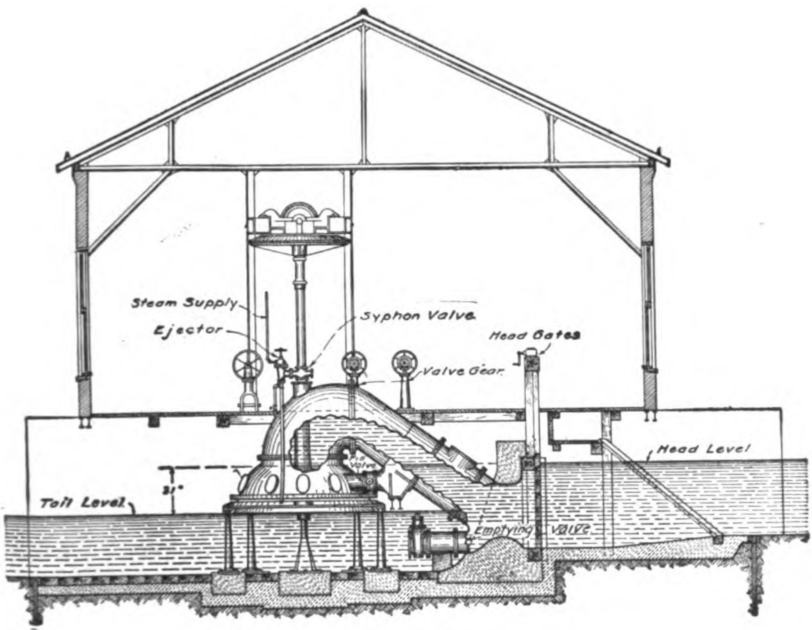


Fig. 309.—Low Head French Water Power Plant.

The adaptability of the vertical shaft turbine to low head is well shown in Figs. 308 and 309. Fig. 308 shows three turbines manufactured by The Trump Manufactnring Company of Springfield, Ohio. These turbines are 61, 56 and 44" respectively, and by suitable gearings are connected with a common shaft. These wheels were installed at Bologna, Italy, and operate under a low water head of 42" and under a high water head of 28". It was necessary to set the wheels considerably below the level of the tail water in order that the turbines should have a sufficient submergence for operation. Fig. 309 is a similar plant installed at Loches, France. In this case the water is conducted to the turbines by means of a syphon supply pipe in order that the turbine might be placed high enough above tail-water that it be accessible at all times without the use of a tail-gate. Air is exhausted from the crown of the syphon by use of a steam ejector whenever the plant is to be started up. This plant operates under the low head of thirty-one inches and is said to work very satisfactorily.

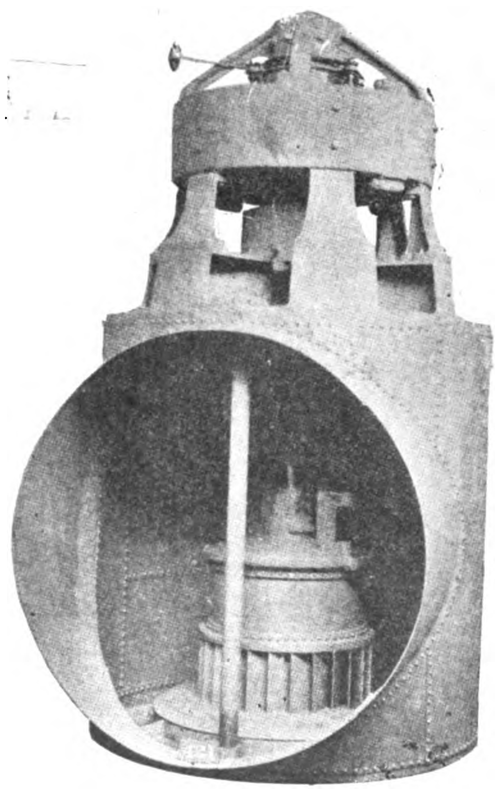


Fig. 310.

Fig. 310 shows a vertical shaft turbine of the Victor cylindrical gate type manufactured by The Platt Iron Works. This wheel is set in an independent case with provision made for the attachment of a cylindrical penstock conducting the water from the head work to the wheel. This figure shows a special design by which the special generator is set on columns resting directly on the wheel case.

Fig. 311 shows the plant of Trenton Falls, New York, of the Utica Gas and Electric Company. The wheel is a Fourneyron turbine, manufactured by The I. P. Morris Company, operating under a 266 foot head, the water being conducted to the wheel through a penstock the length and arrangement of which are shown in Fig. 353. The wheel is provided with a draft-tube and is regularly connected with the generator above. The moving parts of both machines are carried by a vertical shaft bearing, shown in cut.

255. Some Installations of Vertical Wheels in Series.—In the last three illustrations wheels are shown of sufficient size and operat-

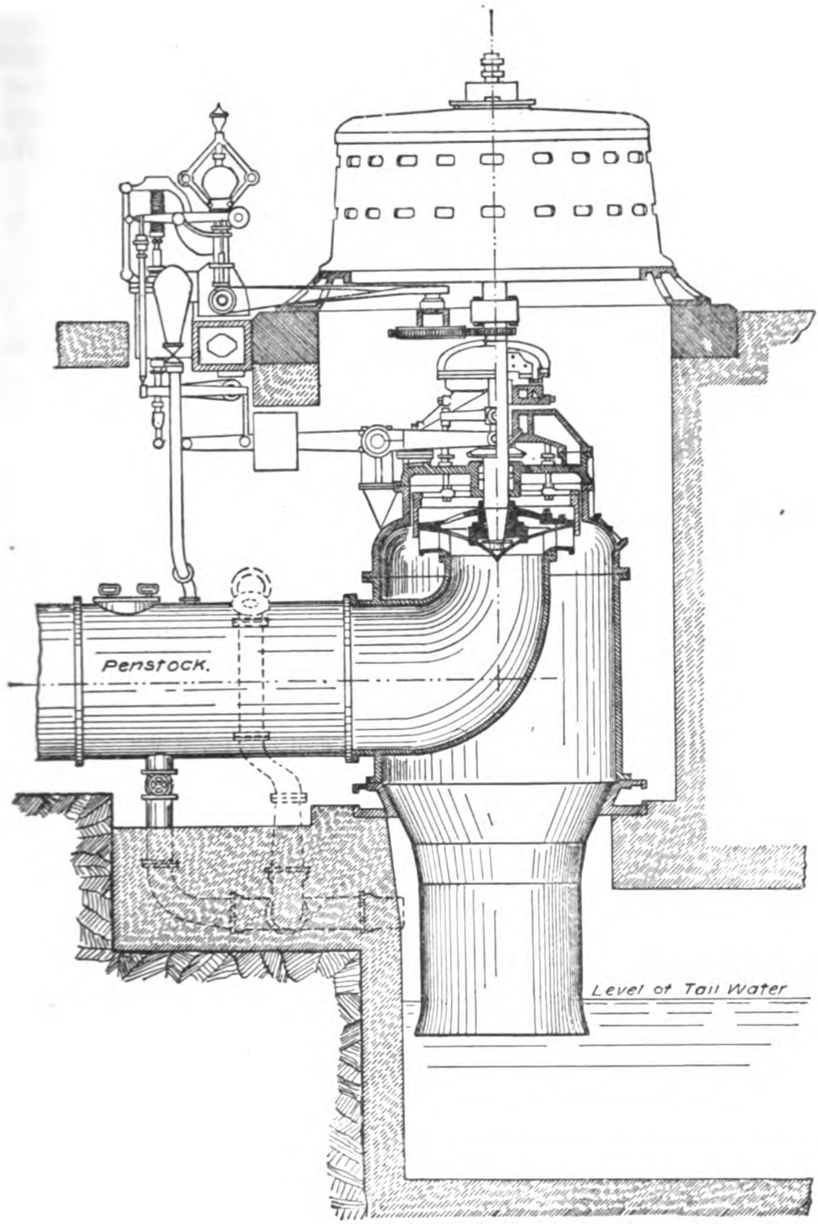


Fig. 311.—The Trenton Falls Plant of the Utica Gas and Electric Co. (I. P. Morris Co.)

ing under sufficient head to be suitable for the independent operation of the machine attached to them. In many cases, however, especially with low head, the arrangement shown in Fig. 308 and in Figs. 325 to 329 inclusive, becomes necessary. In such cases considerable loss is entailed by the use of shafts, gearings, and belts.

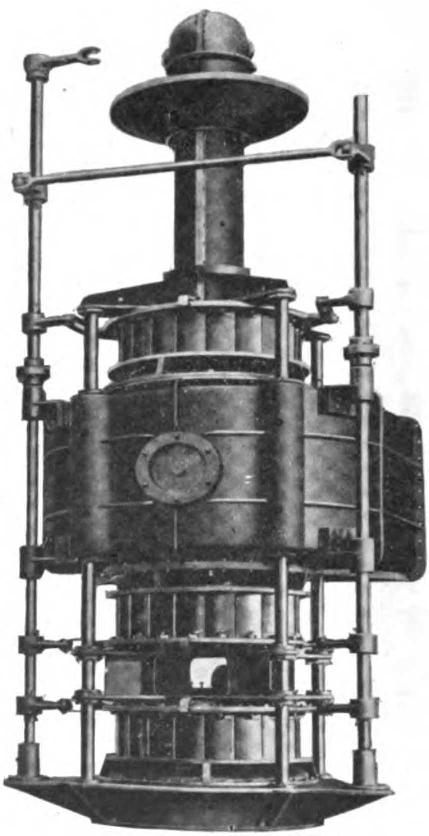


Fig. 312.—Vertical Turbine for Sewall's Falls Plant of the Concord Electric Co.

These losses are so large that it is desirable to avoid or reduce them if possible. For this purpose vertical wheels are sometimes placed tandem as shown in Diagrams C and D, Fig. 306. This type of plant is also illustrated by Figs. 312 and 313 which are illustrative of wheels installed in the plant of the Concord Electric Company, at Concord, N. H.

Fig. 312 shows tandem wheels for this plant as designed and manufactured by The Allis-Chalmers Company of Milwaukee, Wis., and are described in further detail on page

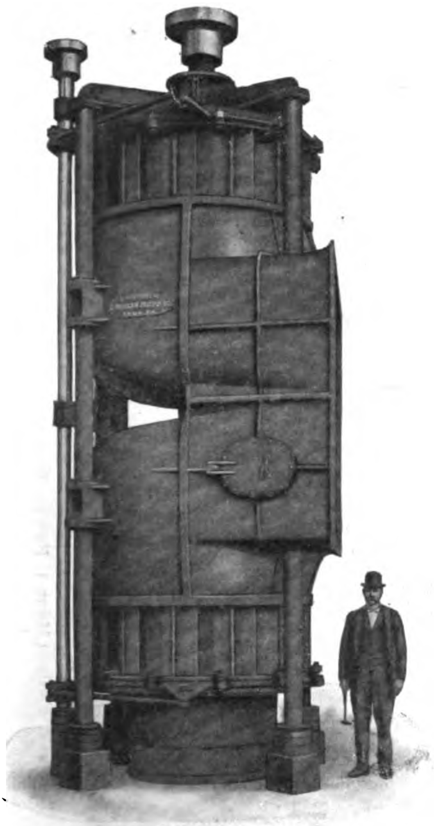


Fig. 313.

Fig. 313 is a view of a double vertical unit, designed and built for the Concord Electric Company by The S. Morgan Smith Company of York, Pa. This form of installation has the advantage of a greater concentration of the machinery This type of installation, while quite common in Europe, is somewhat new in this country and presents several novel and desirable features.

256. Some Installations of Horizontal Water Wheels.—Most machines to be operated by water wheels are built with horizontal shaft, and, as a direct connection of wheels to the machinery to be operated involves a minimum loss in power and consequent greater efficiency than with the various complicated arrangements often necessary with vertical wheels, the horizontal wheel becomes desirable and is adopted whenever practicable in a modern water power plant. The type of such a plant is well illustrated by the power plant at Turner's Falls, Massachusetts, shown by Fig. 314. The single horizontal wheel, direct-connected to the machinery to be operated, is perhaps already snfficiently described in the preceding pages. The arrangement of two or more wheels for such purposes deserves careful consideration. Figs. 315 and 316 show a plan and section of a double unit, for use in an open penstock, as manufactured by The Dayton Globe Iron Works Company of Dayton, Ohio. These figures show a plain,

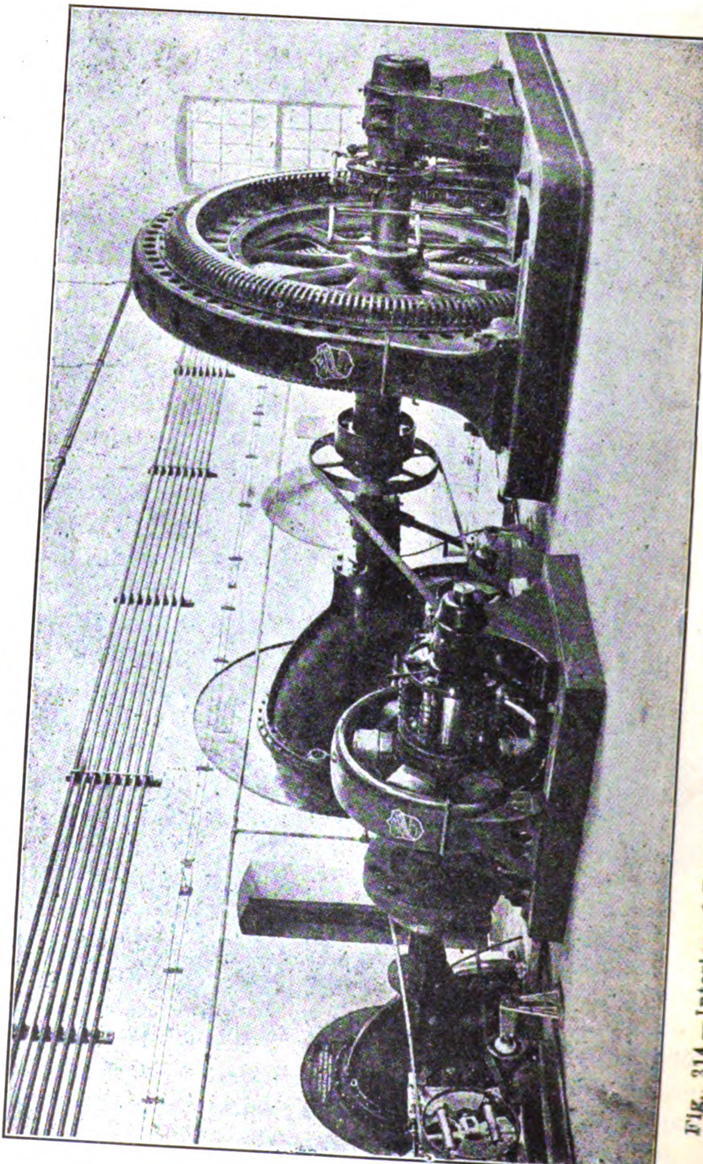


Fig. 314.—Interior of Turner's Falls (Mass.) Power Plant. (Allis-Chalmers Co., Milwaukee, Wis.)

cylindrical, draft-chest connected with a common draft-tube. The details of the arrangement can perhaps be better seen from the half-tone, Fig. 320, which illustrates two of these units conneted together tandem.

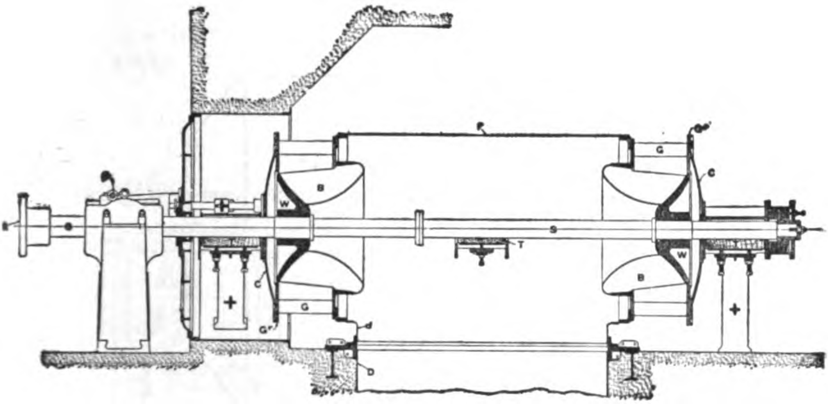


Fig. 315.—Section Double Wheel with Common Draft Tube. (Dayton, Globe Iron Works Co.)

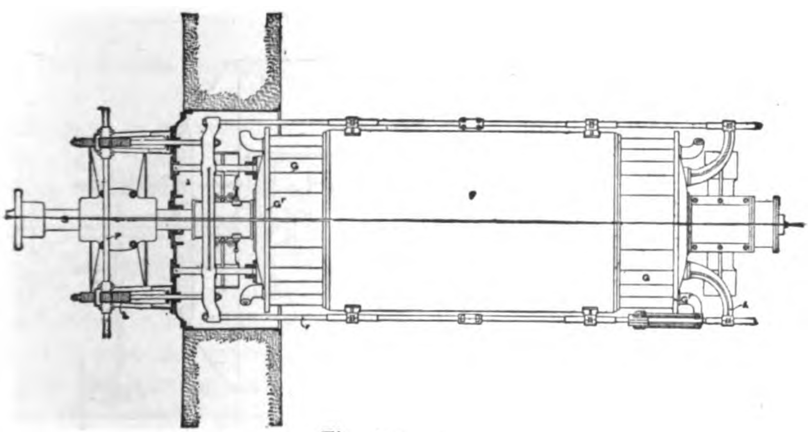


Fig. 316.—Plan.

Figs. 317 and 318 show a similar double unit manufactured by the same company. This unit is shown set in a closed flume for connection by a penstock of suitable size with the head works. In Fig. 318 the chest, into which the turbines discharge, is designed so as to give a certain independence to the discharge of the two turbines until they come to the draft-chest below the wheel. The turbine case, shown in Fig. 316, seems to have more room than necessary in the upper portion of the case in which interference of the two streams and much eddying are possible, all of which is obviated in the the design, shown in Fig. 317. The writer knows of no experiments which show conclusively that such loss actually occurred. More information is needed along this line than is now accessible to the engineer.

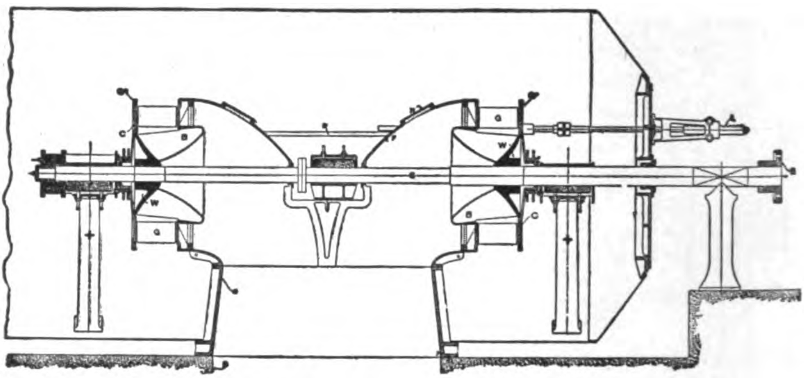


Fig. 317.—Double Horizontal Turbine in Closed Penstock (Dayton Globe Iron Works Co.)

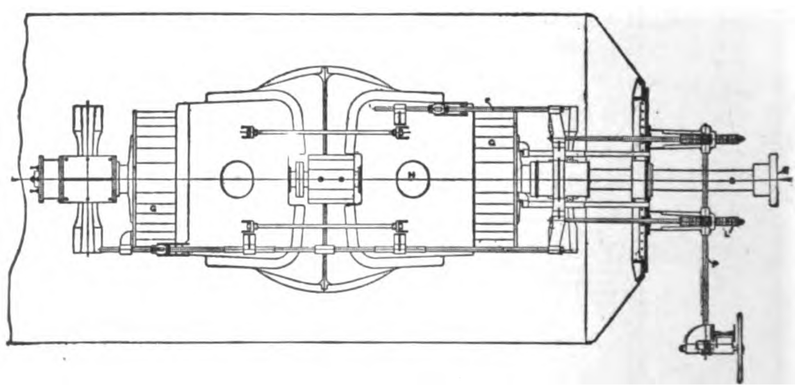


Fig. 318.—Plan.

Fig. 319 is a cross-section of a double unit of the Samson turbine, manufactured by The James Leffel and Company of Springfield, Ohio. This shows a design in which careful attention is given to the maintenance of a uniform and slowly decreasing velocity from the time the water reaches the wheel until it passes from the common draft-chest into the draft-tube below.

257. Some Installations of Multiple Tandem Horizontal Wheels. —Two double units of the wicket gate type, similar to the double units shown in Fig. 315, are illustrated by Fig. 320. These turbines were manufactured by The Dayton Globe Iron Company of Dayton, Ohio, and are shown with the upper portion of the case removed so that the arrangement of the wheels and the gate mechanism are clearly visible. The gates are moved by a cylindrical ring to which

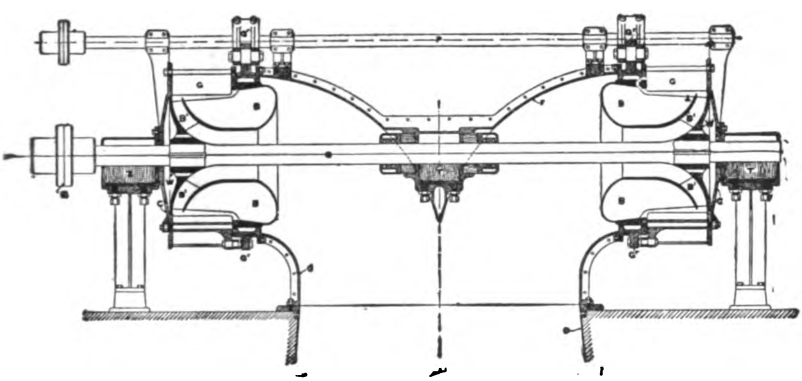


Fig. 319.—Double Horizontal Turbine for Open Penstock. (James Leffel & Co.)

each gate is attached independently. The ring is moved by the link connecting the gate ring to the governor rod which, by its rotating, opens or closes the gate as the power needed requires.

Two double units with cylindrical gate, as manufactured by The S. Morgan Smith Company of York, Pennsylvania, are shown in Fig. 321. The bulkhead casing and the coupling to which the machinery to be operated must be attached, are shown at the left. In this case the governor rods have a horizontal movement, the upper rod moving backward and the lower forward in order to open the cylinder gate.

Figs. 322 and 323 show a section through one of the main units and a plan of the power house and turbines of The Southern Wisconsin Power Company now under construction at Kilbourn, Wisconsin, on the designs and under the supervision of the writer. This plant consists of four main units, each generator having a capacity, at full load, of 1650 kilowatts and an overload capacity of 25 per cent. Each unit is direct-connected to six 57″ turbines now under construction by The Wellman-Seaver-Morgan Com-

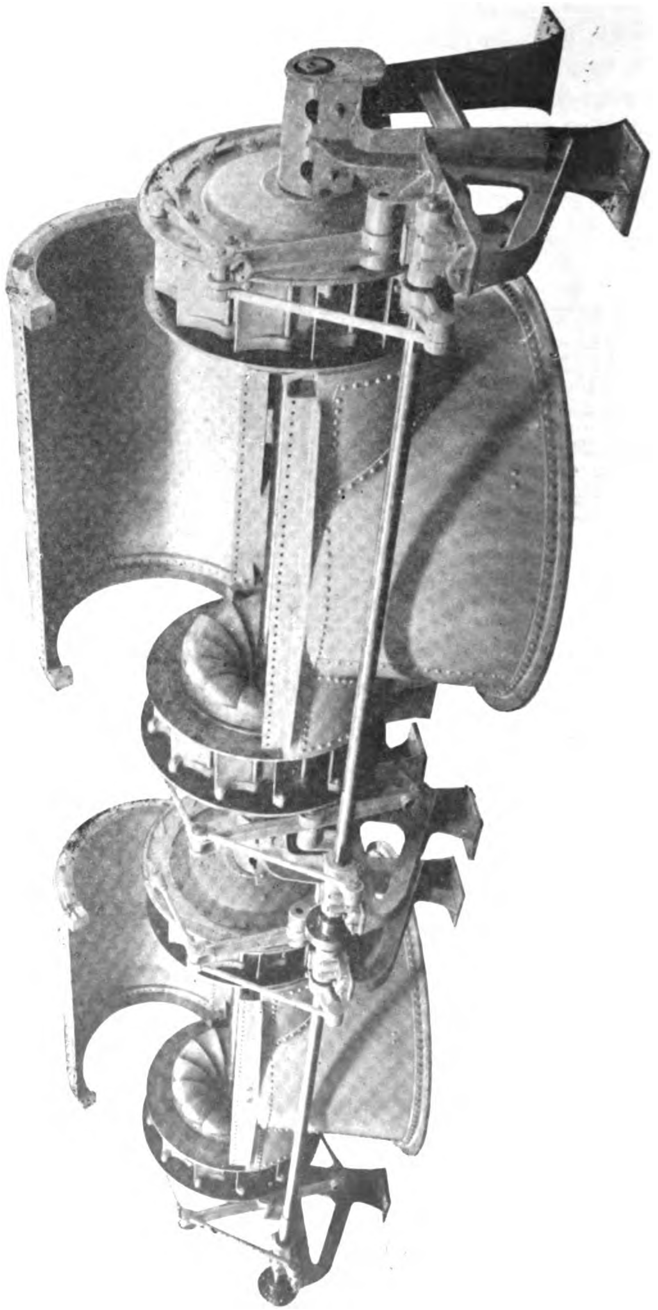


Fig. 320.—Two Double Turbine Units Tandem with Upper Part of Draft Chest Raised. (Dayton Globe Iron Works Co.)

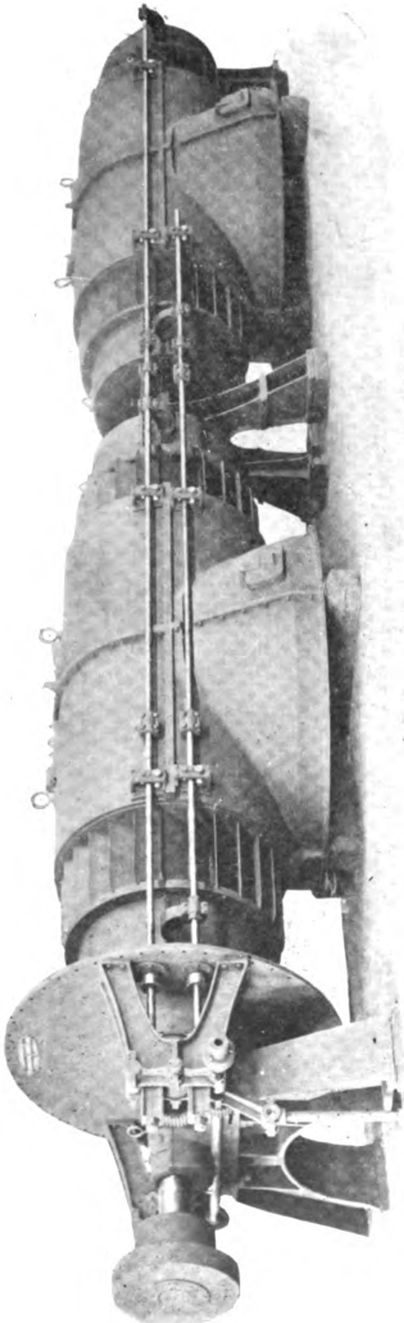


Fig. 321.—Two Doube Turbine Units Tandem. (S. Morgan-Smith Co.)

pany of Cleveland, Ohio. Each turbine unit is set in a separate penstock controlled by three independent sets of gates. The four center wheels discharge in pairs into common draft-tubes, while the two end wheels have independent draft-tubes. All of the hearings within the flume are accessible by independent wrough iron manhole casings.

Fig. 324 shows four pairs of 45" Samson horizontal turbines manufactured by The James Leffel and Company of Springfield, Ohio. These wheels have been installed for The Penn Iron Mining Company of Vulcan, Michigan, where two such units are now in operation. Eight similiar units, designed to deliver 1400 H. P. under 14 foot head, are now under construction by The James Leffel and Company and are to be installed in the plant designed by the writer for The Economy Light and Power Company at Dresden Heights, Illinois, the general arrangement of which is shown by Fig. 350.

When the head increases above 20 or 30 feet, it may become desirable to convey the water from the head-work by means of a closed penstock as shown in the case of the plant of The Winnipeg Electric Railway Company (See Fig. 340).

In this plant are shown four wheels in tandem, direct connected to a generator. The bell-mouthed entrance to the penstock should be noticed, also the air inlet pipe which is designed to admit the air into the penstock when the same is to be emptied, and to admit the water gradually and without shock when it is again filled. When the head becomes still higher the closed pemstock becomes imperative as in the case with The Shawinigan Water and Power Company's plant shown in Fig. 338 where a head of 135 ft. is utilized. Similar arrangements and connections for single and double wheels with penstock are those of The Dodgeville Electric Light and Power Company, shown in Fig. 337, and that of The Hudson River Power Company's plant at Spier's Falls, as shown in Fig. 335.

The plant of the Nevada Power and Mining Company shown in Fig. 341, involves tangential wheels operating with needle nozzle and discharging freely into the tail race below.

In the selection and installation of reaction wheels a considerable latitude in the choice and details of arrangement is possible and it is only after a careful examination and consideration of all the conditions of installation that the correct size, speed, and arrangement of the wheels can be obtained. Numerous failures, more or less serious, in the past have fully shown the fact that

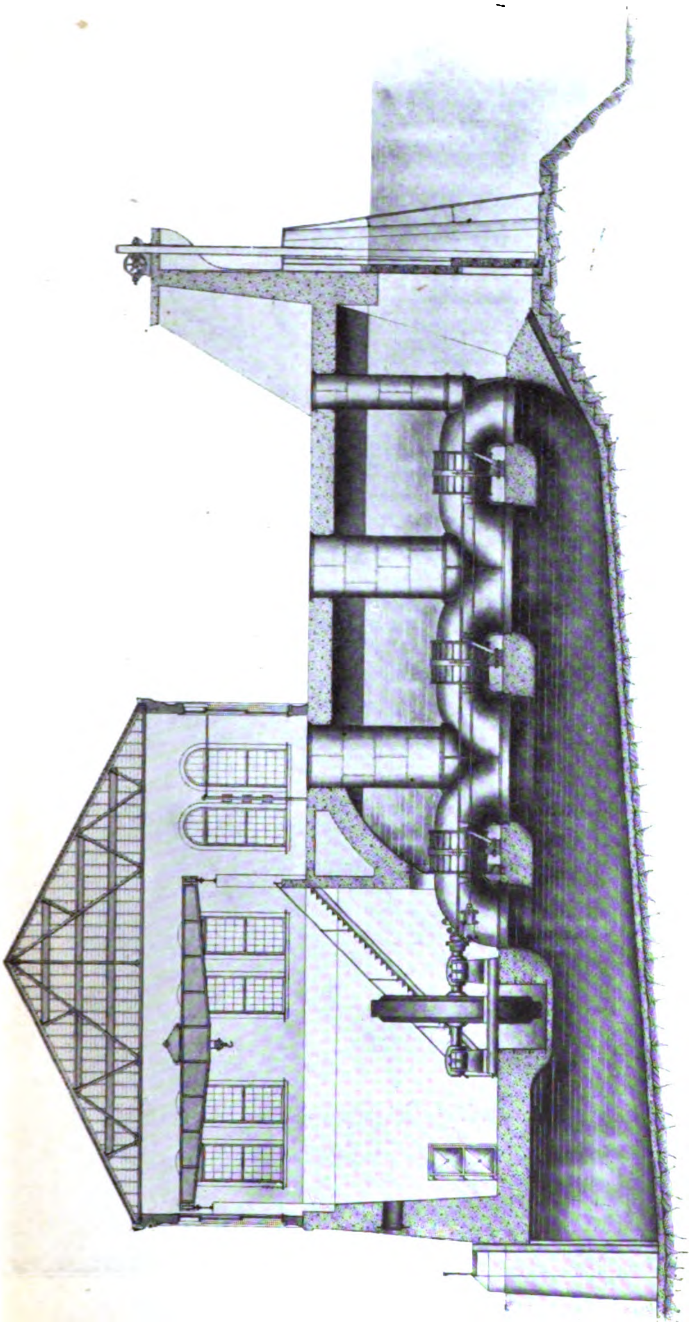


Fig. 322.—Section of Kilbourn Plant, Southern Wisconsin Power Co.

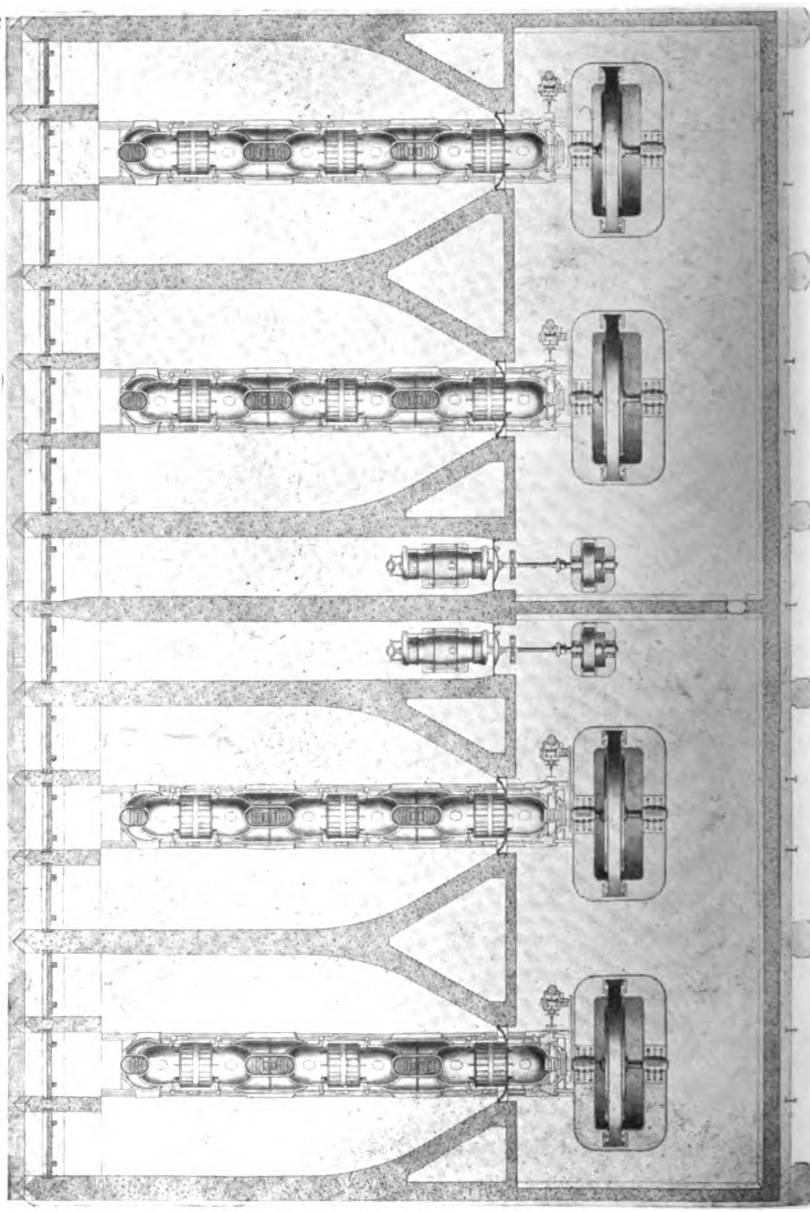


Fig. 323.— Power House of Southern Wisconsin Power Co., Kilbourn. Horizontal Turbines Connected Tandem in Open Penstock.

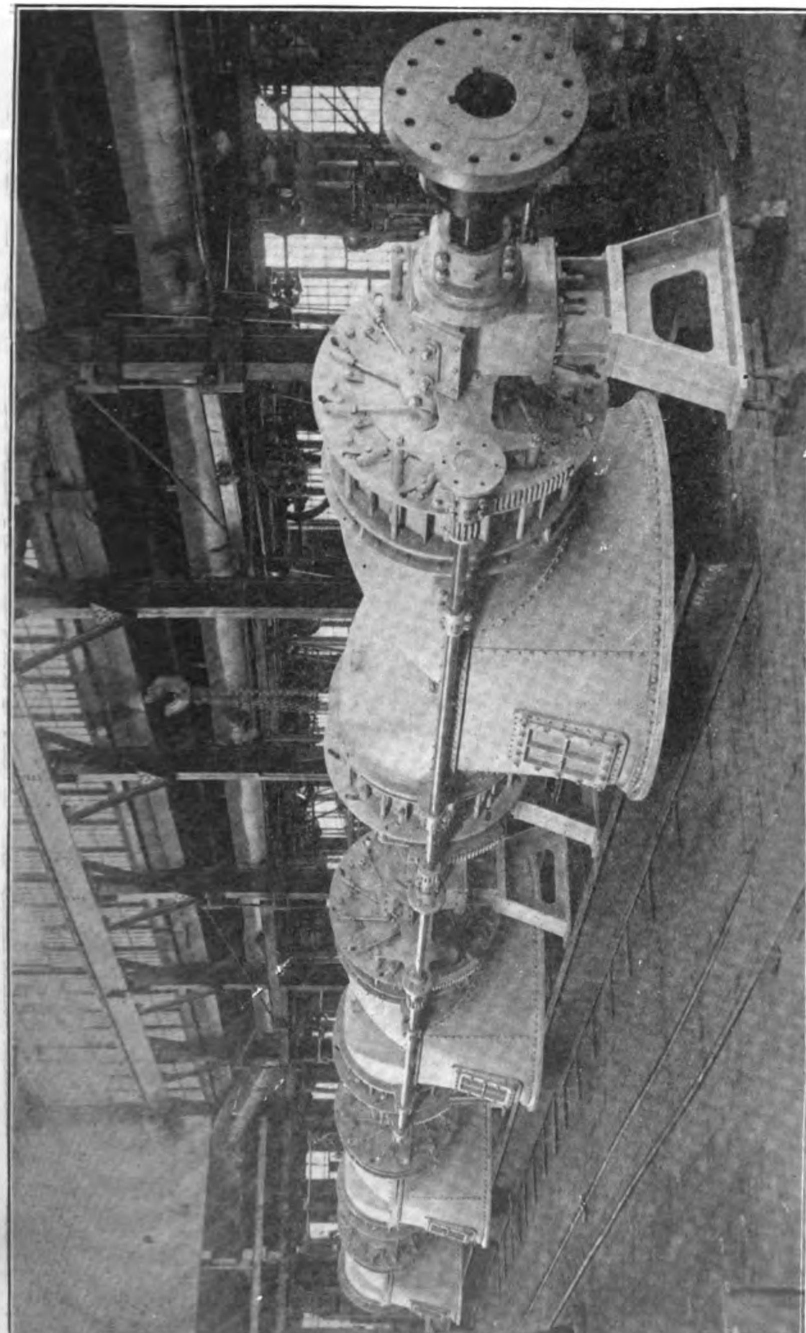


Fig. 324.—Four Pairs 45-Inch Samson Horizontal Turbines. 1400 H. P. Under 14 Foot Head. (James Leffel & Co.)

this work demands the most careful attention and investigation of the engineer and should be attempted only after the most thorough study and mature deliberation.

258. Unbalanced Wheels.—In installing horizontal wheels it is usually desirable to use them in pairs with two, four, six or eight turbines in tandem. It is, of course, possible to introduce an odd number of wheels and this is frequently done where it seems to be desirable. There is an advantage is an even number of wheels for in this case the wheels may be, and should be, so arranged as to balance the thrust by the union of a right hand and left hand wheel in each pair. Where an odd number of wheels is introduced, an unbalanced condition arises which can only be taken care of by a thrust-bearing which, at the best, is an additional complication often unsatisfactory and should be avoided if possible.

There is another cause of unbalanced condition which may be here mentioned. If a pair of wheels is so joined together as to use a common draft-tube then, on starting the wheel, the vacuum formed in the draft-tube is common to both wheels and therefore balanced. If, on the other hand, the wheels have separate draft-tubes, when the wheels are started a partial vacuum is commonly created in one of the draft-tubes in advance of the other, or even when the wheels are in operation the vacuum in one draft-tube is not as great as in the other, creating thereby a thrust in one direction or the other which must be balanced by the connection of the two draft-tubes by an air pipe or must be taken up by a thrust-bearing as in the case of a single wheel.

CHAPTER XXI.

THE SELECTION OF MACHINERY AND DESIGN OF PLANT.

259. Plant Capacity.—The selection of machinery for a power plant depends upon numerous conditions. In the first place, for permanent and constant operation, the machinery must be so selected that its total capacity shall be great enough to take care of the maximum load and have at least one unit in reserve so that if it becomes necessary to shut down one unit for examination or repairs, the plant will still be capable of carrying the maximum load for which it was designed.

The desirable reserve capacity of any plant depends on the contingencies of the service or the degree of liability to disabling accident involved in the operation of any plant, and on the relative cost of such reserve capacity and the damages which might be sustained if the plant should at any time become disabled as a whole or in part and incapable of furnishing all or any part of the power for which it was designed. In many manfacturing plants the occasional delays caused by the entire suspension of power on account of high or low water, or for the necessary repair to machinery, are not serious if cheap power is available for the remainder of the year. For the operation of public utilities, and the furnishing of light and power for diverse municipal and manufacturing purposes, the matter becomes more serious and necessitates a sufficient duplication of units to practically assure continuous operation.

For paper mills and other manufacturing purposes water powers are utilized in which the head and consequent power is practically destroyed during high water conditions. For continuous and uninterrupted service such powers are available only with auxiliary power that can be used during such periods. In the same manner reserve capacity may be unnecessary, desirable or absolutely essential as the importance of maintaining uninterrupted power increases.

260. Influence of Choice of Machinery on Total Capacity.—A study of the week day load curve of The Hartford Electric Light

Company as shown by Fig. 257, page 422, will show that the load for December, 1901, represents the maximum load which that plant was called upon to carry during the year, and, consequently, was the maximum load for which the machinery must have been selected. A considerable variety of unit sizes would be possible which would fill the requirements of this load curve to a greater or less extent. The maximum or peak load shown in December, 1901, was about 3,000 k. w. If a single machine were selected of 3,000 k w. capacity for regular operation, then, in order to have one unit in reserve, it would be necessary to purchase two 3,000 k. w. machines or a total capacity of about 6,000 k. w. If, on the other hand, machinery should be purchased with units of 500 k. w. capacity each, it would be necessary to have six of such units in order to carry the maximum load of 3,000 k. w., and a seventh unit of 500 k. w. capacity would be all that would be needed for the reserve. This would give a total capacity to the plant of 3,500 k. w., giving the capacity of the machine purchased some 2,500 k. w. less than the plant first discussed.

261. Effect of Size of Units on Cost.—The cost of machinery is not in direct proportion to its capacity. The larger machinery is somewhat less in price per kilowatt capacity than the smaller machinery. Hence the cost of the last plant suggested would be more than 35/60 of the cost of the first plant. On the other hand, the installation of such a large number of units complicates the plant and is undesirable. For this plant it would therefore be desirable to select five units of 750 k. w. capacity each, or four units of 1,000 k. w. capacity each, giving in one case a total plant capacity of 3,750 k. w. and in the other case of 4,000 k. w.

A plant having units of 750 k. w. or 1,000 k. w. capacity each would have a less total kilowatt capacity and, consequently, a less first cost compared with a plant having units of 3,000 k. w. capacity. Such a plant would also have a less number of units and consequently less complication in the arrangement than a plant having units of 500 k. w. capacity.

262. Overload.—In the above consideration no mention is made of overload capacity. The ordinary direct-current machinery can be operated at about 25 per cent. overload for short periods of perhaps one hour at a time without danger to the machinery. Alternating machinery can be operated at 50 per cent. overload at similar times or at 25 per cent. overload for two hour periods. In consequence of this condition it is frequently possible to purchase machinery of considerable less capacity than the total load would indicate, depending on the overload capacity of the machine for short periods of maximum load. Unless, however, the estimated load curve covers all possible contingencies for maximum power it is desirable to retain this overload capacity as a provision for a second condition which has not been fully covered in the estimate of the daily load curve; or, in other words, it is desirable to retain the overload capacity as a factor of safety.

263. Economy in Operation.—A second matter that needs the careful consideration of the engineer in the selection of machinery is the question of economic operation under variation in load. A reference to the efficiency curve of most machines will show that the machine will operate most efficiently at some particular load, usually some .75 to full load, and will perhaps give the best results at from .75 to 1.25 load, or to 25 per cent. overload. It therefore becomes important to so select machinery that it will operate efficiently at all conditions of load.

An examination of the load curve of The Hartford Electric Light Company for the full week day load in March, June, September and December, will show that for securing the most efficient results at all times in the day, and at all times in the season, units of 500 k. w. capacity would apparently be the best. Such units would take care, efficiently, of the minimum loads that occur at 6:00 A. M., between 12:00 and 1:00 P. M., and at about 7:00 P. M. At such times one of these units would operate efficiently; but in most cases the period at which it could be operated singly would be for a few minutes only, or perhaps for an hour at the most, when the additional unit would have to be cut in. A 750 k. w. generator would operate with almost as great an efficiency at these times and it would, with its overload capacity, take care of the load for a much greater period of time each day. The 1,000 k. w. machine would perhaps fulfill these requirements even to a greater degree. While it would be less efficient at the minimum point of the load, it would have the advantage of operating singly for a much wider range of load and the additional advantage that, as a rule, the larger the machine the higher the full load efficiency curve.

The complications resulting from the numerous machines, and the losses entailed thereby, have also to be considered and must be carefully weighed in this connection.

The circumstances of operation and many local conditions, which appertain particularly to the plant in question, must be weighed in connection with the selection of this machinery. There is no definite law by which the selection of machinery for any plant can be reduced to an exact science, and several combinations of machinery are possible in almost any plant and will give reasonable satisfaction.

In the above discussion only units of a uniform capacity have been considered and it is usually desirable, other things being equal, to have similar machines so that a minimum number of repairs and duplicate parts may be kept in stock. On the other hand, if a long, low night load is probable, it may be desirable to install one or more units of a capacity suitable to carry such load efficiently.

264. Possibilities in Prime Movers.—A third matter for the careful consideration of the designing engineer is the possibility of a prime mover that is to be used for operating the machines in question. If a steam or gas engine is to be used as the motive power, there is a wide range of selection in speed, capacity, and economy of such machinery, and, as a general rule, the prime mover may be selected to conform to the generator or other machine that is to be operated thereby. In the selection of water wheels for prime movers the conditions are radically different and the selection of the size and capacity of the units to be operated is often modified or controlled by the waterwheels and the conditions under which they will be obliged to operate.

In the selection of the water wheel one of the most important matters is the head and the range of heads under which the wheel will be called upon to operate. While it is possible to select a wheel so that it will operate at almost any reasonable speed under a considerable head, yet the capacity or power of the wheel rapidly decreases in amount with the speed, and if the speed be too high it will be necessary to join two or more wheels in tandem in order to furnish the power necessary to operate the machinery selected. This is perfectly feasible and is done in a great many cases.

265. Capacity of Prime Movers.—It is important to note that if the generator or other machinery to be operated is to be operated under overload conditions, the maximum power to be generated must be kept fully in mind in the selection of a prime mover. In the case of steam engines, these engines can be commonly operated under overload conditions. They are usually rated at their most efficient capacity and can sometimes be operated to 50 per cent. above their normal rating, although their economy under such conditions is apt to materially decrease. Gas engines, on the other hand, are commonly rated at very nearly their full capacity and hence the machinery which they are to operate can be operated only to about the normal rated capacity of the engine.

Water wheels are commonly rated in the catalogues of manufacturers at very nearly full gate and consequently at full power. In some cases they are rated at about seventh-eighths gate so that a small margin of additional power is availalble. In the selection of a water wheel, therefore, it is important that a careful study be made of the actual power that the wheel can generate under full gate and at minimum head. This should be sufficient to operate the machinery at its maximum load.

266. The Installation of Tandem Water Wheels.—The installation of two wheels set tandem, either horizontally or vertically, and directly connected with the machine by a common shaft, is very common and this may be increased to four, six, or occasionally to eight turbines. Every additional machine, however, involves the introduction of increased diameter in the shaft, of additional bearings which must be set and held in alignment, and a complication in the design and construction of the machinery which should be avoided wherever possible. The excuse for the attachment of a number of turbines in tandem arrangement, and the complexity of the plant of water wheels installed, lies in the simplification of the machinery to be operated by them, and in the design and arrangement of other portions of the plant. The extent to which the application of any principle is to be carried is a matter of judgment and can be answered only by experience and the consideration of all of the conditions involved in each particular case.

267. Power Connection.—With the turbine, as with every other prime mover, it is important to convey the power to the machine or machinery to be operated as directly as possible. The turbines should be connected as directly as possible to the machinery to be driven without any unnecessary intervention of gearing, shafting, bearings, belts, cables, or other still more complicated methods of power transmission. Every shaft, every gear, every belt, every bearing and every other means of transmission that intervenes between the power generated in the wheel and the machine in which the power is to be utilized means an extra loss and a decrease in the efficiency of the plant. The machine to be operated should, therefore, whenever practicable, be direct connected to the shaft of the turbine instead of being connected with the turbine by any intermediate mechanical means. (See Figs. 310, 314 and 322–

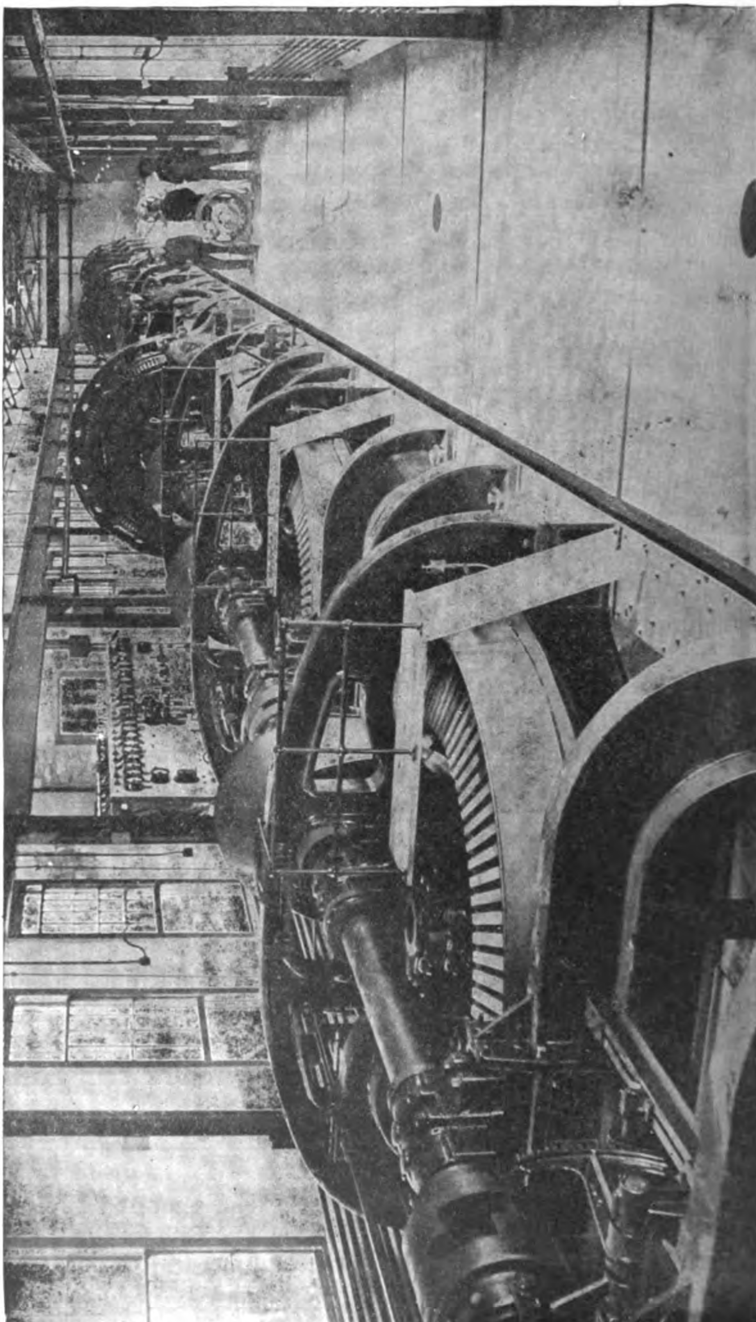


Fig. 325—Wheel Harness—Oliver Power Plant, South Bend, Ind.

Direct connection of machinery and turbine involves a careful selection of both machinery and turbine so that both will work satisfactorily at the same number of revolutions per minute. This frequently involves extra expense that may not be justified in plants for many purposes.

Other methods of connection or of power transmission are, therefore, frequently necessary. With many low head installations direct connections are impracticable for a number of reasons. Sometimes various machines with diverse revolutions are to be driven by the same wheel and the revolutions of the turbines installed must differ from some or all of the machinery to be operated and some form of connection other than the direct must be used. Even where the importance of the plant makes it desirable to use direct connection, it frequently happens that a single turbine gives an insufficient power at the speed desirable for connection to a machine of the desired capacity. Under such conditions it is necessary to unite two or more turbines in order to generate sufficient power for the purposes for which the plant is to be designed. The necessity of using a large number of turbines in a single unit may give rise to very long shafts and a large number of bearings, and the loss due to such an arrangement is sometimes considerable, and if poorly arranged will be almost or quite as inefficient as gearings and shafting well maintained.

268. Various Methods of Connection in Use.—The most common form of turbine used is a single vertical turbine, connected by a beveled crown gear and pinion to a horizontal shaft. Several of such turbines are commonly coupled up to the same shaft and may be set in a single or in separate wheel pits. Such types of installation are shown in Figs. 329 to 334. Fig. 325 shows the turbine harness in the plant of The Oliver Plow Works at South Bend, Indiana, installed by The Dodge Manufacturing Company. The arrangement of the wheel is quite similar to that illustrated by Fig. 334. Three or four vertical wheels are here each connected by a gear and pinon with a horizontal shaft, which, in turn, is connected to an electric generator. In all such cases more or less energy is lost in transmitting the power through the gearing and numerous bearings to the generator. Sometimes it is found desirable not to connect the generators directly with the main shaft, but to connect the generator or other machines to be operated by the power plant by belting them to driving pulleys attached to the same horizontal shaft, as shown by Fig. 326, which shows the power

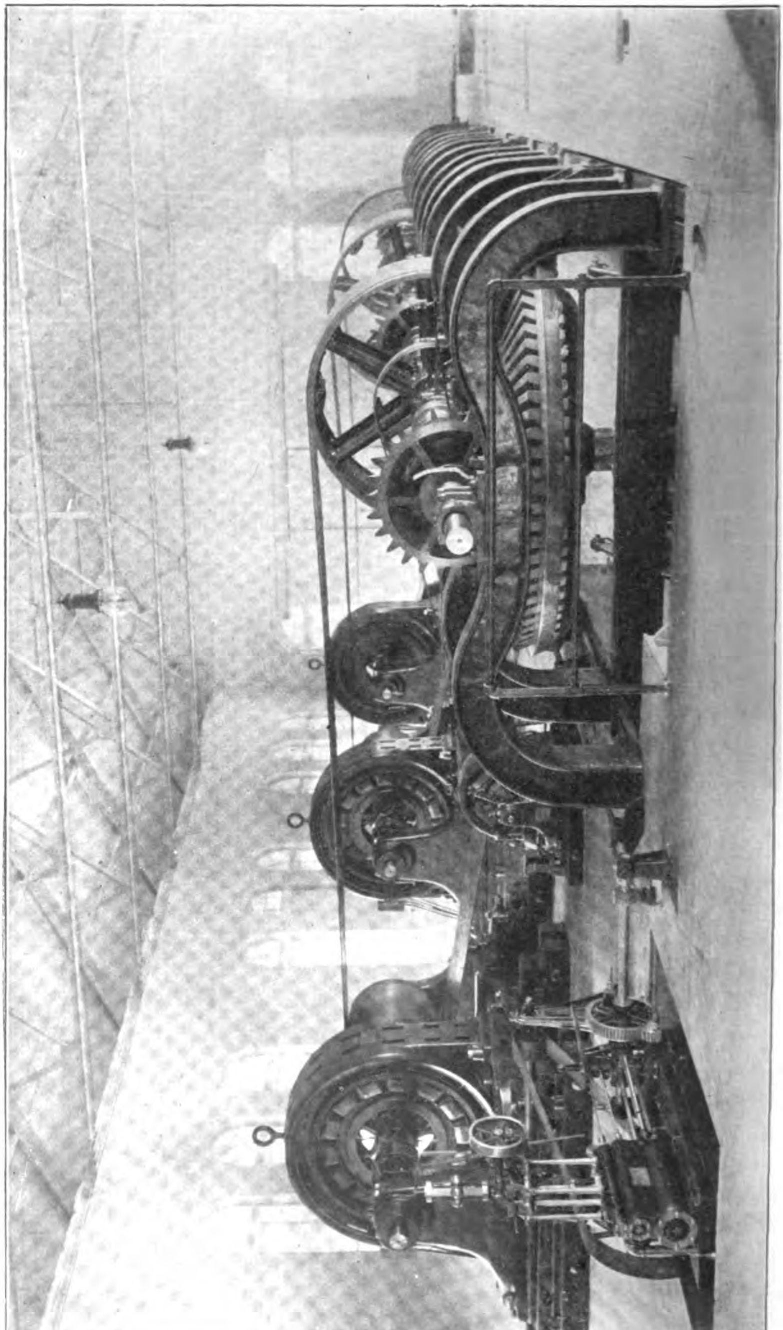


Fig. 326—Power Plant of the Trade Dollar Mining Co., Silver City, Idaho.

plant of The Trade Dollar Mining Company near Silver City, Idaho. This, however, introduces another source of loss through these belts but possesses a certain flexibility due to the ability to thereby drive various small units at a variety of speeds by the simple process of changing the diameter of the pulleys used to drive such machinery. Sometimes rope drives can be used to advantage in place of

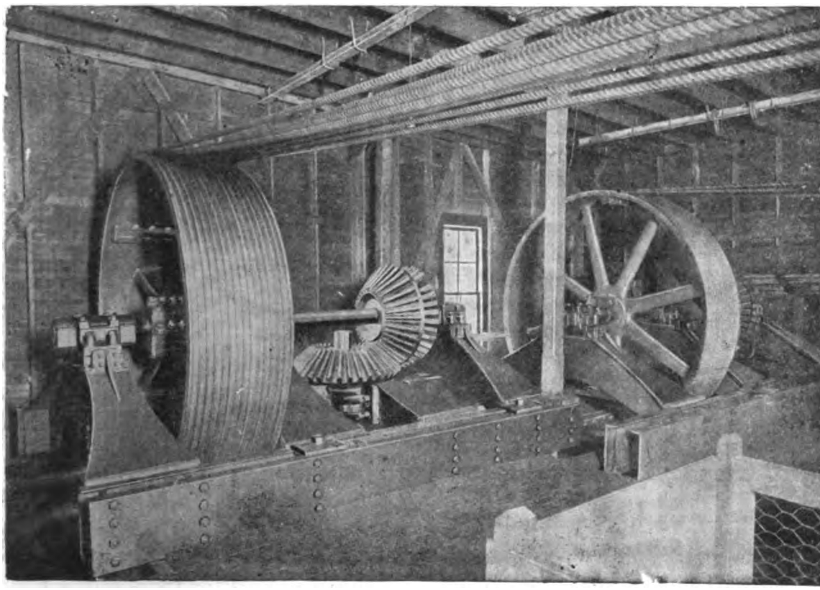


Fig. 327.—Harness and Driving Sheaves, Southwest Missouri Light Co., Joplin, Mo.*

belts. This is especially true where the distance is great or the alignment other than direct. Examples of such connections are shown by Figs. 327 and 328.

Direct connected plants are shown in Figs. 310, 314, 322, 335, etc.

269. Use of Shafting.—A shaft connecting a machine to a prime mover, or imposed in any manner in any power transmission, must be carefully designed and constructed. It must be carefully aligned and have its bearings carefully adjusted. Each bearing may be considered as a point in the alignment of a shaft, and, as two points determine the direction of a straight line, it will be seen that each additional bearing is objectionable for it increases the difficulty of obtaining and maintaining a satisfactory alignment. When more than two bearings are used each must be brought and maintained in

* Dodge Manufacturing Co., Mishawaka, Ind.

the best practicable alignment, both horizonally and vertically. All bearings must be of sufficient size that the limit of bearing pressure shall not exceed good practice and they must be sufficiently adjustable so that the shaft shall have as complete and uniform bear-

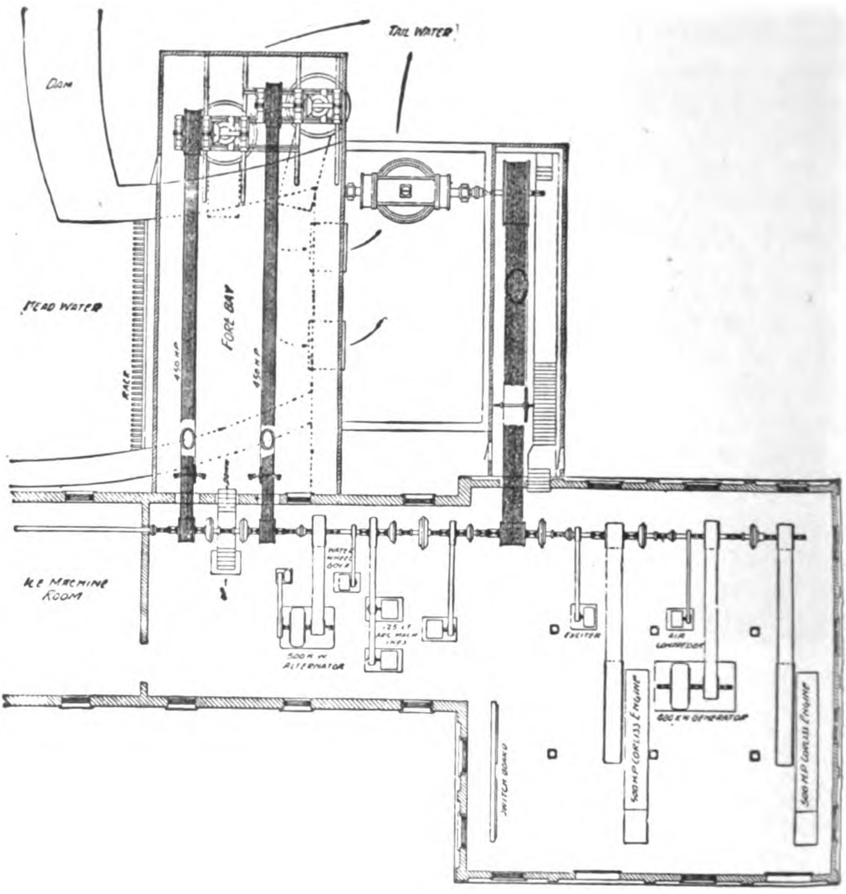


Fig. 328.—Plan Showing Harness, Rope Drive and Jackshaft. Southwest Missouri Light Co.*

ing as possible over the entire surface of the box. Boxes and bearings must be arranged for satisfactory lubrication so that under the hardest service they will not become unduly heated. In order to secure good results the best class of workmanship is necessary and it is also necessary that the plant shall be carefully and prop-

*Dodge Manufacturing

erly maintained. A poor shaft, running in poor boxes, poorly aligned, may consume most of the power generated. Shafting, to be reasonably satisfactory, demands frequent and proper inspection, constant lubrication, and proper maintenance or it will soon become a source of great energy loss.

270. The Wheel Pit.—The wheel is usually set in a chamber called the wheel pit, flume, or sometimes the penstock, which is connected with the head-race from which it can be separated by suitable gates.

The wheel pit in the smaller plants has commonly been constructed of timber but in the larger plants is usually built of a more substantial character,—of concrete, plain or reinforced, stone or iron.

Open pits are commonly used for heads up to 18 or 20 feet, and may be used for considerably higher heads; however, for higher heads, closed flumes of reinforced concrete or steel are commonly used, and such construction is usually connected with the head-race by metal, wood or reinforced pipes, to which the term penstock is commonly applied. This latter form of construction admits of the use of wheels with heads of almost any height.

A number of wheels can be set in the same wheel pit, and are commonly so set, especially where they are used together to operate one machine. It is frequently desirable, however, to separate the turbines and set them in separate pits so that one or more wheels can be shut down at any time without interfering with the operation of the plant. The exent to which this arrangement is carried is a matter of policy and depends upon a variety of conditions which the engineer must settle for each particular case.

271. Turbine Support.—The arrangement and construction of the wheel pit must be such as to furnish a proper support for the turbine in order to secure satisfactory operation. In many of the earlier plants, the wheel pits were built of timber, with the turbine case resting directly on the timber floor, which was often improperly supported. The result of such conditon has been that the turbines settle out of alignment and much energy is expended in undue friction in the transmitting mechanism. The floor or foundation on which the wheel case rests should be of a substantial character and of such a nature that it will not readily deteriorate and allow the wheel to settle. It is usually desirable to support the wheel by a column directly below the wheel case, which should rest upon substantial foundations below the bottom of the tail-race.

(See Fig. 331) In all events settlements and vibrations must be prevented or reduced to a minimum in order to eliminate one of the very important causes of loss which is frequently encountered in water power plants. In many cases, due to defects of this kind, water power plants are giving efficiencies of 50 per cent. and below, where 75 or 80 per cent. should be obtained.

272. Trash Racks.—The water entering the wheel pit from the head-race commonly passes through a trash rack consisting of narrow bars of iron, usually ¼" by 3" in dimension, spaced 1½" to 2" between and reaching from above the head-waters to the bottom of the wheel pit, the purpose of which is to strain out such floating matter as may be brought by the current down the head-race and which, if not taken out at this point, might float into the wheels and if large and heavy enough, might seriously injure the same. These racks have to be raked or cleaned out at intervals depending on the amount of leaves, grass, barks, ice or other floating matter in the stream. In water power plants on some streams where large amounts of such floating matter occurs at certain seasons, it is sometimes necessary to keep a large number of men constantly at work keeping the racks clear.

The accumulation of material on the racks will sometimes shut off the entire flow of water if attention is not given to keeping them clear; hence it is sometimes necessary to so design the racks and their supports that they may sustain the entire head of water.

The racks are usually made of bar iron held apart by spools between each pair of bars and held together by bolts passing through the spools and joining together such a number of bars as may be convenient for handling. The spools should usually be placed near the back of the bar so as to allow the rake teeth to pass readily. The rack should be situated at an angle so as to afford facilities for raking. The deeper the water, the greater should be the inclination, as with long racks, and especially with high velocities, the clearing of the racks becomes more difficult.

Chain racks and automatic mechanical racks have been attempted but without satisfactory results.

Where trouble occurs from ice, involving much winter work, it is frequently desirable to cover the racks with a house in order to protect the workmen.

CHAPTER XXII.

EXAMPLES OF WATER POWER PLANTS.

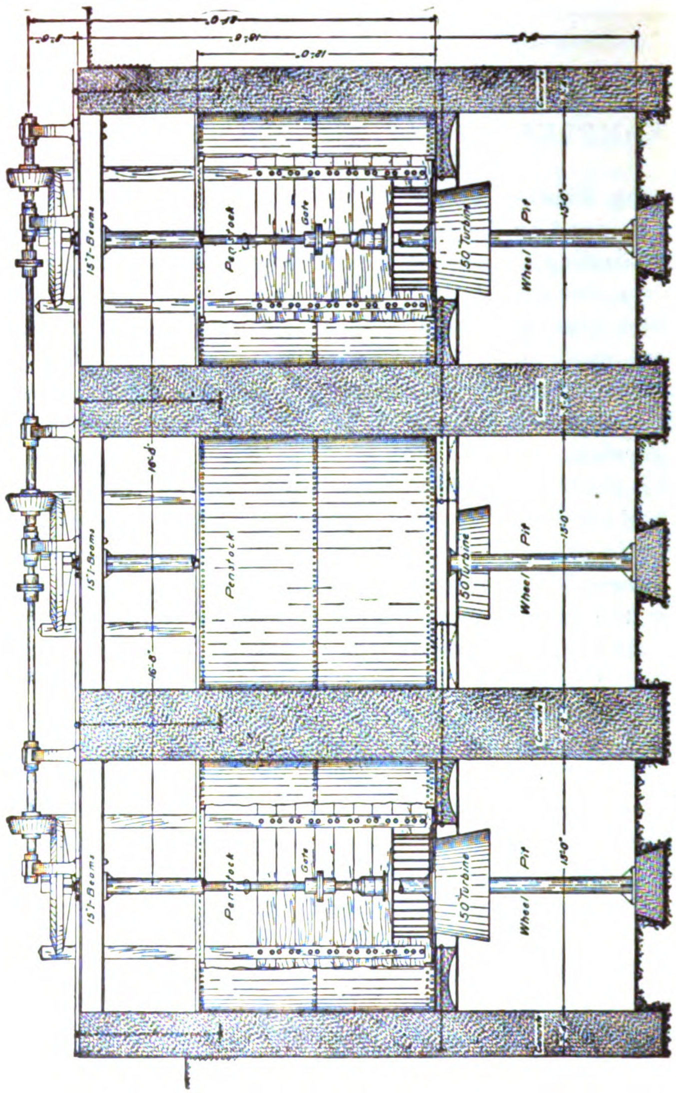
273. **Sterling Plant.**—A rear elevation (Fig. 329) of the plant which was designed by the writer for The Sterling Gas and Electric Company of Sterling, Illinois, shows three 50" vertical Leffel wheels connected to a common shaft by beveled gearings.

The general type of harness used is fully shown in the plan and elevation and needs no further description.

This plant is located on the Sterling race and is next to the last plant on the race on the Sterling side of Rock River. (See Fig. 345.) The head developed at this plant is about 8 feet, and the power of each wheel is about 115 horse power. Each wheel is set in an independent wheel pit which can be closed by means of a gate, as shown in Fig. 332. In order to make repairs on any wheel without interfering with the other wheels, the wheels and harness are well supported from the foundation, a very essential condition for permanently maintaining a high efficiency. The discharge pit is of ample size, so that the velocity with which the escaping waters leave the draft tube is reduced to a practical minimum. A rack, to keep coarse floating material from the wheel, is placed in front of the penstock and is shown in Fig. 331, in section, and in Fig. 332, in partial elevation. The shaft of this plant is extended into the adjacent building and to it are belted the generators which supply electric current for light and power purposes in the city of Sterling. An engine is also connected to this main shaft and may be utilized in case of extreme low water conditions, where sufficient water for power is not available, or for flood conditions where the head is practically destroyed.

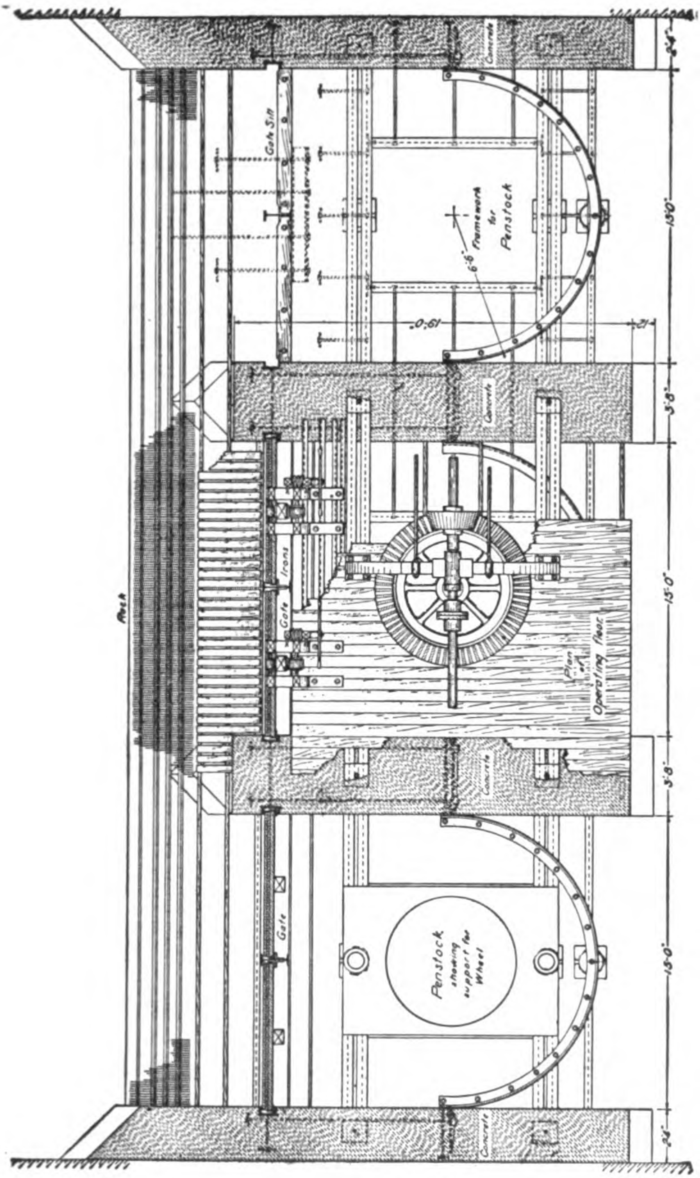
274. **Plant of York-Haven Water Power Company.**—Figure 333 shows the arrangement of the power station of the York-Haven Water Power Company on the Susquehanna River at York, Pa.

The power house is 478 ft. long and 51 ft. wide. The head-race is 500 ft. long and of an average depth of 20 ft. The wheel pits are 19 ft. deep and extend the entire width of the power house, open-



REAR ELEVATION

Fig. 329.—Wheel Pits at The Sterling Gas and Electric Light Co.'s Plant.



PLAN

Fig. 330.—Wheel Pits at the Sterling Gas and Electric Light Co.'s Plant.

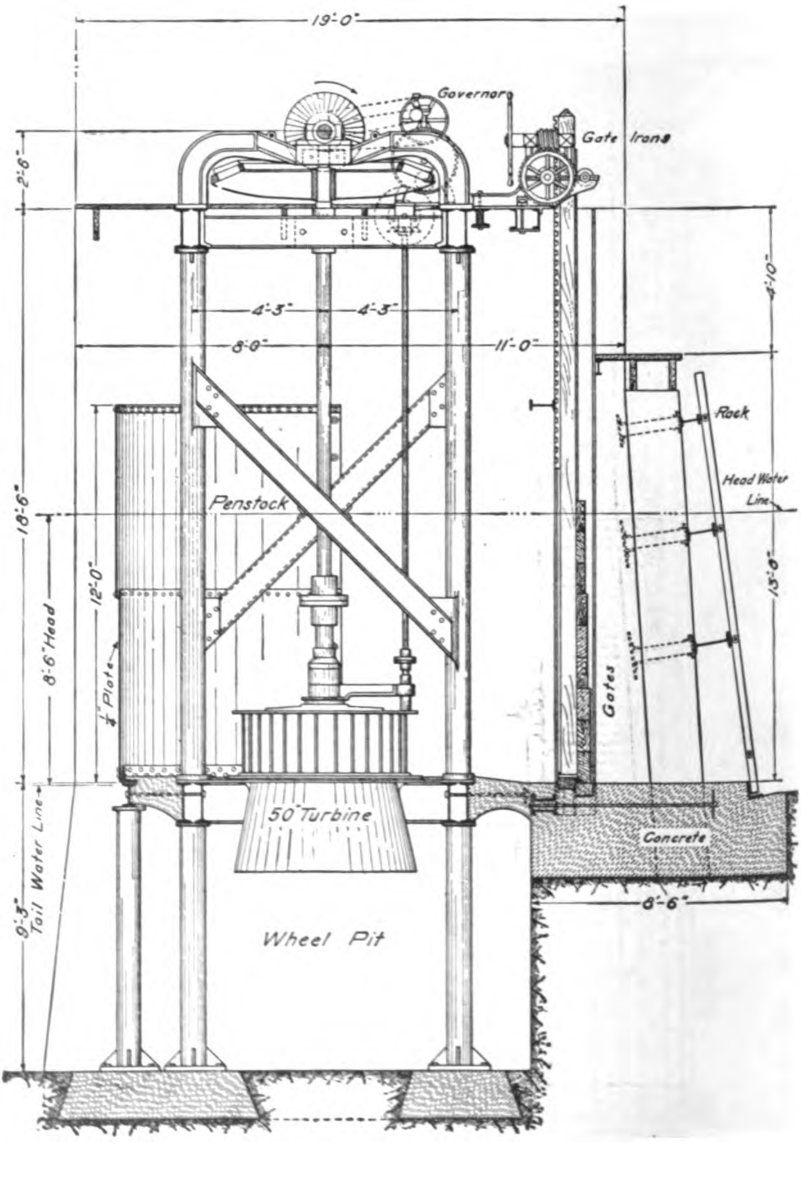


Fig. 331.—Wheel Pit, Sterling Gas and Electric Light Co.'s Plant

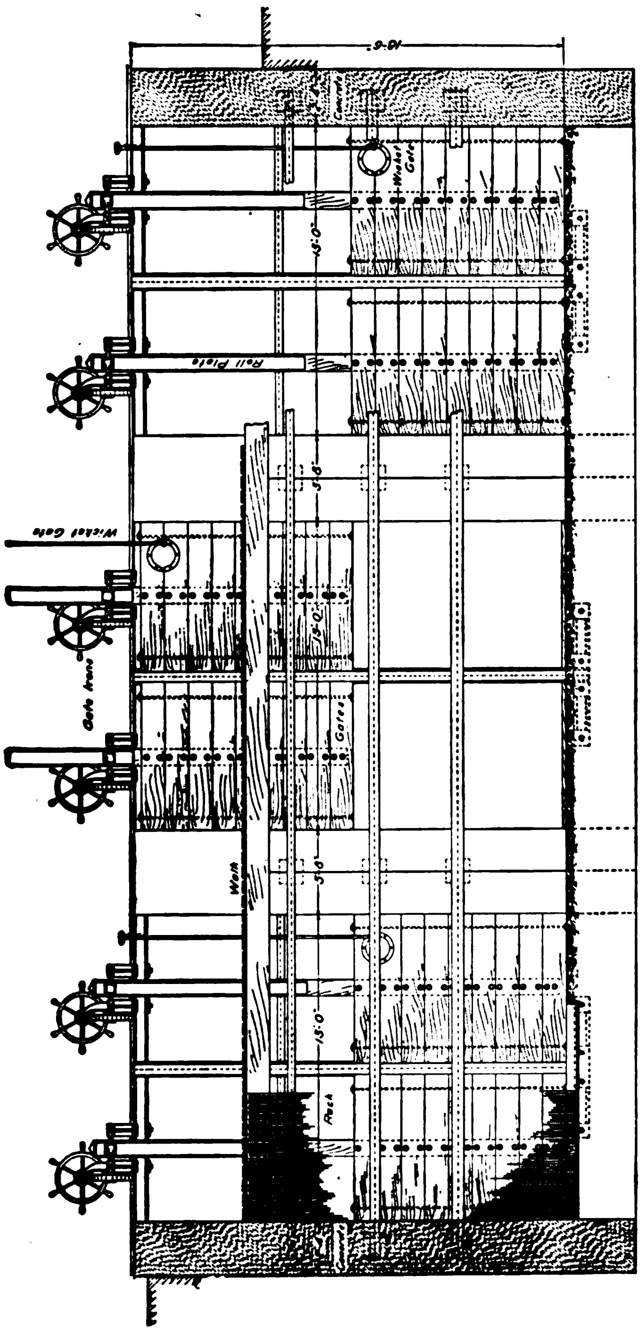


Fig. 332.—Wheel Pits at the Sterling Gas and Electric Light Co.'s Plant.

ing to the forebay. They are protected by iron racks and are made accessible by large head-gates of structural iron which weigh about eleven tons each.

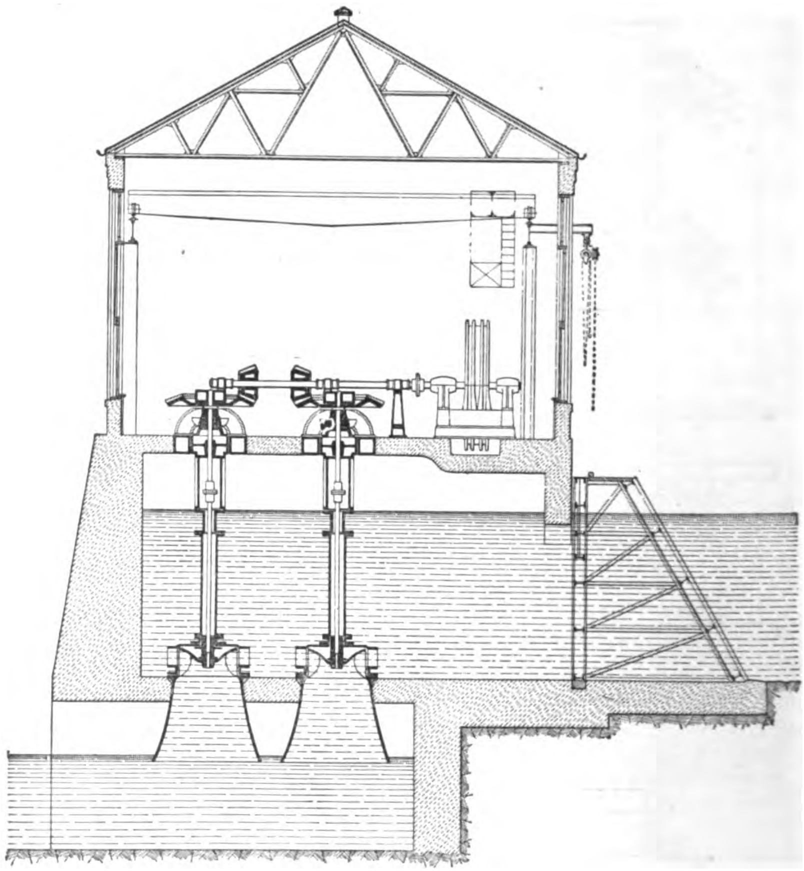


Fig. 333—Plant of York Haven Water Power Co. (Electrical Engines.)

Each pit contains two 78.5″ inward flow turbines, hung from spring bearings just above the runners. The turbines are set on the floor of the pit and are about 6 ft. above the lower water mark.

The draft tubes are 10 ft. long and extend well under water. The net head under normal conditions is about 21 ft. Float gauges on the switch board show at a glance the height of head and tail water.

The turbines were built by the Poole Engineering Company of Baltimore, Mr., and are rated at 550 H. P. each, or 1,100 H. P. per pair.

The turbines are of special design, the buckets being made of pressed steel. The shaft extends vertically from the turbines to bevel gears above the main floor and each is encased in a cast iron tube to protect it from the action of the water and to secure longevity both to the shaft and to the bearings which retain it in line.

The present installation consists of ten pairs of turbines with ten generators, equipped with Sturgess and Lombard governors.

The turbine bearings are supplied with oil from a gravity tank located on the switch-board gallery.

The generators are S. K. C., three-phase, 60 cycle alternators, rated at 875 kilowatts, and generate a 2,400 volt current. The normal speed of the generators is 200 revolutions per minute. Two 250 K. W., 125 volt, S. K. C., compound-wound, direct-current exciters furnish the exciter current to the generator fields.*

275. **Plant of South Bend Electric Company.**—Figure 334 shows the plant of the South Bend Electric Company at Buchanan, Michigan, built in 1901.

The dam, which was constructed in 1895, is of the gravity type, built of wood, with two rows of sheet piling below and one above it. It is about 400 feet long, and affords an average head of 10 feet. This is estimated to furnish a minimum of 2,000 h. p. for from four to six weeks in a year, while the maximum will reach 5,000 h. p. On an average, 2,500 h. p. is available for about three months and 4,000 h. p. for the remainder of the year.

The power house, placed a short distance below the dam, is 273 feet long and 40 feet wide. It is built of stone, with concrete foundations, and slate roof. It parallels the river so that the water from the turbines is discharged directly into the same. The regulating gates are seven in number, and are operated by racks and pinions.

The water wheels are Leffel turbines of 68 inch vertical type, 300 h. p. each. They are geared to a line shaft, which extends nearly the whole length of the building, and to the end of which the generator is coupled. A 40 inch vertical Leffel wheel is used for driving the exciter, which is belted to an intermediate shaft, driven by gears. The line shaft is divided into three units, so that either four, seven or ten wheels can be used for operating the generator, depending

* See Electrical World, vol. 49, March 2nd, 1907.

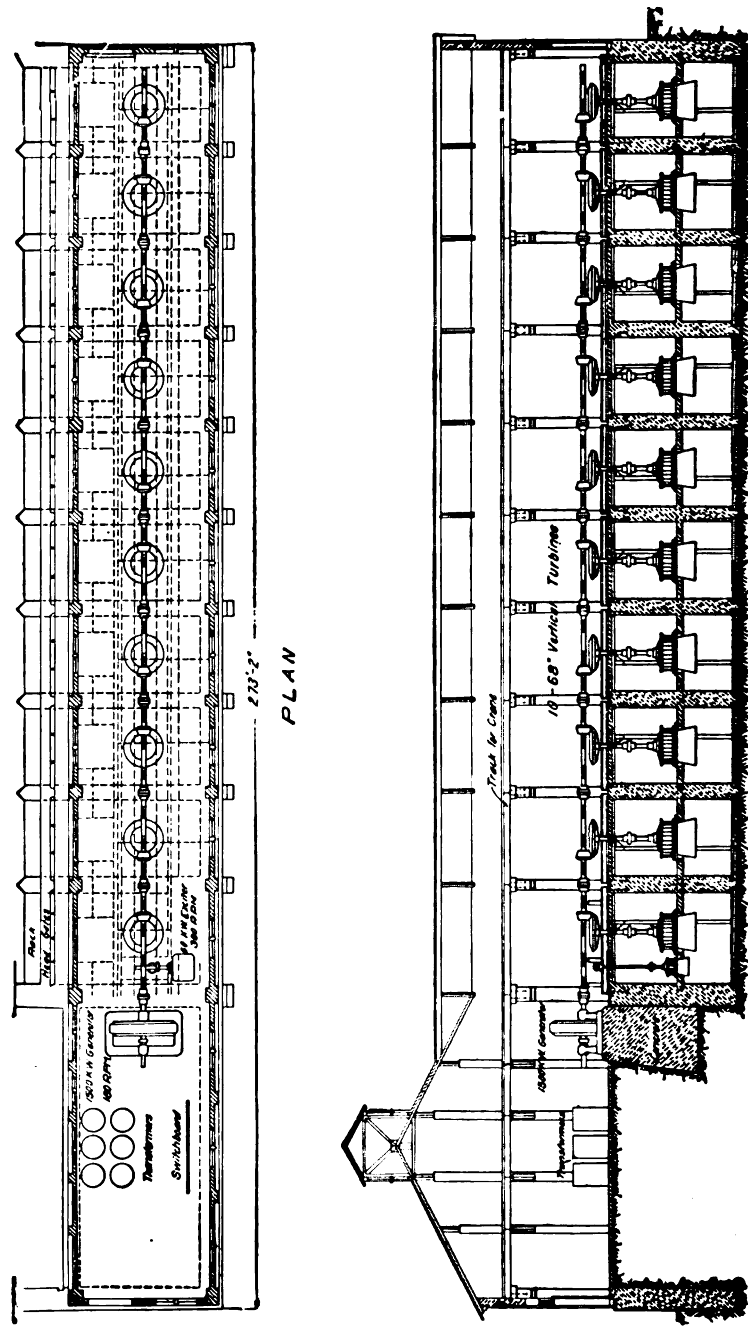


Fig. 433.—Plant of South Bend Electric Co., Buchanan, Mich. Vertical Turbine in Open Penstock Geared to Common Jack Shaft.

upon the load carried. In addition, the gears on the line shaft can be thrown out of mesh, so that any water wheel can be repaired if necessary. The plant is governed by two Lombard water wheel governors driven from the line shaft.

A 20 ton hand-operated crane serves all the apparatus in the building.

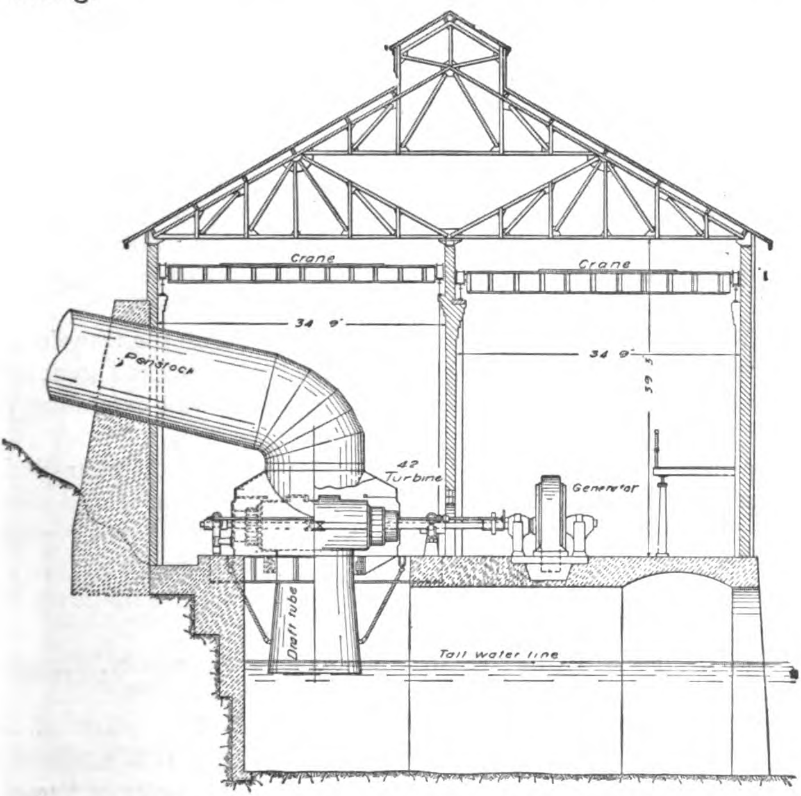


Fig. 335.—Plant of Hudson Water Power Co. Spier's Falls Plant. Double Horizontal Turbines in Steel Penstock. Central Discharge. (Engineering Record.)

The generator is a 1,500 k. w., 60 cycle General Electric revolving field type alternator supplying three-phase current at a pressure of 2,300 volts. The switch-board and transformers are located at one end of the building. There are no high tension switches at the power house.

The power is largely transmitted to South Bend, Indiana, a distance of 16 miles, where the company has a steam power plant which is always kept in such condition as to be put into immediate operation. It is used, however, only in case of extreme low water, at times of a heavy peak, or in case of accident to the transmission line. The steam power house is used as a sub-station and distributing point.*

276. Spier Falls Plant of The Hudson River Power Transmission Company.—A cross section of the Spier Falls Power house is shown in Fig. 335. A head of 75 feet, for operation of this plant, is derived from a granite rubble, ashlar-faced, masonry dam across the Hudson River between Mount McGregor and the Luzerne Mountains. The dam consists of 817 feet of spillway section, the remainder of the dam, 552 feet, being built about 12 feet higher. Water is admitted through arched gateways to a short intake canal designed to carry 6,000 cubic feet per second with a velocity of three feet per second. This canal distributes the water to ten 12′ circular steel penstocks which lead about 150 feet to the wheels.

The power house is divided into three parts with the transformer and switchboard room in one end, the wheel room and generator room being formed by a longitudinal partition wall extending the length of the building, with traveling crane in each.

Each unit consists of a pair of 42″ or 54″ cased S. Morgan Smith wheels, governed by Lombard and Sturgess governors and direct connected to 2,000 and 2,500 k. w. 40 cycle, three-phase revolving field generators, built by The General Electric Company.

The transformer room contains seven 670 k. w. and thirty 833 k. w. General Electric air cooled transformers.

The power is distributed to Glen Falls, Schenectady, Saratoga Springs and Albany. †

277. Plant of Columbus Power Company.—The plant of the Columbus Power Company is shown in Fig. 336. It is situated on the Chattahoochie River just beyond the limits proper of the city of Columbus, Georgia, at a shoal known as Lovers' Leap. At this point a dam of cyclopean or boulder concrete with a cut stone spillway surface was erected giving a head of 40 feet. The length of the dam is 975 feet 8 inches, with a spillway 728 feet long.

The power house is located at one end of the dam, so that no penstocks are necessary. This applies to power house No. 1. Power to drive the plant of The Bibb Manufacturing Company is fur-

*See Electrical World and Engineer, May 30, 1903 and July 14, 1906.

†See Engineering Record, June 27th, 1903.

nished from power house No. 2, being transmitted to the mill by a rope drive system. The power house is supplied with pressure water by means of penstocks let through the bulk-head wall, which extends from house No. 1 to the river bank. In both cases the tail water is discharged into the excavated river bed beneath the power houses. Power House No. 1 is designed to develop 6,000 h. p. is six units, and No. 2 about 3,000 h. p. mainly in two units.

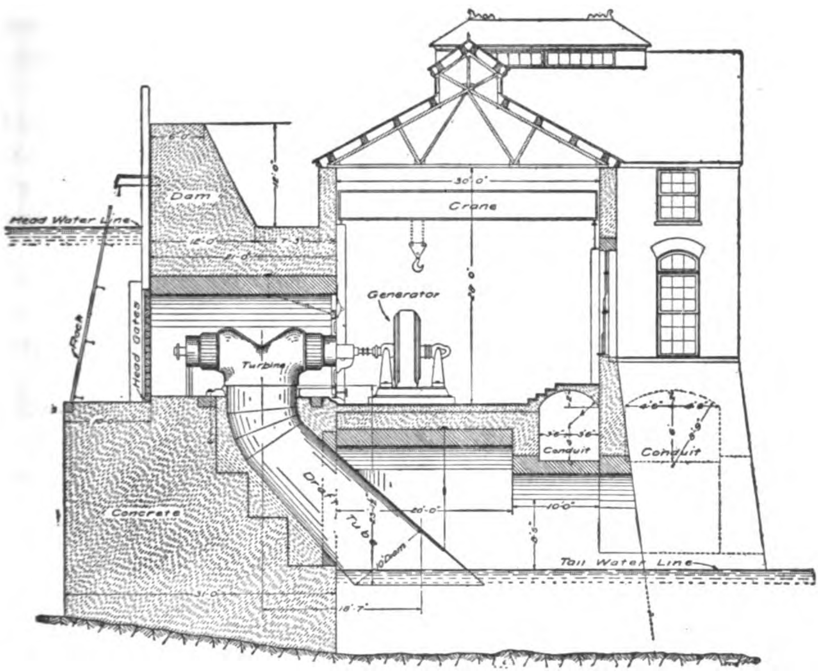


Fig. 336.—Plant of Columbus (Ga.) Power Co. Double Horizontal Turbines in Open Penstock. (Engineering News.)

Power house No. 1 is 137 feet long and 52 feet wide. It rests on heavy stone foundations, the up-stream portions of which form the heavy bulk-head which is pierced by six large openings for plant No. 1, by a smaller opening for the exciter units and a larger one for the penstock leading to power house No. 2.

The openings for power house No. 1 are short flumes or chambers. The back end of each of the wheel chambers is closed with a heavy plate or bulkhead of cast iron and steel separating the wheel chamber from the generator room. The racks are of the usual construction and are supported on a framework of I-beams, giving them an inclination of about 12° with the vertical. The gates to the wheel chambers are of timber and are raised by hand by means of a rack and pinion.

Each of the main wheel chambers contains a pair of horizontal 39 inch Hercules turbines, which discharge into a common draft tube. The center line of the wheels is 15 feet below normal head water level and 25 feet above normal tail water level. Under the total head of 40 feet, each pair of wheels develops 1,484 h. p. at 200 r. p. m. The draft tubes are 7½ feet in diameter at the turbine casing and 10 feet at the discharge end.

Each pair of wheels is direct connected to a two-phase alternator built by the Stanley Electric Manufacturing Company. Each machine has a rated capacity of 1,080 k. w. at 6,000 volts and driven at 200 r. p. m. gives current at 60 cycles. Each is connected to the wheel shaft by a flexible leather coupling.

There are two exciters directly connected to a single 18 inch Hercules wheel. Each exciter is of the Eddy type, having a capacity of 60 k. w. at 75 volts and running at 450 r. p. m. The exciters are under the control of mechanical governors.*

278. Plant of The Dolgeville Electric Light and Power Co.—In Fig. 337 is shown the plant of The Dolgeville Electric Light and Power Company at High Falls, New York, on what is now known as the Auskerada River.

The dam is built of limestone masonry. The height at the spillway is 20 feet, with each abutment 6 feet higher. The total length is about 195 feet. The width at the top is 7 feet and at the bottom 26 feet. The upstream side is perpendicular, the downstream side being curved in order to properly receive and discharge the water. The head gate, 12 ft. square and built in two sections, is fitted with a by-pass gate to relieve the pressure when filling the flume. The steel flume extends from the head gate to the power house, 520 feet away. This flume is 10 feet in diameter, and is made of ¼ inch steel plate, all longitudinal seams being double riveted. Just outside the dam is a vent pipe which assists in relieving the flume from any sudden strains.

There are two 36 inch horizontal Victor turbines, each direct connected to one 450 k. w. 2,400 volt two-phase Westinghouse gen-

* See Electrical World and Engineer, Jan. 23, 1904 or Eng. Record, Jan. 16, 1904.

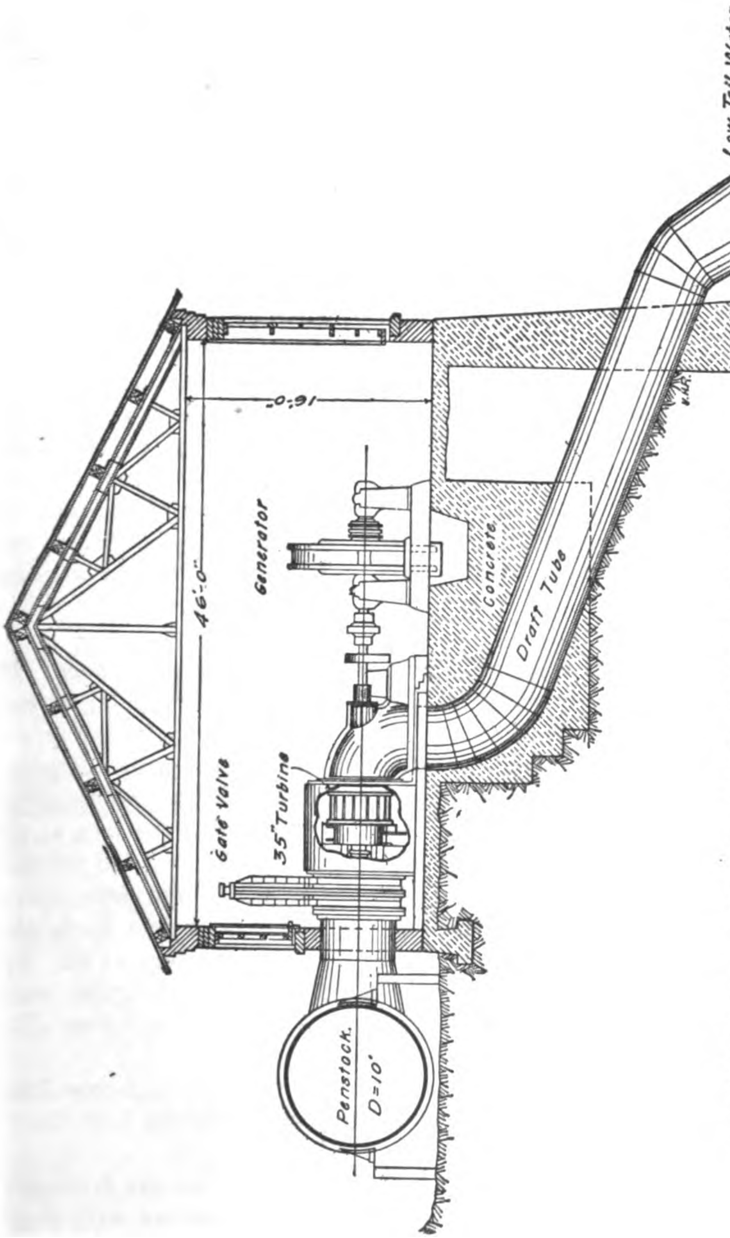


Fig. 337—Plant of Dolgeville Electric Light and Power Co.
Single Horizontal Turbine in Steel Case.
(American Electricain.)

erator. Each of these wheels will develop 600 h. p. at 300 r. p. m. under the working head of the water. which is 72 feet. They are mounted in cylindrical steel casings, and discharge downward through draft tubes, which extend a few inches below the surface of the tail water. Each wheel is supplied with a Giesler electro-mechanical governor.*

279. **Plant of the Shawinigan Water and Power Company.**—The power plant of the Shawinigan Water and Power Company is located on the St. Maurice River, Canada, at a point about 21 miles from Three Rivers, 90 miles from Quebec, and 84 miles from Montreal station. Fig. 338 shows a cross-section of their power station.

The St. Maurice River has a total length of over 400 miles, and is supplied from a great many lakes and streams, the drainage area being about 18,000 square miles. The water flow is very steady throughout the year on account of the dense forest covering this area, and is in the neighborhood of 26,000 cu. ft. per second, seldom going below 20,000 cu. ft. per second. At the crest of the falls the water flows over a natural rock dam and then down over the cascade, making a fall of about 100 feet, then on in a narrow gorge through which the water rushes swiftly and in which there is a further fall of 50 feet.

The intake canal is 1,000 ft. long, 100 ft. wide and 20 ft. deep. Its entrance from the river is located in a rather rapidly flowing stream at the crest of the falls where the water is 20 feet deep, for the reason that at times of rather high water, when the ice is flowing out of the river, the current is expected to carry the ice past the mouth of the canal. The end of the canal where it comes out at the face of the hill is closed by a concrete wall from which the water is led through steel penstock pipes down to the power house 130 feet below. The concrete wall or bulkhead in the canal is 40 feet in height, about 30 feet in thickness at the bottom and 12 feet at the top. On top of this wall are set hydraulic cylinders for lifting the head-gates and on top, covering the cylinders, is a brick gate house. The steel penstocks are 9 feet in diameter.

The electrical apparatus was supplied by the Westinghouse Electric & Manufacturing Company and the turbines by the I. P. Morris Co.

The three turbine units of the original installation are horizontal double units of 6.000 h. p. These are direct connected with single

* See American Electrician, April, 1898, Vol. 10, No. 4.

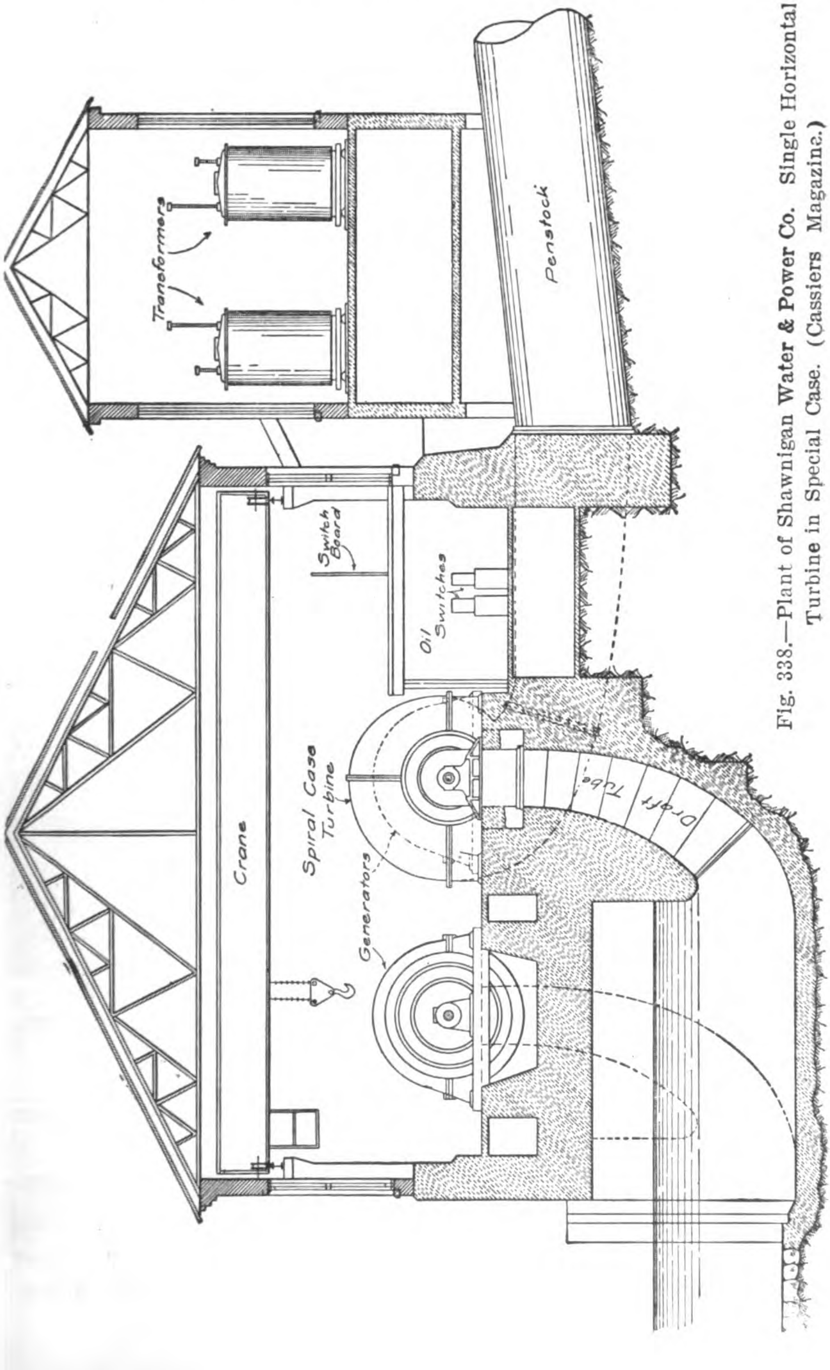


Fig. 338.—Plant of Shawnigan Water & Power Co. Single Horizontal Turbine in Special Case. (Cassiers Magazine.)

5,000 h. p. generator units of the rotating field type, with twenty poles. They are designed to operate at 180 r. p. m. giving two-phase currents at 30 cycles per second and 2,200 volts. A later installa-

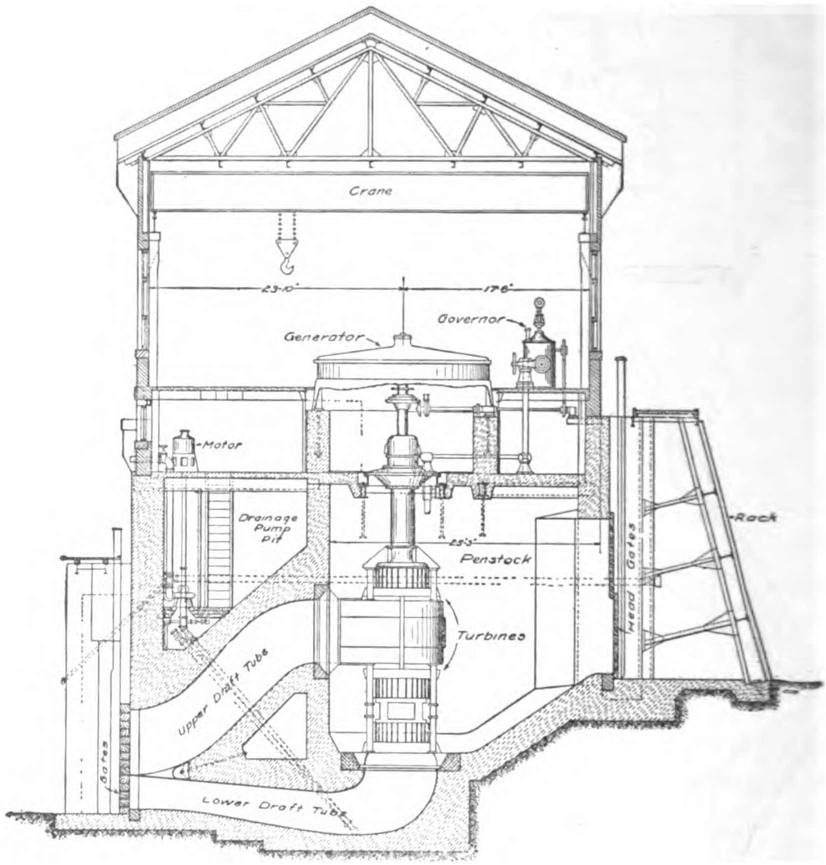


Fig. 339.—Plant of Concord Electric Co. Sewall's Falls Plant. Vertical Turbines Connected in Tandem. (Engineering Record.)

tion consists of two 10,000 h. p. water wheels each driving a 6,600 k. w. generator. (See Figs. 159 and 236.)

A separate penstock is provided for the exciter units which consist of two 400 h. p. turbines direct connected to exciters.*

* See references as given: Eng. Rec. Apr. 28, 1900; Can. Engr., Apr. 1901, May, 1901, and May, 1902; El. Wld. and Engr., Feb. 8, 1902; Cassier's Mag., June, 1904.

280. Plant of the Concord Electric Company.—This plant, shown in Fig. 339, is situated at Sewall's Falls on the Merrimac River about four miles from the State House in Concord, New Hampshire. The dam is a timber crib-work structure about 500′ long and gives a fall varying from 16′ to 17′. The addition to the old plant is the one shown in cross-section by Fig. 339 and is of special interest due to the vertical shaft generating units which were here installed. Comparative estimates showed that all other features of the plant, except the machinery could be built cheaper with the vertical shaft installation and the machinery added only a few thousand dollars to the total cost, while other advantages de-ermined its installation.

The new installation consists of two units, each consisting of 3—55″ bronze runners of the Francis type, mounted on a vertical shaft and hung on a step bearing. The machines are of the Escher-Wyss type built by The Allis Chalmers Company, American representatives of the Escher-Wyss Co. The gates are of wicket pattern, controlled by Escher-Wyss mechanical governors, also built by The Allis Chalmers Company. The generators, which are direct connected to the vertical shaft wheels, are of 500 k. w., 3-phase, 60 cycle, 2,000 volt, 100 r. p. m., revolving field type. Excitation is furnished by one 75 h. p., 3-phase, 2,600 volt induction motor, direct connected to a 45 k. w., 125 volt, compound wound D. C. generator. The exciter unit runs at 680 r. p. m.*

281. Plant of Winnipeg Electric Railway Co.—In Fig. 340 is shown the power plant of the Winnipeg Electric Railway Company. It is situated on the Winnipeg River at a point a few miles from Lac du Bonnet, which is on a branch line of the Canadian Pacific Railroad, 65 miles distant from the City of Winnipeg.

To obtain the necessary water, a canal 120 feet wide and with a clear depth of 8 feet at normal low water was cut to the upper river near Otter Falls. The canal is 8 miles long, with a drop of 5 feet to the mile, equaling a total head of 40 feet. At the point where the dam is located there is a natural fall, and the dam crosses almost at the crest.

With the head and discharge available it is claimed that 30,000 electrical horse power can be developed.

The water wheels are all McCormick turbines regulated by Lombard governors. The turbine pits are protected by racks to keep out ice, logs, etc.

* See Engineering Record, January 6th, 1906.

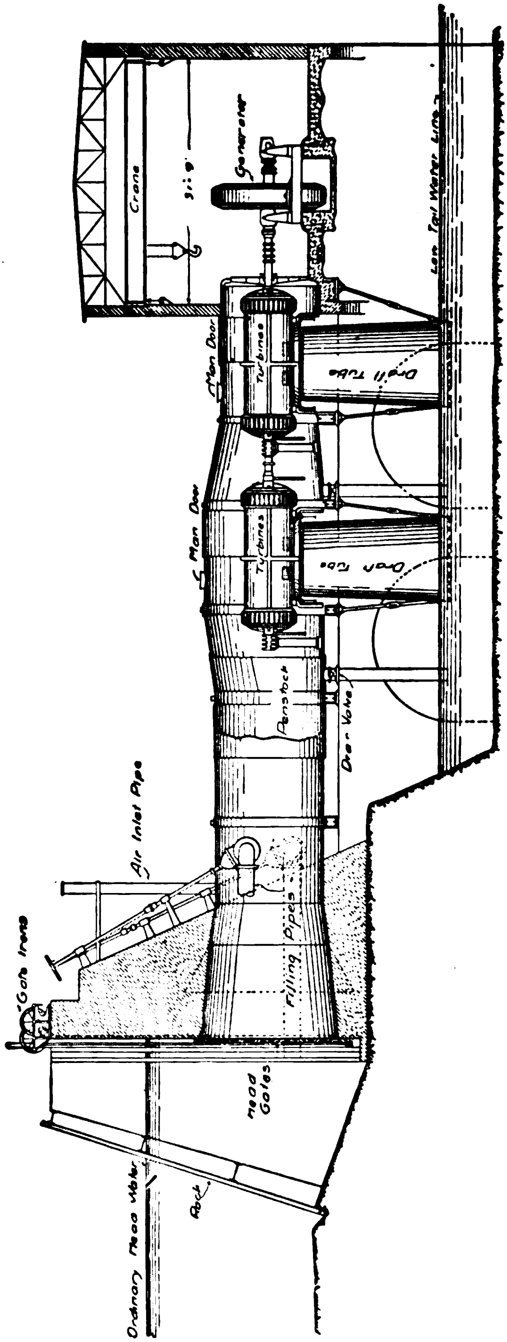


Fig. 340.—Plant of Winnipeg Electric Railway Co. Multiple Horizontal Turbines in Tandem in Steel Penstock. (Electrical World, June 23, 1906.)

The electrical units consist of four 1,000 k. w. and five 2,000 k. w. revolving field, 60 cycle, 2,300 volt, three-phase generators and two 100 k. w. 125 volt, direct-current exciters, all coupled to turbines, and two 175 k. w. 125 volt direct-current exciters, coupled to three-phase, 2,300 volt induction motors.

There are 15 transformers, comprising five banks, by means of which the voltage is stepped up from 2,300 to 60,000 volts for transmission to the sub-station at Winnipeg over a distance of 65 miles.*

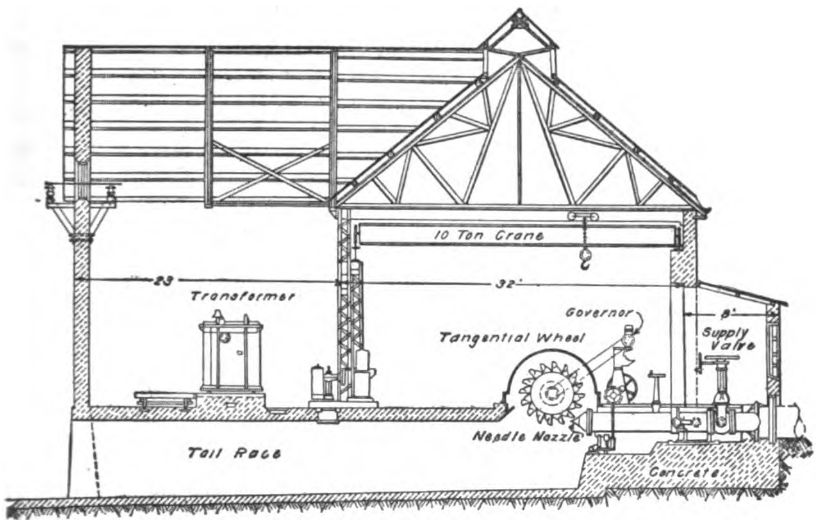


Fig. 341—Plant of Nevada Power Mining and Milling Co. (Engineering Record.),

282. Plant of Nevada Power Mining and Milling Co.—Fig. 341 shows a section through the plant of The Nevada Power Mining and Milling Company on Bishop Creek, near Bishop, Cal. The equipment of the station consists of two 750 k. w., 60 cycle, 2,200 volt, three-phase alternating-current generators, running at 450 r. p. m. and a 1,500 k. w. generator running at 400 r. p. m. This latter generator is shown in the sectional drawing. There are two exciters of 60 k. w. each, delivering current at 140 volts pressure. Both exciters are operated by water wheels, and, in addition, one is provided with an induction motor. The water wheels were made by The Pelton Water Wheel Company of San Francisco. The two

* See Electrical World, June 23, 1906.

750 k. w. machines have Sturgess governors, and the 1,500 k. w. machine has a type Q Lombard governor. Hand-control mechanism is provided for each wheel. Oil is supplied to the governor by two oil pumps operated by water wheels.

Water is taken from the creek at a small diverting dam and conveyed along the mountain-side in a pipe line. The pipe line is about 12,000 feet long, and consists of 6,700 feet of 42-inch wood-stave pipe, 2,150 feet of 30-inch wood-stave pipe, and 3,150 feet of 24-inch steel pipe, all diameters being inside measurements. The 42-inch pipe lies on a nearly level grade, the static head at the lower end being about 30 feet. At this point are placed two 30-inch gate valves, one opening into the 30-inch pipe and the other provided for a future line. The 30-inch pipe descends the hill to a point that gives a static head of 265 feet. Here it joins the 24-inch steel pipe, which descends a steep hill to the power house, the total static head being 1,068 feet.

The power generated at the plant is transmitted, over a line of stranded aluminum, equivalent to No. 0 copper, to Tonopah and Goldfield, Nev., making a total length of line of 113 miles. In crossing the White Mountains the line reaches an elevation of over 10,500 feet.*

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61. Bear River Power Plant and Utah Transmission Systems. Electrical World and Engineer, June 18, 1894.
62. Plant of the Chittenden Power Company, Rutland, Vt. Engineering Record, Dec. 9, 1905.
63. Plant of Vermont Marble Company, Proctor, Vt. Electrical World, Feb. 3, 1906.
64. Water Wheel Equipment in the Puget Sound Power Company's Plant. Electrical World and Engineer, Oct. 22, 1904.
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71. 60,000-volt Hydro-Electric Plant, Winnipeg, Manitoba. Electrical World, June 23, 1906.
72. DeCew Falls Power Plant. Engineer, Apr. 2, 1906.
73. Development of the Montmorency Falls. Electrical World and Engineer, June 17, 1899.
74. The Rheinfelder Power Transmission. Electrician, March 26, 1897.
75. The Bellinzona, Italy, Hydro-Electric Station. Electrical World and Engineer, Sept. 16, 1905.
76. A Norwegian Water Power Plant. Electrical World and Engineer, Apr. 4, 1903.
77. An Italian 40,000-volt Transmission Plant. Electrical World and Engineer, Aug. 19, 1905.
78. Tyrol Hydro-Electric Power Station, Keiserwerke. Electrical World, May 1, 1907.

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81. Hydraulic Station at Cusset, near Lyons, France. Electrical World and Engineer, Dec. 9, 1905.
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85. The Sill Hydraulic Power Plant near Innsbruck. July 1, 1905.
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CHAPTER XXIII.

THE RELATION OF DAM AND POWER STATION.

283. General Consideration.—In any water power plant the water must be taken from some source, conducted to the wheels, and discharged from the same at the lower head. To accomplish this object there must be a head-race leading from the source of supply to the plant which may be of greater or less length and in which more or less of the available head may be lost in order to produce the velocity of flow and overcome the frictional resistance.

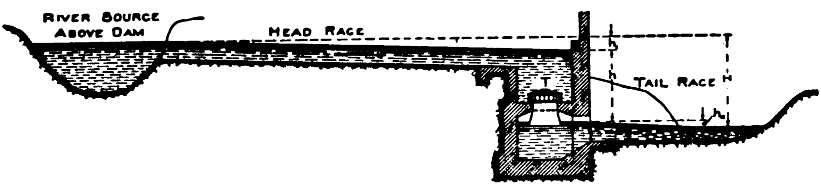


Fig. 342.

After entering the plant the water is discharged through the turbine T into a tail-race of greater or less extent in which there is also a loss caused by friction and velocity of flow, similar to that already expended in the head-race. In Fig. 342 the total head available is H; the head lost in the head-race is indicated by h_1; and the head lost in the tail-race is indicated by h_2. The net energy of the wheel is $h = H - h_1 - h_2$, and a portion of h is also lost in the slip, leakage, and friction of the machinery and transmission.

The power plant should be located with reference to the dam so that (1) the greatest amount of head may be utilized at the least expense; (2) the plant constructed should be as free as possible from interruptions due to floods or other contingencies; (3) the location chosen should be at such a point where security of construction can be accomplished at the minimum expense.

Each of these influences is of importance and the relative location of the power plant and dam must depend upon these and various other conditions which must be carefully considered.

284. Classification of Types of Development.—For the purpose of a clear understanding of the principles involved, the type of development may be grouped or classified into:

First: Concentrated fall, in which the plant is built on the dam or closely adjoining thereto, with a short or no race. In this case the entire fall is concentrated by means of the dam and as a rule this class of development is adaptable only to central power stations where one or two plants only are to be installed on the power.

Second: Diversion type with dam. In this case the fall is developed by means of a dam in the manner conforming to the last type but the water is distributed to one or more plants by means of a long head-race canal through which the water flows to the power station, after which it is discharged either into the stream at some point below the dam or into a tail-race from which it is finally discharged at a point lower down the stream.

Third: Diversion with or without dam. In this case the development is installed with or without a dam at the head of the rapids or fall which is to be utilized and the water is conducted through a long head race, if land of a suitable elevation is available, or, otherwise, through a tunnel to a point immediately above the site of the power station. From the end of the tail-race or tunnel the water is carried to the plant through a metallic penstock.

Fourth: The fourth type is similar to the third except that where the head-race or tunnel is used (the ground being unfavorable to such construction or the expense of the same being unwarranted) a long penstock of metal is provided to conduct the water from the head works to the station.

Fifth: The fifth type is the tunnel tail-race type and involves conducting the water through metallic penstock direct to the wheels located at the minimum level and, after the water is discharged therefrom, the provison of a tunnel tail-race for conducting the water from the turbine to the point where it is to be discharged back into the stream.

It is important to note in this case, as in the case of all other classifications attempted, that such a classification is for the purpose of systematizing the consideration of numerous diversified types and bringing them to a similar basis for examination. In the actual adaptation of plans of development, it is seldom any single type will be found in its simplicity; in most cases modifications of the same become desirable or essential.

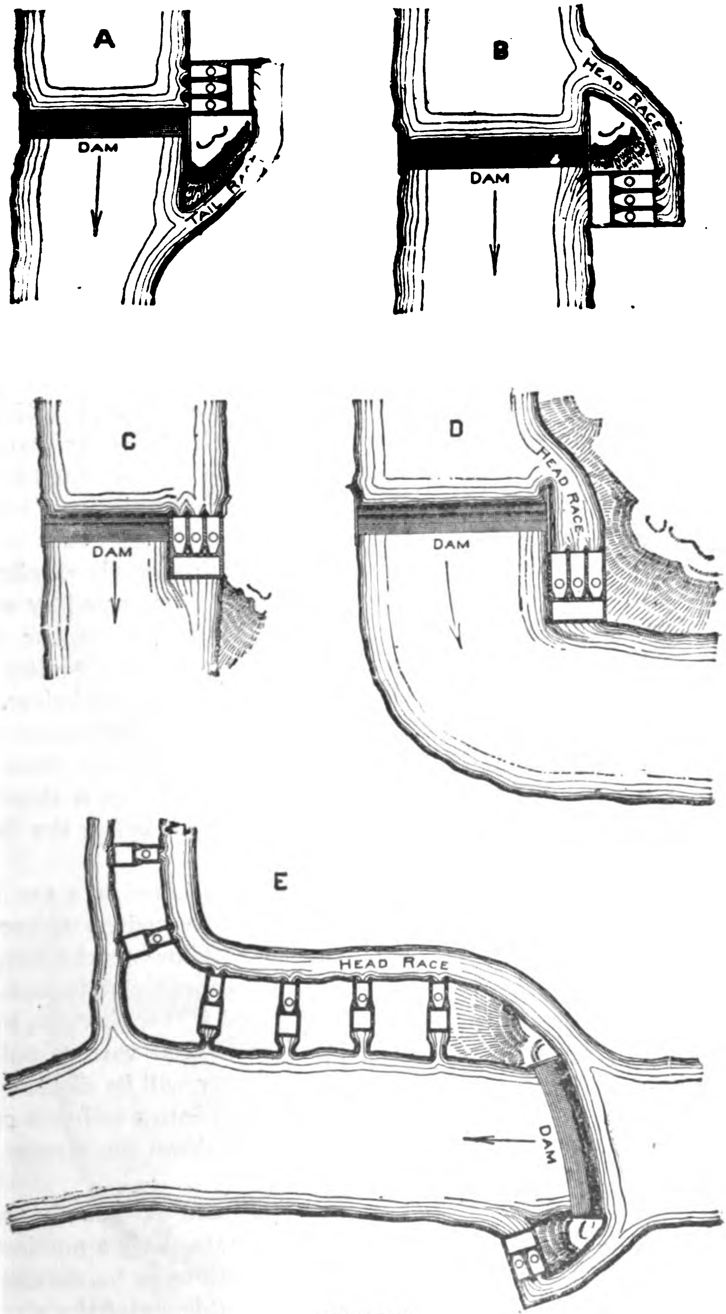


Fig. 343.

285. Concentrated Fall.—In most of the low head water powers the portion of the fall of the river which can be utilized is distributed over minor rapids and small falls and occupies a considerable length of the stream. Where the head is small and the expense of a dam to concentrate the head entirely at one point is permissible, the power house may sometimes be located to advantage in the dam itself. In this case the power house will constitute a part of the dam itself. This is possible only where the length of the spillway remaining is sufficient to pass maximum flood without an undue rise in the head of the water above the dam. In many such cases this plan, which is represented by Diagram C, Fig. 343, meets economical construction as it may both cheapen the cost of the dam and reduce the excavation necessary for the wheel pit and tail-race. The power house built at such point is, however, usually directly in the line of the current and must be so constructed and protected as to prevent its injury or destruction by floods, ice or other contingencies of river flow.

In other cases, where the spillway available by the above plan is not sufficient or where the plant is not properly protected by such forms of construction, the plant may be constructed on one side of the dam, receiving its waters from a head-race which joins the river above the dam and discharges it into the river below, as shown by Diagrams C and D, Fig. 343. Or, where the capacity is suitable, the plant itself may receive the water directly from its head gate from the river above the dam and discharge it through a tail-race which will enter the river at some point below the dam, as shown in Diagram A, Fig. 343.

In other cases, where the power is to be distributed to a number of independent plants, raceways may be constructed on either or both sides of the stream and from the dam, following the stream downward along the bank and more or less approximately parallel thereto as the nature of the conditions demand. The plant drawing the water from this head-race may be distrbuted at various points along the same, and from these plants the water will be discharged after use either directly into the stream itself or into a tail race connecting such plants with a lower point farther down the stream, as shown in Diagram E, Fig. 343.

286. Divided Fall.—An independent tail-race is usually constructed to advantage where the dam concentrates only a portion of the head or fall, leaving certain additional portions to be developed by the use of the tail-race, which may, if desirable, enter the stream

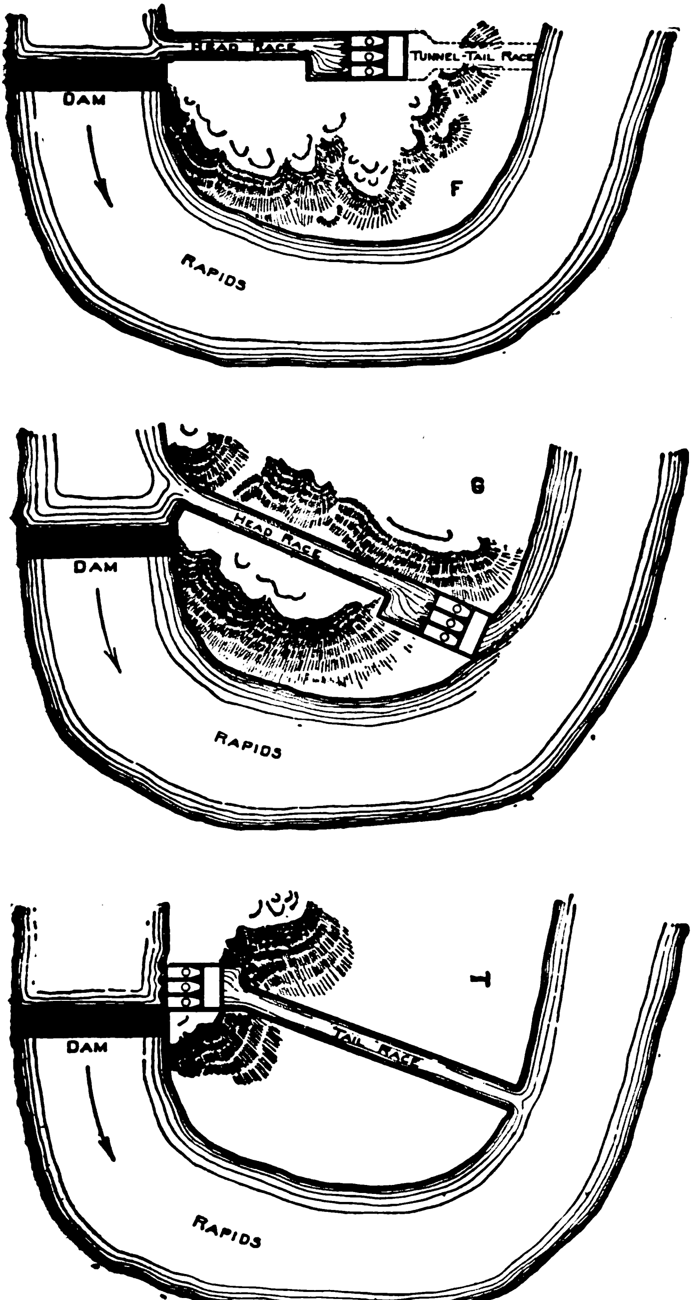

HEAD RACE
TUNNEL-TAIL RACE
DAM
F
RAPIDS
G
HEAD RACE
DAM
RAPIDS
TAIL RACE
DAM
RAPIDS

Fig. 344

at a point much farther down the river and at the foot of the rapids. Where the fall of the stream is considerable, and the expense of construction of the dam to suitable height to concentrate the entire fall at a single point is inadvisable, it is often desirable to build a dam to less height at perhaps considerably less expense and develop at the dam only a portion of the total fall. From this dam a head race may extend to some considerable distance, and the water from this head-race may be delivered to the power plant a mile or two lower down the stream. From this head race, the water, after passing through the wheels, is carried directly into the stream at the lower point, as shown in Diagram G, Fig. 344.

Under other conditions, where the topography of the country is suitable, the head-race may be much less in extent, and a tail-race substituted for receiving the waters after they have been used in the wheel and then conducted to the river at or near the end of the rapids, as shown in Diagram F, Fig. 344.

Under still other conditions the plant itself may be located immediately at the dam and the tail waters may be conducted from the turbine to a tail-race or tail-water tunnel to the lower end of the rapids, as in Diagram H, Fig. 344.

The relation of head-race and tail-race is merely a question of developing the power plant at the least cost and securing the maximum head, and the topographical conditions at the power site will therefore determine which line of development will be best. In a number of cases, where the head or fall is considerable and the power development is large, and where the cost of land for head-races would be almost or quite prohibitive, the stations have been located in the immediate vicinity of the river and have delivered the water into a tail-race tunnel, which frequently empties at a considerable distance down the stream and at the lowest point of delivery that is practicable. In other cases it is more economical to run open raceways for a portion of the distance and then conduct the water under pressure by closed pipes to the wheels at the lower point.

This last method is used particularly under high head and where the water must be conducted for a reasonable distance over an irregular profile.

The quantity of water to be used, the head available, and the value of power modify the arrangements which must be carefully studied in view of the financial, topographical, and other modifying conditions.

287. Examples of the Distribution of Water at Various Plants.—Fig. 345 is a plan of the power development on the Rock River at Sterling, Illinois. The dam at this point is about 940 feet in length. The power is owned by various corporations and private individuals who have combined their interests in the dam and raceways and

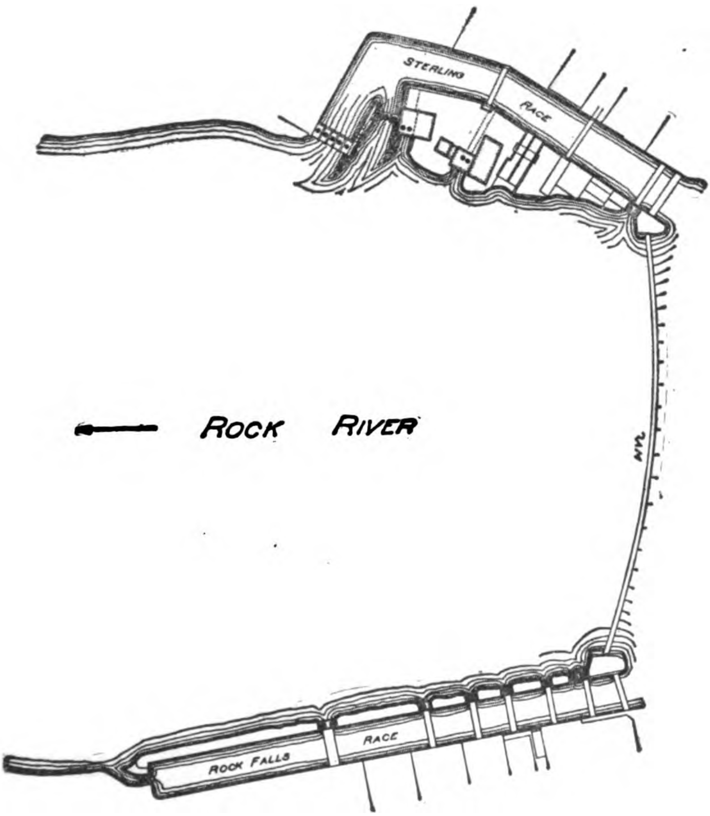


Fig. 345.—Raceways of Sterling Hydraulic Company.

have organized The Sterling Hydraulic Company, whose function is to maintain the same. The individual plants are owned, installed, and operated by the various owners or by manufacturers who lease the power. At this location races have been constructed at the foot of the rapids, but these rapids continue to a point near the lower end of the tail-race, and the plants farthest from the dam have the highest falls. The fall varies from about 8 to 9½ feet

Fig. 346 shows the general arrangement of the canal of The Holyoke Water Power Company at Holyoke, Mass. The total fall of the river at this point, from the head water above the dam to the tail water at the lowest point down the stream, is about sixty feet. The fall is divided into three levels by the various canals, marked: 1st level canal, 2nd level canal, and 3rd level canal.

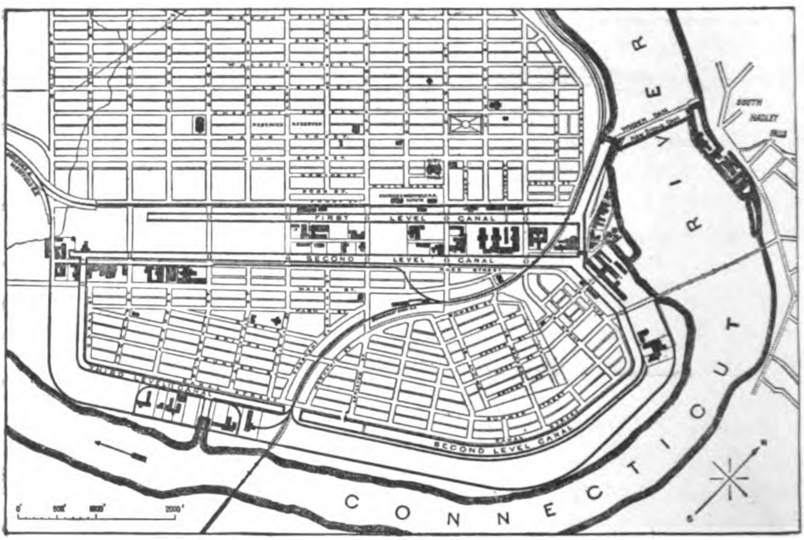
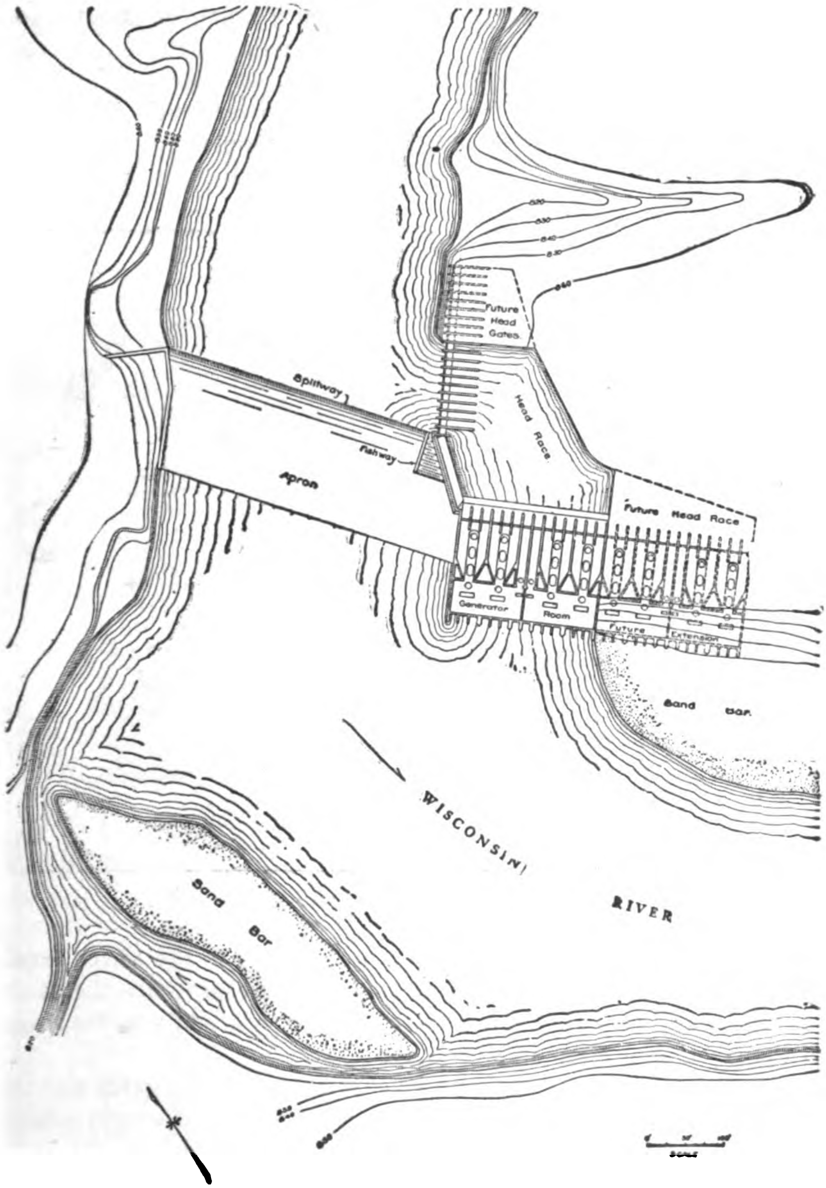


Fig. 346.—Canals of Holyoke Water Power Company.

The first level canal, which has a length of about 6,000 feet, is constructed as a chord across the bend of the river and is approximately some 3,000 feet from the bend. The canal is about 150′ wide near the bulkhead and decreases to about 100′ at the lower end. The water depth is about 20′ at the upper end and about 10′ at the lower. The canals are all walled throughout their length to a height two or three feet above the maximum water surface. The fall from the first level to the second is about 20′. Various mills draw their water supply from the first level as a head-race, and discharge into the second canal as a tail-race. Near the upper end of the canal are a few factories that draw water from the first level and discharge the same into the river with a head of some 35 or 40 feet.

The second level canal is built parallel to the first and at a distance of about 400 feet nearer the river. The main canal is about 6,500 feet in length, but near the left hand of the map is shown to



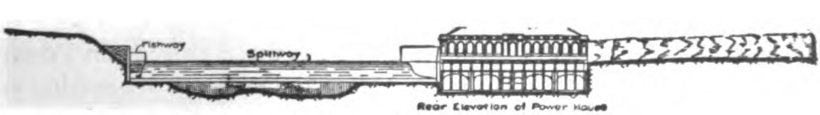


Fig. 347.—Kilbourn Plant of Southern Wisconsin Power Co.

sweep round towards the river and attain a reach of about 3,000 feet in length parallel thereto. The mills drawing their supply from this canal discharge either directly into the third level or into the river. The water supply from each of the lower levels is the tail water from the next level above, but is also supplemented by overflows when the mills fed from the level above are not discharging

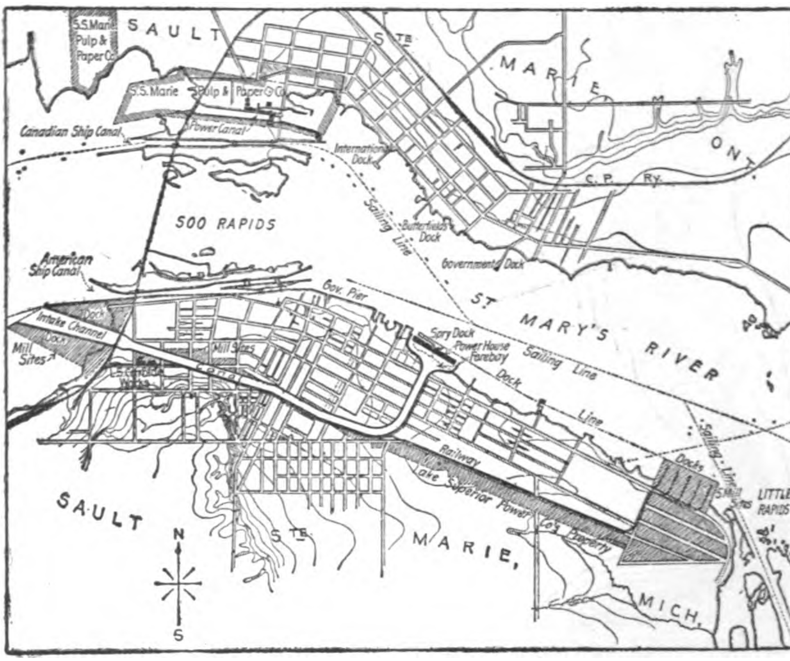


Fig. 348.—Plant of The Lake Superior Power Co.

sufficient water to maintain the quantity needed in the lower level.

The fall from the third level of the river is essentially the same for all the mills drawing water therefrom, but according to the stage of the river ranges from 15 to 27 feet.

The flow of water in the first level is controlled by gates and its height limited by an overflow of about 200 feet in length which acts as a safety overflow and prevents any great rise in the head water during times of flood.

288. Head-Races Only.—Fig. 347 illustrates the general plan of the hydraulic power development of The Southern Wisconsin Power Company at Kilbourn, Wisconsin. Here the entire cross-section of the stream is necessary in order to pass the maximum volume of

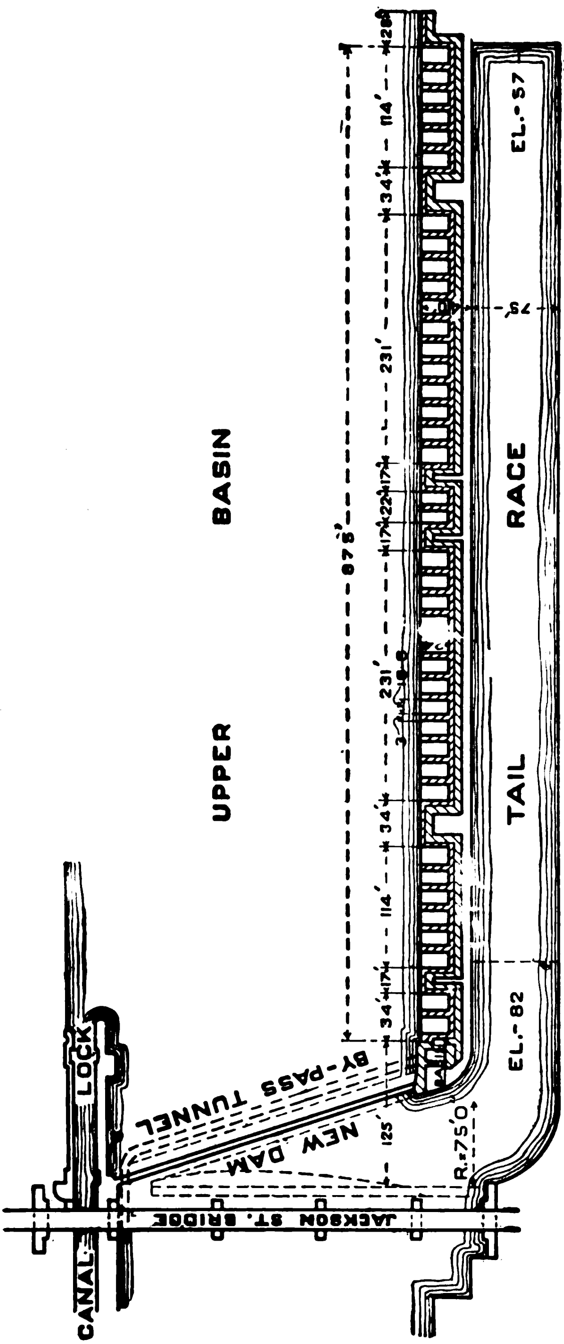


Fig. 349.—Joliet Plant of Economy Light and Power Company.

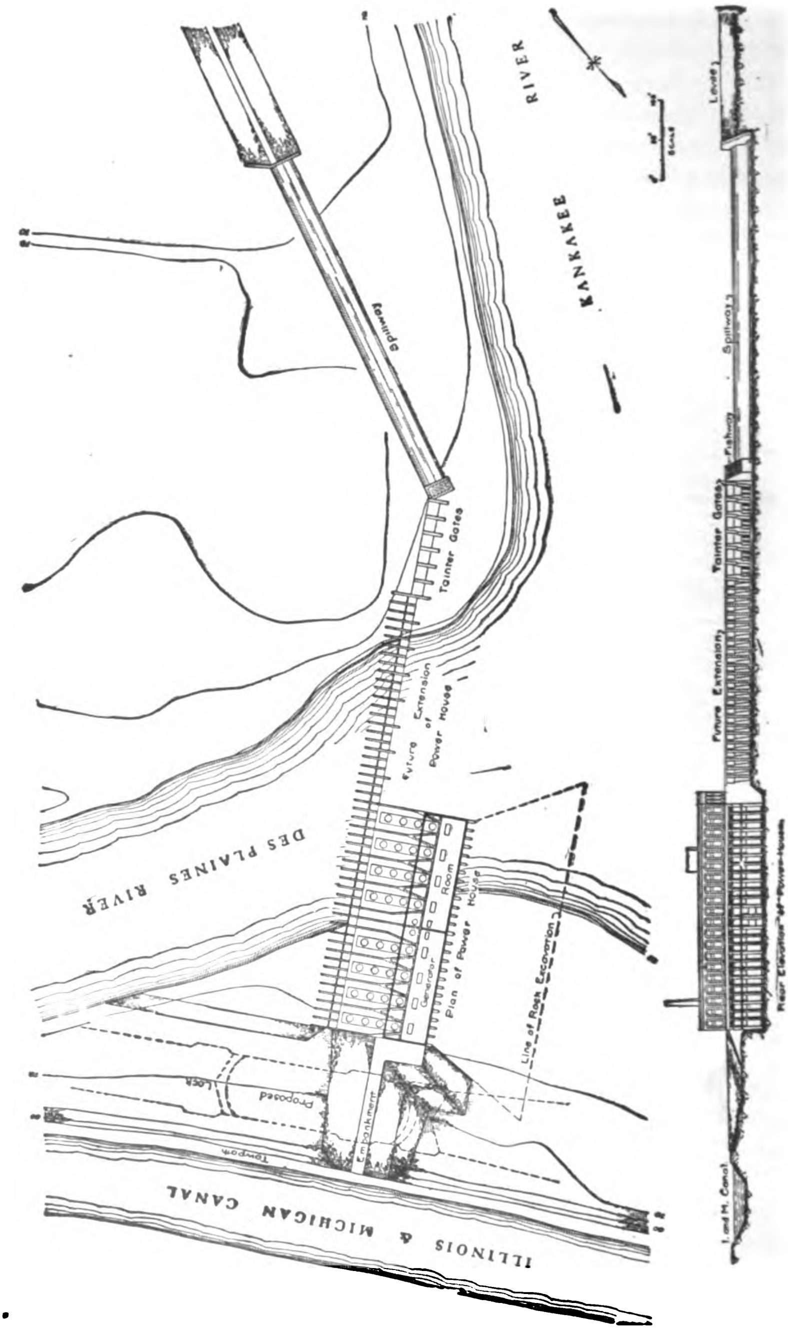


Fig. 350.—Morris Plant of Economy Light and Power Company.

water, which amounts to about 80,000 second-feet. The plant has therefore been constructed at one side of the river, receives the flow through a series of gates built just above the dam, and discharges the water into the river just below the bend in the river, as shown. The plant now under construction is only a portion of that which it is designed to ultimately install. The proposed future extension of the power plant is shown by the dotted lines.

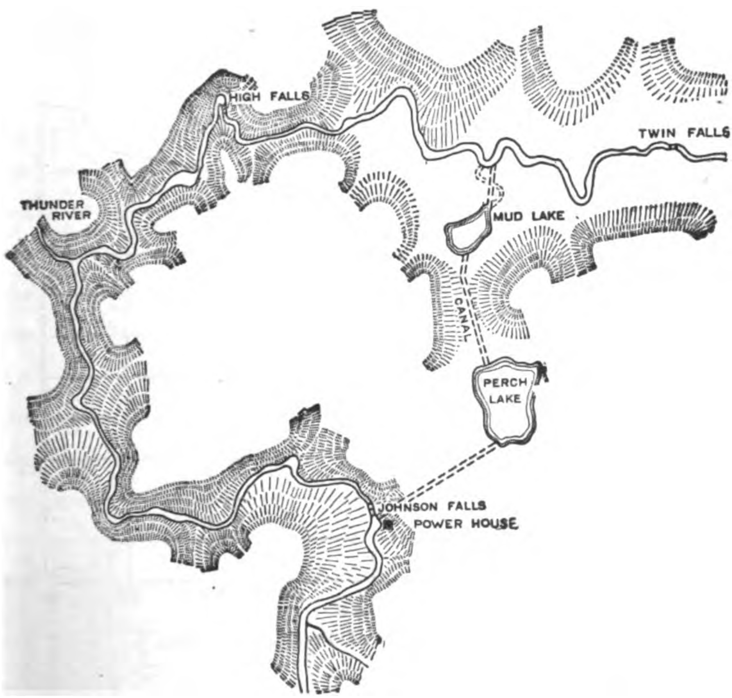


Fig. 351.—Possible Canal for Peshtigo River Development.

Fig. 348 shows the water power plant of The Lake Superior Power Company at St. Mary's Falls, Michigan. The canal on the American side begins just above the entrance to the American ship canal and above the Soo rapids. The water is conducted through this canal to a power house located below the rapids at the point shown on the map. On account of the value of the land this canal was designed for a velocity of flow of about 7½′ per second with full load of the plant, which was designed for about 40,000 h. p. requiring a capacity with available head of 16.2 feet, of about 4,200 cubic feet per second. (See Engineering News of August 4th, 1898.)

Fig. 349 shows the plan of the hydraulic development of The Economy Light and Power Company at Joliet, Illinois. The entire installation as shown is owned by this company. The fall available is about 11 feet and is developed by a concrete dam which creates the upper basin along which the power plant has been constructed. The water flows through the flume gates directly on to the wheels and is discharged into a tail-race built parallel with the river. A

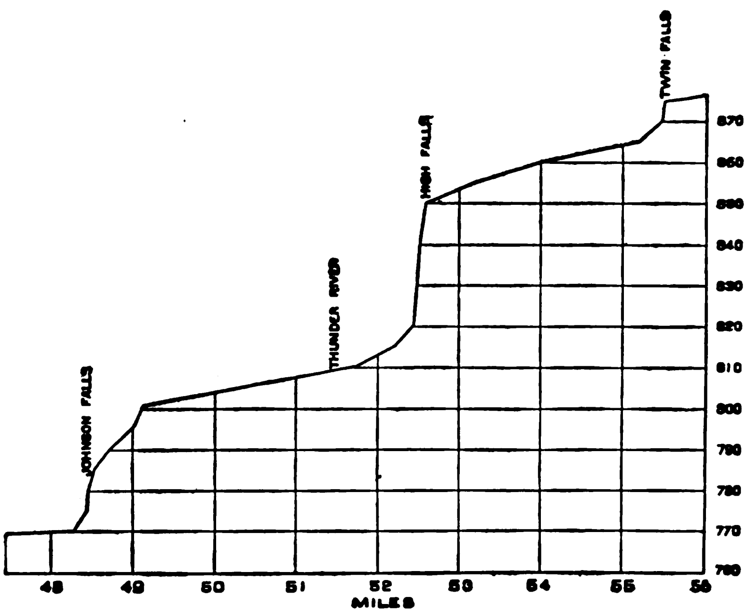


Fig. 352.—Profile of Peshtigo River.

certain amount of water is necessary for feeding the lower level of the canal and this is supplied by a by-pass tunnel shown in dotted line above the dam. This by-pass, which is slightly higher than the elevation of the tail-race, is fed by the discharge of one of the wheels, which operates under a less head than the other wheels in the installation.

289. Plant Located in Dam.—In Fig. 350 is shown the general plan and elevation of the hydraulic plant at Dresden Heights on the Des Plaines River just above its junction with the Kankakee River. These two streams unite at this point to form the Illinois River.

In this case the dam is built across a very wide valley and the length of the dam is much greater than necessary or desirable to

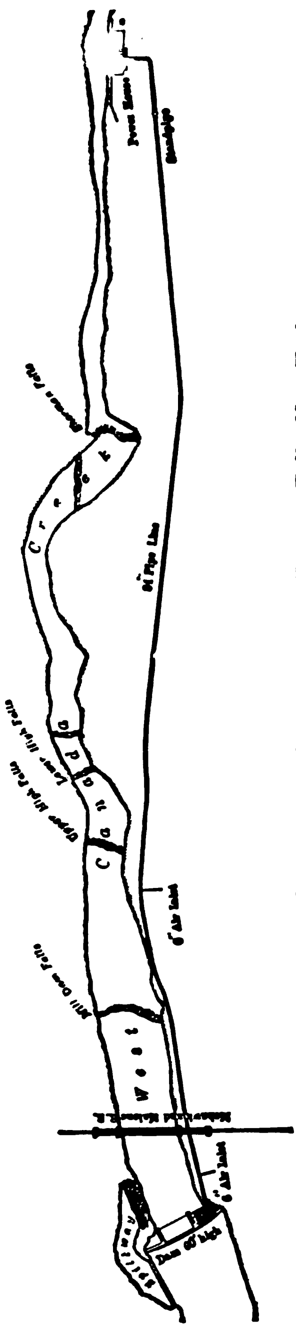


Fig. 353.—Plan of Power Development at Trenton Falls, New York.

accommodate the flood flow of the stream which is approximately 25,000 second-feet. In consequence, the present power plant, as well as the proposed extension to the power station, will form a part of the dam itself and the spillway will occupy only a portion of the entire length of the structure and is so designed as to maintain a satisfactory head at times of flood flow The head of the water above the dam is controlled both by the length of spillway and by six tainter gates by means of which the level of the water above the dam can be controlled at all stages of flow.

290. High Head Developments.—Fig. 351 illustrates the general plan of a possible method of development of the Peshtigo River for The Northern Hydro-Electric Company. The fall available is shown by the profile,—Fig. 352. It is proposed to construct a dam above High Falls of sufficient height to back the water over Twin Falls, and to either develop the power at High Falls and Johnson's Falls independently or conduct the water by a canal to Mud Lake, thence to Perch Lake, thence to the head work to be be built above Johnson's Falls, where a head of about 110′ will be available. If a single development is chosen the water will be be conducted from the head works through penstocks to the power plant to be built at the base of the bluff below Johnson's Falls. The canal in this case will conduct the head waters with very little fall to the immediate site of the plant, thence by penstocks to the tur bine located in the gorge below.

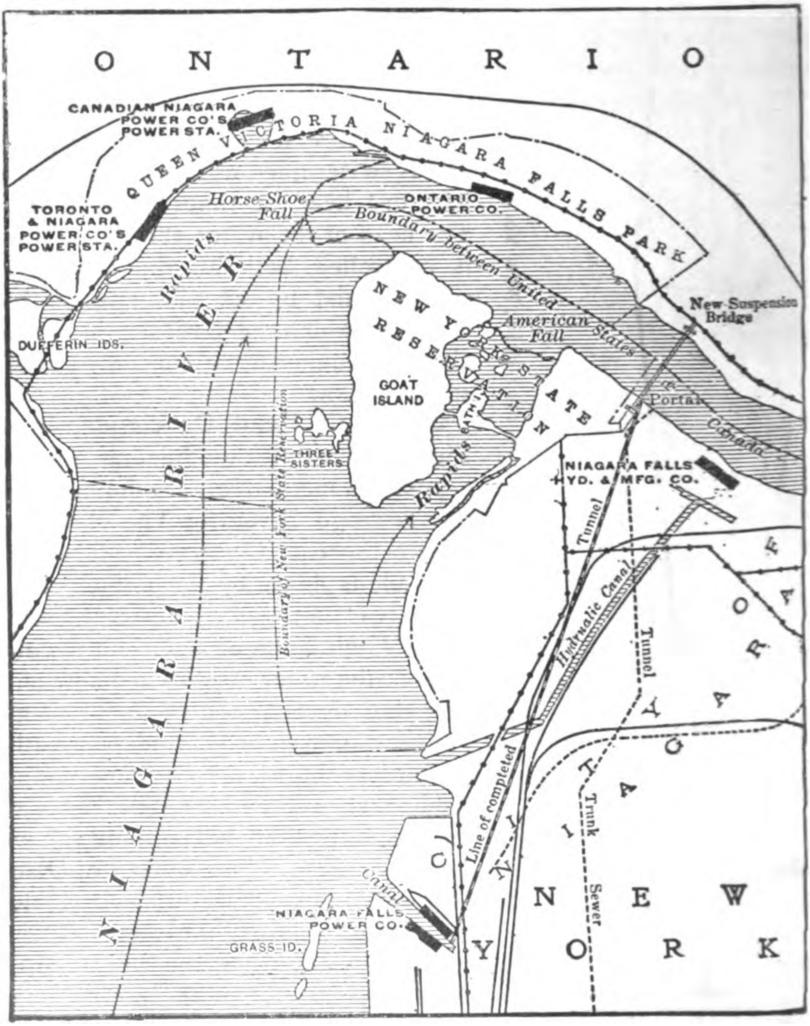


Fig. 354.—Niagara Falls Power Development.

Fig. 353 is a plant of the power development at Trenton Falls, New York. The upper portion of the fall is developed by a dam about 60′ in height, which is connected by an 84″ pipe line with the turbine located in the power house about two miles below. The turbines used in this development are the Fourneyron turbines, which are described in Chap. XIX, and are illustrated by Fig. 311.

Fig. 354 is a general plan of the water power developments at Niagara Falls. The first development was that of The Niagara

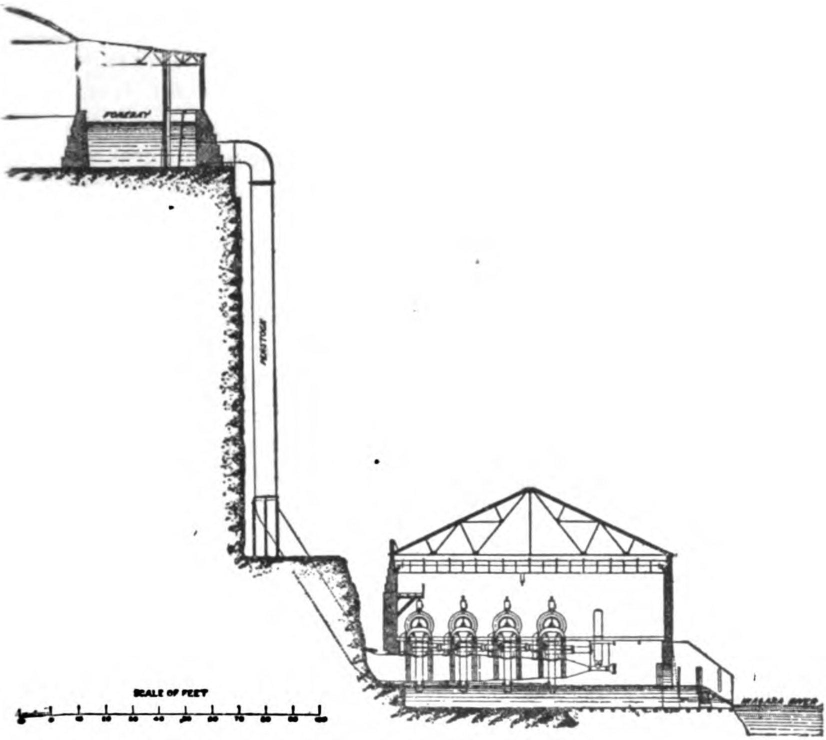


Fig. 355

Falls Hydraulic and Manufacturing Company. By means of a canal the water is taken from the upper end of the rapids and conducted to the lower bluff on the American side, and distributed, by open canals, to various plants located along this bluff..

The second plant constructed was that of The Niagara Falls Power Company, in which power is developed by the vertical shafts connecting with a tail-water tunnel which discharges into the river just below the new suspension bridge.

On the Canadian side are shown three plants.

The Ontario Power Company secures its water supply from the upper portion of the rapids, conducting it through steel conduits to a point above the power house and thence by penstocks to the wheel, located in the gorge below the falls.

In the plants of The Toronto and Niagara Power Company and The Canadian-Niagara Power Company, the water is taken from above the Falls and discharges through penstocks to wheels located at the base of a shaft and thence into tunnels, discharging into the river at a point below the Falls.

Fig. 355 illustrates the plant of The Niagara Falls Hydraulic and Manufacturing Company, which is supplied by water from the hydraulic canal above mentioned. The water is conducted from the forebay by a vertical penstock to which is attached several wheels which deliver the water into a tail-race tunnel and thence into the gorge below.

The plant arrangements above described are typical of many now in use both in this country and in Europe. It is at once obvious that in considering this subject each particular location is a problem by itself which must be considered in all its bearings; but an understanding of the designs and arrangements already in use forms a satisfactory basis from which a judicious selection can be made with suitable modifications to take care of all the conditions of topography and other controlling conditions.

CHAPTER XXIV.

PRINCIPLES OF CONSTRUCTION OF DAMS.

291. Object of Construction.—A dam is a structure constructed with the object of holding back or obstructing the flow and elevating the surface of water. Such structures may be built for the following purposes:

First: To concentrate the fall of a stream so as to admit of the economical development of power.

Second: To deepen the water of a stream so as to facilitate navigation and to so concentrate the fall that vessels may be safely raised from a lower to an upper level by means of locks.

Third: To impound or store water so that it may be utilized as desired for water supply, water power, navigation, irrigation, or other uses.

Fourth: In the form of mine dams or bulk heads to hold back the flow of water which would otherwise flood mines or shafts or cause excessive expense for its removal.

Fifth: As coffer-dams for the purpose of making accessible, usually for construction purposes, submerged areas otherwise inaccessible.

292. Dams for Water Power Purposes.—The primary object of a dam constructed for water power purposes is to concentrate the fall of the stream so that it can be developed to advantage at one point and so that the water thus raised can more readily be delivered to the motors through raceways and penstocks of reasonable length. This object is sometimes accomplished in rivers with steep slopes or high velocities by the construction of wing dams which occupy only a portion of the cross-section of the stream, but cause a heading up of the water and direct a certain portion of the flow into the channel or raceway through which it flows to the wheels. Usually in streams of moderate slope the dam must extend entirely across the stream in order to concentrate sufficient head to be of practical utility.

Wing dams can be used at the head of high falls where only a portion of the volume of flow can be utilized, as at Niagara Falls, or in rapid rivers where a portion of the flow is to be directed into a narrow channel for utilizing low heads by means of undershot or float wheels as is frequently done for irrigation purposes. Where the full benefit of both head and volume is to be utilized the dam must extend from bank to bank and be constructed of as great a height as possible.

293. Height of Dam.—To utilize a river to the maximum extent the highest dam practicable must be constructed.

The height of a dam may be limited by the following factors:

First: The overflow of valuable lands.

Second: The interference with water power rights above the point of development.

Third: The interference with other vested or public rights.

Fourth: The cost of the structure.

The value of the power that can be developed by means of a proposed dam will limit the amount that can be expended in the purchase or condemnation of property affected by backwater from the dam and the cost of its construction. These are among the elements of the cost of the project and must be considered together with other financial elements before a water power project can be considered practicable.

In considering backwater and its effect on riparian rights both high and moderate conditions of flow must be considered. The former condition gives rise to temporary interference, often of little importance when affecting purely farming property, and the real or fancied damages from which can commonly be liquidated by releases at small expense. The latter condition will permanently inundate certain low lands which must be secured by purchase or condemnation. In many states where the laws of eminent domain do not apply to the condemnation of property for such purposes it is necessary to secure such property by private purchases before the work is undertaken, and usually before the project becomes known publicly, for in such cases the owner of a single piece of land may delay the project by a demand for exorbitant remuneration, from which demand there is in such cases no escape. In every case it is desirable that riparian and property rights be fully covered before the construction of the project actually begins.

294. Available Head.—Beside the question of backwater the question of head at the dam is important both in relation to the question of interference and in relation to the question of power. In relation to interference it is an easy matter with a known length and height of dam to determine by calculation from a properly selected weir formula the height of water above the dam under any condition of flow. To determine the head available under all conditions of flow the weir curve must be studied in connection with the rating curve as discussed in Chapter V.

Two conditions of flow often require consideration in this connection:

First: Where a considerable portion of the flow is being utilized by the wheels and therefore does not affect the head of the dam.

Second: Where the water is not being used by the wheels and consequently affects the head of the dam.

Both of these conditions should be studied and determined in relation to their influence on both backwater conditions and power.

295. The Principles of Construction of Dams.—The general principles for the construction of all dams are similar, and are as follows:

First: They must have suitable foundations to sustain the pressure transmitted through them, which must be either impervious or rendered practically so.

Second: They must be stable against overturning.

Third: They must be safe against sliding.

Fourth: They must have a sufficient strength to withstand the strains and shocks to which they are subjected.

Fifth: They must be practically water-tight.

Sixth: They must have essentially water-tight connections with their beds and banks, and, if bed or banks are pervious, with some impervious stratum below the bed and within the banks of the stream.

Seventh: They must be so constructed as to prevent injurious scouring of the bed and banks below them.

The application of the above principles depends on the material from which the dam is to be built and on local conditions.

296. The Foundation of Dams.—The materials used for the construction of dams may be masonry, which includes stone-work and concrete-work, reinforced concrete, timber, steel, loose rock, and earth. Each may be used independently or in combination. Masonry and concrete dams must be built upon foundations which are practically free from possible settlement. Small masonry structures may sometimes be safely constructed on piles or grillage foundation based on softer materials; but the larger and more important structures, if constructed of masonry, can be safely built only upon solid rock. Reinforced concrete is now being extensively used for small structures and is not as seriously affected by slight settlement as in the case of dams of solid masonry. There is, however, little flexibility in structures of this kind, and the foundation

Fig. 356.—Timber Crib Dam at Janesville, Wis.

must be selected in accordance with this fact. Timber and steel possess a flexibility not possible in concrete construction and are much better adapted to locations where the foundation may be subject to settlement.

In construction on rock foundation it is usually desirable to excavate trenches therein in order to give a bond between the structure of the dam and its foundation. It is also essential with rock foundations to determine whether cracks or fissures in the foundation extend below the structure, and if such are found, they must be completely cut off.

On earth, sand or gravel foundations, when such must be used, the flow which would take place through these materials and under the structure of the dam must be completely cut off by the use of steel or timber sheet piling, which, if possible, should be driven from the structure to the rock or to some other impervious stratum. If no impervious stratum is accessible, the sheet piling must be driven to such a distance below the base of the dam that the friction of the flow of water under it will reduce or destroy the head and consequently reduce the flow of water to an inappreciable quantity.

297. Strength of Dams.—A dam to be built in a flowing stream should be designed with a full appreciation of all the stresses to which it may be subjected. Of these, stresses that are due to static pressure can be readily estimated from the known conditions. The strains due to dynamic forces are not so fully understood or easily

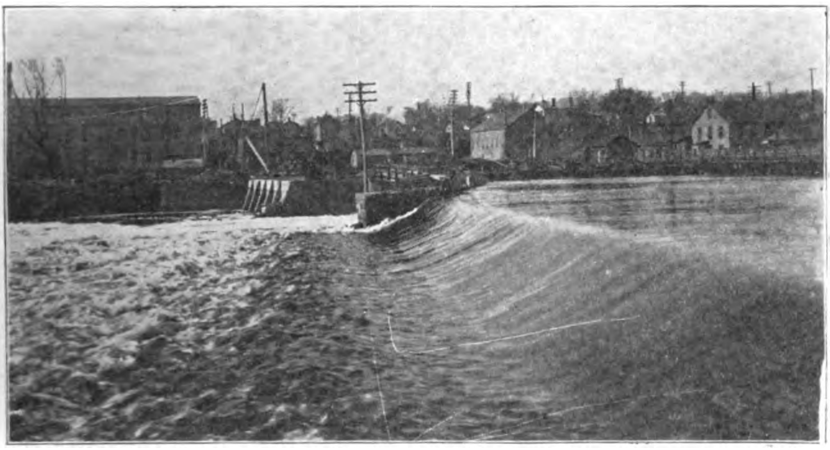


Fig. 357.—Janesville Dam with Moderate Water.

calculated. Where the structure is constructed to retain a definite head of water without overflow, as in the case of reservoir embankments, the problem becomes one largely of statics and the only other stresses to be considered are those due to ice action and the action of waves on the structure. When a dam is constructed in a running stream and is subject to the passage of extensive floods of water over it, frequently accompanied by large masses of floating ice, logs or other material which in many cases may strike the crest of the dam, and bring unknown and violent strains, the problem becomes largely one of experience and judgment.

298. Flood Flows.—The passage of great volumes of water over a dam involves the expenditure of the power so generated upon or immediately adjoining the structure, and unless preparations are made for properly taking care of this immense expenditure of power, the power may be exerted in the destruction of the structure itself.

Figs. 356 to 358 show three views of the timber crib dam at Janesville, Wisconsin, under various conditions of flow. In Fig. 356 the flow of the river is comparatively small and all of the water is being used in the power plant, none passing over the dam. In Fig. 357 the river is at a moderate stage and the greater part of the flow is passing over the crest of the dam. In Fig. 358 some four or five feet of water is passing over the dam and the power that is developed thereby is causing the standing wave and the rough

Fig. 358.—Janesville Dam under High Water.

water shown in the picture below the dam. At this point the power developed by the fall is being expended in waves and eddies, which, unless properly controlled, will attack and injure or destroy the structure. On rock bottom the rock itself will sustain the impact of flow over small dams. But where the rock is soft, or the bottom is composed of material that can be readily disintegrated, it becomes necessary to extend the structure of the dam itself in the form of an apron to cover and protect the bottom.

Fig. 359 shows the preliminary design of a dam for the Southern Wisconsin Power Company, now under construction at Kilbourn, Wisconsin. This dam will be about 17 feet in height above low water and will be subject at times to the passage of floods to a depth of 16 feet above its crest. For section of dam as constructed see Fig. 373. The two ends of the dam will rest upon a rock foundation. Cribs are also carried to the rock at the face of the dam and at the edge of the apron. The center of the dam is sus-

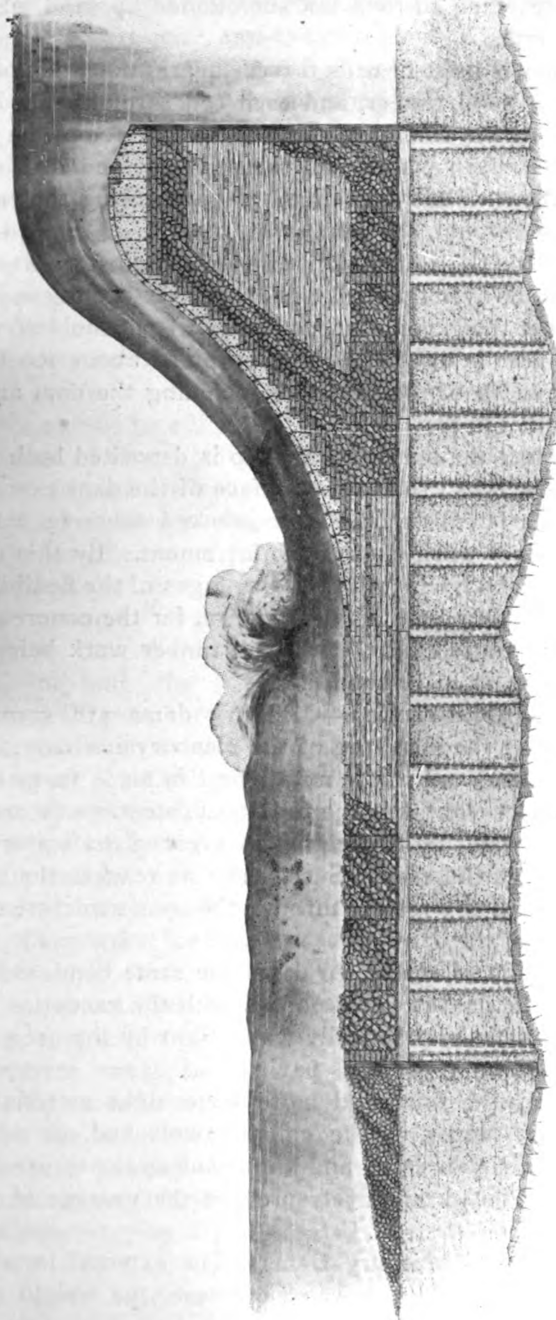


Fig. 359.—Preliminary Study of Dam for Southern Wis. Power Co., Kilbourn, Wis.

tained by piles reaching to rock but surrounded by sand which is retained by the cribs.

The dam proper is built of cells 6 feet square, the walls of each cell being built of solid timber, and each cell carefully filled with stone and sand. At the face of the dam and at the toe of the apron triple sheeting has been placed and sscurely fastened to the dam and cribs from the rock up, thus effectively preventing the passage of water below or through the dam.

During high floods the amount of power which must be wasted in the passage of water over the dam will exceed 100,000 horse power. In order to prevent the expenditure of this power in the destruction of the dam, the dam is extended in an apron of about 100 feet in width, the total width of the structure including the dam and the apron, being about 150 feet.

To further protect the structure, rip-rap is deposited both above and below the structure itself. The surface of the dam exposed at times of low water is constructed of re-inforced concrete, attached directly to the timber work of steel reinforcement. By this design a structure is obtained having all the advantages of the flexibility of timber, with the lasting qualities of masonry, for the concrete only will be exposed at times of low water, all timber work being submerged under every ordinary condition.

299. Impervious Construction—Masonry dams are commonly made impervious by the structure of the masonry itself.

In timber crib dams ordinarily no attempt is made to make the structure itself water-tight, but the top and upstream side are usually covered with water-tight sheeting to prevent the water passing into and through the cribs. Such water as reaches the timber cribs usually passes away readily through the open structure on the down stream side of the dam.

In the construction of rock-filled dams the same condition ordinarily obtains. The dam is fairly porous with the exception of its upper face which is made practically water-tight by the use of concrete, puddle, or some impervious paving.

In earthen dams the finer and more water-tight materials are used on the inner slopes of the embankment, and, in addition thereto, it is customary in large and important works to use a core of concrete or puddle to effectively prevent the passage of water through the structure.

300. The Stability of Masonry Dams.—The external forces acting on a masonry dam are the water pressure, the weight of the masonry, the reaction of the foundation, ice and wave pressure near the top, wind pressure, and back pressure of the water on the down stream side. The action of these forces may cause a dam to fail by:

(1) Sliding on the base or on any horizontal plane above the base.

(2) Overturning.

(3) Crushing the masonry or foundation.

If the dam be built of rubble masonry there will be no danger of failure by sliding on a horizontal joint above the foundation and experience has shown that where a good quality of mortar is used it can be depended upon to prevent sliding in concrete and stone dams having horizontal bed joints. The joint between the dam and its foundation is a more critical point. In rock foundation steps or trenches should be cut so as to afford good anchorage for the dam. In the case of clay, timber or similar foundations the dam will have to be made massive enough so that the tangent of the angle between the resultant pressure on the base and a vertical line is less than the co-efficient of friction between the materials of the dam and the foundation.

It is customary in the design of masonry dams to proportion the section so that the lines of resultant pressure at all horizontal joints, for both the conditions of reservoir full and reservoir empty, shall pass through the middle third points of the joints. If this condition is fulfilled, the factor of safety against overturning at every joint will be 2, and there will also be no danger from tensile stresses developing in the faces of the dam.

Investigation has shown that there is no danger of crushing the masonry except in very high dams, with the consideration of which we are not here concerned.

301. **Calculation for Stability.**—The general conclusion may therefore be stated, that, in the case of ordinary masonry and concrete dams, not over 100 feet in height, to be built on rock foundations, the design can be based upon the condition that the lines of pressure must lie within the middle third of the profile This rule must be modified at the top of the dam to resist the stresses due to waves, ice, etc. The force exerted by ice is an indeterminate quantity and the tops of dams must therefore be proportioned in accordance with empirical rules. Dams are built with top widths varying from 2 to 22 feet, the broader ones usually

carrying a roadway. Coventry suggests the following empirical rules for width of top and height of top above water level·

(1) $$b = 4.0 + 0.07\,H$$

(2) $$y_0 = 1.8 + 0.05\,H$$

Where b is the width of top, y_0 the height above water level and H the greatest depth of water. Both faces of the dam will be vertical until the depth y_1, is reached, where the resultant force passes through the middle third point. Below this depth the general rule will apply. In computing the water pressure against the dam, it

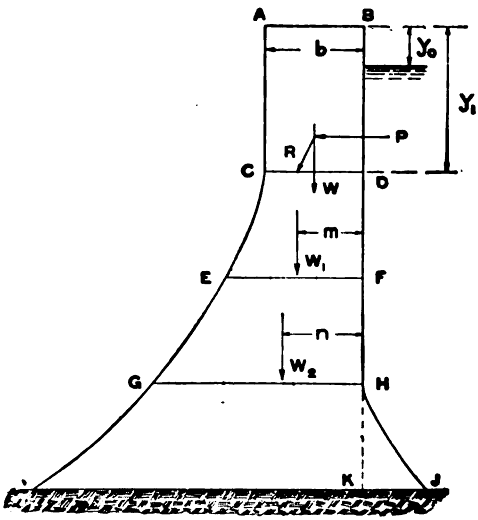


Fig. 360.

is best to consider the water surface level with the top of the dam in order to allow for possible rises due to floods, etc. Having determined the top width, b, and assuming a section of the dam one foot long, the height, y_1, of the rectangular portion can be deduced from the formula

(3) $$y_1 = b\sqrt{s}$$

in which s is the specific gravity of the material of the dam.

The down-stream face of the dam must now be sloped so as to keep the resultant pressure, with the reservoir full, at the limit of the middle third of the length of any joint. Dividing the remainder of the height of the dam into lengths convenient for computation, the length of any joint, (see Fig. 360) as $\overline{GH}$ may be found by the formula

$$(4) \qquad \overline{GH} = \sqrt{B + C^2} - C$$

in which

$$B = \frac{6\,m\,(\text{Area } \overline{ABFE})}{\overline{FH}} + \frac{\overline{BH}^3}{8\,\overline{FH}} + \overline{EF}^2$$

where m = distance from F of the line of action of the weight of masonry above $\overline{EF}$ and

$$C = \tfrac{1}{2}\left[\frac{4\,(\text{Area } \overline{ABFE})}{\overline{FH}} + \overline{EF}\right]$$

The value of n is given by the equation

$$(5) \qquad = \frac{\text{Mom. of } \overline{ABFE} + \text{Mom. of } \overline{EFHG}}{(\text{Area } \overline{ABHG})}$$

moments being taken about the point H.

Equation (4) can be used as long as n is greater than one-third the length of the joint. When this condition can no longer be satisfied with a vertical face, it will be necessary to batter the upstream face also, so that the lines of pressure with reservoir full and empty both lie at the limits of the middle third of the length of any joint.

The length of the joints, as $\overline{IJ}$, may now be found by the formula

$$(6) \qquad \overline{IJ} = \sqrt{\frac{\overline{BK}^3}{8\,\overline{HK}} + \left(\frac{\overline{GH}}{2} + \frac{(\text{Area ABHG})}{\overline{HK}}\right)} - \frac{(\text{Area } \overline{ABHG})}{\overline{HK}} + \frac{\overline{GH}}{2}$$

and the value of $\overline{KJ}$, is

$$(7) \qquad \overline{KJ} = \frac{2\,(\text{Area } \overline{ABHG})\ (\overline{IJ} - 3m) - (\overline{HK} \times \overline{GH}^2)}{6\,(\text{Area } \overline{ABHG}) + \overline{HK}\ (\overline{2GH} + \overline{IJ})}$$

In high dams two more stages, governed by the compressive strength of the masonry, would have to be considered, but, within the limit of height set above, the formulas given are sufficient.

The position of the line of pressure may be readily determined also by graphical methods.

In the case of overfall dams, which are necessarily subjected to dynamic forces, which are more or less indeterminate, the design cannot be so closely figured.

302. **Further Considerations.**—The preceding analysis does not take into account the possibility of an upward pressure from below the dam, due to the previous character of the foundation, or to cracks and fissures, by means of which the pressure of the head water may be transmitted to the base of the dam. This factor is commonly ignored in dam construction, but should be considered,

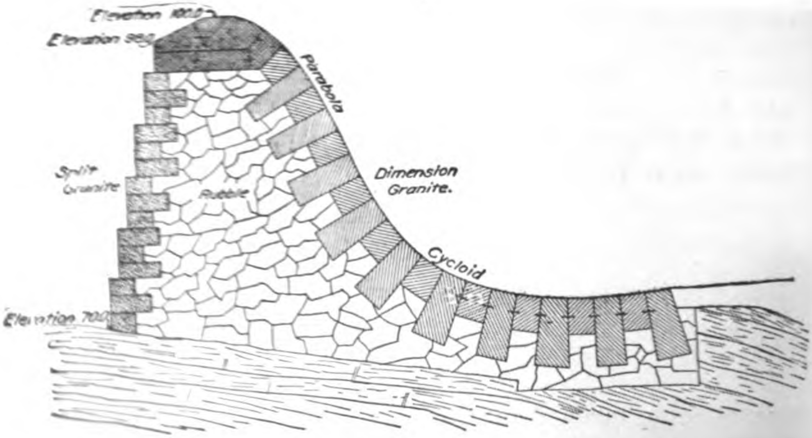


Fig. 361.—Cross-section of Dam of Holyoke Water Power Co.

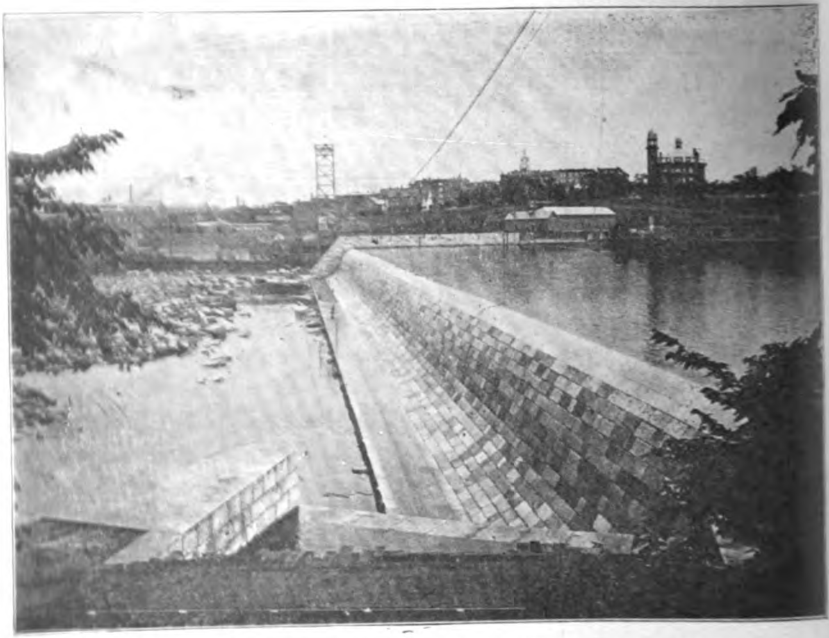


Fig. 362.—Masonry Dam of Holyoke Water Power Co.

and, when occasion requires, the foundation should be so prepared as to obviate or reduce it to a minimum. This may usually be done by the careful preparation of the foundation to prevent inflow, or by the construction of drains from the interior of the foundation to the lower face.

The construction of a dam with a vertical overfall, unless provision is made for the admission of air, will result in the formation of a partial vacuum below the sheet, and a certain extra strain on

Fig. 363.—Holyoke Dam During Flood.

the structure due to the same. The vertical overfall is also frequently objectionable, on account of the action of the falling water on the bed of the stream immediately adjacent to the dam, and on the foundation of the dam itself. It is frequently desirable to give the lower face of the dam a curved outline, in order to guide the water smoothly over the dam, and deliver it approximately tangential to the stream bed. The convex surface of the dam should be of such form that the water will, through gravity, adhere to it.

An example of a dam with a curved face is shown by Fig. 361 which is a section of the dam of the Holyoke Water Power Company. Two views of the dam, one during low water (Fig. 362) and one with about ten feet of water flowing over the crest (Fig.

363) are also shown. A section of the McCall's Ferry dam, built of Cyclopean Concrete (height 53 feet) is shown in Fig. 364 and a section of a small Concrete dam at Danville, Ill., is shown in Fig. 365. The curve for dams of this character should be kept at or above the

Fig. 364.—Section of McCall Ferry Dam (Eng. Rec.).

parabolic path that the water would take in a free fall with the initial horizontal velocity corresponding to the depth of water on the dam.

From equation 50, page 64, the flow over one foot of crest will equal,

$$q = vh = m(\tfrac{2}{3})\sqrt{2g}\, h^{\frac{3}{2}}, \qquad \text{hence,}$$
$$v = m(\tfrac{2}{3})\sqrt{2gh}$$

The abscissa of the parabola is $x = vt$, in which $t =$ time in seconds.

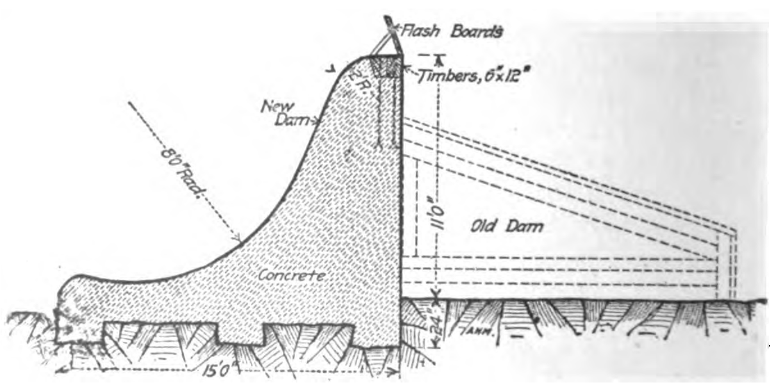


Fig. 365—Concrete Dam, Danville, Ill.

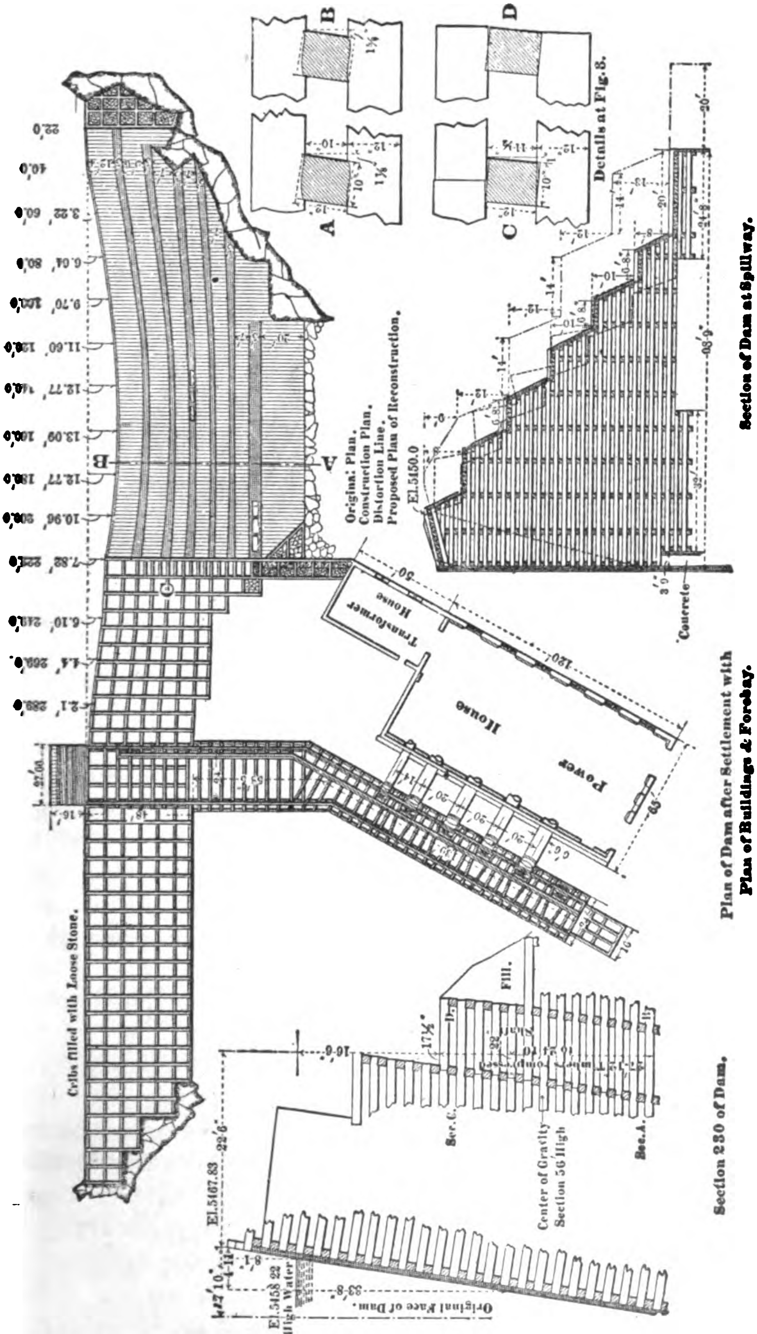


Fig. 366.—Timber Crib Dam of the Montana Power Co., Near Butte, Montana. (Jour. Asso. Eng. Soc., vol. 22, page 175.)

The ordinate is, $y = \frac{1}{2} gt^2$, hence.

$$y = \frac{g}{2v^2} x^2$$

is the equation of the parabola.*

When a curved face is impracticable or undesirable and the bed of the stream, below the dam, is not of suitable material to resist the impact of the falling water, some form of apron must be provided. Sometimes the dam is divided into steps over which the water falls in numerous cascades. Such a dam is shown in Fig. 366. This is the timber crib dam constructed for the Montana

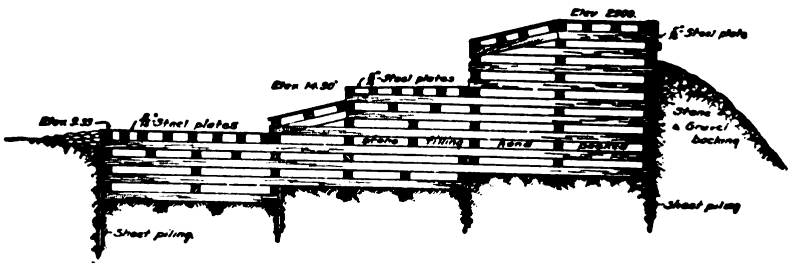


Fig. 367.—Timber Dam at Sewall Falls. (Eng. News, vol. XXXI.)

Power Company, near Butte, Montana. In this case the cells are composed of timber, laid alternately in each direction, with a considerable space left between them, instead of being built solid as in the Kilbourn dam. These cells were filled with broken stone. and the upstream side of the dam was planked with sheeting in order to make the structure water-tight. When the water was admitted behind the dam a portion of the structure was forced out of alignment by the crushing of the timbers, at the points of contact. The amount of this displacement and the cause of the same is quite clearly shown in the cut.

Fig. 367 is a section of the Sewall Falls dam, showing a similar method of resisting the impact of the overflow.

304. Types and Details of Dams.—The types of dams are so numerous, and the details of construction vary so greatly with every locality, that an entire volume would be necessary to adequately cover this subject. As the subject is already well covered in many special treatises and articles, no attempt will be made to discuss this subject in the present edition. Numerous references are given to books and articles in which special forms of construction are discussed and described.

*Turneaure & Russell's "Public Water Supplies," Section 446

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CHAPTER XXV.

APPENDAGES TO DAMS.

305. **Movable Dams.**—The height of a dam is limited in the manner hereinbefore described. It will be noted that the limit is that imposed by high water conditions and that, as a rule, the water surface during low stages could be raised to a considerable amount without interference with the riparian owners, if at the same time flood conditions could be provided for. In order to provide such conditions, movable dams are sometimes constructed which will permit of raising or lowering all or a part of the structure as the

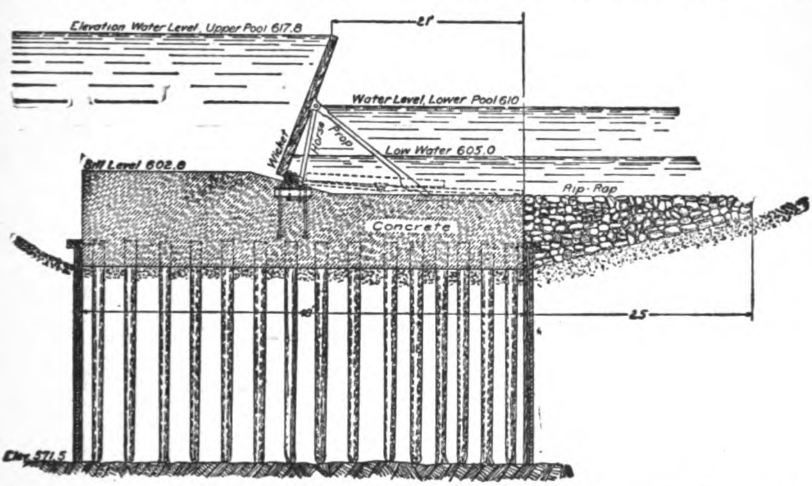

Fig. 368.—U. S. Movable Dam on Pile Foundation, McMechen, W. Va. (Eng. News, vol. 54, page 100.)

stage of the water requires. These flexible portions of the dam may consist of a gate or series of gates which can be raised or lowered. Sometimes a considerable portion of the dam is made flexible by the construction of a bear trap leaf, which is usually raised and lowered by hydraulic pressure, and by means of which the head of water can be readily and rapidly controlled. Sometimes

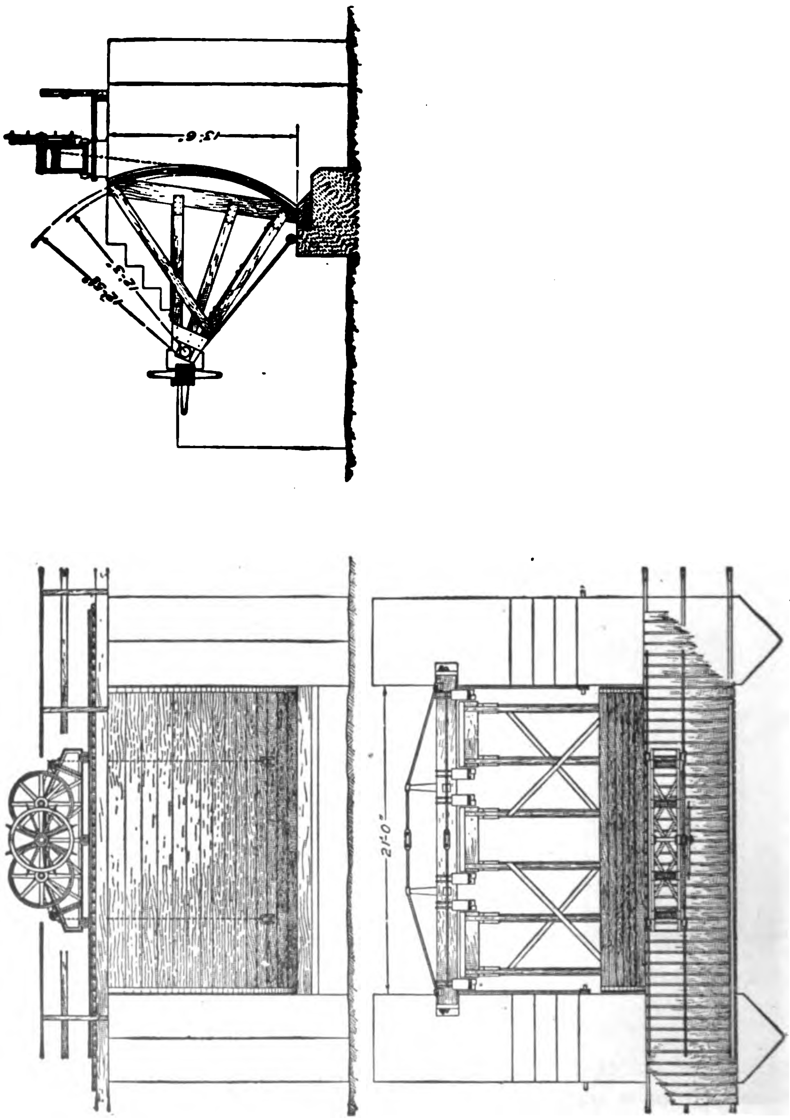

Fig. 869.—Tainter Gates in U. S. Dam at Sterling, Ill.

the entire dam is made movable by the use of Chanoine wickets (see Fig. 368) and similar types of dams, a part of which may be removable while other parts are folded down on the bed of the stream, allowing the flood waters to pass over them. Most of such

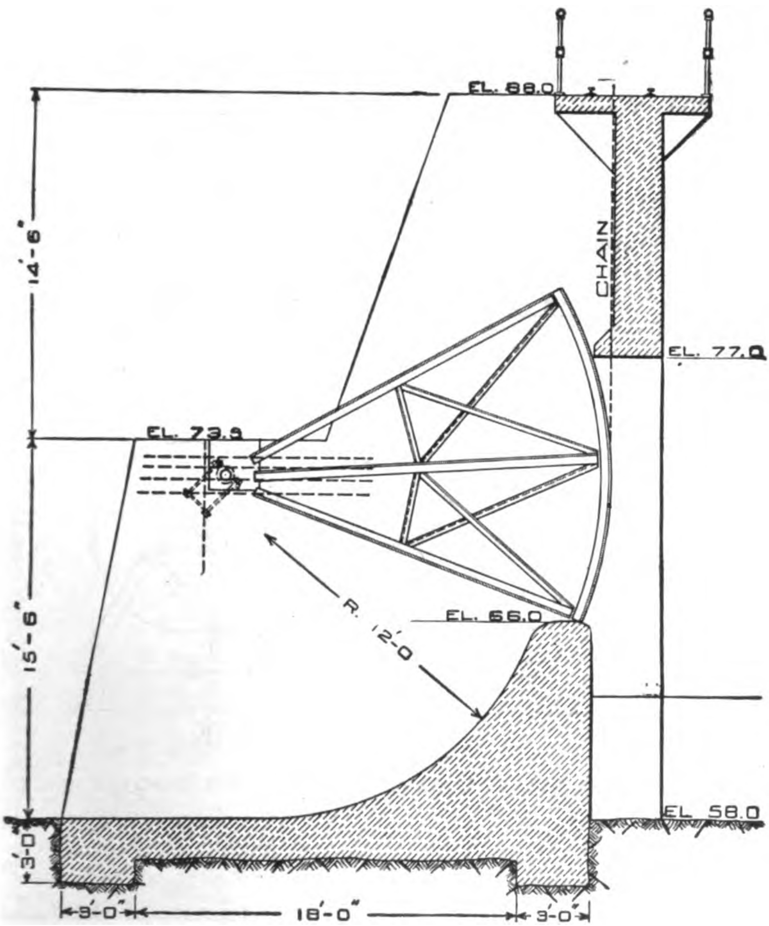

Fig. 370.—Tainter Gates for Morris Plant, Economy Light and Power Co.

constructions are expensive and are used most largely on government works for the control of rivers for navigation purposes.

The objection to movable dams for water power purposes is that the reduction in the elevation of the head water by their use commonly so reduces or destroys the head that the continuity of the power output is interrupted. The same objection also applies to any gate, flash board or other device designed to reduce the head. Such reduction is usually made during conditions of flow under which the natural head that would obtain is already at a minimum.

306. Flood Gates.—Flood gates are quite commonly used in water power dams to control or modify extreme flood heights. These gates are commonly designed to be raised so as to permit of the escape of the water underneath them. The tainter gate, in

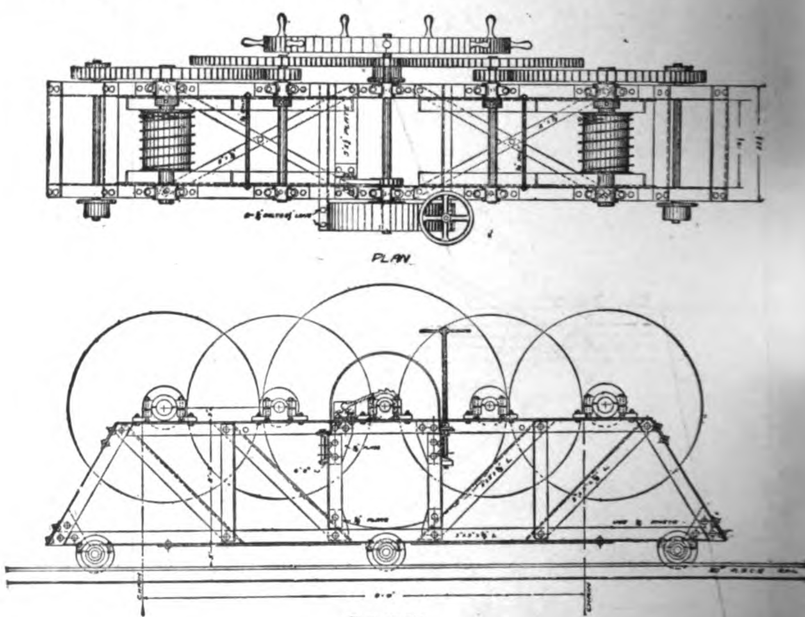

Fig. 371.—Hoist for Tainter Gates of Northern Hydro Electric Power Co.

some of its modifications, is perhaps most widely used for this purpose. Fig. 369 shows a plan, elevation and section of a tainter gate, designed by L. L. Wheeler, resident engineer of the Illinois and Mississippi Canal, for the U. S. Government dam at Sterling, Illinois. This is one of a series of tainter gates designed for the flood control of the Rock River at that point. The gates are operated by an overhead hoist which can be moved from gate to gate when it is desired to manipulate them.

Fig. 370 is a section of one of six gates designed by the writer for the Morris plant of the Economy Light and Power Company.

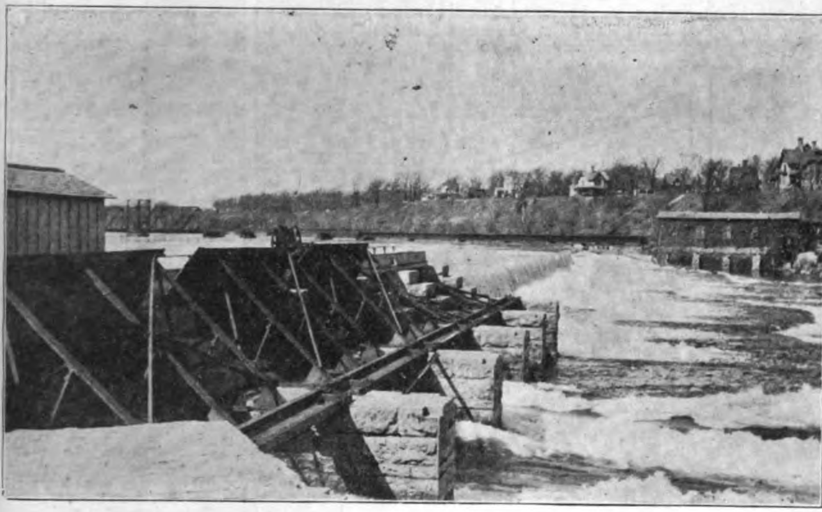

Fig. 372.—Tainter Gates at Upper U. S. Gov. Dam, Appleton, Wis.

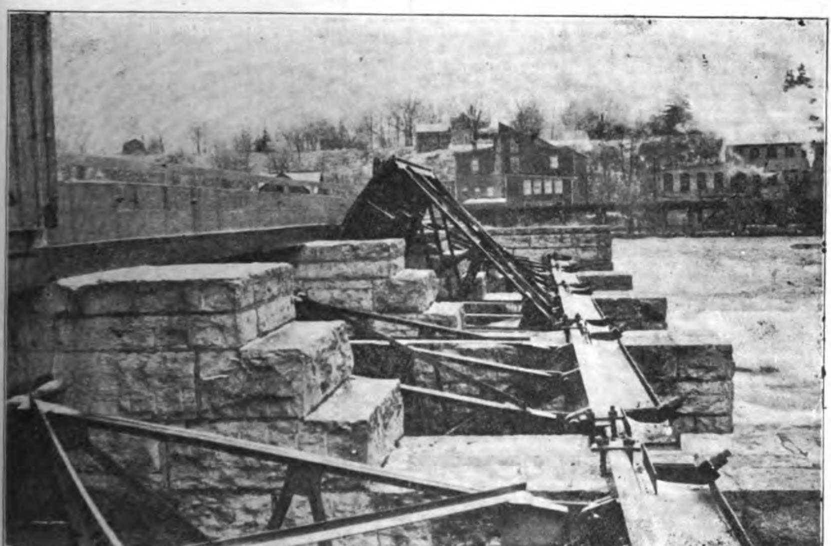

Fig. 373.—Tainter Gates at Lower U. S. Gov. Dam, Appleton, Wis.

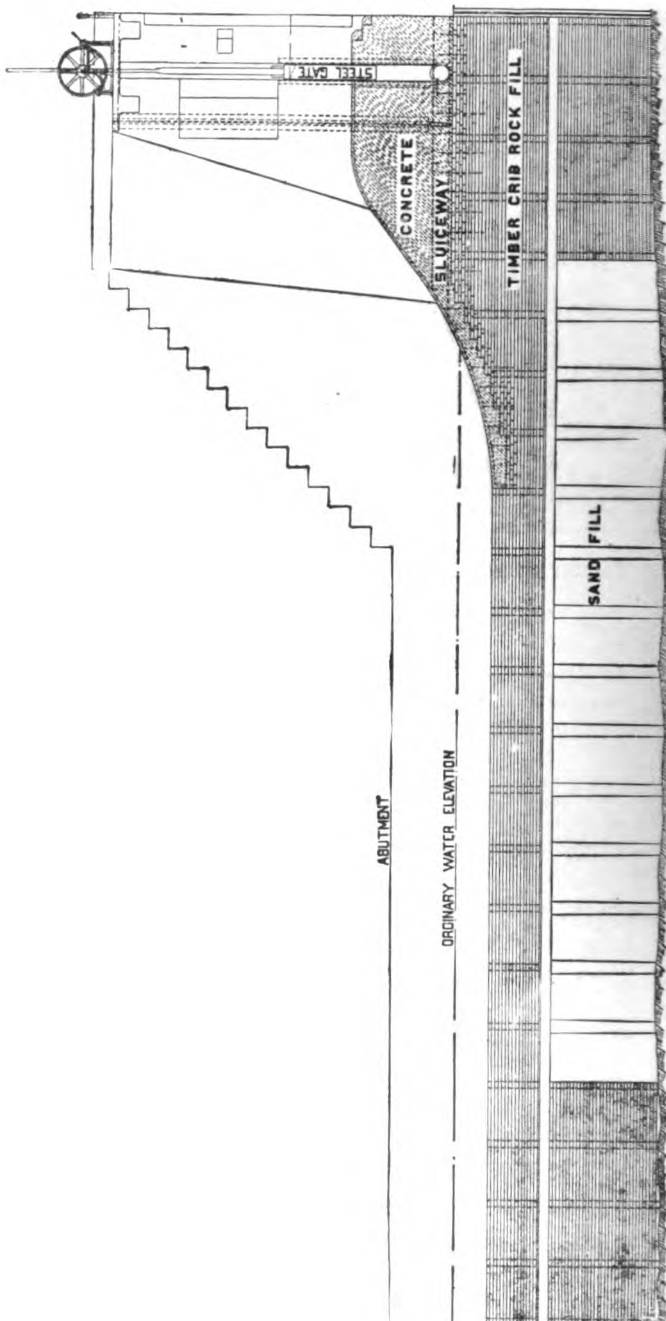

Fig. 374.—Dam at Kilbourn, Wis., with Movable Crest, Southern Wisconsin Power Co.

These gates are operated by a movable hoist, similar to Fig. 371, which travels on a track on the brige above.

Figs. 372 and 373 are views of the steel tainter gates constructed in the upper and lower U. S. Government dams across the Fox River at Appleton, Wisconsin.

In the dam of the Southern Wisconsin Power Company at Kilbourn, Wisconsin, the rise of the flood water is so great (about 16 feet) that it was found impracticable to construct lift gates to reduce the flood heights. In this case the writer has divided the crest,

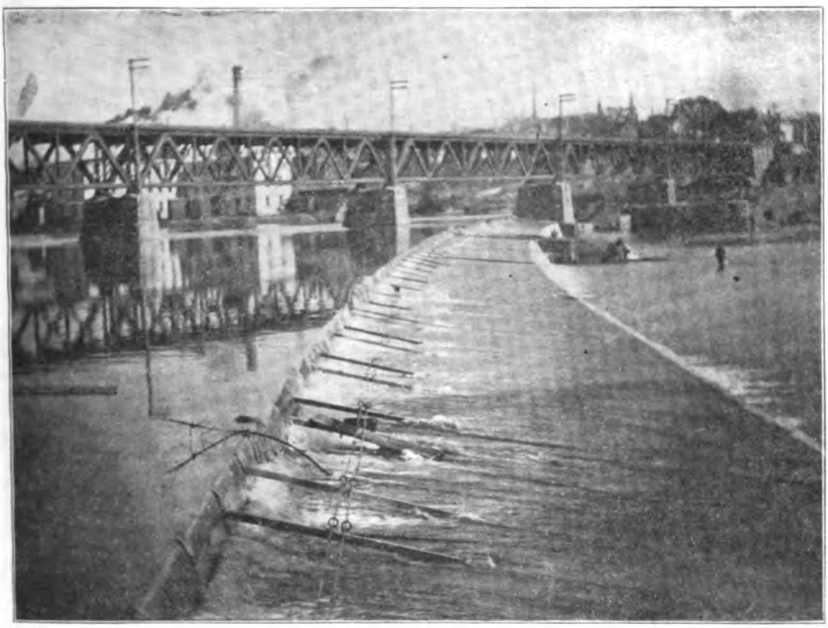

Fig. 375.—Flash Boards and Supports, Rockford Water Power Co.

by piers, into twelve sctions. Between each two piers a twenty-five foot gate is placed (see Fig. 374) which can be lowered into the dam six feet, thus reducing the extreme flood height by that amount. These gates are of steel and weigh about seven tons each. They may be operated by an electric motor or may be manipulated by hand, should occasion require.

307. Flashboards.—The control of limited variations in head is commonly accomplished by means of flash-boards which are widely used for this purpose. The simplest form of flash-board consists of a line of boards placed on the crest of the dam (see Fig. 375) usually held in place by iron pins to which the boards are commonly attached by staples. The object of flash-boards is principally to afford a certain pondage to carry the surplus water from the time of minimum use of power to the time of maximum demand. Incidentally, the head is raised and the power is also increased in this way. The supports of the flashboards should be so arranged that they will withstand only a comparativelly low head of water flowing over the boards, and will be carried away if a sudden

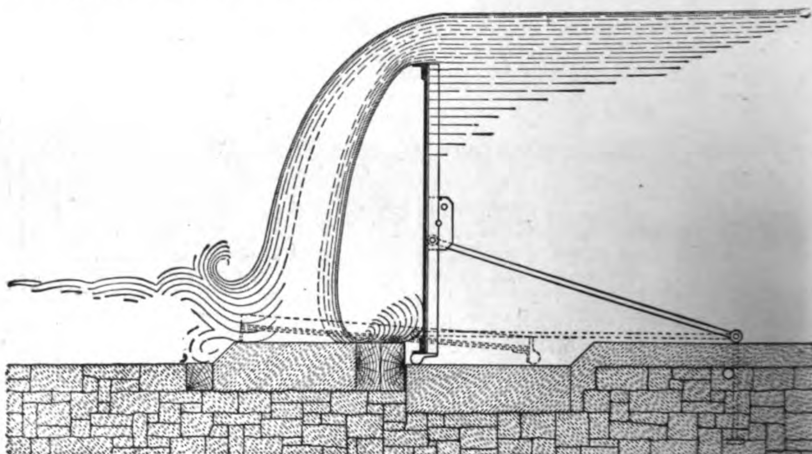

Fig. 376.—Automatic Drop-Shutter for Betiva Dam, India. (Eng. News, June 4, 1903.)

flood should raise the head materially above a safe elevation. If the boards are so supported as to withstand the discharge of heavy floods, they will form a permanent portion of the dam and increase its fixed elevation to such an extent as to create damage which their use is supposed to avoid. Sometimes the pins supporting the boards are made so light that they must be held in position by inclined braces. These braces are sometimes supplied with steel eye-bolts through which is passed a cable. A large steel washer is attached at one end and a winding drum at the other. (See Fig. 375). Commonly, if a flood is anticipated, the boards are removed and stored for future use. If, however, a sudden flood should arise, the inclined braces are removed by winding up the cable and the pressure on the flash-boards bends the pins and the boards are washed away. The expense involved by the loss of flash-boards is not excessive as one set will commonly take care of the entire summer low water period. The expense involved in their use is therefore only the cost of one set of flash-boards per year.

Sometimes the flash-boards constitute a permanent but adjustable part of the dam and are lowered automatically during stages of high water. (See Fig. 376). On some dams, especially at waste weirs of canals and reservoirs where the fluctuations in height are inconsiderable, the dam may be provided with a foot bridge which makes the whole crest of the dam accessible at all times and from which the flash-boards can be readily adjusted. This plan is used on the dam across the Chippewa River at Eau

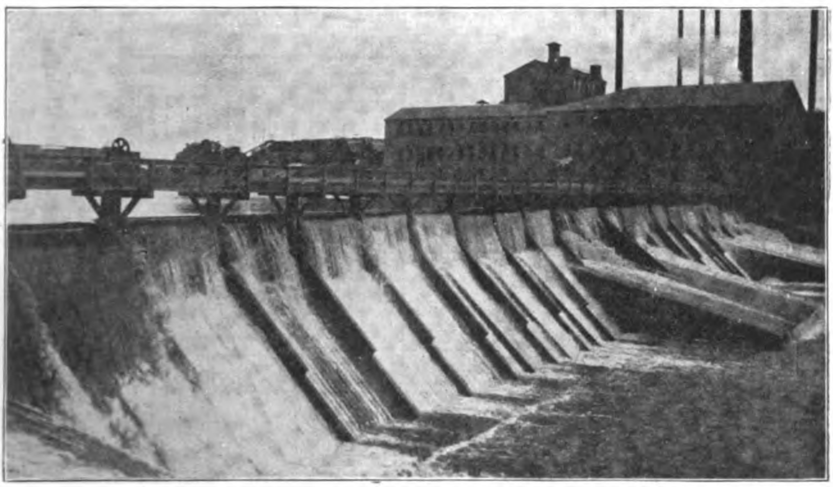

Fig. 377.—Adjustable Flash Boards at Eau Claire, Wis.

Claire, although this river is subject to high floods. (See Fig. 377). Ordinarily, on rivers subject to such conditions, this type of construction is impracticable.

In some dams, instead of gates or flash-boards, vertical stop planks or needles are used. These consist of planks or squared timbers that are lowered vertically into position, stopping off the opening partially or wholly, as desired. They are commonly supported by a shoulder at the bottom of the opening and one or more cross beams above.

308. Head Gates and Head Gate Hoists.—It is usually desirable to control the water at the inlets to the headrace by the use of gates which may be closed in emergencies or for the purpose of making

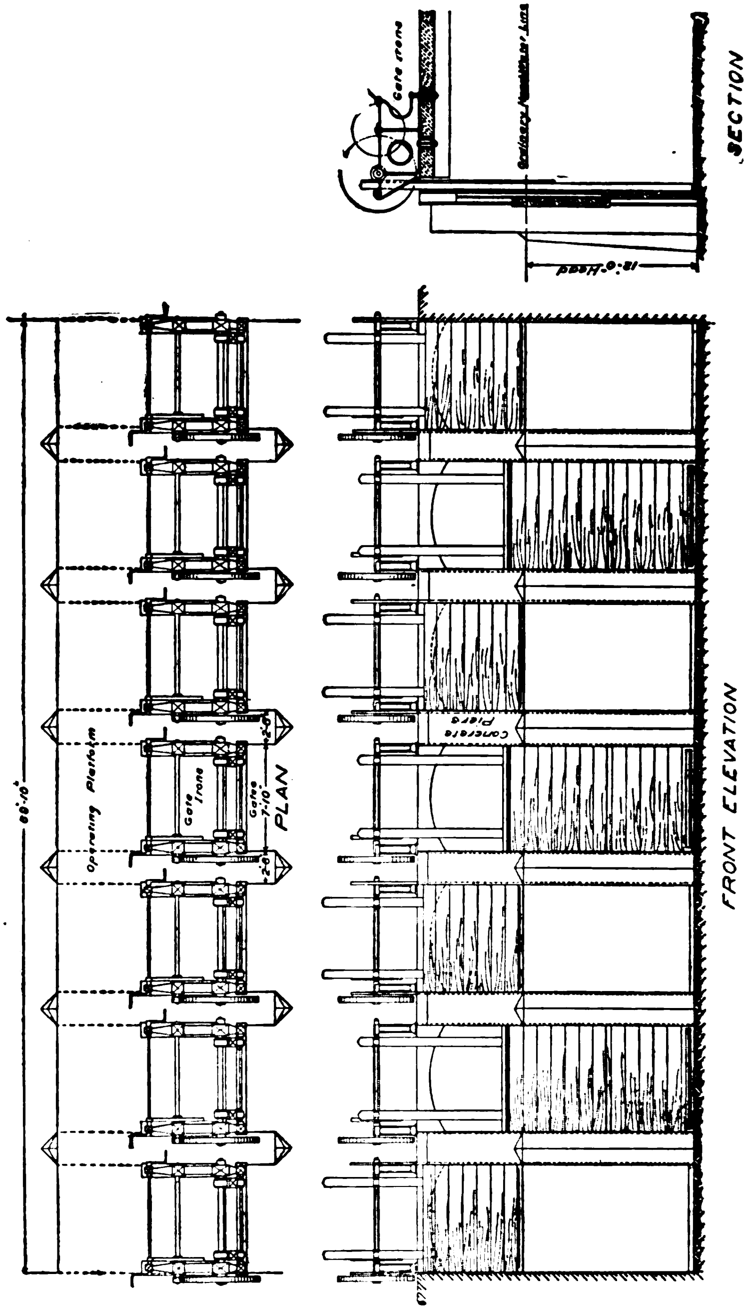

Fig 378.—Head Gates at Constantine, Mich.

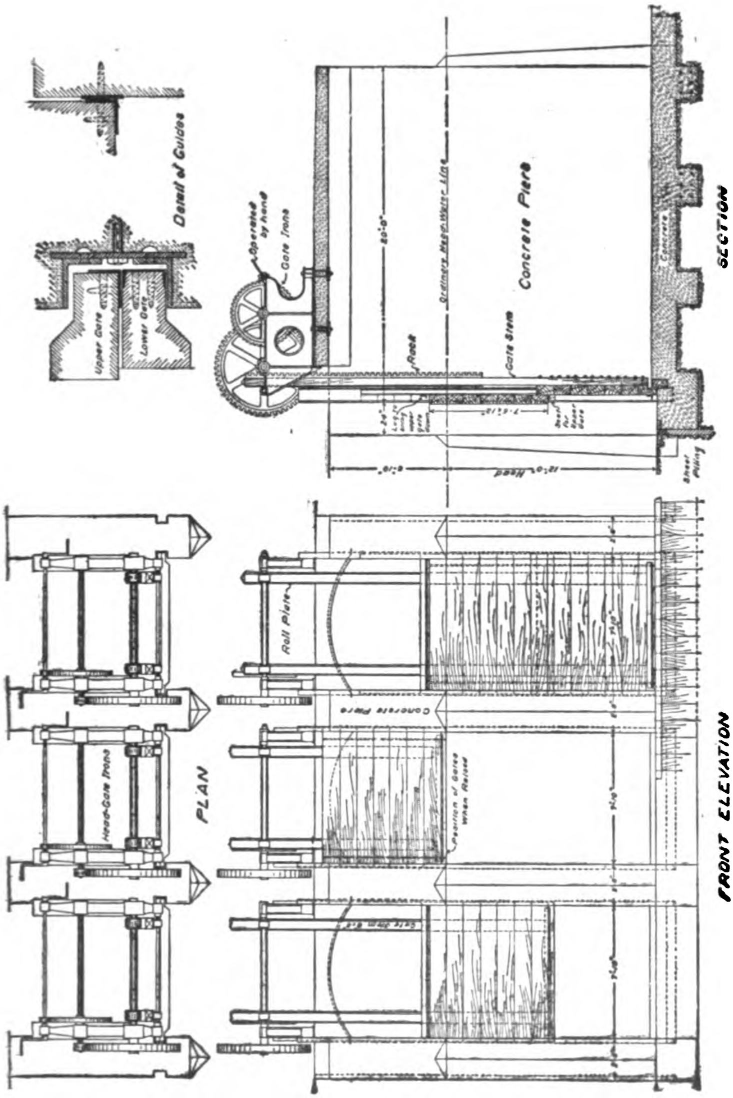

Fig. 379.—Details of Head Gates at Constantine, Mich.

necessary repairs or modifications in the raceway through which the water is diverted to the plant. In northern rivers it is also found desirable to prevent the entrance of ice into the raceway either by the construction of a floating or fixed boom in front of the gates or by constructing a system of submerged arches either in front of, or as a part of, the gateways. By means of such construction the floating ice or other floating material may be diverted from the raceway and passed over the spillway of the dam.

The head gates must be sufficiently substantial to allow the race to be emptied under ordinary conditions of water and to protect the raceway under flood conditions.

Fig. 378 shows an elevation of the head gates, designed by the writer for the power plant at Constantine, Michigan. These are shown in detail by Fig. 379. A rear view of these gates from the race side is also shown in Fig. 380. These gates are double wooden gates with concrete gateways and are arched over between the piers so as to permit the passage of men and teams. These gates are designed to pass about 2,000 cubic feet per second.

Fig. 381 shows a set of double wooden gates, the posts and braces of which are made of structural steel designed by the writer for the power plant of Mr. Wait Talcott, at Rockford, Illinois.

In the Constantine gates the gate mechanism is geared for fairly rapid operation by two men. The Rockford gate apparatus is very simple, the gate being handled with a capstan bar by a single man but at a much slower speed.

Fig. 382 shows the movable head gate hoist designed by the writer for the operation of the head gates at the Kilbourn plant of the Southern Wisconsin Power Company.

309. **Fish-Ways.**—In almost every state fishways are required by law in any dam constructed on natural waterways. These fishways must be so arranged as to permit the free passage of fish up the stream.

Fig. 383 shows a concrete fishway built by the writer in connection with the ogee concrete dam constructed across the Vermillion River at Danville, Illinois. Fig. 384 is a fishway designed by Mr. L. L. Wheeler and constructed in the dam at Sterling, Illinois. The Sterling dam is a timber crib dam and the fishway is constructed of timber. Fig. 385 shows the type of fishway recommended by the Fish Commission of the State of Wisconsin and ordinarily used in that state.

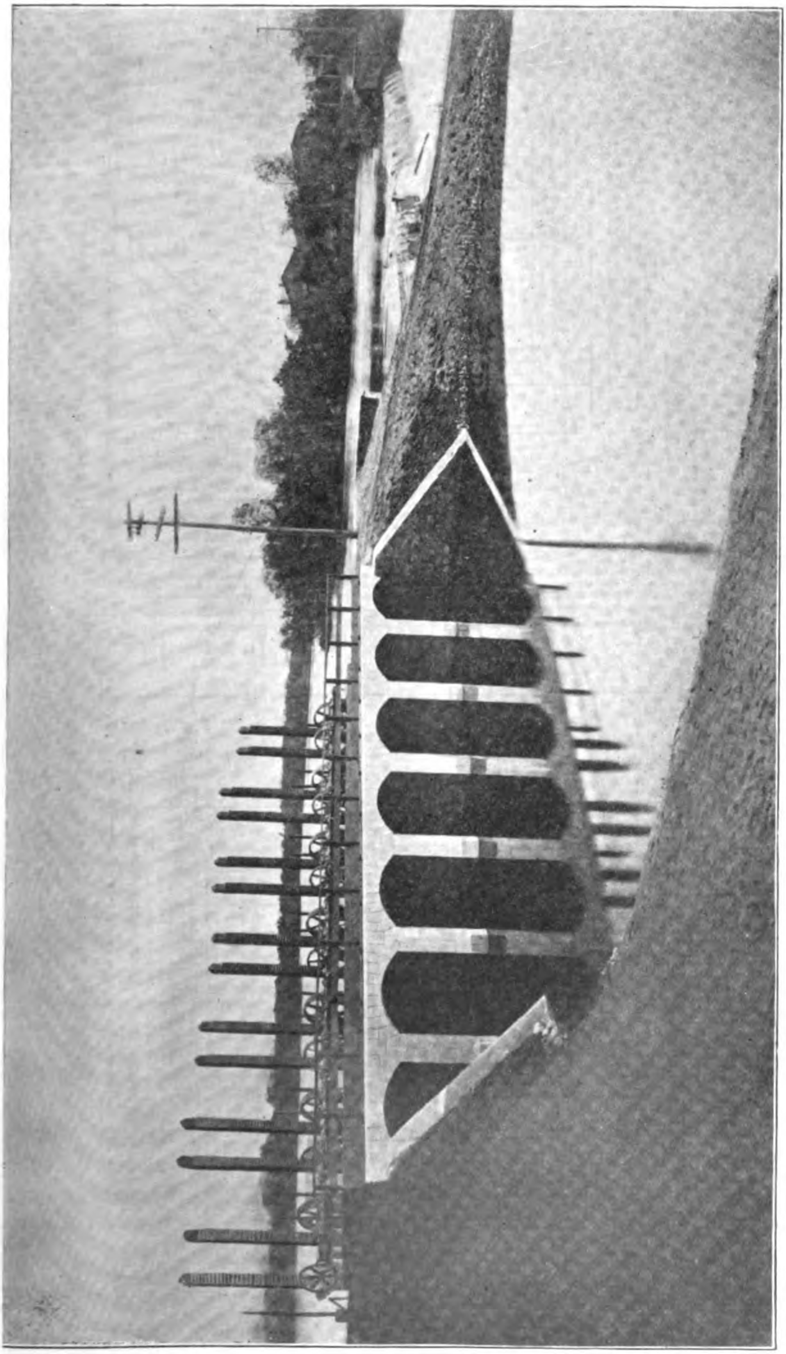

Fig. 380.—Rear View of Head Gates at Constantine, Mich.

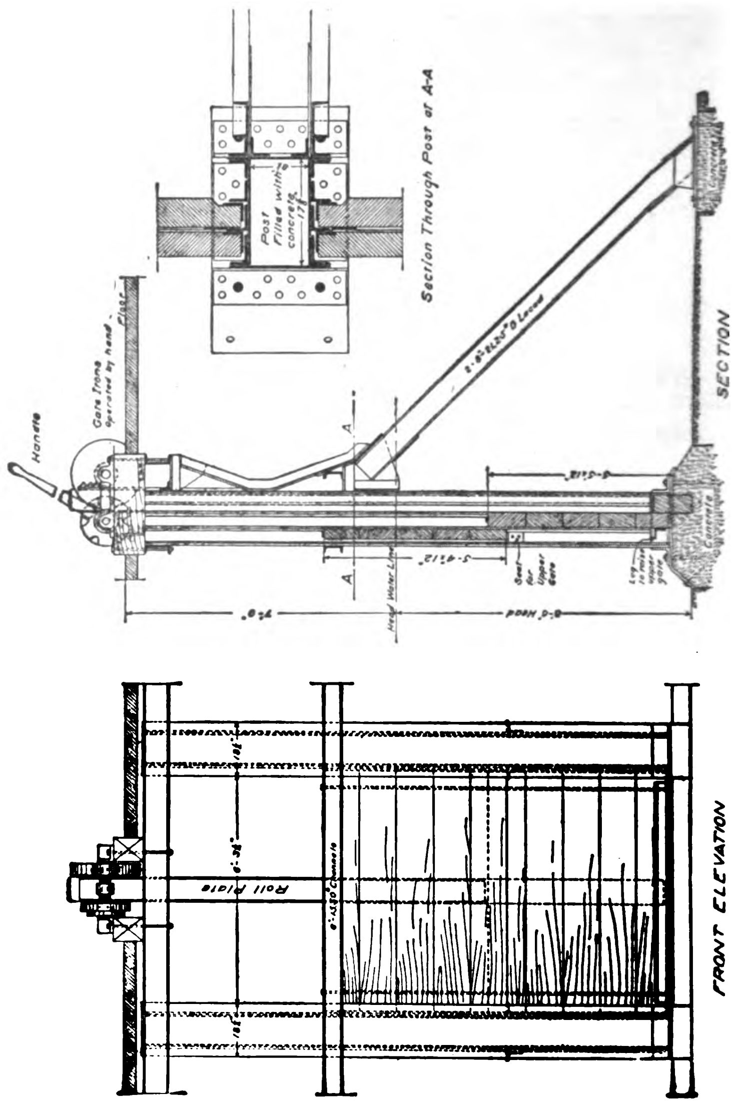

Fig. 381.—Details of Head Gates for Mr. Walt Talcott, Rockford, Ill.

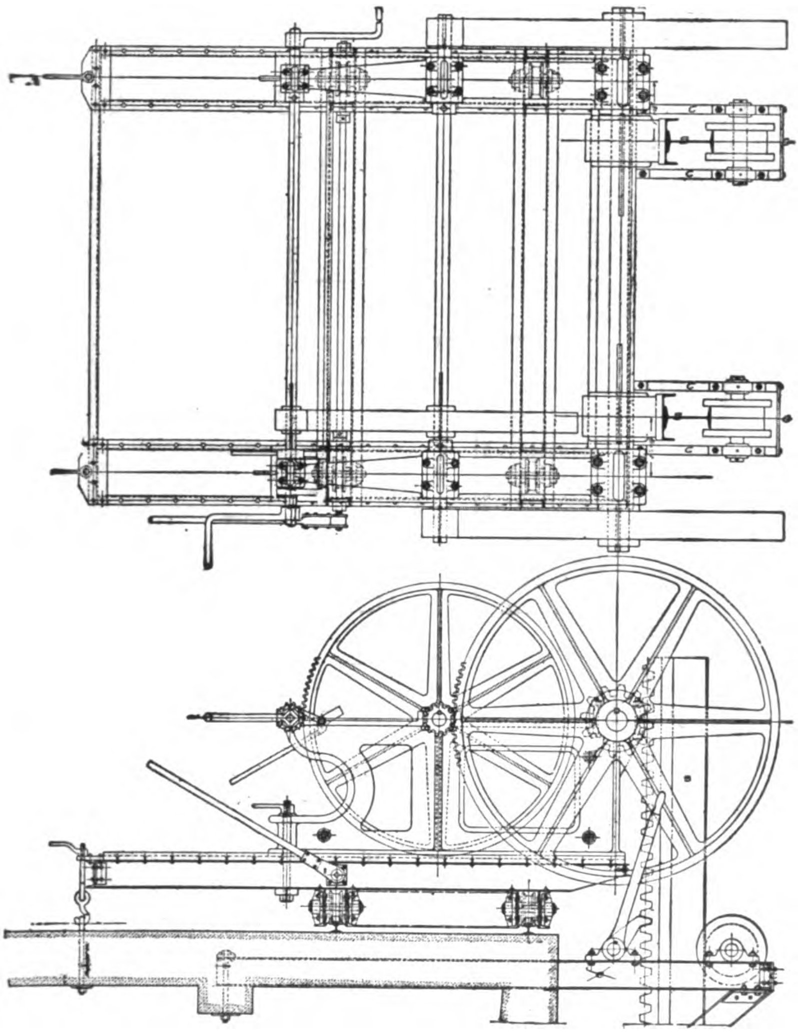

Fig. 382.—Head Gate Hoist, Kilbourn, Wis. (Southern Wisconsin Power Co.)

The purpose of these fishways is to afford a gradual incline through which a continuous stream of water of comparatively low velocity shall flow and against which the fish may readily swim. Both the inlet and outlet should be below low-water and the outlet should be in such a position that the fish, when they ascend the stream and reach the dam, in passing from one side to the other in searching for a passage, are naturally led to the point where the

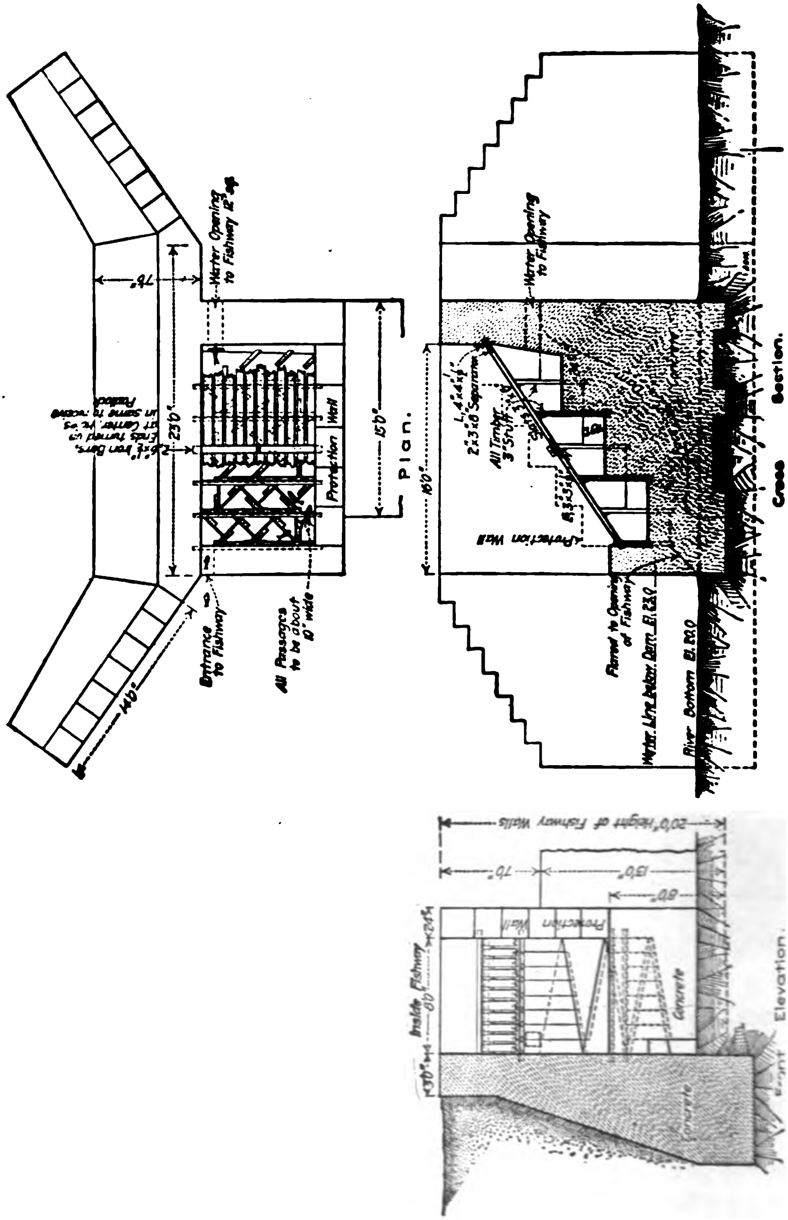

Fig. 383.—Concrete and Timber Fishway in Dam at Danville, Ill.

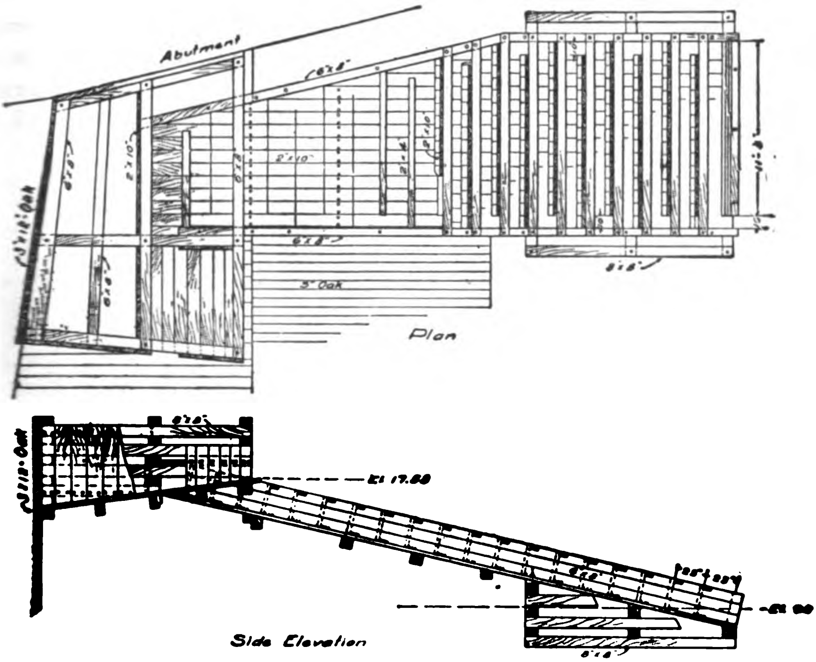

Fig. 384.—Timber Fishway in Dam at Sterling, Ill. (Eng. News.)

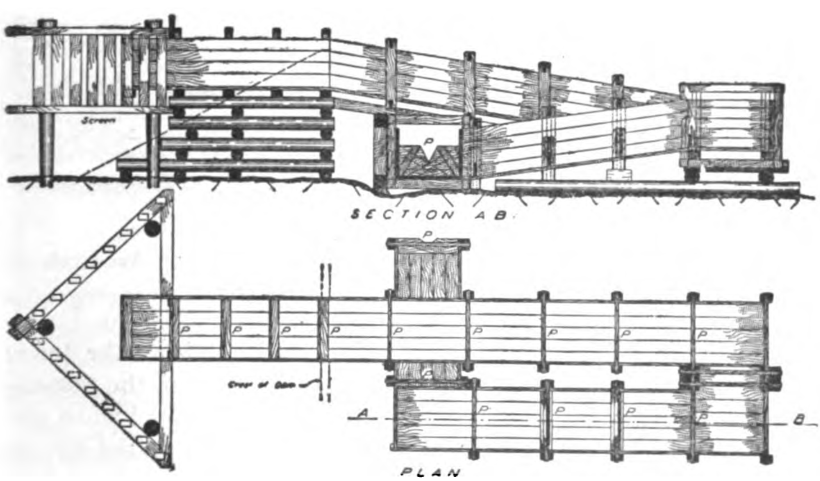

Fig. 385.—Fishway of Fish Commission, State of Wisconsin.

flowing water is encountered. The slope of these fishways should not be steeper than one vertical to four horizontal, and the water should be so deflected that the velocity will be reduced as low as possible. A fishway should be entirely automatic and free from all regulating devices. It is usually desirable for the openings in

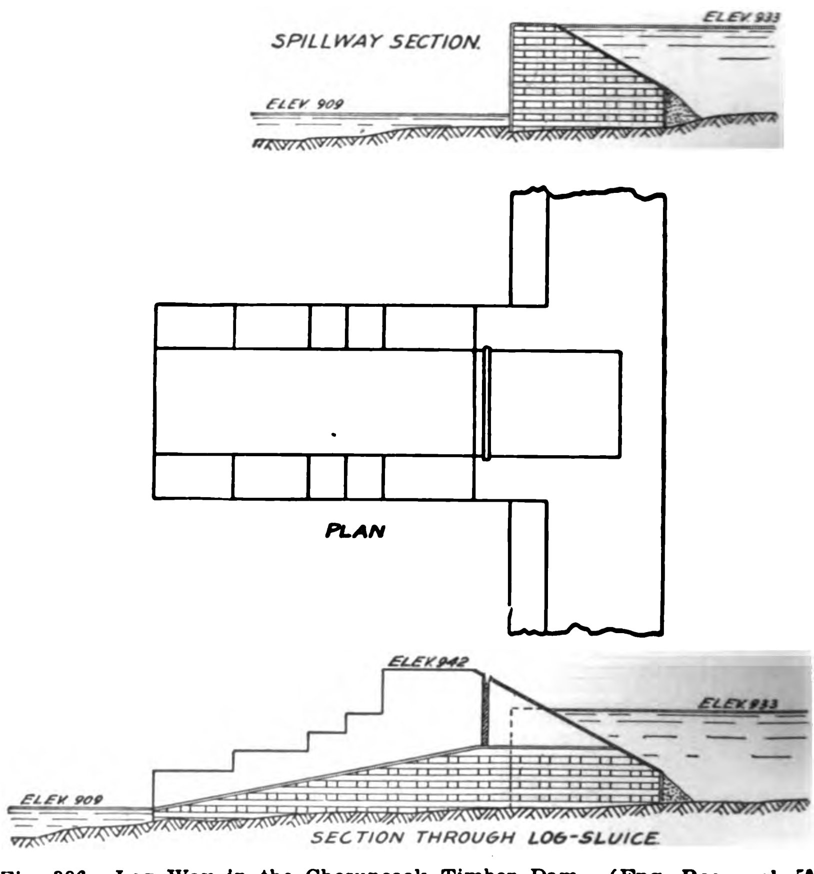

Fig. 386.—Log Way in the Chesuncook Timber Dam. (Eng. Rec., vol. 50, p. 70.)

the bulkheads or baffles to increase progressively from the lower to the upper one in order to insure that the passage of the fishway shall be full of water. The fishway should be so covered as to prevent interference, but must be light or it will not be used by the fish.

310. Log-Ways.—The free navigation of streams for logging purposes is provided by law in most states and it is therefore necessary where logging is practiced to provide ready means for their passage over or through the dam. This is accomplished in the

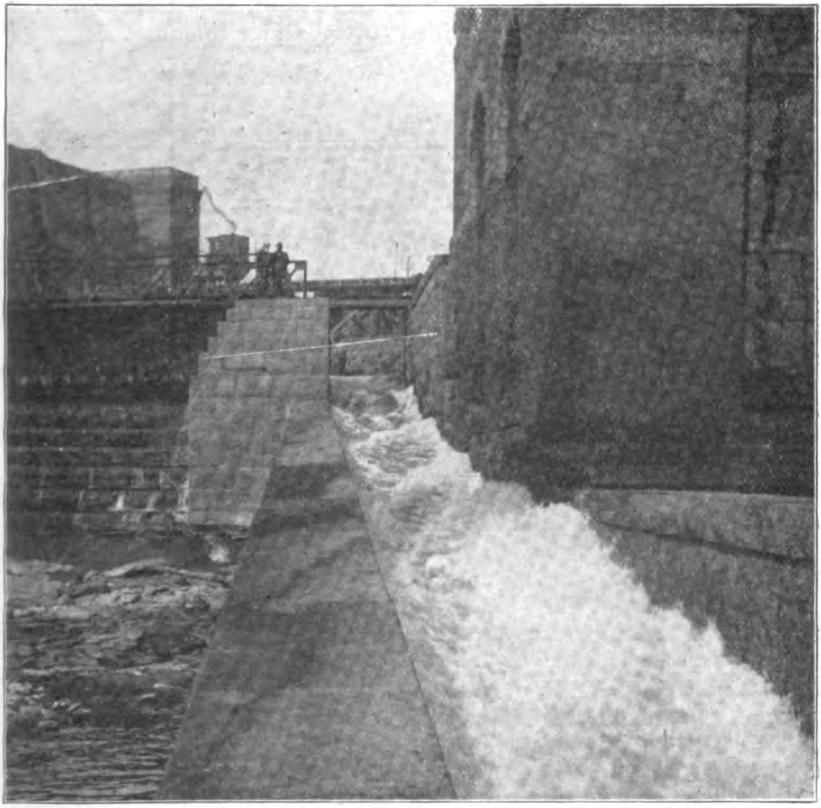

Fig. 387.—Log Way at Lower Dam, Minneapolis, Minn.

Kilbourn dam (see Fig. 374) by the lowering of any one of the flood gates.

Fig. 386 shows a plan and section of the log-sluice constructed in the Chesuncook timber dam on the Penobscot River. A section of the spillway of the dam is also shown in the same figure.

Fig. 387 is a view of the logway in the lower dam at Minneapolis. This sluice is only six or eight feet in width, and the depth and quantity of water flowing is controlled by a bear trap leaf.

In most cases, to avoid an excessive waste of water, it is desirable to build the logway as narrow as possible. Under such conditions it becomes necessary to guide the logs into the sluice by timber booms which, leaving the sluice at a low angle, are strung upstream to such points that the logs in floating down stream shall enter between them and be guided to the sluice opening.

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CHAPTER XXVI.

PONDAGE AND STORAGE.

311. Effect of Pondage on Power.—The terms "Pondage" and "Storage" are quite similar in meaning, both having reference to the impounding of water for future use. The term pondage usually refers to the smaller ponds which permit of the impounding of the night flow for use during the working hours of day. Storage, on the other hand, is usually applied to the larger impounding reservoirs that enable a sufficient quantity of water to be stored to carry the plant, to some extent at least, through the dry season of the year. Between these limits every variation in capacity is of course possible.

In Chapter IV, Section 54, the effect of pondage on the power of a stream is briefly outlined and illustrated by hydrographs shown in Figs. 41 and 42. The pondage illustrated by these diagrams is sufficient to store the entire flow of the river during the parts of the day when the power is not in use and reserve it for those hours of the day when the power is needed. Such a condition can frequently be realized for the low flows during the dry seasons, but the capacity is seldom sufficient to store the larger flows, and if sufficient should be investigated in a different manner to be discussed later. These hydrographs (Figs. 41 and 42) should therefore be examined with these points in view.

In many water power installations practically no pondage is possible and the power of the stream must be utilized as it flows or otherwise it will be wasted. On continuous service, such as is sometimes required by cotton factories, paper mills, and electro-chemical works that run twenty-four hours per day, pondage is not so essential. With most power loads, such as are shown by the various load curves in Chapter XVII, the night load is small and the pondage of the night flow will frequently permit of more than doubling the power that can be otherwise utilized.

312. Effect of Limited Pondage on the Power Curve.—Frequently limited pondage only is possible and its influence on the possible power that can be generated must be carefully investigated.

If power is to be used for a limited number of hours each day, the rate at which power can be used for this time without pondage will be the same as for the continuous power of the stream.

Such proportions of the otherwise unutilized flow of the stream as can be impounded during periods of light load can be added to the daily output. Thus, if power is used for 12 hours per day, and the night flow can be impounded and utilized during the day, the day power will be increased to double what it otherwise would be.

If power is used for only ten hours per day, with perfect pondage the day power will be increased to 2.4 of what it would otherwise be.

In twelve hours there are 43,200 seconds, and in each acre there are 43,560 square feet, it can therefore readily be remembered that for twelve hour pondage there must be practically as many acres one foot deep (or acre feet) in the pond as there are cubic feet per second to be impounded. For ten hour use and fourteen hour storage, the pond area must be increased by one sixth above the capacity needed for twelve hour service. For example: In order to utilize the full flow of the Wisconsin River at Kilbourn in twelve hours, (see Fig. 39) on the day of lowest flow (in August, 1904), a pondage of 3,000 acre feet would have been necessary, and, to utilize this full flow in ten working hours, would have required a pondage of about 3,500 acre feet.

Where the depth of pondage is considerable the effect of the variation in head on the power should receive careful consideration.

313. **Power Hydrograph at Sterling, Illinois.**—In 1903 the writer was retained to investigate the probable effect, on the water power at Sterling, Illinois, of the proposed diversion of water for feeding the Illinois and Mississippi or "Hennepen" Canal.

The pondage formerly available at Sterling, by using eighteen inch flash boards on the dam, was about 42,000,000 cubic feet (almost 1,000 acre feet).

The diversion dam at Sterling has been constructed about one mile above the dam of the Sterling Hydraulic Company and has limited the available pondage to an area of about 5,000,000 sq. ft., and a pondage of about 7,000,000 cubic feet. This change has therefore caused a loss of pondage of about 35,000,000 cubic feet, which represents the night storage (i. e., the storage during the fourteen hours of night), of 700 cubic feet per second, which represents 980 hydraulic horse power for the ten hours of day. That is to say,—the loss of 35,000,000 cubic feet of storage capacity caused by the

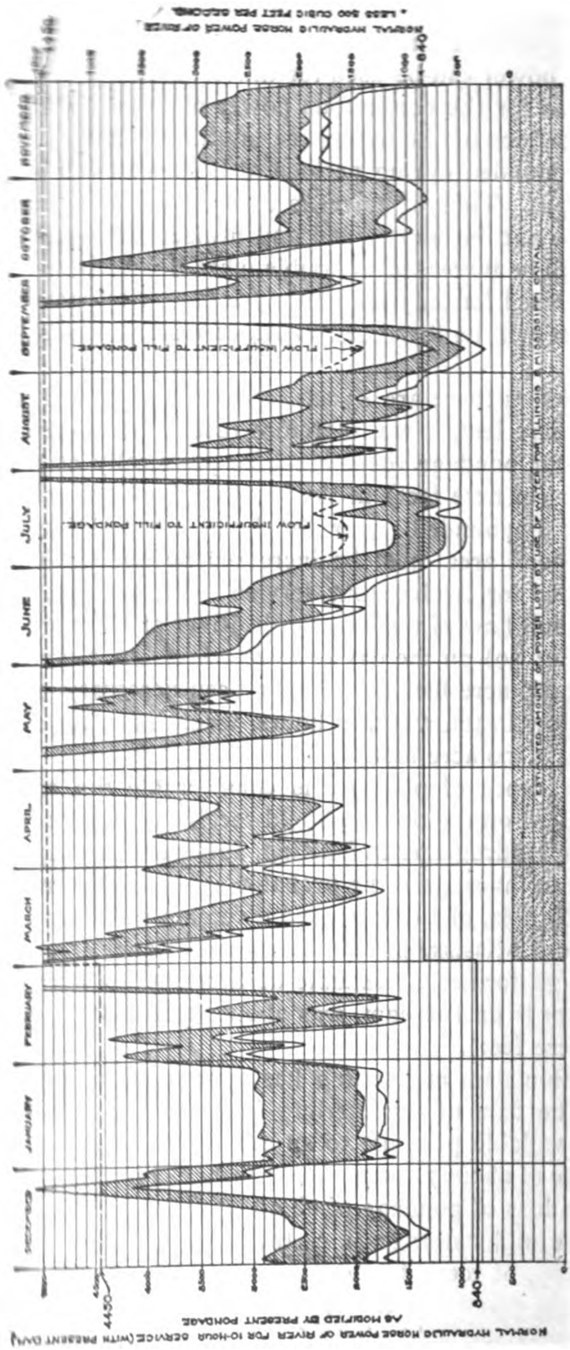


Fig. 388.—Hychograph Showing Effect of Pondage on the Ten-hour Power of the Rock River at Sterling, Illinois.

construction of the U. S. Government dam near the mouth of the Illinois and Mississippi Canal, has lost to the Sterling Hydraulic Company about 980 hydraulic horse power during such periods as the flow of the river is more than 840 cubic feet per second, and less than the capacity of the wheels installed (i. e., 4,450 cubic feet per second).

Fig. 388 gives a graphical illustration of the effects of storage on the normal water power at Sterling and the loss resulting from the loss of storage. The lower flow line is the line of the normal hydraulic horse power of the Rock River for continuous (twenty-four hour) service. It also shows the total power available for ten-hour service without pondage. The flow line just above the line of normal power, and parallel thereto, shows the additional ten-hour power available from a pondage of 7,000,000 cubic feet. The upper flow line shows the ten-hour power made available by the storage of 42,000,000 cubic feet. The hatched area between lines two and three represents therefore the loss in ten-hour power which has been caused by the loss in storage of 36,000,000 cubic feet.

From this diagram it will be noted that when the flow of the river is sufficient to supply the wheels, no loss would be occasioned by the loss in pondage, and, as the flow approaches this point, the actual loss decreases. It should also be noted that when the flow of the river is less than 840 cubic feet per second (above the amount diverted by the canal) the total storage of 42,000,000 cubic feet is more than necessary to store the night flow, hence the loss at such times caused by loss of pondage also decreases.

The approximate total loss of power for the year caused by the loss of 35,000,000 cubic feet of storage, as measured from this diagram, is 980 hydraulic horse power for, approximately, 250 ten-hour days.

314. Effect of Pondage on Other Power.—The pondage of water during the night naturally interferes with the normal flow of the stream and alters the regimen of the river at points below the point of pondage. The effect of such interference on other power, and the effect of other ponds on the plant contemplated, should be carefully considered.

Fig. 389 is a hydrograph of the Fox River taken from observation by the Government Engineers at Rapid Croche, Wisconsin. Above this point are a number of water power dams. Many of the plants run twenty-four daily, but close down on Sundays. The ef-

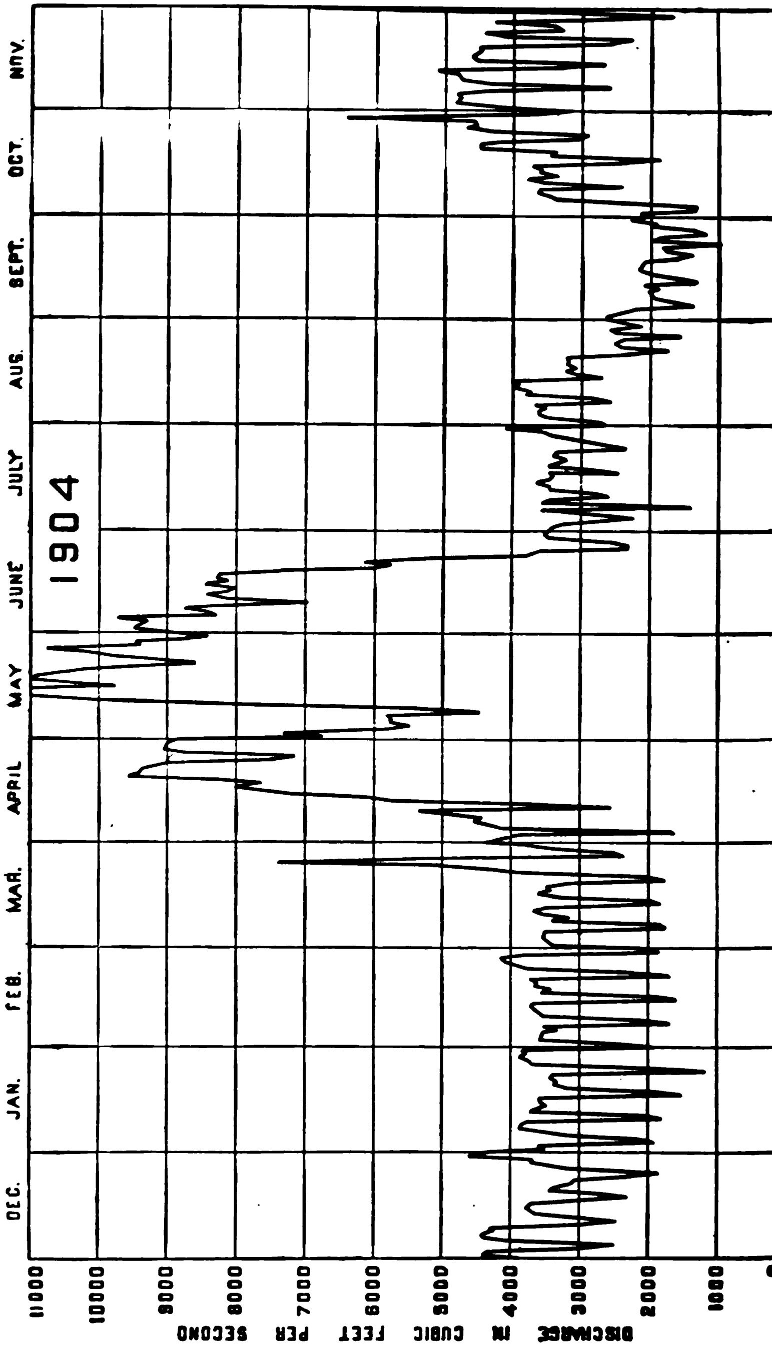


Fig. 889. Hydrograph of the Fox River Showing Effect of Sunday Shut Down of Hydraulic Plants on the Flow at Rapid Croche.

fect of the Sunday shut-down on the stream flow is well shown in the hydrograph and is evident even during flood periods.

315. Effect of Limited Storage.—When the pondage available is more than sufficient to carry the night flow of the low water period over for day use, it becomes possible to equalize, to a greater or less extent, the variation in daily flow and to utilize excess flow to increase deficient flows, thus raising the quantity of available continuous power. The extent of this equalization depends on the quantity of storage and can readily be investigated graphically.

Fig. 390 shows the estimated daily flow of the Wisconsin River at Kilbourn for July, August, and September, the low water period) 1904. From this hydrograph it will be seen that the lowest flow is 3,000 cubic feet per second. From Sec. 312 it is seen that in order to utilize the night flow during the twelve hours of day, a pondage of 3,000 acre feet must be available. With such a pondage the

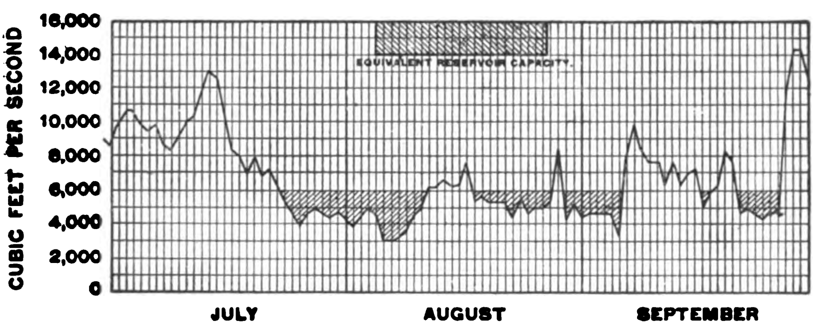


Fig. 390.—Low Water Flow at Kilbourn and Storage Capacity Necessary to Augment it to Various Amounts.

night flow can ordinarily be distributed so as to be available either for twelve hour constant power or to furnish power for any equivalent load curve.

In Fig. 390 the horizontal spaces each represent a flow of 1,000 cubic feet per second, and the vertical spaces, one day. The area of each space therefore, represents 86,400,000 cubic feet, or approximately 2,000 acre-feet.

To increase the low water flow of the river to 4,000 second feet will require a storage capacity equivalent to that represented by approximately three spaces, or a storage of 6,000 acre-feet in addition to the pondage, or a total storage of about 9,000 acre feet. To increase the flow to 5,000 second feet, a total storage of 28,000 acre-feet in addition to the pondage would be required; and a flow of 6,000 second feet, will require a storage of 90,000 acre feet in addition to the pondage. In this latter case the conditions to Sept. 6th must be considered, for the increased flow from August 12th to 17th is not sufficient to fill the reservoir, although it will reduce the capacity required, as will also the increased flow of August 20th.

The reservoir capacity represented by 90,000 acre feet is shown on the diagram both by the curved hatched area above the flow-line and by the rectangular shaded area as well.

If the reservoir capacity is known, and its equivalent represented on the drawing, its effect on the hydrograph can readily be determined by trial. (See also Fig. 393.)

316. Effect of Large Storage.—When large storage is available, the daily flow of a stream can be equalized and its variations therefore becomes less important. In such cases the power of a plant depends on the average weekly or monthly flow of the stream and the possible storage capacity.

S. B. Hill, C. E., has suggested a method of discussing the effect of storage on the flow and power of a stream which is well illustrated by Figs. 391 and 392. These hydrographs were prepared by the writer to illustrate a report on the probable power of a proposed hydraulic development in the South. Figs. 391 represent the mean monthly flow of the river in question for the years 1893 to 1906 inclusive. In this case the scale above the zero line shows both the mean monthly flow of the stream in cubic feet per second and the mean monthly power of the stream in horse power hours per day with the head available. The available storage is here 51,000 acre feet or 2,221,560,000 cubic feet. This storage is equivalent to a flow of 857 second feet for thirty days, or a storage of energy, with the available head, of about 5,000,000 horse power hours.

The maximum daily continuous power (see A-A, Fig. 391) is determined by the effect of the driest year (viz. 1904) on the storage. The effect of the dry periods on the storage is shown by the incisions into the lower or storage line of the diagram. In the year 1904 the reservoir capacity would have been just exhausted in order to maintain the power during the low flows of September, October and November of that year. The amount of available continuous energy (i. e. the position of the line A-A) is determined by equalizing the deficiency in flow during the dry months with the total reservoir capacity.

It is important in the study of storage to see that in the intervening periods of excessive flow, such flows are sufficient to supply the deficiency occasioned by previous demands on the reservoir, otherwise the effect of one dry period must be considered in its relation to subsequent periods in determining the available continuous power (see Fig. 391, 1897 and 1898).

The daily flow of this river for the year 1904 is shown by the hydrograph, Fig. 393, from which it will be seen that with pondage, but without storage, the available power of this stream would be limited to a minimum of 27,000 horse power hours per day.

317. Effect of Auxiliary Power.—In order to maintain a continuous power greater than that due to the minimum flow of the stream plus the pondage, some source of auxiliary power must be available. If it is desired to increase the power of the stream represented in Fig. 391 by 50,000 horse power hours per day, making the total horse power hours delivered 163,400 (represented by line B-B, Fig. 392), auxiliary power, as represented by the shaded areas on this diagram, would be needed. As at all other times water power would be available, the addition of steam auxiliary power would apparently be warranted. The size of the plant needed to furnish such excess power would depend on the method of power utilization. It is evident that during the dry periods in 1899, 1904 and 1905, if the water power was first used to its maximum, and the storage exhausted, an auxiliary plant would be needed of a capacity almost equal to the maximum demand on the plant, and that a plant of less capacity could be utilized satisfactorily only by operating it to a considerable capacity whenever a considerable draft began to be made on the storage. As the extent of the drought, or deficiency of water, could not be anticipated such a use of the auxiliary plant would require a greater expenditure of auxiliary horse power hours than is represented by the shaded areas in Fig. 392.

An investigation of the capacity and amount of auxiliary power needed, without pondage or storage, to maintain a given continuous power, can be readily made from the hydrograph of daily flow as shown by Figs. 394 and 395 which represent such a study of the Rock River at Sterling, Illinois, before the diversion of water for use in the Illinois and Mississippi canal, and the probable addi-

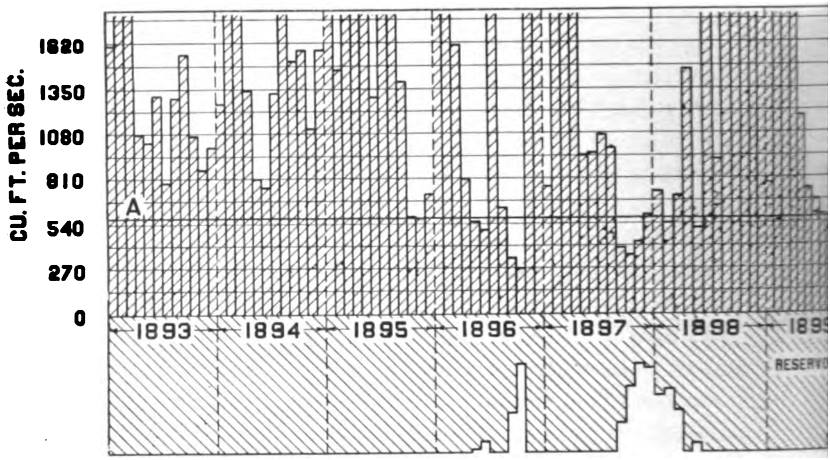


Fig. 391.— Mean Monthly Flow of a Southern

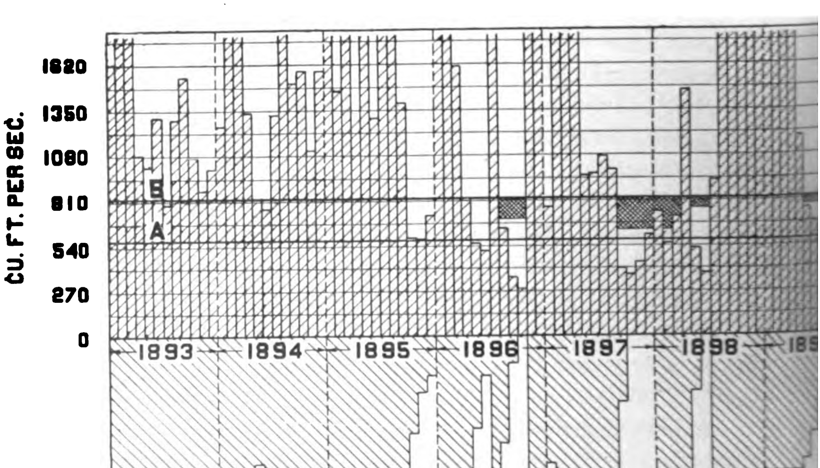
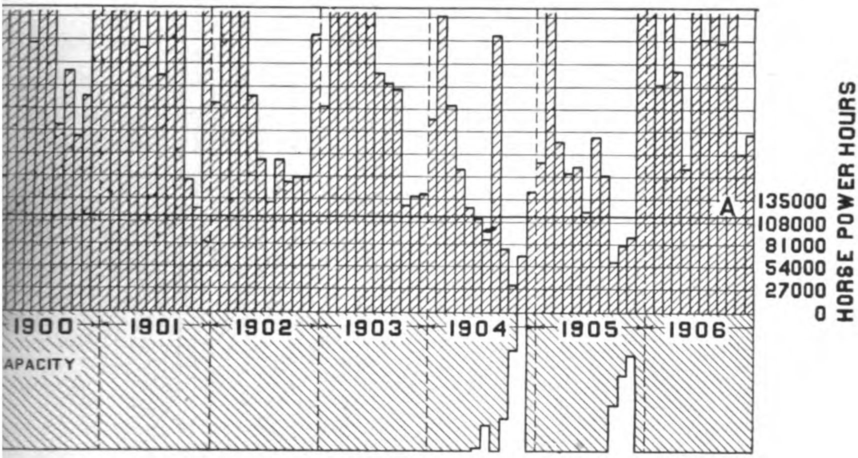


Fig. 392.— Amount of Auxiliary Power



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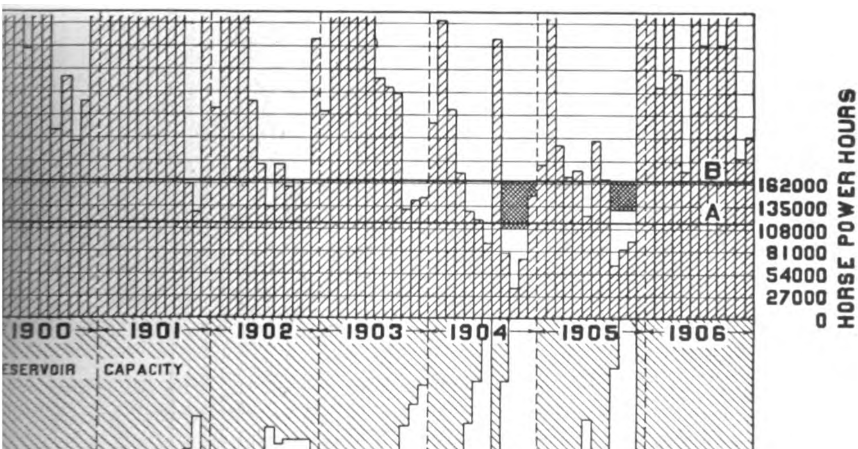


ary to Increase Output by 50,000 H. P. H.

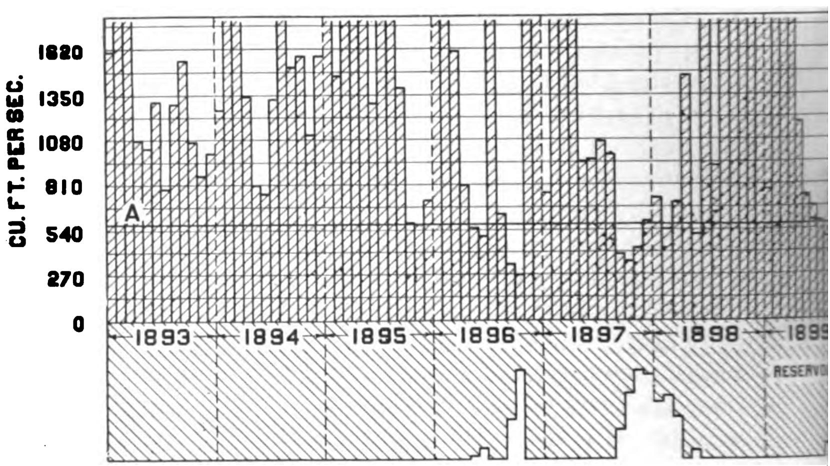


Fig. 391.— Mean Monthly Flow of a Southern R

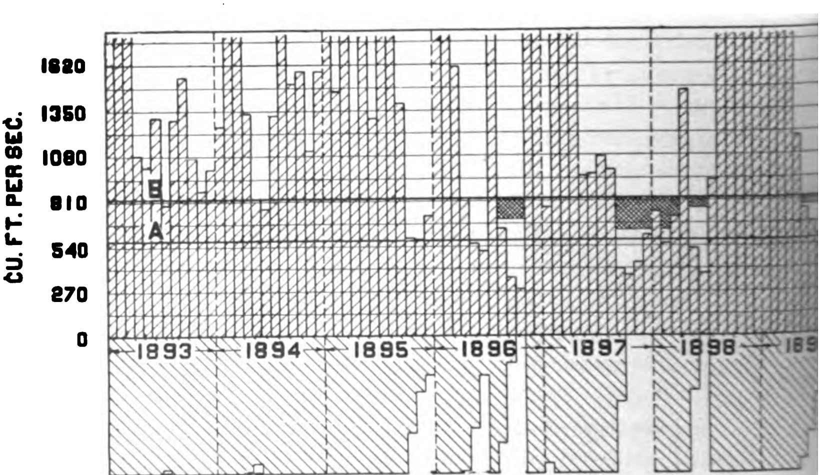
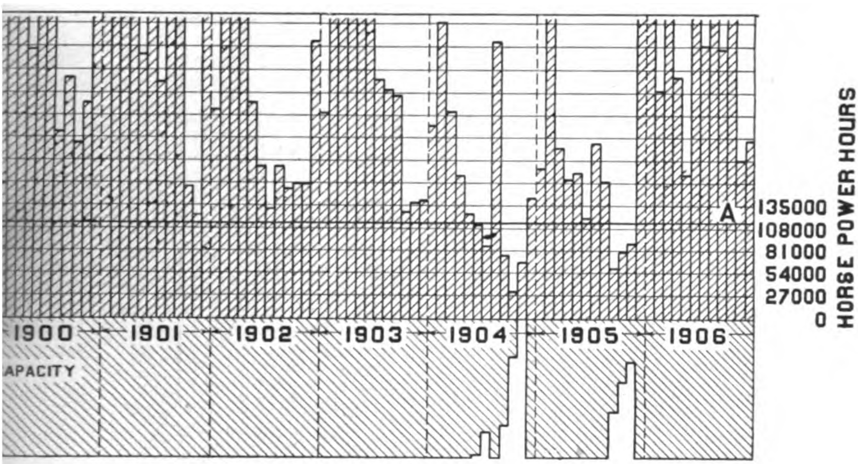


Fig. 392.— Amount of Auxiliary Power



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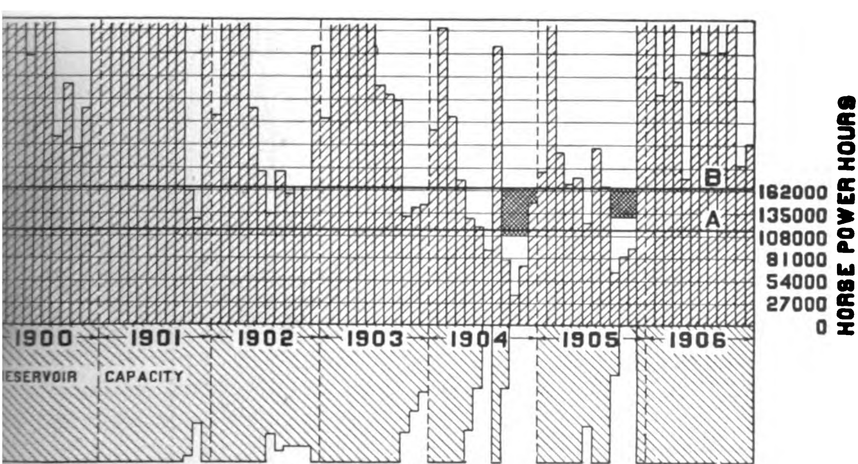

ary to Increase Output by 50,000 H. P. H.

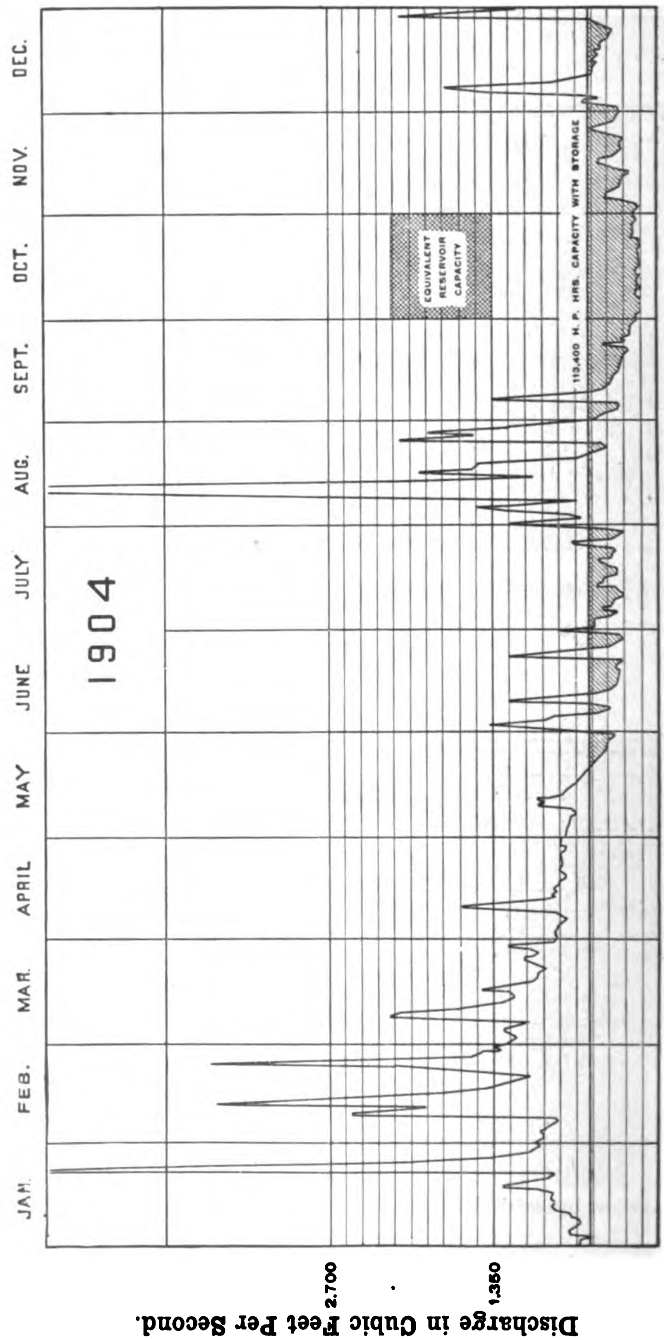

Fig. 303.—Daily Hydrograph of Daily Flow of a Southern River.

tional auxiliary power required to maintain the same power after such diversion.

318. Effect of Maximum Storage.—As the head increases the quantity of water needed to develop a given amount of power decreases, and storage becomes of much greater relative value. The storage of comparatively small quantities of water also becomes a more simple matter, but conditions which need little consideration with larger flows and lower heads, then become more important. In such cases, relatively, large reservoir capacity sometimes becomes

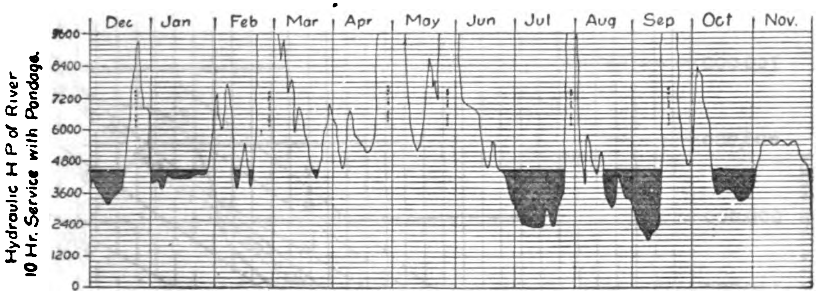

Fig. 394.—Hydrograph Showing Auxiliary Power Necessary to Maintain 4450 Ten-hour Horse Power at Sterling, Ill.

Fig. 395.—Hydrograph Showing Auxiliary Power Needed to Maintain Capacity of Wheels and Probable Increase Due to Diversion of Water for Illinois and Mississippi Canal.

possible and only the questions of desirability and cost limit the extent to which such storage may be carried.

319. Calculations for Storage.—Rippl has outlined a method of computing storage which may occasionally be used to advantage under high head conditions, when it is desired to utilize the average flow of a series of dry months or years by extensive storage. This method consists in graphically representing the net yield of the stream during the period of low flow and from the curve of the net flow estimating the quantity of storage necssary for its full utilization.

The method suggested may be illustrated as follows:

From a study of the hydrographic conditions on the water shed for a considerable term of year, the period of extreme low flow is selected. For this period the observed or estimated flow of the stream for each month is reduced by the loss due to evaporation,

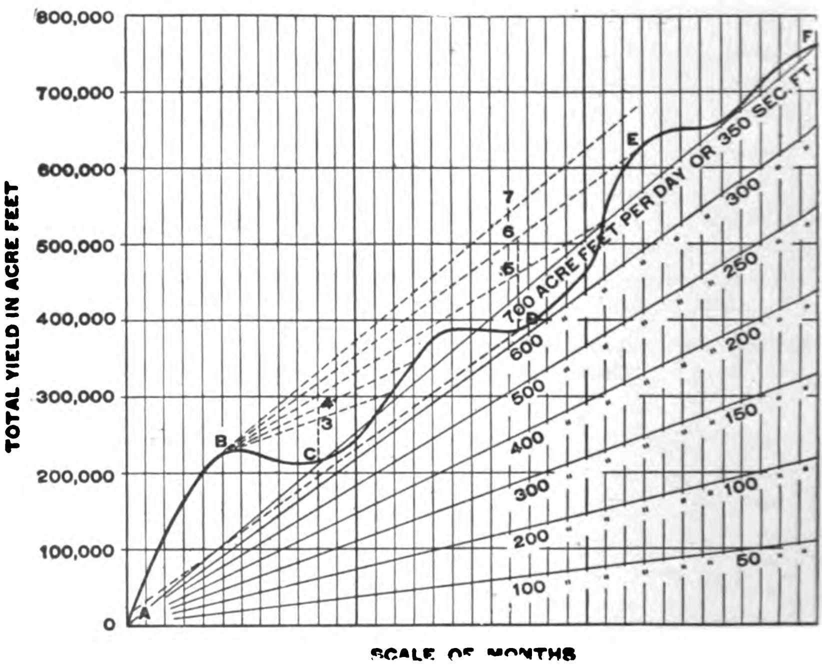

Fig. 396.—Diagram Illustrating Rippl Method of Calculating Storage.

seepage, etc. The remainder represents the net quantity of water available for power purposes, The summation of these monthly balances, added one to the other consecutively can be platted in a curve in which the abscissa of each point represents the total time from the beginning of the period; and the ordinate, the total quantity of water available during the same interval. The scale may represent inches on the drainage areas, cubic feet, acre feet, or such other unit as may be desired. Such a curve is represented in Fig. 396 by the irregular curve A-B-C-D-E-F. The inclination of the curve at any point indicates the rate of the net flow at that particular time. When the curve is parallel to the horizontal axis, the flow at that time will just balance the losses caused by evaporation, seepage, etc. A negative inclination of the supply line shows that a loss from the reservoir is taking place.

In a similar manner the curve of consumption can be platted. For most purposes this can be considered a straight line as the variation in the use of power from season to season is a refinement not usually warranted, unless the uses to which the power is to be put at various times of the year are well established. In Fig. 396 a series of straight lines of consumption are drawn, representing the use of water at rates of 100 to 600 acre feet per day. These rates correspond essentially to rates of from 50 to 300 cubic feet per second.

The ordinate between the supply and any demand line represents the total surplus from the beginning of the period considered, and when inclination of the supply line is less than that of the demand line, the yield of the drainage area is less than the demand and a reservoir is necessary.

The deficiency occurring during dry periods is found by drawing lines parallel to the demand line, or lines, and tangent to the curve at the various summits of the supply curve, as at B.

The maximum deficiency in the supply, and the necessary capacity of the reservoir to maintain the demand during the period, is shown by the maximum ordinate drawn from the tangent to the curve itself. The period during which the reservoir would be drawn below the high water line is represented by the horizontal distance between the tangent point and the first point of intersection of the curve. If the tangent from any summit parallel to any demand line fails to intersect the curve, it indicates that, during that period, the supply is inadequate for the demand. To insure a full reservoir it is necessary that a parallel tangent drawn backward from the low points on the supply curve shall intersect the curve at some point below. For example: The line B-7, representing a daily consumption of 700 acre feet, does not again intersect the curve and is therefore beyond the capacity of the stream. The line B-6 intersects the curve at E and is the limit of the stream capacity. Such a consumption will be provided by a storage of about 150,000 acre feet as represented by the length of the line 6-D, and such a reservoir will be below the flow line for about twenty-two months during the dry period illustrated in this diagram. That this reservoir will fill is shown by the intersection of the lower tangent D-A with the curve near A. The conditions necessary to maintain capacities of 500, 400 and 300 second feet are shown respectively by the tangents B-5, B-4 and B-3, and the verticals 5-D, 4-C and 3-C.

If the amount of storage is known, and it is desired to ascertain the maximum demand, that can be satisfied by such fixed capacity,

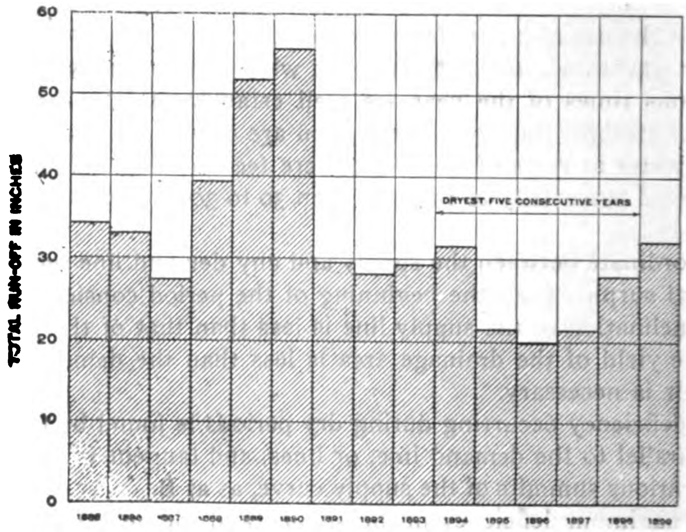

Fig. 397.—Diagram Showing Annual Run-off from Tohickon Creek.

the rate is determined by drawing various tangent lines from the summits, having the maximum ordinates equal to the fixed storage.
the rate is detremined by drawing various tangent lines from the summits, having the maximum ordinates equal to the fixed storage.

320. **Method of Storage Calculations.**—The results of calculations, as outlined in Sec. 319 for various conditions of storage on Tohickon Creek, are shown in Table XXXIX and Fig. 398. Tohickon Creek is one of the possible sources of water supply which has been investigated by the City of Philadelphia for a considerable period. The observed monthly rainfalls and stream flows from the drainage area of this stream (in inches on the drainage area) are given in Tables XL and XLI. The five year period of minimum flow is found by inspection to run from December, 1893, to November, 1898, as shown by Fig. 397. The approximate evaporation during the period is taken from Appendix F.

The calculations of the mass curves are based on the extreme variations in reservoir area of 0 to 100 per cent; that is, on the as-

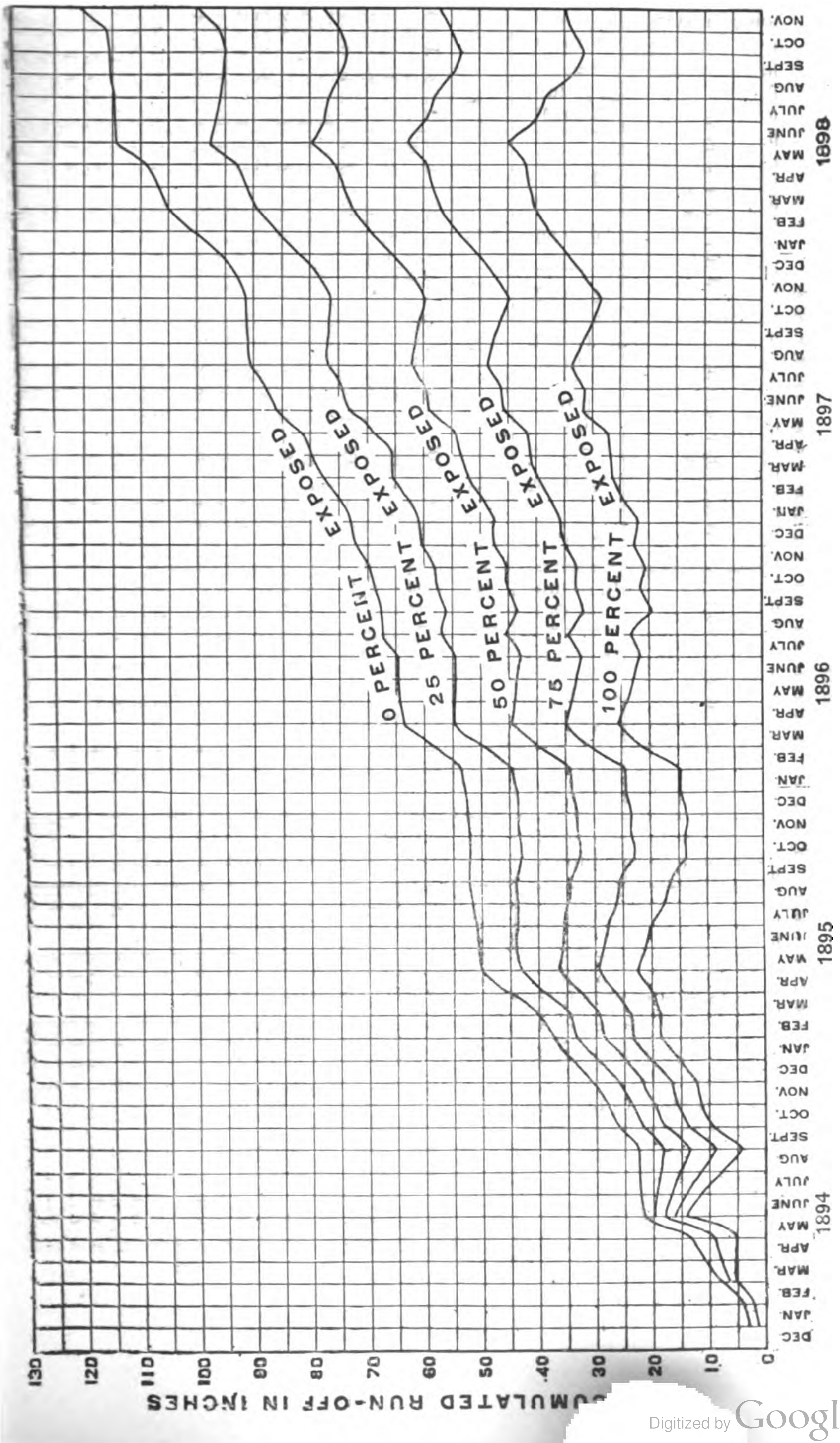

Fig. 398.—Mass Curve of Run-off of Tohickon Creek for five dry years.

TABLE XXXIX.

Calculation of Hydraulic Elements on the Tohickon Creek Water Shed.

Date.	Run-off in inches.	Rainfall in inches.	Evaporation and other losses.	3 — 4	With no reservoir area.	With 25% reservoir area.				With 50% reservoir area.				With 75% reservoir area.				With 100% reservoir area.
					Accum. sums of 2	75% of 2.	25% of 5.	Sum of 7 and 8	Accum. sum of 9.	50% of 2.	50% of 5.	Sum of 11 and 12	Accum. sum of 13	25% of 2.	75% of 5.	Sum of 15 and 16	Accum. sum of 17	Accum. sum of 5.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1894																		
December ..	3.10	3.10	—1.50	+1.60	3.10	2.33	+ .40	2.73	2.73	1.55	+ .80	2.35	2.35	.78	+1.20	1.98	1.98	1.60
January	.80	1.82	—1.00	+ .82	3.90	.60	+ .20	.80	3.53	.40	+ .41	.81	3.16	.20	+ .65	.85	2.83	2.42
February ...	3.79	3.96	—1.00	+2.96	7.69	2.84	+ .74	3.58	7.11	1.89	+1.48	3.37	6.53	.95	+2.22	3.17	6.00	5.38
March	3.08	1.65	—1.70	— .05	10.77	2.31	— .01	2.30	9.41	1.54	— .02	1.52	8.05	.77	— .04	.73	6.73	5.33
April	2.27	2.91	—3.00	— .09	13.04	1.70	— .02	1.68	11.09	1.13	— .05	1.08	9.13	.57	— .06	0.51	7.24	5.24
May	8.57	13.53	—4.50	+9.03	21.61	6.42	+2.26	8.68	19.77	4.28	+4.51	8.79	17.92	2.14	+6.77	8.91	16.15	14.27
June	.53	2.63	—5.50	—2.87	22.14	.39	— .72	— .33	19.44	.27	—1.44	—1.17	16.75	.13	—2.16	—2.03	14.12	11.40
July	.18	2.28	—6.00	—3.72	22.32	.13	— .93	— .80	18.64	.09	—1.86	—1.77	14.98	.04	—2.79	—2.75	11.37	7.68
August	.12	2.04	—5.50	—3.46	23.44	.09	— .86	— .77	17.87	.06	—1.73	—1.67	13.31	.03	—2.60	—2.57	8.80	4.22
September..	3.33	9.44	—4.00	+5.44	25.77	2.50	+1.36	3.86	21.73	1.66	+2.72	4.38	17.69	.83	+4.08	4.91	13.71	9.66
October	2.10	5.18	—3.20	+1.98	27.87	1.58	+ .49	2.07	23.80	1.05	+ .99	2.04	19.73	.52	+1.47	1.99	15.70	11.64
November..	2.67	3.00	—2.30	+ .70	30.54	2.00	+ .17	2.17	25.97	1.34	+ .35	1.69	21.42	.67	+ .52	1.19	16.89	12.34
1895																		
December...	3.57	4.60	—1.50	+3.10	34.11	2.68	+ .77	3.45	29.42	1.78	+1.55	3.33	24.75	.89	+2.33	3.22	20.11	15.44
January	3.96	4.19	—1.00	+3.19	38.07	2.96	+ .79	3.75	33.17	1.98	+1.59	3.57	28.32	.99	+2.38	3.37	23.48	18.63
February ...	1.70	.96	—1.00	— .04	39.77	1.27	— .01	1.26	34.43	.85	— .02	.83	29.15	.42	— .03	.38	23.86	18.59
March	5.37	3.11	—1.70	+1.41	45.14	4.03	+ .35	4.38	38.81	2.68	+ .71	3.39	32.54	1.34	+1.06	2.40	26.26	20.00
April	4.65	5.50	—3.00	+2.50	44.79	3.48	+ .62	4.10	42.91	2.32	+1.25	3.57	36.11	1.16	+1.87	3.03	29.29	22.50
May	.66	2.99	—4.50	—1.51	50.45	.49	+ .38	.87	43.78	.33	— .76	— .43	35.68	.16	—1.13	— .97	28.32	20.99
June	.27	4.40	—5.50	—1.01	50.72	.20	— .25	— .05	43.73	.14	— .50	— .36	35.32	.07	— .76	— .69	27.63	19.98

July........	.81	3.53	-6.00	-2.47	51.53	.61	- .62	- .01	43.72	.41	-1.24	- .83	34.49	.20	-1.85	-1.65	25.98	17.51
August.....	.36	4.43	-5.50	-1.07	51.89	.27	- .27	.00	43.72	.18	- .53	- .35	34.14	.09	- .80	- .71	25.27	16.44
September..	.04	.67	-4.00	-3.33	51.93	.03	- .84	- .81	42.91	.02	-1.67	-1.65	32.49	.01	-2.49	-2.48	22.79	13.11
October	.09	3.86	-3.20	+ .66	52.02	.07	+ .16	+ .23	43.14	.05	+ .33	.38	32.87	.02	+ .49	.51	23.30	13.77
89																		
November..	.14	2.11	-2.30	- .19	52.16	.10	- .05	+ .05	43.19	.07	- .09	- .02	32.85	.04	- .16	- .12	23.18	13.58
1896																		
December ..	.66	2.51	-1.50	+1.01	52.82	.50	+ .25	+ .75	43.94	.33	+ .51	.84	33.69	.16	+ .76	.92	24.10	14.59
January	.54	1.18	-1.00	+ .18	53.36	.41	+ .04	+ .45	44.39	.27	+ .09	.36	34.65	.13	+ .13	.26	24.36	14.77
February...	4.59	7.90	-1.00	+6.90	57.95	3.46	+1.72	+5.18	49.57	2.30	+3.45	5.75	39.80	1.15	+5.18	6.33	30.69	21.67
March	5.40	5.44	-1.70	+3.74	63.35	4.05	+ .94	+4.99	54.56	2.70	+1.87	4.57	44.37	1.35	+2.80	4.15	34.84	25.41
April.......	.73	1.48	-3.00	-1.52	64.08	.55	- .38	+ .17	54.73	.37	- .76	- .39	43.98	.18	-1.14	- .96	33.88	23.89
May........	.30	3.18	-4.50	-1.32	64.38	.22	- .33	- .11	54.62	.15	- .66	- .51	43.47	.07	- .99	- .92	32.96	22.57
June.......	.18	4.07	-5.50	-1.43	64.56	.13	- .36	- .23	54.39	.09	- .72	- .63	42.84	.05	-1.07	-1.02	31.94	21.14
July........	2.54	8.06	-6.00	+2.06	67.10	1.91	+ .52	+2.43	56.82	1.27	+1.03	2.30	45.14	.63	+1 57	2.20	34.14	23.60
August.....	.18	1.63	-5.50	-3.87	67.28	.13	- .97	- .84	55.98	.09	-1.93	-1.84	43.30	.04	-2.90	-2.86	31.28	19.83
September..	1.12	5.83	-4.00	+1.83	68.40	.84	+ .46	+1.30	57.28	.56	+ .92	1.48	44.78	.28	+1.38	1.66	32.94	21.16
October.....	1.06	2.67	-3.20	- .53	69.46	.79	- .13	+ .66	57.94	.53	- .27	.26	45.04	.27	- .40	- .13	32.81	20.63
November..	2.34	4.08	-2.30	+1.78	71.80	1.75	+ .44	+2.19	60.13	1.17	+ .89	+2.06	47.10	.58	+1.34	1.92	34.73	22.41
1897																		
December ..	.81	.94	-1.50	- .56	72.61	.60	- .14	.46	60.59	.40	- .28	+ .12	47.22	1.00	- .42	+ .58	35.31	21.85
January	1.81	2.20	-1.00	+2.20	74.42	1.36	+ .55	+1.91	62.50	.90	+1.10	2.00	49.22	.45	+1.65	2.10	37.41	24.05
February...	2.92	3.10	-1.00	+2.10	77.34	2.19	+ .52	+2.71	65.21	1.46	+1.05	2.51	51.73	.73	+1.57	2.30	39.71	26.15
March......	2.19	2.46	-1.70	+ .76	79.53	1.64	+ .19	+1.83	65.14	1.09	+ .38	1.47	53.20	.55	+ .57	1.12	40.83	26.91
April.......	1.55	3.20	-3.00	+ .20	81.08	1.16	+ .05	+1.21	68.25	.78	+ .10	.88	54.08	.39	+ .15	.54	41.37	27.11
May........	4.65	8.90	-4.50	+4.40	85.73	3.48	+1.10	+4.58	72.83	2.32	+2.20	4.52	58.60	1.16	+3.30	4 46	45.83	31.51
June.......	1.71	5.10	-5.50	- .40	87.44	1.28	- .10	+1.18	74.01	.86	- .20	.66	59.26	.43	- .30	.13	45.96	31.11
July........	2.68	8.46	-6.00	+2.46	90.10	2.01	+ .62	+2.63	76.64	1.34	+1.23	2.57	61.83	.67	+1.84	2.51	48.47	33.57
August.....	.73	3.75	-5.50	-1.75	90.85	.55	- .44	+ .11	76.75	.36	- .87	- .51	61.32	.18	-1.31	-1.13	47.84	31.82
September..	.12	1.92	-4.00	-2.08	90.97	.09	- .52	- .43	76.32	.06	-1.04	- .98	60.34	.03	-1.56	-1.53	45.81	29.74
October	.08	1.82	-3.20	-1.38	91.05	.06	- .35	- .29	76.03	.04	- .69	- .65	59.69	.02	-1.04	-1.02	44.79	28.36
November..	1.78	5.03	-2.30	+2.73	92.83	1.34	+ .68	+2.02	78.05	.89	+1.37	2.26	61.95	.44	+2.04	2.48	47.27	31.99
1898																		
December ..	4.08	4.63	-1.50	+3.13	96.91	3.06	+ .78	+3.84	81.89	2.04	+1.57	3.61	65.56	1.02	+2.34	3.36	50.63	34.22
January....	3.70	4.20	-1.00	+3.20	100.61	2.77	+ .80	+3.57	85.46	1.85	+1.60	3.45	69.01	.92	+2.42	3.32	53.95	37.42
February...	4.05	3.38	-1.00	+2.38	104.66	3.03	+ .59	+3.62	89.08	2.03	+1.19	3.22	72.23	1.01	+1.79	2.80	56.75	39.80
March......	1.83	2.84	-1.70	+1.14	106.49	1.37	+ .28	+1.65	90.73	.91	+ .57	1.48	73.71	.46	+ .85	1.31	58.06	40.94
April.......	2.50	3.73	-3.00	+ .73	108.99	1.87	+ .18	+2.05	92.78	1.25	+ .36	1.61	75.32	.62	+ .55	1.17	59.23	41.67
May........	5.01	7.62	-4.50	+3.12	114.03	3.78	+ .78	+4.56	97.34	2.52	+1.56	4.08	79.40	1.26	+2.34	3.60	62.83	44.79
June.......	.19	.76	-5.50	-4.74	114.22	.14	-1.18	-1.04	96.30	.09	-2.37	-2.28	77.12	.05	-3.55	-3.50	59.33	40.05

TABLE XXXIX—Continued.

Calculation of Hydraulic Elements on the Tohickon Creek Water Shed.

Date.	Run-off in inches.	Rainfall in inches.	Evaporation and other losses.	3 — 4.	With no reservoir area.	With 25% reservoir arear.				With 50% reservoir area.				With 75% reservoir area.				With 100% reservoir area.
					Accum. sum of 2.	75% of 2.	25% of 5.	Sum of 7 and 8.	Accum. sum of 9.	50% of 2.	50% of 5.	Sum of 11 and 12.	Accum. sum of 13	25% of 2.	75% of 5.	Sum of 15 and 16.	Accum. sum of 17	Accum. sum of 5.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
July........	.07	4.06	—6.00	—1.94	114.29	.05	— .49	— .44	95.86	.04	— .97	— .93	76.19	.02	—1.45	—1.43	57.90	38.11
August.....	.74	6.05	—5.50	—4.89	115.03	.55	—1.22	— .67	95.19	.37	—2.44	—2.07	74.12	.18	—3.65	—3.43	54.47	33.22
September..	.08	2.03	—4.00	—1.97	115.11	.06	— .49	— .43	94.76	.04	— .99	— .95	73.17	.02	—1.48	—1.46	53.01	31.25
October	.61	5.21	—3.20	+2.01	115.72	.46	+ .50	+ .96	95.72	.30	+1.01	+1.31	74.48	.15	+1.50	1.65	54.66	33.26
November..	4.50	3.56	—2.30	+1.26	120.22	3.37	+ .31	+3.68	99.40	2.25	+ .63	+2.88	77.36	1.12	+ .94	2.06	56.72	34.52

TABLE XL.

Tohickon Creek—Monthly Rainfall in Inches

Year.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Precip-itation.
1886	4.15	6.01	4.76	3.42	7.14	4.53	5.47	1.08	1.30	2.59	5.16	3.83	49.45
1887	4.24	5.47	3.07	2.42	2.59	5.77	8.13	5.30	3.36	1.93	1.42	6.53	50.22
1888	5.31	4.34	5.23	4.08	3.03	1.69	3.20	8.07	8.32	4.06	3.66	4.35	55.34
1889	4.23	2.37	3.67	4.90	5.41	6.94	12.33	4.63	7.92	4.57	8.86	1.99	68.04
1890	2.82	4.73	6.77	2.48	6.30	3.93	5.81	5.75	2.98	6.31	1.07	2.75	51.60
1891	6.14	4.58	4.79	1.97	2.83	3.38	7.49	8.90	1.37	3.81	1.97	5.09	52.32
1892	5.49	1.23	4.13	1.95	5.55	3.20	4.27	3.76	2.91	.64	7.10	1.57	41.80
1893	2.96	5.88	2.46	4.96	4.98	4.05	2.10	8.67	3.20	3.72	4.37	3.17	50.52
1894	1.82	3.96	1.65	2.91	13.50	2.63	2.28	2.03	9.44	5.18	3.01	4.60	53.01
1895	4.19	.96	3.11	5.50	2.99	4.49	3.53	4.43	.68	3.86	2.11	2.57	38.24
1896	1.18	7.90	5.44	1.48	3.18	4.07	8.06	1.63	5.83	2.67	4.08	.94	46.46
1897	2.20	3.10	2.46	3.20	8.90	5.10	8.47	4.75	1.92	1.83	5.02	4.64	50.59
1898	4.19	3.38	2.84	3.73	7.62	.76	4.00	6.05	2.03	5.21	3.56	3.49	46.92
1899	3.68	4.75	6.60	2.19	2.23	2.74	3.29	5.05	6.70	1.39	2.55	2.34	43.51
Average	3.64	4.08	4.04	3.33	4.74	3.95	5.42	4.93	4.16	3.71	3.33	3.78	49.11

TABLE XLI.

Tohickon Creek—Monthly Discharge in Inches on Drainage Area.

Year.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Dis-charge.
1886	4.36	9.19	4.28	4.75	3.43	1.41	.77	.09	.03	.05	1.91	2.38	32.65
1887	5.04	5.25	3.84	1.02	.93	1.21	1.63	1.96	.40	.25	.25	3.20	24.98
1888	6.38	6.72	6.27	4.28	.52	.15	.06	1.77	5.50	1.54	3.11	3.47	39.77
1889	4.38	1.51	3 86	2.88	1.70	2.29	6.41	3.75	3.40	2.33	7.97	1.92	42.40
1890	2.06	3.78	6.37	1.79	8.09	.75	.87	.92	1.22	3.54	.69	1.51	26.59
1891	6.15	5.68	5.03	1.58	.28	.17	.90	8.92	.94	.46	.63	4.27	30.01
1892	6.53	1.19	4.87	.84	2.05	.70	.51	.30	.19	.09	3.19	1.67	22.13
1893	2.22	6.64	4.54	3.22	3.79	.45	.10	1.56	.83	.60	2.62	3.10	29.67
1894	.80	3.80	3.09	2.28	8.58	.53	.19	.12	3.37	2.10	2.67	3.57	31.10
1895	3.95	1.70	5.37	4.65	.66	.27	.80	37	.03	.09	.13	.67	18.69
1896	.54	4.59	5.48	.73	.30	.18	2.54	.19	1.12	1.06	2.34	.80	19.87
1897	1.81	2.92	2.19	1.55	4.63	1.71	2.68	.73	.12	.07	1.79	4.08	24.28
1898	3.70	4.05	1.83	2.50	5.04	.19	.07	.74	.08	.60	4.50	4.23	27.55
1899	4.72	5.56	8.99	1.57	.25	.07	.08	1.02	2.26	.19	1.02	1.28	27.01
Average	3.59	4.25	4.70	2.50	2.08	.76	1.15	1.19	1.36	1.20	1.89	2 89	27.58

sumption that the reservoir may occupy from nothing to the entire drainage area.

The conditions on the reservoir area are those due to the equalization of the rainfall with the evaporation, seepage and other losses. The conditions on the balance of the water shed are given by the run-off and its summation.

Table XXXIX shows these calculations in detail and the mass curves drawn from columns 6, 10, 14, 18 and 19 are platted in Fig. 398. The maximum continuous power which could be maintained throughout this period without storage is shown by the lowest slopes of the zero per cent. mass curve. The possible maximum development of the stream with various percentages of reservoir area can be determined by an analysis of the lower curves similar to that described in Sec. 319.

321. Analytical Methods.—Graphical methods of computation have been heretofore suggested as a means of investigating pondage and storage conditions. Such methods are believed to be advantageous in most cases on account of presenting visible evidence which can usually be more clearly understood than an abstract analysis.

Analytical methods for the consideration of these questions are usually obvious after the graphical methods discussed are understood, and such methods should usually be used to check up the graphical deductions. Such methods may be illustrated by the following analysis of the effect of low water conditions on a proposed water power on a Western river on which the writer recently furnished a report.

In this case daily guage readings were available for about ten years, and the rainfall records were available for a considerably longer period.

From these records it appeared that the year 1905 was the driest year on record, and that the power available during the low water period of that year would have been equailed at least at all times during every year in the past twenty years, and with a probable like result in the future.

At the proposed plant each cubic foot per second, flowing during a day of twenty-four hours, will, at 80 per cent. efficiency, produce 3.63 continuous horse power. In order to develop 8,000 twenty-four hour horse power, it would be necessary, therefore, to have available a continuous flow of 2,200 second feet, while the minimum flow in 1905 was only 1240 second feet. An examination of the gaugings shows that during the dry period of 1905 the water was deficient in quantity for sixty-eight days. The average flow for this period was 1,700 second feet, causing an average deficiency of 500 second feet. To impound sufficient water to maintain 2,200 second feet would require, therefore, a storage capacity of about 1,000 acre feet for each day of the dry period, or a total reservoir capacity of about 68,000 acre feet. Above the proposed dam site is a lake having an area of about 60 square miles or 38,400 acres. By raising the level of this lake two feet a storage of 76,800 acre feet would be attainable which, with careful manipulation would be sufficient to maintain the desired power.

If no storage were possible, and auxiliary power was to be established, the maximum capacity of the auxiliary plant would be determined by the day of lowest flow. During this day there was a deficiency of 960 second feet, equivalent to about 3,500 horse power The average deficiency for the period was 500 second feet, representing a necessary average of auxiliary power of 1815 horse power, or 43,560 horse power hours per day. The total auxiliary power for this period (68 days) would therefore be about 3,000,000 horse power hours.

In the same manner the total amount of auxiliary power necessary during each year could be estimated and the interest and depreciation on the cost of the plant, plus the average annual operating expenses of the auxiliary plant, when considered in connection with similar elements of the water power installation, would furnish the basis for an estimate of the first cost and operating expenses of the combined plant to develop the required power.

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CHAPTER XXVII.

COST, VALUE AND SALE OF POWER.

322. **Financial Considerations.**—Every engineer who is called upon to advise as to the commercial feasibility of a proposed water power development must carefully consider all financial aspects of the project, for on its financial feasibility the entire commercial success depends. It is not enough that the power be constant and sufficient in quantity, that the plant be well designed, and that the cost of the same be reasonable; but there must also be a market in which the power can be utilized to advantage and the price at which the power can be sold in competition with all other sources of power must be sufficient to pay all expenses involved in the construction and operation of the plant and afford a fair return to those who assume the risk of the undertaking.

It is a common belief that any water power development must be profitable. Knowing that an undeveloped water power is a continual waste of energy, it is commonly assumed that the saving of this waste is bound to result in a profit to those who acquire the property and develop the power. That many water powrs can not be developed at a profit under present conditions is a fact which in many instances is learned by its owner only after a large and unwarranted expense is entailed.

323. **Purpose of Development.**—Any water power project must be examined in the light of the purposes for which it is to be used or the market it is to supply. The supply must be constant and continuous not only for every day in the year but for every year of its operation unless its use will permit of the discontinuation of the power during droughts, high water, or other contingencies that will decrease or temporarily suspend the generation of power by the plant.

If its use or market will permit of such interruption, a temporary power may sometimes be developed to advantage. Where the power furnished must be continuous in order to avoid losses or great inconvenience, precautions must be taken to so design the plant with duplication of parts, extra units and suitable pondage or storage or with such sufficient auxiliary sources of power that interruptions shall be essentially obviated.

In some cases considerable losses have been entailed by hydraulic developments constructed without sufficient study or consideration of these questions. In such cases, the plants after completion, were unable to maintain continuous power, without the installation of auxiliary steam plants for use during the temporary interruptions to which the plant was subject, and the income from the sale of power would not warrant the extra expense and hence the plants were commercial failures.

324. Cost of Water Power,—The cost of water power depends on:

First: The investment in real estate, water rights, power plant and equipment, transmision lines, sub-stations, distribution system, etc., and the interest which must be paid thereon.

Second: On the loss from the depreciation of the various elements of the plant, the cost of maintenance and repairs, the cost of contingent damages from floods or other accidents.

Third: The operating expenses, including labor, oil, waste, and other station supplies and expenses, including also in, hydro-electic plants, the patroling and maintenance of the transmission lines and distribution system.

Fourth: The expenses for taxes, insurance, etc.

The total annual cost due to the above sources of expense is the annual cost of the power to be furnished by the plant, be the quantity of that power much or little.

The investment charge should be liberally estimated and should include the entire expense of development including auxiliary power plant, if needed. All contingencies should be carefully considered and estimated. A serious error in the estimate of cost caused by large and unexpected contingencies in construction may mean a commercial failure of the enterprise. The same consideration should be given to the estimate of contingent expenses, depreciation and operating expenses, and each other factor on which the financial life of the plant depends.

325. Cost of Development.—The various conditions under which water power is developed greatly affect the cost of development. As a general rule, other things being comparatively equal, the larger the power developed the smaller the cost of development per unit capacity. This is particularly true when developments of various capacities are considered on the same stream. Many of the features of the development must be essentially the same regardless of the ultimate capacity of the plant. This is especially true of dams and river protection work. The variation in cost per unit capacity of various sized plants is well illustrated by Table XLII.

TABLE XLII.

*Estimate of the cost of a Hydro-Electric Plant at Niagara Falls.**

Items.	24-Hour Power Capacity.		
	50,000 H. P. Development.	75,000 H. P. Development.	100,000 H.P. Development.
Tunnel tail-race	$1,250,000	$1,250,000	$1,250,000
Headworks and canal	450,000	450,000	450,000
Wheel pit	500,000	700,000	700,000
Power house	300,000	450,000	600,000
Hydraulic equipment	1,080,000	1,440,000	1,980,000
Electric equipment	760,000	910,000	1,400,000
Transformer station and equipment	350,000	525,000	700,000
Office building and machine shop	100,000	100,000	100,000
Miscellaneous	75,000	75,000	75,000
	$4,865,000	$5,900,000	$7,255,000
Engineering and contingencies 10 per cent	485,000	590,000	725,000
	$5,350,000	$6,490,000	$7,980,000
Interest, 2 years at 4 per cent	436,560	529,584	651,168
Total capital cost	$5,786,560	$7,019,584	$8,631,168
Per horse-power	$114	$94	$86

*First report of Hydro-Electric Power Commission of the Province of Ontario, page 15.

Other things being comparatively equal, the cost of development varies inversely, although not in the same ratio, as the head. The reason of this is evident from the fact that while the power of a stream is directly proportional to the head, the capacity of a turbine increases as the three-halves power of the head. With double the head the power of a wheel is increased almost three times.

For moderate changes in head, the cost of the turbines will vary in proportion to their size and not their capacity; so that the cost per unit of capacity will usually decrease considerably with the head. The cost per unit of capacity of other features of water power plants will also frequently decrease as the head increases. This is particularly true of pondage capacity which increases in value directly as the head increases, although the cost per unit of land overflowed may remain constant. The relative cost of high and low head developments may be illustrated by the comparative cost of two plants recently designed by the writer which were of approximately the same capacity but working under different heads. The comparison is as follows:

TABLE XLIII

Comparative Cost of Water Power Plants.

Capacity.	Head.	Cost of Water Power Development.			
		Without dam.	With dam.	With dam and electrical equipment.	With dam, electrical equipment and transmission line.
8,000	18	63.50	86	115	150
8,000	80	21	39	60	90

TABLE XLIV.

Estimates of the cost of developing various Comachian power from Reports of Ontario Hydro-Electric Power Commission.

Location of Proposed Development.	Natural head.	Available head.	Power developed, H. P.	Estimated capital cost.	Cost per H. P.	Cost per H. P. per ft. head.
(1) Healey's Falls, Lower Trent River..	....	60	8000	$675000	$84.38	....
Middle Falls, Lower Trent River....	....	30	5200	475000	91.37	....
Rauney's Fall..........................	....	35	6000	425000	69.67	....
Rapids above Glen Miller...........	....	18	3200	3500 0	109.38	....
Rapids above Trenton...............	....	18	3200	370000	115.63	....
(2) Maitland River.........................	....	80 (5)	1600	325000	203.12	....
Saugeen River..........................	....	40	1333	250000	187.53	....
Beaver River (Eugenia Falls).......	....	420	2267	291000	128.23	....
Severn River (Big Chute)...........	....	52 (6)	4000	350000	87.50	....
South River..............................	....	85	750	115000	153.33	....
(3) St. Lawrence River, Iroquois, Ont.	....	12	1200	179000	149.16	....
Mississippi River, High Falls, Ont. A	....	78 (7)	2400	195000	81.25	....
Mississippi River, High Falls, Ont. B	....	78	1100	128000	181.82	....
Montreal River, Fountain Falls, Ont.	....	27	2400	214000	89.16	....
(4) Dog Lake, Kaministiquia River.....	347	310 (8)	13676	832000	61.00	....
	347	310	6840	619700	91.00	....
Cameron rapids..........................	39	....	16356	815000	50.00	....
	39	....	8250	600000	73.00	....
Slate Falls...............................	31	40	3686	357600	97.00	....
	31	40	1843	2600.0	141.00	....

Third Report; (5) Dam rather expensive. (6) Head works and canal less expensive than ordinary. (7) With storage developed. (8) Including 3500 feet of head water tunnel.

TABLE XLV.

Development costs of various American water power plants.

Name or location of plant.	Reference.	Head in feet.	Horse power capacity at turbine shaft.	Cost.	Cost per H. P.	Notes see page 651.
1. Chicago Drainage Canal, Lockport, Ill.	Elec. World, 1906, Vol. 47, page 398	28	15,500	$3,500,000	$225 80	d
2. Columbus, Ga.	Elec. World and Engr., 1904, Vol. 43, page 165	40	9,000	450,000	50 00	c and e
3. Catawba, S. C.	Eng. Record, 1904, Vol. 50, pages 114, 129	25	10,000	1,100,000	110 00	d and f
4. Tariffuille, Conn.	Amer. Electrician, 1900, Vol. 12, page 107	31	2,300	....	125 00	d
5. Delta, Penna.	Eng. News, 1898, Vol. 39, page 250	42	550	30,000	54 00	d and g
6. Lachine, Montreal	Elec. World, 1898, Vol. 31, page 744	16	6,600	957,200	145 80	d and h
7. Winnipeg, Manitoba	Elec. World, 1906, Vol. 47, page 1291	40	25,600	4,000,000	156 25	d and i
8. Manchester, N. H.	The Engineer, 1902, Vol. 39, page 64	30	6,000	....	66 00	a and j
9. Lowell, Mass.	The Engineer, 1902, Vol. 39, page 64	13	....	....	110 00	a and j
10. Lowell, Mass.	The Engineer, 1902, Vol. 39, page 64	18	....	....	57 00	a and j
11. Big Cottonwood, Utah.	The Engineer, 1902, Vol. 39, page 64	370	3,000	325,000	108 25	d and k
12. Lawrence, Mass.	The Engineer, 1902, Vol. 39, page 64	....	1,000	....	67 50	a and j
13. Spier Falls, N. Y.	Scien. Amer. Sept. 12, 1903	90	50,000	2,100,000	42 00	c

TABLE XLVI.—*Development costs of various foreign water power plants.*

Name or location of plant.	Reference.	Head in feet.	Horse power capacity at turbine shaft.	Cost.	Cost per H. P.	Notes see below.
Zürich, Switzerland.........	Electricity (N. Y.), 1899, Vol. 16, p. 148	Very low.	25,300	$4,650,000	$183 90	d and l
Rhinefelden, Germany......	Electrician (London), 1897, Vol. 38, page 716.........................	10 to 16	15,000	1,225,000	81 70	c
Paderno, Italy..............	The Engineer, 1902, Vol. 39, page 64.	90	13,000	....	120 00	b
Champ, France.............	Eng. Record, 1905, Vol. 52, page 648.	104	6,750	1,000,000	148 00	d
Dep't. de l'Isére, France..	Elec. Review (London), 1898, Vol. 43, page 475.........................	330	4,000	136,000	34 00	b
Dep't. de Jura, France....		6.5	300	45,000	150 00	d
Upper Savoy, France......		450	11,000	182,000	165 50	c and m
Chèdde, France...........	Die Ausnutzung der Wasserkräfte, page 198—E. Mattern.............	455	10,000	....	30 00 42 50	a c and n
Chèvres, Switzerland......		14 to 27	9,600	1,044,000	109 00	b
Kubel, Switzerland		296	5,000	1,074,000	215 00	d and o
Schaffhausen, Germany...		13.8 to 15.8 11.5 to 14.8	2,700	365,000	135 00	d and p
Gersthofen, Germany.....		32.8 to 34.4	6,000	812,500	135 00	b
Augsburg, Germany.......		....	9,100	1,875,000	206 00	d
Heimbach, Germany......		230 to 360	16,500	2,125,000	130 00	d and q
Lyon, France.............		33 to 40	22,750	6,500,000	287 50	d and r
Mühlhausen, Germany. ..		24 to 30	23,000	3,075,000	132 50	b

Notes in tables XLV and XLVI.

a—The cost of water power development, not including dam.
b—The cost of water power development, including dam
c—The cost of complete water power development, including electric station equipment.
d—The cost of complete water power development., including electric station equipment and transmission lines.
e—Mostly 12-hour H. P. distributed to adjacent mills at the generated voltage.
f—Severe climatic and river conditions during construction.
g—Very favorable location; cheap timber dam; transmission line only 5 miles long.
h—Includes extra real estate investment.
i—Expensive canals in rock, and very extensive concrete construction.
j—Factory installation.
k—Pelton wheels and 1500 ft. wood-stave pipe line.
l—Four interconnected plants; including also steam auxiliary.
m—Not including 5000 H. P. necessary steam auxiliary.
n—Not including dam.
o—With 1000 H. P. steam auxiliary.
p—Two interconnected plants.
q—15 mile transmissien line.
r—12 mile feeder canal.

The estimates of The Ontario Hydro-Electric Power Commission of the cost of various hydro-electric plants proposed in Ontario, furnish a good example of the variations in the cost, per unit of power, of various plants under various conditions. These estimates are shown in Table XLIV.

The actual costs per horse power capacity of various complete American and foreign plants are shown in Tables XLV and XLVI, respectively.

326. **Depreciation.**—In every operating plant there is in the course of time a certain deterioration or reduction in value due to ordinary operation and the effect of the elements. In the consideration of any power plant as an investment, allowance must be made in the annual charges for a sum sufficient to keep the original investment intact. In order to accomplish this an allowance should be made on each feature of the plant for the annual reduction in value or deterioration. The amount of depreciation will vary with the character and use of the machinery or structure and should be estimated with the best possible knoweldge of the conditions under which the plant will be operated, fully in mind. Such estimates should be sufficiently large to fully cover this item in order that the feasibility of the project may be correctly estimated.

The allowance for depreciation in an operating plant should be placed in a sinking fund which should be used to replace the various portions of the plant at the expiration of their useful life.

327. **Annual Cost of Developed Power.**—As already pointed out the annual cost of operating a plant includes:

a. Administration and operating expense.

b. Maintenance and repairs.

c. Depreciation.

d. Interest, insurance and taxes.

Each of these items will vary with the duration and the conditions under which the power plant is installed and operated. The method of estimating these charges in shown in the following estimates of the cost of operation of the Chicago Sanitary District Hydro-Electric Plant (see Electric World, Feb. 28, 1906).

Total cost of development and transmission $3,500,000.00

ESTIMATE OF COST.

Interest on investment at 4 per cent	$140,000.00
Taxes on real estate buildings, etc.	7,260.00
Depreciation on buildings at 1 per cent	3,650.00

Depreciation on water wheels at 2 per cent...........	2,027.32	
Depreciation on generators at 2 per cent..............	1,824.60	
Depreciation on pole line at 3 per cent...............	2,020.50	
Depreciation on other electrical appliances at 3 per ct.	3,995.52	
Total fixed charges............................		$161,137.94
OPERATING EXPENSES.		
Power and sub-station labor..........................	63,240.00	
Repairs to machinery and buildings..................	3,700.00	
Incidental expenses..................................	1,200.00	
Operating Lawrence avenue pumping station.........	43,960.00	
Operating 39th avenue pumping station..............	120,380.00	
Interest on investment 39th avenue pumping station..	15,599.76	
		248,079.76
Total cost to sanitary district..............................		$409,217.70
Capacity 15,500 H. P. Cost per H. P. per annum................		$26.40

An interesting comparison of the estimated yearly cost of various Hydro-Electric generating plants is given in the various reports of the Ontario Hydro-Electric Power Commission which are reproduced in Table XLVII.

328. **Cost of Distribution.**—Having estimated the annual cost of the development of power at the plant, the cost of distributing the power to the customer must also be considered. In many power plants the power is generated at or near the point where it is to be used and the transmission losses and costs will include its transmission through shafting, cables, and belts, or by electrical means, to the machine or appliances in which it is to be utilized. In other cases the power has to be transmitted for miles by high voltage electric currents. The units of power for which the power company will receive compensation may or may not include these various transmission losses. Where the power is distributed to a factory, the losses in transmission though shafting, belting, etc., is usually at the consumer's expense; but the transmission loss in long distance lines is ordinarily assumed by the power company and must be taken into account in the determination of the cost of furnishing power to the consumer The losses in any system of distribution are a considerable element of the cost of the delivered power and must be carefully estimated. (Sec. 20, page 24, et seq.)

The losses in the distribution of power in various mills, factories, etc., as determined by Prof. C. H. Benjamin, are given in Table XLVIII. The reports of The Ontario Hydro-Electric Power Commission, to which references have already been made, furnish numerous clear analyses of the cost of electrical distribution. Table

XLIX shows such an estimate for the delivery of power from a proposed Niagara plant to a proposed sub-station at Hamilton, Ontario. Table L shows the estimate of the Commission on the cost of distributing power from a sub-station to an individual consumer not within the local distribution. The variations in the cost of power from the generating plant to the consumer is also well shown by Table LI, taken from the same source.

TABLE XLVII.

Estimated yearly operating expenses of generating plant from Reports of Ontario Hydro-Electric Power Commission.

Location of Plant.	Horse-power.	Net H. P. transformed for transmisson.	Operating expenses including administration.	Maintenance and repairs.	Depreciation.	Interest at 4 per cent.	Water rental.	Yearly charge.	Yearly cost of transformed 24-hour power.
(1) Niagara plant...........	50000	48750	$57900	$115700	$86800	$231400	$52000	$544300	$11.16
	75000	73125	70200	140400	105300	280800	65000	661700	9.05
	100000	97500	86300	172600	129500	345200	77500	811100	8.32
(2) Middle Falls............	5200	4990	11875	9500	9500	19000	....	49875	10.00
Healey's Falls...........	8000	7680	16875	13500	13500	27000	....	70875	9.10
Two above combined....	13200	12670	23000	23000	23000	46000	....	115000	9.08
(3) Maitland River..........	1600	......	5665	2754	2755	13000	....	24174	......
Saugeen River..........	1333	......	4840	3247	3247	9984	....	21318	......
South River.............	750	......	4100	2620	2620	4534	....	13874	......
Severn River (Big Chute)	4000	......	17483	8571	8571	14000	....	48575	......
Severn and Beaver Rivers combined.............	6267	......	23713	13968	14000	25640	....	77289	......
(4) St. Lawrence River.....	....	1200	6864	5119	5118	7151	....	24252	......
Mississippi River High Falls..................	....	2400	9391	3840	3841	7777	....	24849	......
Mississippi River High Falls..................	....	1100	6390	2491	2491	4908	....	16280	......
Montreal River Fountain Falls.............	....	2400	9850	3903	*21622	8539	....	43914	......
(5) Dog Lake................	....	13675	13760	16427	15927	35278	....	79392	......
	....	6840	11296	10632	10132	24787	....	56847	......
Cameron Rapids.........	....	16350	16375	17327	16727	32561	....	82990	....
	....	8250	14390	11478	10978	24008	....	60854	......
Slate Falls..............	....	3686	6000	6634	6334	14303	....	33272	......
	....	1843	6000	3868	3669	10400	....	23957	......

*Including 10-year sinking fund.

To make the delivered current available for power, a motor must be installed. This is commonly furnished by the consumer. Table LII shows the estimated cost of induction motor service per horse power per year.

329. Effect of Partial Load on Cost of Power.—The maximum amount of work that any plant can accomplish will be done only when the plant works to its full capacity for twenty-four hours per

TABLE XLVIII—*Data and Results of Factory Friction Tests.*

Nature of Work.	Total length line shafting, feet.	Diameter line shaft, inches.	Revolutions per minute.	Number of bearings.	Number of counters.	Number of machines.	Total horse power.	Percentage to drive shafting and engine friction.	Horse power per 100 sq. ft. of shafting per minute.	Horse power per bearing.	Horse power per counter.	Horse power per belt.
Wire drawing and polishing	1,130	2½ 2¾ 4 6	170	115	69		400.	39.2	.10	1.37	2.28	1.76
Steel stamping and polishing	580	3 3½	200	68	27	18	74.	77.	.059	.84	2.11	2.40
Average	.....	....	....	...	...	...	......	......	.08	1.10	2.19	2.08
Boiler and machine work	530	2½ 3	150	46	47	43	38.6	65.6	.04	.550	.538	.477
Bridge machinery	1,460	2½ 3 4	110	142	79	69	59.2	80.7	.04	.337	.606	.521
Heavy machine work	1,120	3	190	110	96	68	112.	57.	.038	.581	.665	.453
Heavy machine work	1,065	2 3 4	180 150 150	114	152	123	168.	54.2	.06	.799	.600	.475
Average	.....	....	....	...	...	...	......	62.3	.044	.567	.602	.481
Light machine work	748	1½ 1¾ 2 3	135 135 135 150	101	133	250	40.4	51.2	.034	.204	.155	.095
Manufacture of small tools	500	2 3	114	58	314	313	74.3	53.8	.09	.689	.127	119
Manufacture of small tools	990	2½ 1¾	175 136	102	202	258	47.2	51.8	.03	.240	121	.113
Sewing machines and bicycles	2,490	2 6	150	274	403	454	190.	56.9	.05	.397	.269	.208
Sewing machines	1,472	2 3 4	160 160 125	184	435	179	107.	69.7	.034	.406	.172	.154
Screw machines and screws	1,800	2 2¼ 2½ 3	180	180	392	428	241.	47.3	.05	.633	.291	.235
Average	.....	....	....	.. .	...	...	......	55.1	.048	.428	189	.154
Steel wood-screws	674	1½ 2 3	175–160 175	96	89	392	117.	14.5	.02	.178	191	.130
Manufacture of steel nails	988	2½	200	74	175	184	91.6	49.9	.035	.615	260	.244
Planing mill	165	3	267	19	40	53	39.2	73.	.08	1.52	.715	.636
Light machine work	275	2	175	37	27	30	8.28	48.6	.015	.109	.749	.084

TABLE XLIX.

Showing investments, annual charges, and cost of low tension power at sub-station. (Sub-station included.)

	Full load.	¾ load.	½ load.
Total horse-power distributed...........	16,000	12,000	8,000
Total investment, including step-down stations and interswitching............	$450,879	$404,879	$358,379
Investment per H. P. delivered.........	28 18	33 73	44 80
Total annual repairs, depreciation, patrolling and operation..................	22,496	19,092	15,651
Administration, 10 per cent of repairs, etc.	2,250	1,909	1,565
Annual interest, 4 per cent of investment	18,035	16,195	14,335
Total annual charges..............	$42,781	$37,196	$31,551
Cost of 24-hour power, including line and step-down sub-station losses...........	$12 69	$12 49	$12 35
Cost of transmitting and transforming....	2 67	3 10	3 94
Total cost of power................	$15 36	$15 59	$16 29

The above costs of power are based on an assumed rate of $12.00 per 24-hour horse-power per annum for high-tension power at Niagara Falls.

TABLE L.

Showing cost of distribution from municipal sub-station to an individual consumer, not covered by local distribution.

Distance in miles from municipal sub-station.	Cost per Horse-Power per Annum for the Delivery of Various Amounts of Power.							
	50 H. P.	75 H. P.	100 H. P.	150 H. P.	200 H. P.	250 H. P.	300 H. P.	
2............	$5 58	$4 20	$3 53	$2 92	$2 74	$2 60	$2 51	
3............	6 89	5 20	4 41	3 60	3 25	3 10	3 03	
4............	7 92	6 18	5 20	4 27	3 93	3 72	3 86	2200 volts.
5............	8 87	7 18	5 98	4 96	4 55	4 32	4 17	
6............	10 20	8 24	6 77	5 38	5 13	4 60	4 43	
8............	14 10	10 14	8 40	6 97	6 24	5 79	5 58	11000 volts.
10............	16 12	12 13	9 54	8 31	7 68	6 96	6 17	
12............	18 76	14 03	11 12	10 12	8 42	7 96	7 22	16500 volts.
15.....	22 74	17 08	13 48	10 89	9 35	8 84	8 32	

TABLE LI.

Amount of Power Delivered.	Cost of 24-Hour Power per H. P. per Annum.		
	At Niagara Falls including line and step-down sub station losses.	At sub-station.	At customer.
Full load, 2,000 H. P.	$18 54	$21 89	$26 03
¾ load, 1,500 H. P.	13 18	23 54	29 06
½ load, 1,000 H. P.	12 85	27 21	34 48

TABLE LII.

Capital cost and annual charges on motor installations.

Polyphase 25-cycle, induction motors.

Capacity H. P.	Capital cost per H. P. installed.	Annual Charges.			
		Interest 5 per cent.	Depreciation and repairs, 6 per cent.	Oil, care and operation.	Total per H. P. per annum.
5	$41 00	$2 05	$2 46	$4 00	$8 51
10	39 00	1 95	2 34	3 00	7 29
15	35 00	1 75	2 10	2 50	6 35
25	28 00	1 40	1 88	2 00	5 28
35	25 00	1 25	1 50	1 75	4 50
50	24 00	1 20	1 44	1 50	4 14
75	21 00	1 05	1 26	1 25	3 56
100	20 00	1 00	1 20	1 00	3 20
150	17 00	85	1 02	80	2 67
200	16 00	80	96	70	2 46

day. Thus, if a plant has a capacity of one thousand horse power and is operated continuously during the twenty-four hours, the total output will be twenty-four thousand horse power hours of work. Under such conditions the plant can be built at a minimum expense per unit of output and the cost of operation, fixed charges, interest, etc., will be less per unit of work done than under any other condition of operation.

For example: If a plant of one thousand horse power be installed at a cost of one hundred thousand dolars, the annual cost of operation, including fixed charges and all other legitimate expenses, may be estimated as follows:

Interest on $100,000 at 6 per cent.	$ 6,000
Repairs and depreciation	5,300
Operating expenses	10,000
Miscellaneous and contingent expenses	4,250
Total annual cost of power	$25,550

On the above basis the annual cost for each horse power of maximum load will be $25.55. If the plant works at its maximum capacity for twenty-four hours per day, the cost per horse power hour will be .292 cts. If, however, the plant is operated to its full capacity for 12 hours per day only, the total cost of power may be reduced to say $23,000 per annum. In this case the cost per horse power of maximum load will be reduced to $23.00 per year, but the cost per horse power hour of energy generated will be increased to .526 cts. In many cases the plant will be used for ten hours per day and for six days per week. Its maximum capacity may be utilized only occasionally, and the demand for power will vary greatly from hour to hour resulting in a load factor of perhaps 50 per cent. or less. In this case the annual cost per maximum horse power will still not exceed twenty-three dollars ($23)) per year, but the annual cost of average ten hour power will be forty-six dollars ($46), and the cost per horse power hour of useful work will be increased approximately to 1.5 cents. The cost of each unit of power under the last condition is over five times as great as in the first case mentioned, and about three times as great as in the second case discussed. It is therefore obvious that unless the conditions of use are carefully studied and conservatively estimated, they may lead to unfortunate investments and financial losses.

330. **Cost of Auxiliary Power, or Power Generated From Other than Water Power Sources.**—It frequently becomes necessary to estimate the cost of power plants and of power developed from other than water power sources. This is necessary in order to determine the probable cost of auxiliary power plants and such auxiliary power as may be needed to assist a water power plant at times when the hydraulic power is deficient. It is also necessary to determine the cost of power with which the hydraulic plant may be called upon to compete.

For a correct estimate of such cost, it is necessary to determine the efficiency of the various parts of the plant (see page 31) under all conditions of operation in order to correctly determine the actual cost of power due to the conditions of operation. The conditions for maximum efficiencies are seldom met in actual operation, and the cost of generating the power is increased by the irregularities of operating conditions. In all power plants the effect of partial or irregular load affects the cost of power in the same manner as already described in Section 328.

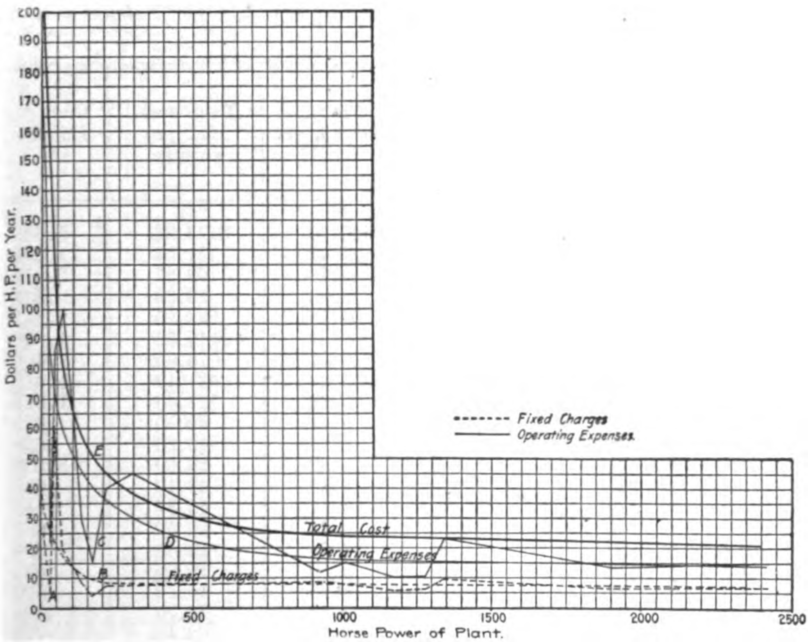

Fig. 399. Cost of Steam Power per Horse Power per Annum in Various Plants.

By far the largest amount of power generated is from fuel and by steam plants. The cost of the development of steam power is modified by the cost and character of coal used; the size and character of the machinery operated; the character of the load (that is, the load factor); the number of hours during which the plant is used per year; and the skill and ability of the engineer and fireman who have charge of the plant. Observations of the actual cost of developing power must therefore form the basis of any accurate estimate of the cost of power production.

Mr. H. A. Foster* made actual tests of twenty-two different power plants, including manufacturing establishments, electric light stations, pumping plants, etc., and determined for each plant the power consumption per annum and its cost, including not only running expenses but fixed charges. The cost per horse power per annum varied from a minimum of $15.69 to a maximum of $233.95.

TABLE LIII.

Showing average power developed and its cost per HP. in 22 steam power plants.

Output.		Operating expenses, per HP.	Fixed charges, per H. P.	Total cost, HP. per annum.	Cost per HP. hr., cts.
Average HP. developed.	No. of days per annum.				
12.4	361	$147.93	$25.40	$173.33	5.648
20.9	365	123.12	28.42	151 54	1.868
21.5	361	90.47	17.80	108.27	2.918
32.9	330	22.56	5.83	28.39	.832
36.7	365	137.25	96.70	233.95	2.811
42.4	365	86.38	63.20	149.58	1.708
53.	309	56.94	19.51	76.45	1.596
58.8	365	97.30	33.82	131.12	1.613
70.4	365	101 69	20.78	122.45	1.641
129.3	365	30.14	9.41	39.55	.871
166.7	313	15.19	4.47	19.66	.639
173.	313	22.66	5.83	28.39	3.333
210.9	290	40.33	7.86	48.19	.693
296.7	297	45.56	7.81	53.37	.749
926.	307	11.73	8.77	20 50	.691
1,010.8	306	15.70	7.74	23.44	.794
1,174.8	306	10.19	5.50	15.69	.531
1,278.7	293	10.49	6.23	16.72	.590
1,345.5	365	23 28	9.42	32.70	.820
1,352.	365	33.03	29.41	62.44	.713
1,909.7	306	13.40	6.63	20.03	.677
2,422.	306	15.67	6 73	22 40	.757

A summation of the results of these observations is shown in Table LIII and the plotted results of the table are shown in Fig. 399.

Mr. R. W. Conant** determined the operating expenses of various street railway power stations and compiled a table (see Table LIV which gives important information bearing on this question.

* See Trans. Am. Inst. E. E. Vol. 14, p. 385.

** See Engineering News, Vol. 40, p 181

TABLE LIV.

Coast of operation of various street railway power stations.

Stations.	Capacity in 1,000 K-W.	Type.					Period averaged, days.	Load factor, per cent.	Labor.						Fuel.				General expenses per K-W. hour.	Total operating expenses per. K-W. hr.	Fixed charges.
		Engines.	Generators.	Belted or D. C.*	Simple or compound.	Non-condensing or condensing			No. of shifts.	Lenght of shifts hours.	Total shifts, hours.	No of men per 1,000 K-W.	Rate of pay, per hour.	Per K-W. hour output.	Lbs. per K-W. hour output.	Price per net ton.	Anthracite or bituminous.	Cost per K-W. hour.			
														Cents				Cents	Cents	Cents	Cents
Standard	3.6	3	3	D. C.	C.	C.	365	33½	3	8	8,760	1.94	$0.27	0.157	2.2	$3.00	B.	0.33	0.093	0.58	0.405
10........	0.4	2	2	D. C.	S.	N.	1	24	......	......	......	7.5	.18	......	......	2.00	.5 A.	......	......	1.59	......
11........	1.4	5	8	B.	S.	N.	365	23	2	12	8,760	3.7	.20	.32	6.7	2.24	A.	.75	.33	1.40	......
12........	1.0	3	7	B.	C.	C.	365	23	2	12	8,760	3.7	.20	.31	4.3	2.93	B.	.63	.11	1.05	.44
13........	2.6	4	4	D. C.	S.	N.	365	42	2	12	8,760	3.0	.22	.16	6.5	1.44	B.	.47	.09	.72	.24
14........	1.6	4	4	Both	S. C.	N. C.	365	19	2	12	8,760	1.87	.15	......	......	......	......	......	......	......	......
15........	1.6	10	16	B.	S. C.	N. C.	183	41	2	12	4,380	6.3	.16	.25	6.5	2.10	B.	.68	.14	1.07	.24
16........	1.6	8	8	B.	C.	C.	365	33	3	8	8,760	3.7	.20	.23	5.0	3.30	B.	.83	.09	1.15	......
17........	0.8	2	4	B.	C.	C.	365	24	2	12	8,760	3.7	.18	.28	5.0	1.43	B.	.36	.06	.70	......
18........	0.4	2	3	B.	S.	N.	365	......	2	12	8,760	3.7	.12	......	......	.75	B.	......	......	......	......
19........	3.0	6	6	D. C.	......	......	31	......	2	12	744	3.0	.17	......	......	.60	......	......	......	.68	......
23........	0.6	3	3	B.	C.	C.	183	16	2	12	4,380	3.3	.25	.51	6.0	3.44	B.	1.03	.16	1.70	1.00
25........	4.0	10	19	B.	S.	N.	365	33	2	12	8,760	......	......	......	5.6	1.05	......	.29	......	.56	......
26........	0.8	7	12	B.	S.	N.	365	19	2	12	8,760	......	......	......	6.8	1.15	......	.39	......	1.20	......
28........	1.4	4	8	B.	S.	C.	31	23	2	12	744	5.0	.17	.37	7.0	1.63	......	.57	.08	1.02	......
29........	1.9	5	5	B.	C.	C.	365	27	2	10	7,300	3.2	.21	.25	3.9	2.80	......	.55	.23	1.03	......
30........	1.7	5	5	B.	C.	C.	365	23	2	10	7,300	3.0	.21	.27	3.3	3.00	B.	.49	.15	.91	......
31........	0.9	3	3	B.	C.	C.	365	32	2	10	7,300	4.5	.21	.29	4.4	2.84	B.	.62	.19	1.10	......
32........	5.3	5	30	B.	C.	C.	365	31	2	12	8,760	3.5	.20	.23	3.5	3.00	B.	.52	.09	.84	......
33........	1.5	4	12	Both	C.	C.	365	16	2	12	8,760	5.0	.20	.62	4.3	3.08	B.	.66	.22	1.50	......
34........	2.0	4	4	D. C.	C.	C.	183	57	2	12	4,392	5.8	.20	.18	5.3	1.00	B.	.27	.12	.57	......
35........	1.2	1	1	D. C.	C.	C.	365	37	2	12	8,700	2.1	.26	.15	4.7	1.24	B.	.29	.04	.48	......
37........	1.5	3	11	B.	S. C.	N. C.	151	11	2	12	3,624	8.0	.16	1.10	7.3	1.75	.5 A.†	.63	.12	1.85	......
40........	9.2	6	6	D. C.	C.	C.	365	36	3	8	8,760	2.7	.23	.17	3.0	1.60	A.	.24	.11	.52	......
42........	1.8	3	6	B.	C.	N.	365	45	3	8	8,760	5.0	.26	.29	5.7	1.60	A.	.45	.18	.92	......
43........	5.3	5	10	Both	C.	C.	365	15	2	12	8,760	3.7	.16	.39	3.7	2.70	.6 B.‡	.50	.14	1.03	......
44........	6.0	6	12	B.	C.	C.	365	30	3	8	8,760	3.3	.24	.26	3.5	1.60	A.	.28	.15	.69	......
45........	1.0	4	7	oth	C.	C.	31	28	2	10	600	3.0	.15	.16	4.7	1.90	B.	.45	.05	.66	......
47........	0.75	4	4	D. C.	C.	C.	183	20	2	12	4,380	8.0	.12	.41	5.1	2.12	.5 A.†	.54	.21	1.16	......
48........	0.6	2	2	D. C.	C.	C.	365	35	2	10	7,300	6.7	.15	.29	3.3	2.75	B.	.45	.28	1.02	......

*D. C.—Direct-connected. †.5 anthracite and .5 bituminous. ‡.6 bituminous and .4 anthracite. §Includes repairs, all supplies other than coal, superintendence and office expenses.

An important discussion of the effect of the load factor on the cost of power was recently made by Mr. E. M. Archibald.† This discussion was accompanied by various diagrams which illustrate clearly the principles involved. Two of these diagrams are reproduced in Figs 400 and 401. The diagrams are so complete as to need no further description. The additional diagrams and

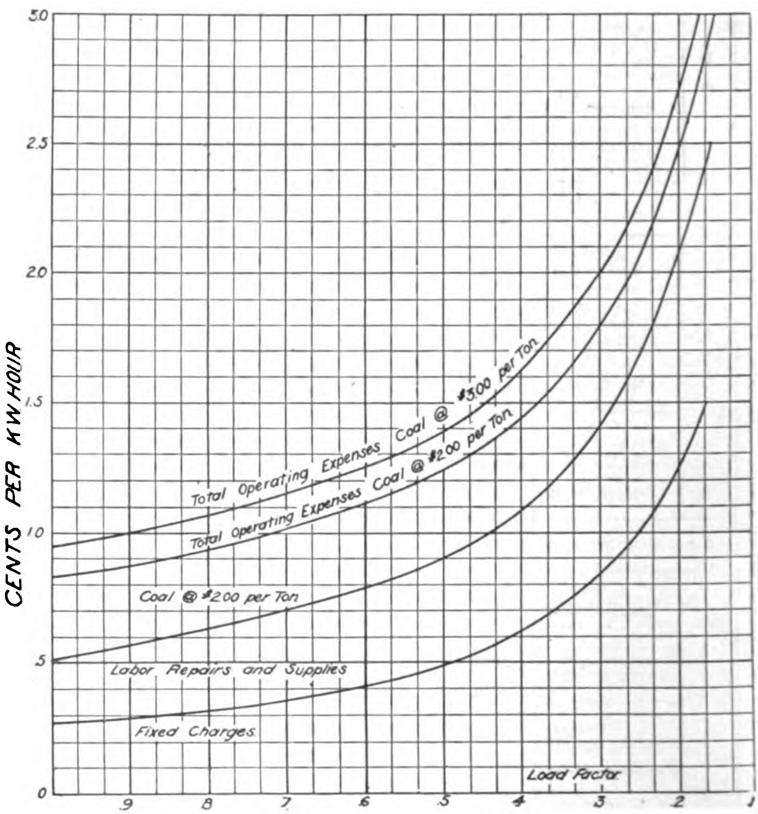

Fig. 400.—Operating Expense of a 900 K. W. Condensing Steam Plant with a 750 K. W. Peak.

the descriptive matter in the paper itself should be carefully studied in this connection.

Table LV shows the capital costs of steam power plants of various capacities and the annual cost of power per brake horse power as estimated by The Ontario Hydro-Electric Power Commission.

† See Electrical Age, Nov., 1906.

Similar costs for producer gas power are shown in Table LVI from the same source, and the Commission's estimate of the effect on the cost of power of variations in the price of coal, is shown in Table LVII.

331. Market Price of Water Power.—The market price of water power must be predicated on two considerations: First: The price at which the Power Company can afford to furnish power and insure a fair return of its investment, and, Second: The price that the consumer can afford to pay for the power. The latter amount

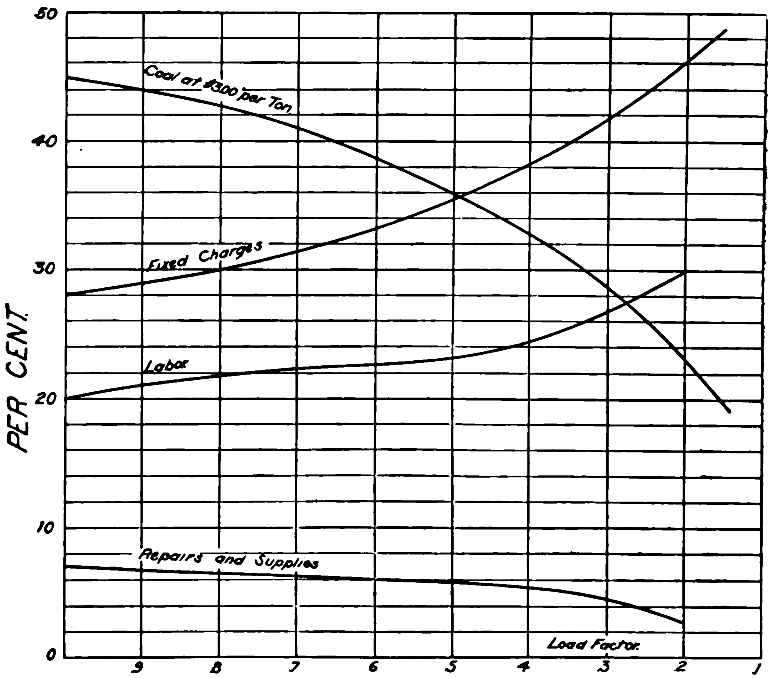

Fig. 401.—Ratio of individual Items of Expense to Total Operating Expense of a 900 K. W. Condensing Steam Plant With a 750 K. W. Peak.

is commonly fixed by what it will cost to produce the power by some other means.

If, in the preliminary investigation of a water power project, it is found that the cost to the Power Company of generating power will be greater than the price at which the power can be sold, it is, of course, evident that the plant will be a financial failure, and the scheme should at once be abandoned. In introducing a new

TABLE LV.

Showing capital costs of steam plants installed and annual costs of power per brake horse-power.

Size of Plant H. P.	Capital Cost of Plant per H. P. Installed. Engines, boilers, etc., installed.	Buildings.	Total.	Annual cost of 10-hour power per B. H. P.	Annual cost of 24-hour power per B. H. P.
Class I.—Engines: simple, slide-valve, non-condensing. Boilers: return tubular.					
10..........	$66 00	$40 00	$106 00	$91 16	$180 76
20..........	56 00	37 00	93 00	76 31	151 48
30..........	48 70	35 00	83 70	66 46	131 68
40..........	44 75	33 50	78 25	59 46	117 74
50..........	43 00	31 00	74 00	53 95	106 46
Class II.—Engines: Simple, Corliss, non-condensing. Boilers: Return tubular.					
30..........	$70 70	$35 00	$105 70	$61 14	$117 70
40..........	62 85	33 50	96 35	55 50	107 10
50..........	59 00	31 00	90 00	50 70	97 78
60..........	56 00	30 00	86 70	47 42	91 34
80..........	50 00	27 50	77 50	43 86	85 41
100..........	44 60	25 00	69 60	40 55	79 19
Class III.—Engines: Compound, Corliss, condensing. Boilers: Return tubular, with reserve capacity.					
100..........	$63 40	$28 00	$91 40	$33 18	$60 05
150..........	53 70	24 00	77 70	29 83	54 63
200..........	50 10	20 00	70 10	28 14	51 72
300..........	45 90	18 00	63 90	26 27	48 83
400..........	43 55	16 00	59 55	24 84	46 12
500..........	41 25	14 00	55 25	23 73	44 21
750..........	40 50	13 00	53 50	23 56	44 02
1,000..........	39 00	12 00	51 00	23 26	43 71
Class IV.—Engines: Compound, Corliss, condensing. Boilers: Water-tube, with reserve capacity.					
300..........	$55 20	$18 00	$73 20	$25 77	$46 32
400..........	51 50	16 00	67 50	24 18	43 61
500..........	49 40	14 00	63 40	23 19	42 03
750..........	46 80	13 00	59 70	22 88	41 56
1,000..........	44 30	12 00	56 80	22 47	41 11

Note.—Annual costs include interest at 5 per cent, depreciation and repairs on plant, oil and waste, labor and fuel, (coal at $4.00 per ton).

Brake horse-power is the mechanical power at engine shaft.

TABLE LVI.

Showing capital costs of producer gas plants installed and annual costs of power per brake horse-power.

Size of Plant, H. P.	Capital Cost of Plant per H. P. Installed. Machine'y etc.	Buildings.	Total.	Annual cost of 10-hour power per B. H. P.	Annual cost of 24-hour power per B. H. P.
10	$137 00	$40 00	$177 00	$53 48	$90 02
20	110 00	36 00	146 00	44 47	75 22
30	93 00	33 00	126 00	38 73	65 99
40	84 50	29 00	113 50	85 05	59 85
50	80 00	26 00	106 00	32 27	55 22
60	79 00	24 00	103 00	30 49	52 03
80	78 20	22 00	100 20	28 70	48 95
100	77 50	20 00	97 50	27 05	45 40
150	76 00	19 00	95 00	25 87	43 17
200	74 00	17 00	91 00	24 95	41 78
300	73 00	16 00	89 00	24 24	40 40
400	71 50	14 00	85 50	23 41	39 03
500	70 00	12 00	82 00	22 54	37 54
750	67 50	10 00	77 50	21 55	35 99
1,000	65 00	8 00	73 00	20 46	34 66

NOTE.—Annual costs include: interest at 5 per cent, depreciation and repairs on plant, oil and waste, labor and fuel (Bituminous coal at $4.00 and Anthracite coal at $5.00 per ton).

TABLE LVII.

Showing the effect on the cost of power of a variation in the price of coal of one-half dollar per ton.

Size of Plant.	Suction Producer Gas. 10 Hour.	Suction Producer Gas. 24 Hour.	Steam.	Steam. 10 Hour.	Steam. 24 Hour.
10	$1 15	$2 53	Simple slide valve	$6 14	$13 47
20	1 13	2 46		5 25	11 56
30	1 10	2 40		4 71	10 35
40	1 07	2 33	Simple automatic non-condesing	3 56	7 84
50	1 04	2 29		3 37	7 41
60	1 01	2 24		3 26	7 16
80	98	2 18		3 15	6 97
100	96	2 12		3 12	6 87
150	94	2 07	Compound condensing	1 75	3 85
200	92	2 02		1 69	3 71
300	90	1 98		1 62	3 60
400	88	1 94		1 56	3 44
500	86	1 89	Compound condensing water-tube boilers	1 39	3 05
750	82	1 81		1 39	3 05
1000	76	1 72		1 39	3 05

source of power into any community where the power introduced will be obliged to compete with other sources, it can seldom be expected that the power to be so furnished can be sold at the same price as the power already on the market. It is at least only safe to estimate that the power must be sold at a somewhat lower figure. If the power already in use is sold or generated at a profit, a cut in price may be anticipated from the competing company; and, in the second place, as a considerable expense is necessarily involved in the change of apparatus, etc., necessary to utilize a new source of power, consumers will be slow to make such changes unless they can do so to a considerable financial advantage.

In calculating the cost of power to a consumer, if he undertakes to generate it himself, the fair cost should be based upon interest, depreciation, operation, etc., of the plant which is necessary to be installed. If, however, the consumer has such a plant already installed, no further investment is necessary, and as the machinery installed can not be sold to advantage, the investment charges or the fixed charges on such a plant can not be considered, and the consumer will make a change in power only provided the power can be furnished from the new source at or below the actual cost of generation in his own plant, or at such additional cost as the convenient reliability of other desirable features of the new source of power will warrant.

In estimating the price at which the consumer can afford to purchase power, not only the price at which power is now sold but any possible decrease in the sale price, due to competition or to other and more economical developments, must be considered. Better and more economical machinery in local plants, or water powers that are nearer the market and that can be developed or operated at less expense, may so reduce the market price as to seriously affect the value of power, and hence the probability of the development.

332. **Sale of Power.**—Attention has already been called to the fact that if the capacity of a plant can be used for only a portion of the time, the cost of the development per unit of power, and therefore the cost per unit, is very greatly increased. This is a matter of the greatest importance which should be kept clearly in mind in the sale power. The load factor of many users is comparatively low. Most companies organized for the general sale of electrical power in municipalities have a load factor of 35% or less. A sale of power to such consumers, to be used under such conditions, is

liable to very greatly increase the cost of power to the power company, especially if the maximum power to be furnished is large as compared with the total capacity of the plant. For example: If, in a 3,000 horse power plant, power is sold on a horse power hour basis, with a peak load of 1,000 horse power and a load factor of 30%, the average twenty-four hour power furnished to the consumer will be only 300 horse power, while the total peak that the power plant will be called upon to carry at any time will be 1,000 horse power or one-third of the total capacity of the power plant. With such sale of power the power plant is likely to be seriously handicapped. With power sold in such large blocks, the overlappings of the peak loads can not reasonably be expected to compensate for each other. The net results of such a sale will be that the company has tied up one-third of the capacity of its plant but will receive payment for only one-tenth of its capacity. It is evident that unless such conditions are realized and such a charge is made for power as will compensate the power company for the same, the power company may readily tie up its entire out-put and yet not receive 50% of the income that should be reasonably anticipated. If, on the other hand, the sale of power is made in small blocks, or to small consumers, it is frequently possible to greatly over-sell the total capacity of the plant and yet take care of the consumers in a satisfactory manner. That is, on account of the overlapping of the peak loads and the equalization of the load carried throughout the twenty-four hours, the total connected load sold may often considerably exceed the capacity of the plant. For example: In one water power plant, having a total capacity of about 4,000 horse power, the actual connected load is over 10,000 horse power. In many power plants the actual connected load is two or three times the plant's capacity. It is evident, however, that such a condition can exist only with small consumers, and that where a single consumer's load is a large fraction of the plant's capacity, it will not only be impossible to overload the power plant, but in addition extra machinery must always be installed to supply the demand should any accident happen to the regular installation.

Mr. E. W. Lloyd has compiled some valuable data concerning the power loads on various central states from various classes of consumers. This data is given in Table LVIII.

The increase in the charges for power to consumers on account of the variation in power factor is well illustrated by Fig. 402 taken from the paper of Mr. Archibald before referred to.

TABLE LVIII.

Actual conditions under which power is furnished to consumers from Central Stations.

Character of Installations.	Average kw.-hours per month.	Average connected motor load, hp.	Individual or group drive.	Average number of motors.	Percentage of average load to connected motor load.	Total number of installations.
Bakeries	1582	32.8	Group	2.7	27.8	17
Bakeries	705.3	22.5	Individual	3.1	19.5	8
Boiler shops	326.7	51.4	G	2.8	33.3	11
Boiler shops	1172	32.2	I	5.2	20.7	5
Boots and shoes	3050	39.7	G	5.8	42.8	13
Box making	1555	18.1	G	4.3	45.4	20
Blacksmiths	586	9.4	G	2.2	34.2	12
Brass finishing	5736	40.5	G	7.4	45.0	9
Butchers and packers	1990	24.8	G	2.0	36.4	13
Butchers and packers	1049	36.9	I	6.7	18.8	10
Breweries	12310	94.0	G	4.6	33.0	8
Carpet cleaning	644	14.5	G	1.6	30.1	12
Cement mixing	2009	37.5	G	1.0	24.9	4
Candy manufactory	1893	26.6	G	3.5	33.6	10
Candy manufactory	796	29.9	I	7.5	16.3	8
Cotton mills	11829	99.0	G	3.0	60.1	3
Carriage works	2091	24.8	G	3.5	35.5	22
Chemical works	4802	109.0	G	5.5	23.5	6
Clothing manufacturing	1181	23.0	G	4.0	44.5	33
Grain elevators	3842	114.4	G & I	3.8	32.6	19
Feather cleaners	2447	54.4	G & I	5.5	25.7	2
General manufacturing	6133	67.6	G & I	6.4	33.9	181
Engraving and electrotyping	863	12.4	G	2.5	46.9	8
Engraving and electrotyping	2369	46.3	I	26.7	22.5	7
Glass grinding	2760	33.5	G	3.0	36 6	6
Foundries	2057	27.7	G	2 3	43.7	15
Foundries	2419	81.1	I	7 0	21.3	18
Furniture manufacturing	1750	35.7	G	3.6	35.6	9
Flour mills	41276	148.5	G	3.1	48.1	13
Hoisting and conveying	2905	70.5	G	6.4	28.3	5
Hoisting and conveying	6562	253.0	I	20.0	13.0	9
Ice cream	596	31.0	G & I	5.4	35.9	7
Refrigeration	4645	36.7	G & I	2.5	53.4	17
Jewelry manufacturing	2526	31.7	G	4.6	31.6	5
Laundries	676	10.8	G	2.1	34.0	19
Marble finishing	1464	19.8	G & I	1.3	51.3	2
Machine shops	4006	57.6	G	4.5	34.5	51
Newspapers	3150	47.4	G	4.8	38.0	24
Newspapers	4975	137.0	I	17.3	15.1	21
Ornamental iron works	2771	38.4	G	3.6	41.6	9
Paint manufacturing	2814	60.4	G & I	4.6	26.5	11
Printers and bookbinders	1147	20.4	G	2.6	39.5	54
Printers and bookbinders	6215	76.8	I	24.0	26.0	39
Plumbing, manufacturing	3020	42.4	G	4.8	21.5	15
Rubber manufacturing	1051	26.0	G & I	15.0	24.7	2
Sheet metal manufacturing	1321	38.8	G	3.7	27.3	17
Soap manufacturing	3434	73.0	G	10.0	27.6	2
Seeds	2917	55.1	G & I	5.8	24.4	5
Structural steel	6514	176.0	I	16.1	18.5	6
Structural steel	77704	552.1	G	35.6	31.1	6
Stone cutting	7425	76.5	G & I	3.8	34.4	20
Tanners	2466	28.6	G	2.6	54.6	5
Tobacco working	3441	62.3	G	7.0	37.5	4
Wholesale groceries	2005	47 0	G & I	4.5	26.0	17
Wood working	2306	39.5	G & I	3.6	33.3	64
Woolen mills	20985	150.0	G	3.0	71.0	1
Averages	3500			6.08	33.9	951

333. An Equitable Basis for The Sale of Power.—It is therefore essential, in order to establish an equitable basis for the sale of power, that some additional factor besides the units of power furnished be considered in determining the basis for the prices charged. One of the most equitable bases for the sale of power

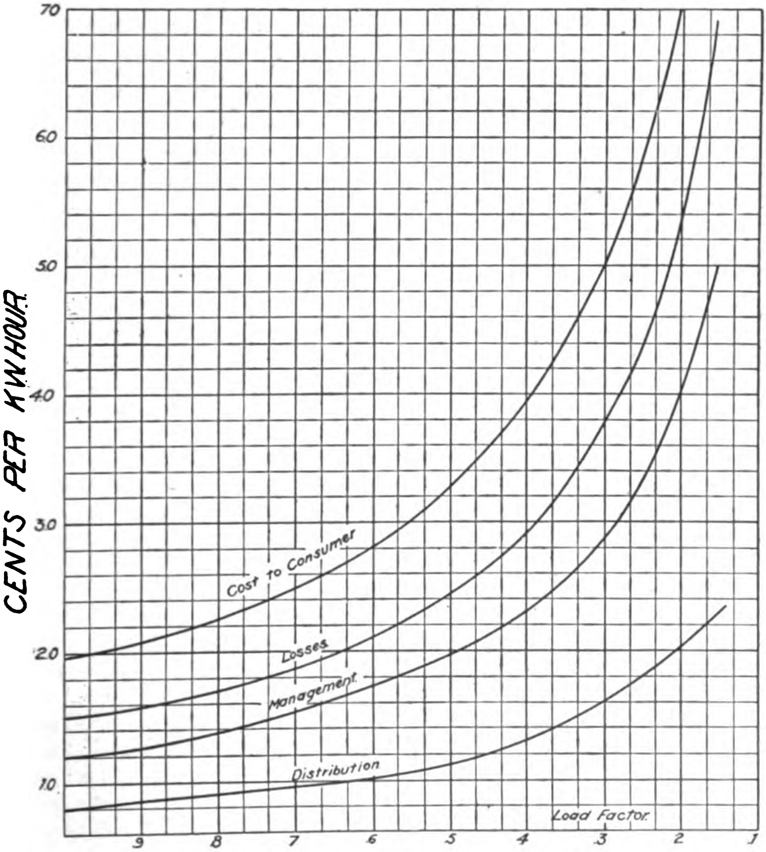

Fig. 402.—Cost of Steam-Generated Electric Power to the Consumer.

is apparently: First: A service charge to the consumer of a fixed price, based on the peak load carried; Second: To this should be added a price for the units of power actually furnished. The fixed price should equal the interest, depreciation, etc., on the capacity that is to be provided or set aside to carry the peak load of the customer. The unit price for power should be an equitable charge for the quantity of power which will actually be sold. Where both of these quantities are fixed, a net price per horse power per year, or a total price per annum for the power to be furnished, can, of course, be arranged equitably. The main idea in establishing a price for power is to keep clearly in mind the factors that enter into the sale of power, so that in making a contract for the use of power the rights of both power company and consumer shall be duly considered. The sale of power at a profit is one of the most essential features in the management of the power plant, and many plants have been wholly or partially financial failures on account of the ignorance of the basic principle on which power should be sold.

The method of charging for power outlined above is illustrated by the charges for Electric Current furnished from Niagara Falls, by the Cataract Power & Conduit Co. of Buffalo, as given in the Engineering News (May 26th, 1898) as follows:—

"All payments for power are to be made monthly and the amount of each monthly payment will consist of a charge for service, and in addition thereto, a charge for power. The charge for service is $1 per kilowatt per month, and this charge will depend only upon the amount of power which the user may require the Cataract Power & Conduit Company to keep available and ready for his use. The monthly charge for power will depend upon the aggregate amount used, as determined by integrating meters installed by the Conduit Company upon the premises of the consumer. The charge for power will be determined from the following schedule:—

Units (K-W. hrs.) used per month.	*Charge per unit* For current up to	*Charge per unit* For the excess
Up to 1,000	1,000 units, 2.0 cts.	2.0 cts.
1,000 to 2,000	1,000 units, 2.0 cts.	1.5 cts.
2,000 to 3,000	2,000 units, 1.5 cts.	1.2 cts.
3,000 to 5,000	3,000 units, 1.2 cts.	1.0 cts.
5,000 to 10,000	5,000 units, 1.0 cts.	0.8 cts.
10,000 to 20,000	10,000 units, 0.8 cts.	0.75 cts.
20,000 to 40,000	20,000 units, 0.75 cts.	0.70 cts.
40,000 to 80,000	40,000 units, 0.70 cts.	0.66 cts.
Over 80,000	80,000 units, 0.66 cts.	0.64 cts.

334. Value of Improvements Intended to Effect Economy.—In many plants the first cost of an installation is an important matter and must sometimes have a greater effect than the interest and depreciation charge would seem to warrant. In most cases the plan should be studied in detail and improvements introduced or rejected on the basis of their true financial value. Such consideration should usually be made on the following basis:

Dr.		
Investment required to effect improvements	$.............	
Interest on investment..................		$..............
Depreciation on improvements		$..............
Extra expense of operation and maintenance............		$..............
Total annual cost of improvement..		$..............
Cr.		
Saving in power (or in expense) effected by improvement	$..............	
Annual value of saving effected..........		$..............
Net annual gain or loss due to improvement..........................		$..............
Capitalized value of power (or expense) effected by improvement		$..............
Net capitalized loss or gain effected		$ ....

335. **Value of a Water Power Property.**—It has frequently become necessary in this country to condemn water power privileges on account of the necessity of securing public water supplies or for other public purposes. Under such conditions it frequently becomes necessary to estimate the value of the water power property. When such matters are brought into court and various witnesses are heard on the subject, it is commonly found that very great differences of opinion exist as to the value of power. These differences of opinion are largely the result of entirely different points of view.

To those who have carefully followed the discussion of the hydrograph, and the estimate of power based thereon, the great variations that occur in the potential power of streams at various times in the season, and in the various years, are obvious.

It is apparent that different engineers, even if they take carefully into account these variations in power, may differ very greatly indeed as to the extent to which the power can be economically developed.

The question of pondage as discussed in Chap. XXVI also has a very important bearing on this matter. It is only by a careful study of the whole question and the consideration of the power market that even an approximately correct answer to this question can be given. The value of such a plant may be considered in a variety of ways: First: Its value if intelligently and recently designed, may be represented by the cost of its reproduction plus a certain value for the water power rights; Second. Its value may be computed on the capitalized net income that the plant can or does earn; or, Third: The value of the plant may be considered equal to the capitalized value of the most economical plant that can be installed to furnish power at the point at which the power is to be used. By the term "most economical" is meant not necessarily the one lowest in first cost, but the plant that, when considered in the broadest sense, will furnish power, all things considered, at a less cost than from any other source of power. The subject is a very broad one and one that needs careful consideration and study. A number of references are given to discussions of this subject before various engineering societies, to which the engineer is referred for further information on this important subject.

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CHAPTER XXVIII.

THE INVESTIGATION OF WATER POWER PROJECTS.

336. **The Extent of the Investigation.**—The investigation of any water power project should include a careful study of all available data relating to the physical and meteorological factors that affect the water supply and that obtain on the drainage area of the stream on which the water power development is projected. The present condition of these factors is readily obtainable by careful observations and surveys but the most difficult and yet the most important information needed for the correct understanding of the project is the variations from present conditions that have occurred in the past and that are therefore liable to re-occur in the future. On the correct interpretation of the available data the success of the project or at least the economy of the installation depends, especially if, as is usually the case, it is desired to develop the plant to its economical maximum.

The extent of the investigation must be governed by the importance of the project, and will also depend on whether the investigation and report are to be of a preliminary character, or are to be the basis of a final report on which the feasibility of the project may be decided.

337. **Preliminary Investigation and Report.**—An examination of the data available in any first class engineering library will generally give the information necessary to form an approximate judgment of the probable feasibility of the project in so far as it depends on the flow of the stream. The approximate area drained by the stream can be determined by referenec to such maps as may be available and the probable flow and the variations in the same that will occur from day to day and from month to month, can usually be determined by the construction of comparative hydrographs made from either the measured flow of the stream, if such information is available, (see Literature page 194) or otherwise on the comparative flow of similar and adjacent streams as described in Sec. 51, page 83, and Sec. 100, page 184.

From such an investigation together with an appropriate knowledge of the available head, an estimate of the probable power of the stream can be made, and from such information an opinion can be formed as to whether it is desirable to carry the investigation further. Frequently such an investigation will show beyond question the futility of the project, and even an examination of the locality becomes unnecessary. If the preliminary investigation shows that sufficient power is probably available on the stream in question the investigation can be carried into sufficient detail to warrant an opinion as to whether or not the project is feasible in all of its phases.

338. Study of Run-off.—The information of primary importance in a water power project is the amount and variation in the run-off of the stream itself. If this is not available the run-off of neighboring streams that have similar physical and meteorological conditions prevailing on their drainage area is next in importance.

As already pointed out, (see Sec. 99, page 181), the hydrograph of the actual flow of the stream itself is the best information for studying its variations in flow. Such hydrographs must be available for a considerable term of years, and it is desirable that they should cover all extremes of rain-fall and drought, and other physical and meteorological conditions that influence run-off.

In the investigation of the hydrographical condition of any stream, a single gauging of the stream is of little or no value. It is however, desirabble to establish a guaging station as early as possible and to take daily gauge readings. It is also important, both for the purpose of an understanding of the guage reading and for the purpose of the study of head, to make stream flow measurements (see Chap. XI) under all large variations in flow, as early as possible in order that a rating curve may be established.

When no local hydrographs are available, or when such available hydrographs are limited to a few years, it becomes desirable to gather together the flow data of all adjacent and similar streams and to construct comparative hydrographs therefrom, as described in Sec. 100, page 184. A long continued series of hydrographs of a neighboring stream where similar conditions prevail is important and should usually be utilized even if local observations have been made for a few years. The value of comparative hydrographs is dependent on the similarity of conditions, a question that demands careful consideration and a considerable amount of data to determine, and even then can be regarded only as indicative. It is also essential to make careful comparisons of the relations that exist between the hydrograph of the river under discussion and those of adjoining rivers, for such period as such data may be mutually available on both streams, in order that variations between the areas compared may be determined.

339. Study of Rain-Fall.—The rainfall records of the United States Weather Bureau and, previous to these, the records of the observations of the United States Signal Service (see Literature, page 130) are available from various stations throughout the United States for a long term of years. It is desirable to collect the rainfall data for the drainage area of the stream under consideration, and also on such other drainage areas as may be used for comparative purposes. This information should be classified and studied as outlined in Chapter VI. In investigating rain-fall it is usually especially desirable to make a study of both the annual rain-fall and the periodical rain-fall of the divisions of the water year. (See Sec. 77, page 126.) The distribution of the rain-fall of these periods has a greater effect on the low water flow than the total rain-fall for the year.

The relations between rain-fall and run-off for the period for which complete data is available should be investigated and such relations established as clearly as possible for the drainage areas under conideration. (See Chapter VIII.) With the information concerning run-off commonly available and the rainfall records for a considerable term of years, it will be possible to draw fairly accurate conclusions as to the probable variation and average flow of the stream. The probability of a larger maximum or a smaller minimum than the stream flow observations themselves indicate can also be determined from such an investigation.

340. Study of Topographical and Geological Conditions.—The topographical and geological conditions may ordinarily be investigated from data available in the publications of the United States Geological Survey, or of the Geological Surveys of the state in which the drainage area may lie. The information sought from this investigation is a knowledge of the conditions that will effect run-off, consequently, such a study is not of particular importance provided sufficient rain-fall and run-off data is available for the purpose of the investigation.

If, however, the hydrographical condition of the areas under consideration, or of other adjacent and similarly located areas have not been previously investigated, and if few or no local observations of stream flow have been made, the topographical and geological data may form the basis of a more intelligent opinion in regard to the probable run-off than can be obtained without such consideration. In any event, this source of information should be utilized to the full extent warranted, as should all other sources of information that will in any way assist the engineer in an intelligent understanding of the problem before him, and the formation of a correct opinion as to the possibilities and probabilities of the case in question.

341. **Study of Flood-Flow.**—It is important to establish both from information that is usually available in the stream valley under consideration, and from information which may be available from adjoining streams, the probable maximum flood-flow of the stream. This must be determined, or at least a safe approximate estimate must be made in order that the dam and other works for the control of the flow can be intelligently designed. (See Sec. 93, page 163.)

After the rating curve has been established the elevation of the high water marks in the immediate vicinity and the relation of the same to guage heights will usually give a safe basis for the estimate of extreme flood-flows.

342. **Study of the Back-Water Curve.**—A topographical survey of the proposed site of the dam and of the stream valley above the dam site, to the probable practical limit of the back-water effect, should be carefully made. In order to investigate the probable height of the back-water under all conditions of flow it will be necessary to make cross-sections of the river at such intervals and under such conditions as will permit of the division of the river into lengths or divisions having comparatively uniform sections. Gages should then be established at the various stations and observations should be made of the gage heights at each station during various stages of flow (see Chapter X). From the quantity of water flowing at any stage, together with the cross sections of the river on the various divisions, the value of the hydraulic elements and especially of the friction coefficients for each division and their variations under such condition of flow, can be calculated. (See Secs. 37 to 40, page 44.) After this has been done it is possible to calculate the back-water curve (see Sec. 42, page 58) and to establish the probable limit of the back-water flow line under any other conditions of flow in a fairly reliable manner.

Study of Head.—The consideration of these conditions, the height of the water surface at the dam due to various sections and length of the spill-way and the practicable limit to which flood height in the valley above must be restricted, will usually establish the limit of the height to which the dam can or should be built and will indicate whether it is necessary or desirable to construct flood gates or to use an adjustable crest, flash boards, or means for regulating and limiting the flood height. When these conditions are established the variations in head under various conditions of flow can be determined (see Chap. V, page 93) and the effect of such variations on the power which may be developed can be calculated. (See Sec. 62, page 103.)

343. **Study of Storage and Pondage.**—The topographical survey will also give information concerning the storage and pondage condition immediately above the dam. In special cases, reservoirs beyond the limit of the back-water effect may be desirable and special surveys under such conditions will be necessary. As the conditions of pondage and storage materially effect the amount of power available, these questions frequently become of great importance and should receive the attention of the engineer that their importance in each particular case seems to warrant. After definite information is obtained concerning the extreme permissible limit of flood-flow, and the possibilities of storage and pondage, an estimate of the power of the stream under various conditions of use can be readily made. (See Chap. XXVI.)

344. **Study of Probable Load Curve.**—It is important in considering the power of the stream and especially the desirable condition of pondage, to ascertain as far as practicable the probable necessary distribution of the demand for power throughout the day. The way in which the power is to be used, whether on 10 hour, 12 hour, or 24 hour service, and its probable variation during the hours of use, has a most important bearing on the design of the plant. (See Chapters XVII and XXI.) If variations in the demand for power throughout the year are also likely to occur, and such variations are likely to effect the requirements for storage, they must also receive consideration.

A census of the power used in the district, to be supplied from the proposed water power development, is important and should be made in as great detail and with as great care as practicable. An accurate estimate of the amount of power used by a factory or manufacturing plant is a matter of considerable difficulty. In some plants where power is electrically distributed, the use of indicating, and sometimes of recording instruments, make it very easy to determine the energy output of the power plant. In most manufacturing establishments where power is distributed by belts, shafting, and other than electrical means, the amount of power actually developed and utilized is seldom definitely known. The use of the steam engine indicator, if opportunity for such use is offered, will give a knowledge of the indicated power of the engine at the time observations are made; and if the probable variations are investigated, a fairly close estimate of power used can often be made by this means.

The annual quantity of coal used, and a careful study of the condition and character of the boiler service, requirements for heating, condition of the engine used, together with a careful examination of the machinery operated, will form the basis of a fairly approximate estimate of power used. Even where the estimate of power used is fairly accurate, it must be remembered that when such power is used and transmitted through a multitude of shafts, belts, etc., that if the electric power is substituted and individual motors used on the machine to be operated, the power then used will be very greatly reduced in amount.

345. Study of Power Development.—Having established the probable load curve, the head under all conditions of flow, and the flow as modified by the pondage or storage conditions, the extent of the power development can be determined. All of the questions that have been previously discussed lead up to the consideration of the question of the desirable capacity or extent of the proposed power development. This capacity should always be estimated on a conservative basis. If, as is usually the case, uncertainties exist as to the probable demand and distribution of power, or the probable minimum flow of the stream, it is desirable to develop the project to a point below the probable commercial maximum but to keep in mind the probability of the desirability of future enlargements and to consider the plans with the future in view. In this connection the question of auxiliary power, and the capacity of the plant as modified by such power, should receive attention.

346. Study of Auxiliary Power.—The necessity of auxiliary power in connection with the proposed water power development can be determined by an intelligent study of the hydrograph and an investigation of the effects thereon of the storage and pondage available. (See Sec. 317.) As a general principle, it may be stated that a stream can often be developed to commercial advantage to the extent of the power which will be uniformily available for eight or nine months of the dryest year, and that auxiliary power is usually warranted to furnish the power needed for the remainder of the season. This is a general rule which must be applied with caution. Every proposed development must be carefully investigated for itself, and no general conclusion should form the basis of a final report on the feasibility of such a project. The greater the demand for power, and the greater the cost of development from other than water power sources, the more expense is warranted for auxiliary service, pondage, etc., and the greater the capacity to which the water power should be ultimately developed.

347. Study of Site of Dam and Power Station.—In addition to the topographical survey previously mentioned, it is necessary to examine in considerable detail the bed and banks of the stream and make all necessary soundings to fully establish all conditions on which the character of the construction recommended must depend. It is important that all conditions be carefully investigated and the type of construction to be recommended carefully considered. The storage of energy almost always involves a hazard which must be met with economical but safe design and construction. The prevention of flow under and around the structure requires a detailed knowledge of the local conditions and is one of the most uncertain conditions which, unless carefully and correctly estimated, is apt to result in considerable extra expense. The flood flow is a condition which needs the most careful consideration for it is often the condition of greatest danger and, to assure safe construction during the short period when such conditions obtain, requires special attention and intimate knowledge of the local conditions, and often involves considerable expense.

348. Study of Plant Design.—The study of plant design requires an extensive study of the various types of development that are in practical use and the adaptability of such designs to the conditions of the particular locality under consideration. It is seldom that plans, no matter how successfully carried out in one place, can be duplicated to advantage in another. Each plant should be built to meet the particular conditions under which it is to be installed and operated, and the best ideas from all sources that will apply to the local conditions should be correlated and embodied in the proposed plant. Extensive experience, observation, and study are each desirable and each essential for the best results. For his own, as well as for his client's good, the engineer should endeavor to secure the very best results possible when all things are carefully weighed and considered. No reasonable amount of conscientious work, painstaking thought, study, labor or expense should stand in the way of such results; and anything less than this is a detriment to future professional attainments which no engineer, young or old, can afford.

In the previous chapters the general principles underlying the design of the various elements of the plant have been considered. The consideration of these matters has been very brief and the engineer must extend his study in all cases to the extensive literature on each subject, reference to some of which has been given at the end of most chapters. Additional references can be found in the Engineering Index and in the indexes to the various technical publications and the publications of the various engineering societies. A personal visit to and a detailed examination of successful plants is a method for the acquisition of exact knowledge which should not be neglected. New novel and untried designs are frequently described in engineering publications. If they are successful their success is often heralded in a similar manner. Their failure is seldom mentioned by the technical press and the only method of ascertaining their true value is by personal and confidential inquiry on the ground.

349. **The Estimate of Cost.**—In order that the preliminary estimate shall be made on a safe basis, reasonable allowances should be made for unforeseen and possible contingencies. This is especially desirable in preliminary estimates on which the feasibility of the entire project may be based. If a safe estimate of the actual cost of construction,—that is an estimate which will surely not be exceeded and will undoubtedly be reduced in construction,—makes the feasibility of the project doubtful, then, as a general proposition, the project is not worthy of further consideration. If the project is predicated on the basis of an estimate that is known to be safe, it can lead to no unfortunate investments. The owners of a development are always satisfied if the cost of development is less than the engineer's estimate; but an increase in cost is often a serious matter.

The desire to develop a project is sometimes apt to give an optimistic coloring to the engineer's report. This is a tendency which, both on account of the interest of his client and his own future reputation, he should carefuly guard against.

If the feasibility of the project is reasonably well established by the preliminary examination, the examination should be still further extended and made fairly complete. Preliminary plans should be outlined in order that a safe detailed estimate may be made. The expense involved in such preliminary work is well warranted by the results obtained. In many cases plants have been recommended on insufficient examination, and the estimates made with too optimistic a view of the conditions to be met. The latter development of the necessity of increased expense, has made the project less attractive and has resulted in great disappointment both to the owners and to the engineer on whose opinion the work has gone forward.

350. **The Report.**—As far as practicable the engineer, in making a report on a water power project, should furnish his client with all of the data on which his deductions are based. He should discuss this data and its bearing on the project and point out as clearly as possible the reasons for the opinions he expresses. In a well drawn report the engineer can usually so illustrate and describe the conditions by which a project is modified and controlled, that any good business man will understand the basis on which his opinion rests and the degree of probability of any departure from the expected result. While this is not true in regard to the technical details, it is entirely true with the general consideration on which the feasibility of a project rests. If a report can not be so drawn it is due either to insufficient data or to the fact that the engineer himself does not fully understand and appreciate the logic of the situation.

In general, a complete report on a water power project should include a careful consideration and discussion of the following:

First: A general description of the drainage area, including the size and the topographical, geological, and other physical conditions that may have a direct bearing on the feasibility of the project.

Second: The run-off data available on the streams in question, if any such data exists.

Third: If local run-off data is available, but only for a brief term of years, the rainfall of the district for as long a period as possible should be collected, and its relations to the available run-off data established. From this the probable modification of the run-off during other years during which the rainfall is found to vary, should be carefully and fully discussed.

Fourth: The run-off data on adjoining streams, having drainage areas with similar physical, topographical and geological conditions, and where the hydrographical conditions of the rainfall and run-off are apparently similar, when the difference therein can be determined and estimated, should be graphically presented and discussed.

Fifth: The relations of the rainfall and of other conditions on the comparative areas considered, and their variations from the particular location under consideration, should be fully illustrated.

Sixth: The conclusion in regard to the probable flow from the drainage area, considered on the basis of its run-off, and the run-off of comparative areas should be fully considered.

Seventh: A general description of the locality at which the dam and power stations are to be constructed, and the physical conditions there existing, also the effect of such conditions upon the construction of the plant, should be described and the methods of meeting them should be carefully and fully outlined.

Eighth: The head available and the variations under various conditions of flow should receive careful consideration.

Ninth: The probable power available with and without pondage, or with the pondage found by the preliminary survey to be available, should be carefully and fully treated, as this is one of the essential features of the report.

Tenth: The auxiliary power, if any, necessary to maintain the plant at all times to the capacity recommended, often needs specific discussion.

Eleventh: An estimate should be made of the probable cost of the development, the probable operating expenses, and the probable cost of maintenance.

Twelfth: The probable market for the power to be generated, and the probable distribution of the demand for the power through the day and year, and the basis on which such estimates are made, should be given.

Thirteenth: The sources of power used in the territory which it is proposed to supply, the cost of developing the same, and the probable price at which power can be sold, are of primary importance.

Fourteenth: The report should be accompanied by hydrographs, preliminary plans, and such other drawings as will, with the data furnished, show conclusively that the facts are as the report sets forth.

Fifteenth: In general it is advisable that the report itself should be clear, concise and definite in its statements and recommendations. Any elaborate discussion of voluminous data should be furnished in the form of an appendix to which the main report should refer for confirmation of its findings and recommendations

APPENDIX—A.

WATER HAMMER.

In Chapter XVIII, Section 213, it is shown that the pressure head due to a change of velocity in a water column is expressed by the formula

$$(1) \qquad h_a = \frac{l}{g} \times \frac{dv}{dt}$$

It is evident that the water hammer head produced by the rapid closing of a gate at the end of a pipe line will be maximum for the maximum possible value of $\frac{dv}{dt}$, or that obtained by closing the gate instantly. Were it not for the elasticity of water and pipe, instantaneous gate closure would produce an infinite rate of retardation, $\frac{dv}{dt}$, and hence infinite pressure. In reality the water near the gate first compresses and the surrounding pipe expands, due to the water hammer pressure, the flow meanwhile continuing undiminished in the remainder of the pipe in order to fill the additional space thus obtained. The point up to which this compression of the water has taken place, as shown by Joukowsky * travels along the pipe from gate to reservoir as a wave with a velocity, λ,† equal to that of

* See the "Memoires of the Imperial Acadamemy of Sciences of St. Petersburg," vol. IX, No. 5. Ueber den Hydraulischen Stoss in Wasserleitungsröhren, by N. Joukowsky; published in German and Russian. See also the synopsis of same by O. Simin in The Trans. of the American W. W. Ass'n, 1904.

† λ varies from about 4,500 to 3,000 feet per second as the size of the pipe increases, and can always be obtained by the formula (due to Joukowsky):

$$\lambda = \frac{12}{\sqrt{\frac{w}{g}\left(\frac{1}{K} + \frac{d}{eE}\right)}}$$

where:

λ = velocity of the wave in feet per second.
K = volumnar modulus of elasticity of the water = 294,000 pounds per square inch.
e = thickness of the pipe walls in inches.
E = modulus of elasticity of the material of the pipe.
w, g, and d = as previously defined in Chapter XVIII.

sound in the same column of water. The water has not all been brought to rest until the wave reaches the reservoir, which evidently requires a time $\frac{l}{\lambda}$. Although only an elementary length of the water column is brought to rest at a time, the effect upon the pressure is the same as would result from retarding the whole column as a unit in a time $\frac{l}{\lambda}$. The maximum possible rate of retardation is hence

$$\text{Max.}\frac{dv}{dt} = v \div \frac{l}{\lambda} = \frac{v\lambda}{l}$$

From Equation (1)

$$(2) \qquad H_m = \text{maximum } h_a = \frac{l}{g} \cdot \frac{v\lambda}{l} = \frac{\lambda v}{g} \text{ *}$$

The pressure-head given by this formula varies from about 140 to 100 feet per foot of extinguished velocity as the pipe increases in size from 2″ upwards. If the gate is only partially closed by this instantaneous motion, the pressure head is given by the same formula in which case v represents the amount of the velocity which is instantaneously extinguished.

Thus, in the case of instantaneous gate movement, the pressure is not produced at the same instant along the entire pipe, but travels as a wave with a velocity λ from the gate to the origin of the pipe and back again to the gate. It then reverses and becomes a wave of rarefaction which travels twice the length of the pipe in the same manner. This continues until the energy of the moving column of water has been dissipated by friction, and the wave gradually subsides. This phenomenon is identical with that of the vibrating sound wave in an organ pipe.

Although equation (2) gives the maximum possible pressure head which can result from the extinction of a given velocity v in a pipe it does not, however, represent the maximum pressure which could be obtained as the result of several successive gate movements; in fact, no limit can be assigned to the pressure which might result in case several water hammer waves were to be produced at intervals differing approximately by multiples of the vibration

* This formula is the same as that obtained by Joukowsky by two other methods of analysis. His discussion of water hammer phenomena includes all that is known upon the subject and it, or Simin's spnopsis, should be read especially by every engineer interested in high head developments as the subject can only briefly be touched in this book.

period of the water column, in which case they are known to "pile up" to enormous indeterminable pressures.

When the flow in a pipe is shut off by the gradual closure of a gate then equation (1) and also the following equation

$$\text{(3)} \qquad \frac{dv}{dt} = \frac{gH}{l}\left(1 - \frac{v^2}{V^2}\right)$$

from Chapter XIX, sections 213 and 217, apply as before except that in this case not only v but also V is a variable, its value being different for each successive position of the gate, and its law of variation depending upon the law and rate of gate movement. The integration of equation (3) in its general form, to obtain the velocity curve is then very difficult if not impossible.

An approximate curve of v, and hence also of h can be plotted by assuming the gate closure to take place by means of a great many small instantaneous movements, according to any law which may be chosen. The value of V for each of the many gate positions can then be computed from the known hydraulic data of the wheels and penstock.

Now, in equation (3), substitute for v the initial velocity in the pipe, and for V the normal velocity (above determined), after the gate has received its first small instantaneous movement. The result will be the initial slope of the v–t curve $= \frac{dv}{dt_0}$. Assume this rate of decrease in velocity to continue constant for the short interval between successive gate movements; then the actual velocity, v, at the instant of the next gate movement will be

$$\text{(4)} \qquad v = v_0 - i\frac{dv}{dt_0}$$

where i is the interval between the two movements.

Assume this new value of v, to be v_0 and using the value of V for the corresponding (or second) gate position, again apply equations (3) and (4), until the gate is completely shut.

Having thus determined the v–t curve, the head curve can be readily found from equation (1), which gives the excess of head above static or so called water hammer head.

Substituting the value of $\frac{dv}{dt}$ from (3) in (1) give

$$\text{(5)} \qquad h_a = H\left(1 - \frac{v^2}{V^2}\right)$$

Church has investigated this problem by a method described in the Journal of the Franklin Institute for April and May, 1890.

APPENDIX—B.

SPEED REGULATION, A MORE DETAILED ANALYSIS THAN IN CHAPTER XVIII.

In Chapter XVIII, Section 217, the following equation was shown to express the rate of acceleration of water in the penstock subsequent to an instantaneous change in gate opening of the wheel.

$$\text{(1)} \qquad \frac{dv}{dt} = \frac{gH}{l}\left(1 - \frac{v^2}{V^2}\right)$$

Separating the variables v and t, gives

$$dt = \frac{l\,V^2}{gH} \cdot \frac{dv}{V^2 - v^2}$$

Integrating we have:

$$\text{(2)} \qquad t = \frac{lV}{2gH} \log_e \frac{V - v}{V + v} + C$$

To determine the constant of integration, C, assume that $v = v_0$ when $t = 0$, hence

$$C = -\frac{lV}{2gH} \log_e \frac{V - v_0}{V + v_0}$$

Let

$$\text{(3)} \qquad k = \frac{2gH}{Vl} \text{ and } k' = \frac{2\,g\,H}{2.3\ Vl}$$

$$\text{(4)} \qquad B = \frac{V + v_0}{V - v_0}$$

Substituting these values of C, B and k in (2), gives,

$$\text{(5)} \qquad t = \frac{1}{k} \log_e \frac{1}{B} \cdot \frac{V - v}{V + v}$$

From the definition of a logarithm: if $X = \log_e N$, then $e^x = N$ hence

$$\text{(6)} \qquad e^{kt} = \frac{1}{B} \cdot \frac{V - v}{V + v}$$

Solving for v we obtain:

$$\text{(7)} \qquad v = V\frac{B\,e^{kt} - 1}{B\,e^{kt} + 1}$$

From the principles of logarithms we have:

$$^{kt} = 10^{\frac{kt}{2.3}} = 10^{k't}$$

hence

(8) $$v = V \cdot \frac{B \times \text{antilog}\ k't - 1}{B \times \text{antilog}\ k't + 1}$$

Equation (8) is very readily applied to finding the curve of velocity increase or decrease in any pipe line subsequent to a sudden change of gate opening. It has been experimentally demonstrated

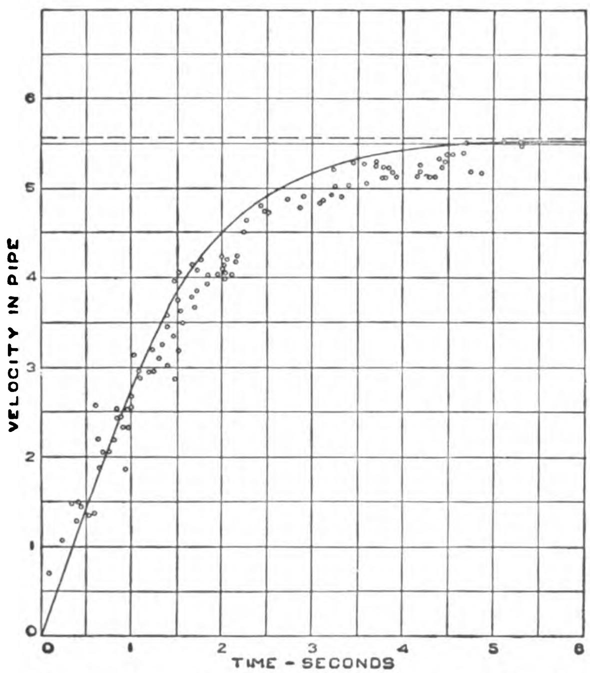

Fig. 403.—Curve Showing the Acceleration of Water in a Pipe Line After a Sudden Opening of the Gate.

for the acceleration of water in the drive pipe of an hydraulic ram, as shown by Fig. 403 which is taken from Bulletin No. 205, University of Wisconsin, Engineering Series, Vol. 4, No. 3, "An Investigation of the Hydraulic Ram," by the writer.

The curve is the plot of equation (8) and the experimental points were determined by an especially designed instrument. The fact that they fall commonly below the theoretical curve is due to a systematic friction error in the instrument. The agreement is sufficiently close, however, to entirely verify the form of equation (8).

Fig. 404 shows the curves determined from equation (8) for

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the wheel used for illustrative problems in Chap. XVIII, Section 228. Acceleration curves are shown for changes from 0 to the velocities of $\frac{1}{4}$, $\frac{1}{2}$, .9 and full loads; retardation curves from an initial velocity of 5′ per sec. to the above velocities. It will be observed that in each case the actual velocity approaches, but theoretically never equals, the normal value, V, for the given gate position.

The values of the constants used in computing these v–t curves are given below. B, for the accelerating from an initial velocity of zero, is:

$$B = \frac{V + v_0}{V - v_0} = \frac{V}{V} = 1$$

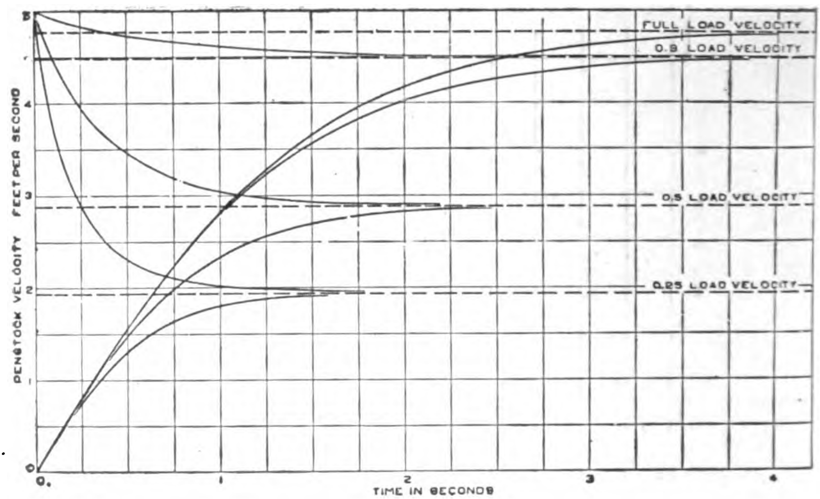

Fig. 404.—Curves of Acceleration and Retardation of Water in Penstock for Various Gate Movements.

The other constants are: H = 50′, l = 500 , and v_0 = 5′ for retardation curves; also for the retardation curves B is negative, since v_0 is greater than V. If we always use the positive value of

$$B = \frac{V + v_0}{V - v_0}$$

we will obtain two equations:

For increasing velocities or *acceleration*

$$v = V \frac{\text{antilog } k't - 1}{\text{antilog } k't + 1} \tag{9}$$

For decreasing or *retarding* velocities,

$$v = V \frac{B \text{ antilog } k't + 1}{B \text{ antilog } k't - 1} \tag{10}$$

From equations (3) and (4) we obtain the table,

Load.	V.	B	k′
1.0	4.77	41.3	.585
.9	4.49	19.1	.623
.5	2.88	3.71	.975
.25	1.94	2.27	1.444

The computations of v, by equations (9) and (10), for various assumed values of t is very simple if tabulated as below. The computation of the curve of acceleration and retardation of water in the penstock from 0, and from 5 feet per second, respectively, to its value 2.88 ft. per sec. for ½ load is shown. It is assumed that the gate opens instantly from 0 to its position at ½ load, and closes to this position instantly when the velocity is 5′ per sec., giving the values of velocity in columns v and v′, (4) and (6), respectively.

*Computation of v-t curve.**

H = 50′, l = 500′, d = 8′, k′ = .975, B = 3.71, v_0 = 0 and 5′, V = 2.88′,

(1)	(2)	(3)	(4) = v	(5)	(6) = v′
t	k′t	antilog of k′t	$\frac{(3)-1}{(3)+1}2.88$	(3) × 3.71	$\frac{(5)+1}{(5)-1}2.88$
.0	.0	1.	.0	3.71	5.0
.1	.0973	1.251	.321	4.64	4.17
.2	.1946	1.565	.635	5.81	4.077
.4	.3892	2.45	1.210	9.10	3.59
.6	.5838	3.835	1.690	14.23	3.31
.8	.7784	6.003	2.055	22.27	3.15
1.0	.973	9.397	2.327	34.85	3.05
1.2	1.168	14.72	2.513	54.70	2.99
1.4	1.362	23.01	2.64	85.5	2.95
1.7	1.654	45.08	2.753	167.3	2.91
2.0	1.946	88.31	2.81	328.0	2.897

* A number enclosed in parenthesis refers to the value given in the column of that number.

Referring again to Figure 404 we see that the acceleration curves thus computed all have a common tangent at the origin showing an initial rate of acceleration in each case of,

$$\frac{dv}{dt} = \frac{gH}{l}$$

The initial rate of retardation, however, depends upon the gate opening.

As shown by equations (9), (10) and the curves in Figure 404 the velocity never equals, but approaches indefinitely near, to its normal value, V, for a given gate opening.

To show the application of the foregoing discussion to the change of penstock velocity, power, speed, etc., at a change of load, refer to Figure 405. Here the line A B represents ¼ load, line C C represents full load, line D D .8 load and line H H 45 per cent. load for the same wheel discussed above. Lines A′ B′, C′ C′ and D′ D′ represent the corresponding hydraulic power input lines. Line abccba represents the line of gate movement from its initial position at ¼ to its position at full load and back again to ¼ load. Line O C_v C is copied from Figure (404) and represents the curve of velocity increase which would result from a sudden complete opening of the gate. At b the gate begins to open, and the velocity to increase along an estimated curve B_v C_v. This curve could be more accurately determined by the process outlined in Appendix A, but was not so determined here. In the same way curve F B'_v A_v was taken from Figure 404 and the velocity curve during gate movement, C'_v B'_v was estimated.

Having thus obtained the velocity curve A_v B_v C_v C C'_v B'_v A_v. the curve of effective head at the wheel can be readily determined from equation (11) Chapter XVIII, or

$$\text{(11)} \qquad h = \frac{v^2}{V^2} H'$$

While the gate is in motion from b to c the valve of V changes, but can be readily estimated by interpolation from the values at ¼ and full gates. From c to c (gate curve) V is constant, and equal to 4.77 ft. per second. Since the friction loss in the penstock is slight in the problem under discussion H′ is assumed to equal H = 50′. The resulting curve for h is A_h B_h C_h C_h C'_h B'_h A_h

The curve of hydraulic horse power or *input* was then determined by applying the equation below to several points along the v and h curves obtaining curve A′ B′ C_1 Y′ X′

$$P_1 = \frac{9h}{8.8} = \frac{A\,v\,h}{8.8}$$

The output power curve A B C_0 Y X was then computed by

$$P = \frac{q\,h\,E}{8.8}$$

E or efficiency for each point was obtained from the characteristic curve of the wheel, Figure 245, by first computing from the known values of q, h, and S (= 180) at each point the values of the discharge under one foot head and ϕ.

Many interesting facts can now be seen from a study of Figure 405 It will be seen that the opening or closing of the gate in order to increase, or decrease, the power of the wheel has an immediate effect directly opposite to that intended and that in the output curve the

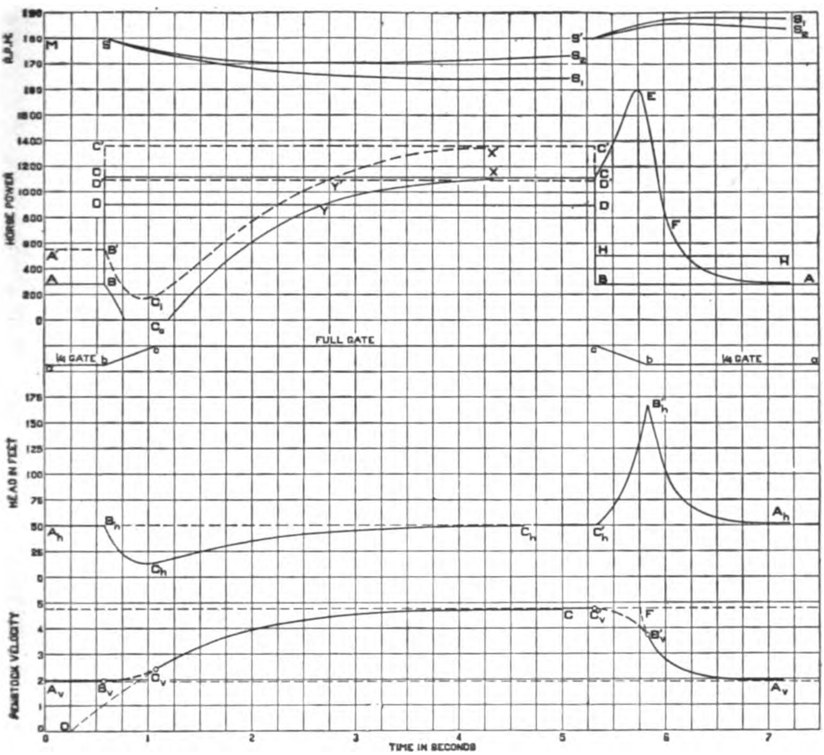

Fig. 405.—Graphical Analysis of Speed Regulation.

power reduces to practically, if not quite, zero for nearly one-half second. The effective head drops very greatly during acceleration, and rises during retardation. It is evident that the rate of gate movement here used (½ second) is too fast for closure, since the head rises to about 165 feet, over three times its normal value.

Now, since the product of power and time gives energy or work, it is evident that the areas of the figures generated by the ordinates to the various load curves are proportional to the demand for energy and the areas of the output curves are proportional to the supply. The area between the two curves, therefore, represents a deficiency or excess of work accomplished by the wheel, and can be measured by means of a planimeter or otherwise. The value of one square is $\frac{1}{4} \times 200 = 50$ horse power seconds $= 50 \times 550 = 27{,}500$ foot pounds.

It was found in this way that the deficient hydraulic energy supplied to the wheel, assuming the load demand to increase from $\frac{1}{4}$ to full is

$$\begin{aligned} &27{,}500 \times \text{area } B' \, C_1 \, Y' \, X' \, C' \, B' \\ &= 27{,}500 \times 36 \\ &= 990{,}000 \text{ foot pounds.} \end{aligned}$$

The deficient load output is

$$\begin{aligned} &27{,}500 \times \text{area } B \, C_0 \, Y \, X \, C \, B \\ &= 27{,}500 \times 35 = 963{,}000 \text{ foot pounds.} \end{aligned}$$

This deficiency of input over output must be supplied from the energy stored in the rotating parts, or from the fly-wheel effect, and can be accomplished only by a drop in speed of the power unit. Furthermore, in the case considered, the speed can never return to normal as long as the load remains at full value, but suffers a permanent drop due to the fact that v, q, h and power theoretically approach, but never equal the normal values for the new gate opening.

The excess energy, when the load again drops to its $\frac{1}{4}$ value is.

$$\begin{aligned} &27{,}500 \times \text{area } C \, E \, F \, A \, B \, C \quad \text{or} \\ &27{,}500 \times 18 = 495{,}000 \text{ foot pounds.} \end{aligned}$$

It is evident that this excess energy at decreasing load will always be less than the deficient energy at time of increasing load, since the low efficiency of the wheel during the velocity-change tends to decrease the former and increase the latter.

It is also possible to dissipate the excess energy through a by-pass or relief valve, while no method is available for supplying the deficiency during load increase except at a sacrifice of the kinetic energy of the rotating parts and consequent reduction of speed.

In Section 226, Chap. XVIII, it was shown that the percentage departure of the speed from normal is

$$\delta = 294{,}000 \frac{R \times \Delta K}{I S^2}$$

Since the deficient energy ΔK is actually measured in this case, the estimated co-efficient R becomes unity. The normal speed, S, of the wheel is 180, and I will be assumed as 1,000,000 ft.^{2} lbs., or 1,000,000 pounds at one foot radius, then

$$\delta = 294,000 \frac{963,000}{1,000,000 \times 180^2}$$
$$= 8.7 \text{ per cent.}$$

This is a permanent drop in speed.

In order for the speed to pick up again to normal, the gate must therefore overrun. The condition then is best illustrated by assuming in Figure 405 that the load increases only to 0.8 of full load value, following the line A B D D, while the gate movement follows the same line as before. In this case the v, h, wheel imput, and wheel output curves will be unchanged.

The deficiency of input or of energy in the delivered water is then (by means of planimeter) represented by area $B' \; D' \; Y' \; C_1 \; B'$ or

$$= 27,500 \times 21.8 = 600,000 \text{ foot pounds.}$$

The deficiency of output, represented by area $B \; D \; Y \; C_0 \; B$, is

$$27,500 \times 21.3 = 586,000 \text{ foot pounds,}$$

giving a speed regulation of

$$= 294,000 \frac{586,000}{1,000,000 \times 180^2} = 5.32 \text{ per cent}$$

The two quantities will probably always agree as closely as the accuracy of the problem demands, and much labor can be saved in an analysis if hydraulic horse power, or input, only is considered.

At Y the power curve crosses the demand line, D D, and the speed begins to pick up, due to an excess of developed power. The time required for return to normal can be obtained by continuing the two curves until the excess area equals the former deficiency. In this case 8½ seconds is required.

By the successive application of equation (41) Chapter XVIII to narrow vertical strips of the excess or deficient energy area, we may plat the speed curve of the unit. In this way curve MSS_1, Figure 405, for increase from ¼ to full load; curve MSS_2 for increase from ¼ to .8 load but simultaneous full gate opening; curve $S' \; S_1$, for decrease from full to ¼ load, and curve $S' \; S_2$ for decrease from full to 45 per cent. load, were platted. Curves MSS_1 and $S' \; S_1$ never returned to normal (180 R. P. M.), but curve MSS_2 returns in 8½ seconds, and curve $S' \; S_2$ in 4 seconds.

It is the belief of the writer that this method of analysis is not too long for a problem in practice and, if not, is therefore better than the method given in Chapter XVIII since the conditions before and during gate movement can be readily included.

APPENDIX—C.

THE STAND-PIPE.

It was shown in Section 223, Chapter XVIII that the following equations apply to the operation of a plant with standpipe:

$$(1) \qquad \frac{dv}{dt} = \frac{g}{l} \text{(accelerating head)} = \frac{g}{l} h_a$$

$$(2) \qquad \frac{dy}{dt} = \frac{dh}{dt} = \frac{Av - q}{F}$$

The value of h_a in a plant with penstock, is

$$h_a = y - h_f$$

Hence
$$= y - (1 + f\frac{l}{d} + \text{etc.})\frac{v^2}{2g} = y - cv^2$$

$$(3) \qquad \frac{dv}{dt} = \frac{g}{l}(y - cv^2)$$

Equation (2) gives the instantaneous rate of fluctuations of water level in the stand-pipe.

Equation (3) gives the rate of increase of penstock velocity in terms of the then existing values of y and v.

The quantity, q, in equation (2), represents the water used by the wheel. This may remain practically constant if the head fluctuation is not too large, in which case the speed of the wheel will suffer: or, by means of an ideal action of the governor, it may be made to fluctuate inversely as the head h, thus maintaining a constant value of the product, qh, and hence of the power input of the wheel. In case this latter assumption is made, then:

$$q\,h = q_1\,h_1$$

or
$$q\,(H - y) = Av_1\,(H - c\,v_1^2)$$

Substituting this value of q in equation (2) gives:

$$(4) \qquad \frac{dy}{dt} = \frac{A}{F}\left[v - \frac{v_1\,(H - c\,v_1^2)}{(H - y)}\right]$$

The solution of the two simultaneous differential equations 2 and 3, or 3 and 4, depending upon which assumption is made, is necessary in order to determine the exact curve of variation of head and velocity. Their general solution is however, very difficult if not impossible in this form. The equations may be applied successively to short portions of the arc by considering the curves to consist of a great many short straight lines. This method is not too long for application to a problem in practice, and will assist in obtaining approximate formulas which will be seen to coincide very closely with the true curves.

Assume an installation where $d = 8'$, $l = 500'$ $H = 50$, $F = 8A$. Let the velocities on the penstock at fractional loads be the same as given in the problem considered in Section 228, Chapter XVIII. If the load suddenly increases from ¼ to full, the velocity in the penstock must accelerate from 1.94 to 4.77 feet per second, or q from 97.8 to 240 cu. ft. per sec.

Estimating $f = .018$, equation (3) gives

$$\frac{dv}{dt} = \frac{32.15}{500}\left[y - \left(1 + .018\,\frac{500}{8}\right)\frac{v^2}{64.3}\right] \quad \text{or}$$

$$(5) \qquad \frac{dv}{dt} = .0643\,(y - .0331\,v^2)$$

and equation (4) gives:

$$\frac{dy}{dt} = \frac{dh}{dt} = \frac{v}{8} - \frac{4.77 \times 49.25}{8\,(H - y)} \quad \text{or}$$

$$(6) \qquad \frac{dy}{dt} = \frac{v}{8} - \frac{29.4}{H - y}$$

Curves A_v and A_h, Figure 406, show the curves of velocity, v, and head, h, respectively, obtained by applying equations (5) and (6) alternating to the two curves, considering them to remain straight for the time interval between consecutive points which were taken from ¼ to one second apart depending upon the curvature. The closer these points are taken the more accurate would be the resulting curves.

If friction in the penstock, and the action of the governor, in compensating for the fluctuations of h, be neglected then equations (1) and (2) become

$$(7) \qquad \frac{dv}{dt} = \frac{g}{l}\,y$$

$$(8) \qquad \frac{dy}{dt} = \frac{A}{F}\,(v_1 - v)$$

Dividing (8) by (7):

$$\frac{dy}{dv} = \frac{Al}{Fg} \cdot \frac{v_1 - v}{y}$$

Integrating:

$$(9) \qquad \frac{y^2}{2} = \frac{Al}{Fg}\left(v_1 v - \frac{v^2}{2}\right) + C$$

To determine the constant of integration, C; let $v = v_0$ when $y = 0$, whence:

$$C = \frac{Al}{Fg}\left(\frac{v_0^2}{2} - v_1 v_0\right)$$

Substituting this value in (9) gives:

$$(10) \qquad y^2 = \frac{Al}{Fg}\left[(v_1 - v_0)^2 - (v_1 - v)^2\right]$$

Substituting this value of y in (7) and solving for dt gives:

$$(11) \qquad dt = \sqrt{\frac{lF}{Ag}} \cdot \frac{dv}{\sqrt{(v_1 - v_0)^2 - (v_1 - v)^2}}$$

The integral of (11) is:

$$(12) \qquad t = -\sqrt{\frac{lF}{Ag}} \sin^{-1} \frac{v_1 - v}{v_1 - v_0} + C$$

When $t = 0$, $v = v_0$, hence

$$C = \frac{\pi}{2}\sqrt{\frac{lF}{Ag}}$$

after which (12) becomes:

$$t = \sqrt{\frac{lF}{Ag}}\left[-\sin^{-1}\frac{v_1 - v}{v_1 - v_0} + \frac{\pi}{2}\right] \quad \text{or}$$

$$t = \sqrt{\frac{lF}{Ag}} \cos^{-1}\frac{v_1 - v}{v_1 - v_0}$$

Solving this equation for v gives:

$$(13) \qquad v = v_1 - (v_1 - v_0)\cos\sqrt{\frac{Ag}{lF}}\, t$$

If this value of v be now substituted in equation (8) the equation for y in terms of t can be obtained as follows:

$$\frac{dy}{dt} = \frac{A}{F}\left(v_1 - v_0\right)\cos\sqrt{\frac{Ag}{lF}}\, t$$

$$y = \frac{A}{F}(v_1 - v_0) \cdot \sqrt{\frac{lF}{Ag}} \sin\sqrt{\frac{Ag}{lF}}\, t + C$$

When $y = 0$, $t = 0$, hence $C = 0$ and

$$(14) \qquad y = -\sqrt{\frac{Al}{Fg}}(v_1 - v_0)\sin\sqrt{\frac{Ag}{lF}}\, t$$

Since this equation is that of a true sine curve it will be readily seen that the maximum ordinate and hence the maximum departure of the head from normal is

$$(15) \qquad Y = \pm\sqrt{\frac{Al}{Fg}}(v_1 - v_0),$$

and return to normal head occurs when

$$\sqrt{\frac{Ag}{lF}}\, t = \sqrt{\frac{Ag}{lF}}\, T = \pi$$

Whence

$$(16) \qquad T = \pi\sqrt{\frac{lF}{Ag}}$$

Equations 13 and 14 may now be revised to read

(17) $$v = v_1 - (v_1 - v_0) \cos \frac{\pi}{T} t \quad \text{and}$$

(18) $$y = Y \sin \frac{\pi}{T} t$$

These equations, (17) and (18), are shown for a particular problem, by the dotted lines B_v and B_h in Figure 406. The closeness of their agreement with the curves A_v and A_h which involve the effect of both friction and governor action shows that the values

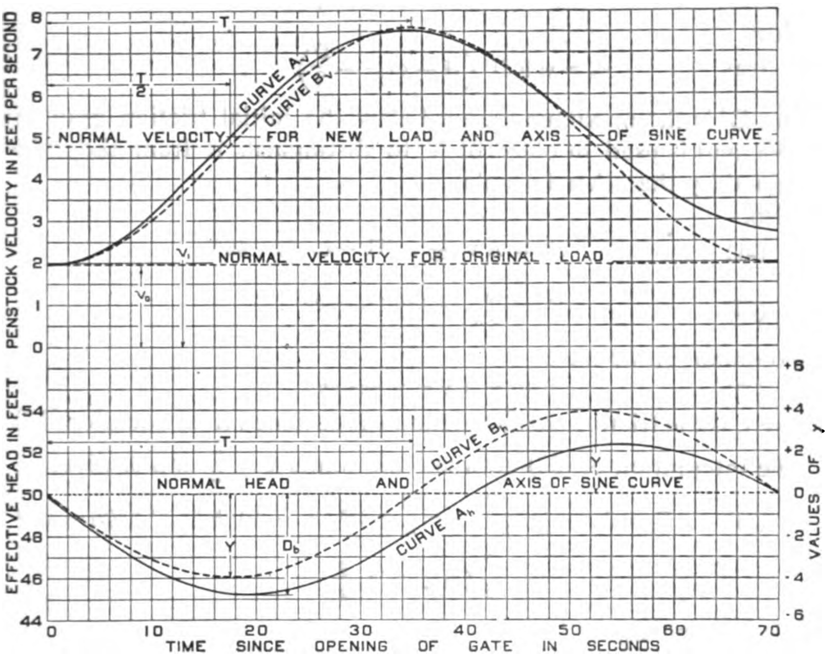

Fig. 406.—Curves Showing Fluctuations of Head and Penstock-Velocity in a Plant with Standpipe.

T and Y would commonly be as close to the truth as the estimate could be made of the probable load change ($v_1 - v_0$), for which the stand pipe should be designed.

More exact formulas can be derived, however, from the stand point of energy as follows:

Let the time required to reach D′ and hence to approximately reach the valve v_1, under exact conditions, be $\frac{T'}{2}$.

The time $\frac{T'}{2}$ will be slightly greater than $\frac{T}{2}$, when friction and governor action are involved, and the method of determining it will be given later (equation 30).

It is evident that the number of foot pounds of energy which must be supplied by the standpipe in this time $\frac{T}{2}$ is equal to the energy required by the wheel plus that required to accelerate the water in the penstock plus that necessary to overcome the friction of the penstock minus that supplied through the penstock,

$$(19) \qquad \text{Or} \qquad E_s = E_w + E_a + E_f - E_p$$

Now,

$$(20) \qquad E_s = w F D' \left(H - cv_0{}^2 - \frac{D'}{2}\right)$$

where D' is the maximum surge below the initial friction gradient for V_0, and is used in place of Y to distinguish it from the value obtained by the other formula:

Also,

$$(21) \qquad E_w = A\, v_1 \frac{T'}{2} w\, (H - cv_1{}^2) \quad \text{and}$$

$$(22) \qquad E_a = \frac{w}{2\,g}\ A\, l\, (v_1{}^2 - v_0{}^2)$$

To obtain E_f we have

$$(23) \qquad d\, E_f = A\, v\, w \times cv^2\, dt$$

where c is the friction coefficient and v is obtained from equation (17).

The integration of (23) between the limits $t = \frac{T'}{2}$ and 0, gives,

$$(24) \qquad E_f = A\, w\, c \left[\frac{v_1{}^2\, T'}{2} - \frac{3\,T'}{\pi} v_1{}^2 (v_1 - v_0) + \tfrac{3}{4}\, T'\, v_1 (v_1 - v_0)^2 - \frac{T'}{6} (v_1 - v_0)^2 \right]$$

Also to find E_p we have

$$d\, E_p = HAwv\, dt,$$

where v is obtained from equation (17) as before. Integrating between the limits $\frac{T'}{2}$ and 0, gives

$$(25) \qquad E_p = HAwT'\left(\frac{v_1}{2} - \frac{v_1 - v_0}{\pi}\right)$$

Combining and simplifying:

$$(26) \qquad D'^2 - 2\,(H - cv_0{}^2)\, D' = -\frac{2A}{F}\left\{\frac{1}{2g}(v_1{}^2 - v_0{}^2) + c\left[-\frac{3T'}{\pi} v_1{}^2 (v_1 - v_0) + \tfrac{3}{4}\, T'\, v_1 (v_1 - v_0)^2 - \frac{T'}{6}(v_1 - v_0)^2\right] + \frac{HT'}{\pi}(v_1 - v_0)\right\}$$

$$(27) \qquad D_b = D' + cv_0{}^2$$

The upward surge can be found by the same equation by a proper change of signs, but is unimportant since it is always less than the downward surge D_b for the same change of velocities.

If friction be omitted and T′ be changed to T for reasons mentioned later, equation (26) reduces to

$$(28) \qquad D^2 - 2HD = -\frac{2A}{F}\left\{\frac{1}{2g}(v_1^2 - v_0^2) + \frac{HT}{\pi}(v_1 - v_0)\right\}$$

To derive an equation for the maximum upward surge D_a, when full load is rejected, we may equate the original kinetic energy in the penstock to that expended in friction plus that used in raising water in the standpipe.. The energy lost in friction is found from equation (24) by putting $v_1 = o$

$$\text{or} \quad E_f = \frac{A\,w\,c\,T\,v_0^3}{6}$$

The other quantities are evident. This gives:

$$\frac{W\,A\,L}{2\,g}v_0^2 = \frac{A\,w\,c\,T\,v_0^3}{6} + \frac{w\,F\,D_a^2}{2} \quad \text{or}$$

$$(29) \qquad D_a^2 = \frac{A}{F}v_0^2\left(\frac{1}{g} - \frac{c\,T\,v_0}{2}\right)$$

Equations (21), (24), (25) and (26) are all theoretically exact except for the assumption that the velocity change takes place along a simple harmonic in time $\frac{T'}{2}$. The true curve for a half cycle, as used, is scarcely distinguishable from a simple harmonic but its period T′ or time for return of water in standpipe to normal level is greater than the value T, given by equation (7). In three cases which the writer has solved by successively applying the differential equations to short positions of the arc he has found that the true value T′ may be closely approximated by the following formula:

$$(30) \qquad T' = \frac{D}{Y}T$$

where T is found from equation (16),
Y from equation (15), and
D from equation (28).

The quantity T′ is useful in itself as the true time for return to normal head, but its use in formula (26) for determining D′ is not advisable, as the writer has found by solving a number of problems that the value of D′, thus found, agrees almost exactly with the value of D found from equation (28), in which equation the value of T from equation (16) is used. Equation (28) is therefore offered as a much simpler substitute for equation (26) and equation (27) becomes:

(31) $$D_b = D + c\,v_0^2 \;*$$

Like all wave motions, these surge waves are liable to pile up, one upon another, in case several gate movements occur at proper intervals and, in fact, no limit can be placed upon the possible amplitude of the surge which can occur in this way. In a plant where large frequent load changes are anticipated the danger from this source should receive careful attention. Some means should be adopted for causing the wave, due to a given gate movement, to rapidly subside in order to lessen the probability of its combination with another wave. One method of accomplishing this result is by arranging the standpipe to overflow at a definite elevation above the forebay. This limits the upward surge and thereby the maximum possible downward surge which could occur under any assumption of gate movements. This method necessitates a waste of water.

Another method† is that of imposing a resistance between penstock and standpipe. This not only causes the waves to subside more rapidly but also, if properly designed, reduces the amplitude of a single wave. This is of greatest advantage near full load where the downward surge is apt to lower the head sufficiently to make it impossible for the unit to deliver the required power. Another effect of the resistance, however, is to change the form of the curve of effective head so that, instead of a slow sinuous pressure drop after an increase of load, a sudden drop is obtained. This is evidently opposed to good speed regulation as it adds to the effective sudden load for which the governor must compensate by requiring a greater q to make up, not only for the increased load, but also for the suddenly decreased head.**

*Mr. Raymond D. Johnson in Am. Soc. M. E. 1908 has derived an equation for D as follows:

$$D'^2 = \frac{Al}{Fg}(v_1 - v_0)^2 + c^2(v_1^2 - v_0^2)^2$$

The results obtained by this equation agree quite closely with those obtained by the writer's method and the two entirely independent analyses of the problem are mutually corroborative.

†See paper on "Surge Tanks for Water Power Plants" by R. D. Johnson with discussions by the writer and others in the Trans. Am. Soc. of M. E. 1908.

**For further discussion of this subject and a mathematical analysis of the problem see Mr. R. D. Johnson's paper with discussions as previously referred to.

APPENDIX—D.

TEST DATA OF TURBINE WATER WHEELS.

TABLE LIX.

Test of a 113-inch Boott Center Vent Turbine. Built in 1849 for the Boott Cotton Mills, Lowell, Mass., after designs by James B. Francis.

Number of experiment.	Gate opening (proportional part.	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
1..........	0.25	0.600	14.60	4	35.6	67.53	42.2	37.7
2..........	0.25	0.577	14.67	7	52.7	64 89	21.94	20.3
3..........	0.25	0.590	14.57	8	43.4	66.43	36.4	33.2
4..........	0.25	0.592	14.16	17	32.7	66.61	40.8	38,2
5..........	0.25	0.596	14.20	12	30.0	67.03	41.1	38.1
6..........	0.25	0.595	14.14	7	27.0	66.89	40.0	37.3
7..........	0.25	0.598	14.24	7	25.25	67.37	29.3	27.0
8..........	0.25	0.544	14.30	8	60 8	61.08	0.0	0.0
9..........	0.50	0.756	14.29	11	60.6	85.0	41.7	30.3
10..........	0.50	0.767	14.23	10	57.6	86.35	52.4	37.6
11..........	0.50	0.775	14.20	10	55.9	87.08	57.8	41.3
12..........	0.50	0.780	14 19	8	54.1	87.69	62.7	44.4
13..........	0.50	0.785	14.19	14	51.4	88.29	69.5	48.9
14..........	0.50	0.802	13.78	11	41.6	90.17	82.0	58.3
15..........	0.50	0.815	13.61	11	35.9	91.70	84.4	59.6
16..........	0.50	0.685	13.95	8	70.6	77.11	0.0	0.0
17..........	0.75	0.852	13.52	7	66.3	95.76	35.9	24.5
18..........	0.75	0.876	13.37	2.5	59.6	98.49	65.3	43.7
19..........	0.75	0.893	13.37	7	54.1	100.42	87.6	57.6
20..........	0.75	0.910	13.40	9	49.5	102.42	102.2	65.7
21..........	0.75	0.913	13.38	12	47.2	102.82	107.7	69.0
22..........	0.75	0.920	13.34	9	44.8	103.52	111.2	71.0
23..........	0.75	0.922	13.32	9	43.5	103.77	112.8	72.0
24..........	0.75	0.921	13.33	9	42.6	103.69	114.0	72.8
25..........	0.75	0.925	13.30	10	41.9	104.23	114.9	73.1
26..........	0.75	0.818	13.70	8	75.2	92.02	0.0	0.0
27..........	1.00	1.000	13.40	9	42.5	112.53	136.4	79.7
28..........	1.00	1.005	13.43	6	41.9	112.99	137.0	79.6
29..........	1.00	1.000	13.33	6	40.7	112.56	135.6	79.7
30..........	1.00	1.005	13.38	7	40.3	113.00	136.6	79.7
31..........	1.00	1.006	13.39	8	39.6	113.07	136.8	79.7
32..........	1.00	1.007	13.38	8	38.9	113.16	136.9	79.7
33..........	1.00	1.006	13.36	9	38.1	113.09	136.6	79.8
34..........	1.00	1.010	13.38	8	37.4	113.67	136.5	79.2
35..........	1.00	1.017	13.40	8	36.8	114.29	136.7	78.7
36..........	1.00	1.013	13.32	8	35.5	113.97	134.5	78.1
37..........	1.00	0.982	13.54	4	0.0	110.45	0.0	0.0
38..........	1.00	0.980	13.57	2	0.0	110.32	0.0	0.0
39..........	1.00	0.887	13.60	7	77.0	99.8	0.0	0.0

TABLEL LX.

EXPERIMENTS MADE FOR THE TREMONT AND SUFFOLK MILLS AT HOLYOKE TESTING FLUME, DECEMBER 3–5, 1890, ON A 48-INCH VICTOR TURBINE, WITH CYLINDER GATE.-

1. Duration of the experiment.		2. Height of speed-gate opening.	3. Number of revolutions of wheel		4. Weight in the scale.	5. Useful effect.		6. Depth of water on the weir	7. Quantity of water discharged by the wheel.	8. H. Fall at the wheel.	9. Total power of the water passing the wheel.	10. Efficiency or ratio of the useful effect to the power expended	11. Ratio of the velocity of exterior circumference of wheel to that due the fall. H.	12. Horse-power of wheel under a head of 13 feet	13. Discharge of wheel under a head of 13 feet
Seconds.		Inches	Total during the experiment.	Per minute.	Pounds (Avoirdupois).	Foot-pounds per second.	Horse-power.	Feet.	Cubic feet per second.	Feet.	Foot-pounds per second.				Cubic feet per second.
360.	Full Gate.	20.58	856.	142.7	None	....	....	1.255	94.49	13.185	....	....	1.1127	....	93.82
300.		"	609.	121.8	100.	40738.	74 07	.335	103.75	.019	84251.	0.4365	0.9558	73.91	103.67
300.		"	523.	104.6	200.	69971.	127.22	436	115.87	12.951	98602.	7475	.8230	127.94	116.09
360.		"	602.	100.3	230.	77184.	140.33	.465	119.44	13.200	98341.	7849	.7817	137.15	118.58
300.		"	480.	96.0	250.	80272.	145.95	.479	121.18	167	99524.	.8066	.7491	143.18	120.41
360.		"	561.5	93.6	265.	82917.	150.81	.491	122.67	.262	101475.	8174	7277	146.36	121.45
360.		"	547 5	91.2	275.	83931.	152.60	497	123.41	265	102110.	.8220	.7090	148.05	122.17
360.		"	527.	87 8	285.	83726.	152.23	.499	123.67	.157	101492.	8249	.6854	149.51	122.93
180		"	253.	84.3	295.	83210.	151.29	.502	124.05	139	101665.	.8184	.6585	148.90	123.39
360.		"	498.	83 0	300.	83283.	151.42	.503	124.17	.140	101771.	.8183	6483	149.01	123.51
420		"	571 5	81.6	305.	83286.	151.43	.506	124.55	157	102214.	.8148	.6370	148.73	123.80
360.		"	479.	79.8	310.	82775.	150 50	.507	124.67	.130	102103.	.8107	.6235	148.27	124 05
540.		"	695.5	77.3	317.	81935.	148.97	.509	124.93	095	102043.	.8029	.6048	147.35	124.48
300.		"	375.	75.0	330.	82781	150.51	.518	126.05	.221	103948.	7964	5840	146.75	124.99
300.		"	350.5	70.1	345	80889.	147 07	.523	126.67	201	104802.	.7755	.5463	143.72	125.70
420.		"	Still.	Still.	....	....	....	546	129.60	.061	....	......	.. ..		129.29
300.	7/8 Gate (nearly).	17.90	541.	108.2	175	63332.	115.15	1.374	108.38	13.463	91018.	0.6959	0.8349		108.50
300.		"	516	103.2	200.	69034.	125.52	.395	110.90	.389	92617.	.7454	.7966		109.28
360.		"	586.	97.7	225.	73499.	133.64	.410	112.71	.324	93672.	.7846	7578		111.33
360.		"	549.5	91.6	250	76579.	139.24	.426	114.65	.252	94769.	.8081	7125		113.55
360.		"	535.	89.2	260	77541.	140.98	.431	115.26	.235	95151.	.8149	6942		114.23
300.		"	431.	86.2	270.	77844	141.53	.436	115.87	.195	95365.	.8163	.6719		115.01
480		"	666.	83.2	280.	77965.	141.75	.440	116.36	.169	95580.	.8157	.6492		115.61
360.		"	464.	77.3	300.	77597.	141.08	.451	117.72	.342	97233.	.7980	6015		116 64
240.		"	280.5	70.1	320.	75055.	136.46	.456	118.34	.199	97428.	7704	.5463		117.44
300.		"	Still.	Still.	......	......	....	.459	118.70	.161	....	......	....		117.97
300		"	725.5	145.1	None.	.. . ..	....	.245	98.35	14.028	.. . ..	.. . ..	1.0971		[illegible]

300.	¾ Gate (nearly). 15.58	510.	102.0	170.	57997.	105.45	1.298	98.88	13.043	80420.	0.7212	0.7997		98.69
360.	"	476.	96.2	200.	63683.	116.79	.316	101.58	12.985	82288.	.7744	.7480		101.59
240.	"	362.	90.5	220.	66593.	121.08	.331	103.28	13.049	84063.	.7922	.7093		103.09
300.	"	432.5	86.5	235.	67989.	123.62	.343	104.70	.049	85219.	.7978	.6780		104.50
300.	"	420.	84.0	245.	68834.	125.15	.348	105.29	.029	85567.	.8044	.6589		105.17
300.	"	405.	81.0	255.	69084.	125.61	.354	106.00	.029	86145.	.8020	.6354		105.88
300.	"	387.5	77.5	265.	68691.	124.89	.359	106.59	.017	86544.	.7937	.6062		105.52
300.	"	365.	73.0	275.	67144.	122.08	.364	107.18	.025	87077.	.7711	.5727		107.06
180.	"	206.5	68.8	285.	65614.	119.30	.366	107.42	.024	87265.	.7519	.5396		107.82
300.	⅝ Gate (nearly). 12.90	516.	103.2	140.	48324.	87.86	1.192	87.40	13.193	71923.	0.6719	0.8045		86.76
360.	"	588.	98.0	160.	52445.	95.35	.206	89.19	.159	73206.	.7164	.7649		88.65
180.	"	280.	93.3	175.	54630.	99.33	.218	90.80	.110	73842.	.7398	.7296		89.92
300.	"	456.5	91.3	185.	56498	102.72	.229	91.54	.165	75170.	.7515	.7125		90.96
360.	"	528.5	88.1	195.	57449.	104.45	.236	92.38	.139	75669.	.7592	.6882		91.84
180.	"	253.	84.3	205.	57824.	105.13	.243	93.12	.097	76072.	.7601	.6595		92.77
240.	"	317.	79.2	220.	58315.	106.03	.252	94.15	.062	76708.	.7602	.6205		93.98
300.	"	365.5	73.1	235.	57457	104.47	.259	94.94	12.998	76943.	.7467	.5742		94.97
300.	½ Gate (nearly). 10.34	507.	101.4	115.	39002.	70.91	1.077	74.98	13.239	61917.	0.6299	0.7891		74.30
300.	"	479.5	95.9	130.	41698.	75.81	.089	76.25	.195	62757.	.6644	.7475		75.68
360.	"	540.	90.0	145.	43648.	79.36	.101	77.52	.137	63521.	.6871	.7031		77.11
300.	"	431.	86.2	155.	44688.	81.25	.107	78.16	.113	63929.	.6990	.6740		77.82
360.	"	418.	82.6	165.	45585.	82.88	.114	78.91	.063	64296.	.7090	.6471		78.72
360.	"	471.	78.5	175.	45948.	83.54	.121	79.66	.027	64728.	.7099	.6158		79.58
180.	"	222.	74.0	185.	45789.	83.25	.129	80.52	.010	65342.	.7008	.5809		80.49
300.	"	360.	72.0	190.	45755.	83.19	.131	80.73	12.999	65457.	.6990	.5654		80.73
240.	"	Still.	Still.	......	....	....	.118	79.88	13.254	....	....	....		78.57
300.	"	645.5	129.1	None.	.....	....	0.986	65.62	.487	....	....	.9972		64.54
300.	⅜ Gate. (nearly). 7.80	512.5	102.5	70.	23998.	43.63	0.925	59.59	13.215	48119.	0.4886	0.7983		59.10
360.	"	590.5	98.4	80.	26334.	47.88	.932	60.27	.200	49623.	.5307	.7668		59.81
360.	"	544.5	90.7	95.	28835.	52.43	.944	61.45	.147	50392.	.5722	.7083		61.11
360.	"	517.	86.2	105.	30261	55.02	.950	62.04	.084	50632.	.5977	.6747		61.84
360	"	488.	81.3	115.	31284.	56.88	.957	62.73	.039	51019.	.6132	.6375		62.64
360.	"	457.	76.2	125.	31844.	57.90	.964	63.48	.007	51461.	.6188	.5962		63.41
300.	"	357.5	71.5	135.	32285.	58.70	.970	64.03	12.967	51780.	.6235	.5622		64.10
240.	¼ Gate (nearly). 5.28	410.	102.5	25.	8571.	15.58	0.741	42.65	13.094	34834.	0.2460	0.8020		42.50
300.	"	485.	97.0	35.	11355.	20.65	.747	43.18	.059	35172.	.3228	.7600		43.08
300.	"	456.5	91.3	45.	13742.	24.98	.754	43.79	.019	35560.	.3864	.7164		43.76
360.	"	428.5	88.1	50.	14781.	26.78	.756	43.96	.005	35660.	.4131	.6917		43.96
360.	"	511.	85.2	55.	15667.	28.49	.759	44.28	12.972	35788.	.4378	.6698		44.28
240.	"	327.5	81.9	60.	16431.	29.87	.761	44.40	.939	35834.	.4585	.6447		44.50
300.	"	404.	80.8	65.	17566.	31.94	.770	45.19	13.118	37244.	.4717	.6294		44.82
360.	"	446.	74.3	75.	18647.	33.90	.776	45.73	.175	37580.	.4962	.5796		45.43
300.	"	338.	67.6	85.	19219.	34.94	.781	46.17	.138	37821.	.5061	.5282		45.94

TABLE LXI.

Test of a 96-inch Fourneyron Turbine Built in 1851 for the Tremont Mills, Lowell, Mass., after designs by James B. Francis.

Number of experiment.	Gate opening (proportional part.	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
1	1.0	1.01	12.86	8	53 62	139 42	161.4	78.4
2	1.0	1.01	12.86	10	53.5	139.42	159.2	78.3
3	1.0	1.01	12.87	10	53.6	139.47	159.5	78.4
4	1.0	1.18	12.55	6	95.8	156 65	60.6	27.2
5	1.0	1.12	12 61	8	91.9	154.39	77.9	35 3
6	1.0	1.10	12.65	7	87.7	152.27	94.0	43.0
7	1.0	1.08	12.70	7	83.0	149.46	109.2	50.7
8	1.0	1.07	12.72	5	78.5	147.29	121.5	57.2
9	1.0	1.06	12.78	5	77.4	146.02	131.7	62.2
10	1.0	1.05	12.80	8	71 0	144.87	140.2	66.7
11	1 0	1.04	12.82	9	67.5	143.91	147 2	70.3
12	1.0	....	....	9	107.0	....	0.0	0.0
13	1.0	1.18	12 51	9	107.0	163.43	0.0	0.0
14	1.0	1.03	12.86	9	64.0	142.52	152.6	73.5
15	1.0	1.03	12.89	9	61.4	142.04	155.8	75.0
16	1.0	1.03	12.89	8	60.0	141.98	157.0	75.6
17	1.0	1.02	12.90	9	58.2	141.28	158.3	76.6
18	1.0	1.02	12.88	7	56.7	140.47	158.9	77.4
19	1.0	1.01	12.88	10	55.4	140.08	159.5	77.9
20	1.0	1.01	12.87	9	54.7	140.01	159.7	78.0
21	1.0	1.01	12.90	10	54.1	139.90	160.0	78.1
22	1.0	1.01	12.90	14	53.8	139.67	160.2	78.4
23	1.0	1.17	12.43	9	106.8	161.69	0.0	0.0
24	1.0	1.01	12.90	9	53.6	139.01	160.5	78.9
25	1.0	1.01	12.90	13	53.1	139.03	160.4	78.8
26	1.0	1.0	12.89	5	52.5	138.76	159.5	78.6
27	1.0	1.0	12.90	14	52.8	138.85	160.5	79.0
28	1.0	1.0	12.91	13	52.4	138.87	160.5	78.9
29	1.0	1.0	12.91	12	52.0	138.51	160.6	79.2
30	1.0	1.0	12.90	12	51.1	138.19	160.5	79.4
31	1.0	1.17	12.54	9	106.8	162.32	0.0	0.0
32	1.0	1.0	12.91	6	50.2	138.27	160.6	79.3
33	1.0	1.0	12.93	10	48.8	138.23	160.6	79.2
34	1.0	1.0	12.94	11	47.1	38.09	160.0	78.9
35	1.0	1.0	12.94	11	44.5	137.71	158.4	78.3
36	1.0	0.99	12.96	11	41.7	136.49	156.2	77.9
37	1.0	0.98	12.94	10	38.7	135.14	152.6	77.0
38	1.0	1.17	12.5	9	107.1	161.69	0.0	0.0
39	1.0	0.98	12.96	12	38.8	135.34	153.0	76.9
40	1.0	0.97	12.97	8	36.0	134.80	149.3	75.3
41	1.0	0.97	12.98	11	31.9	133.75	142.7	72.5
42	1.0	0.97	12.95	9	27.3	133.43	133.0	67.9
43	1.0	0.98	12.80	1.5	0.0	135.65	0.0	0 0
44	1.0	0.98	12.77	2.5	0.0	135.62	0.0	0.0
45	1.0	1.17	12.47	9	106.9	162.02	0.0	0.0
46	1.0	1.00	12.95	11	49.9	138.62	161.1	79.1
47	1.0	1.00	12.93	10	49.0	138.50	160.7	79.1
48	1.0	1.00	12.95	11	48.2	138.47	160.5	78.9
49	1.0	1 00	12.95	12	47 4	138.37	160.3	78.9
50	1.0	1.00	12.95	11	46.2	138.16	159.8	78.7
51	0.75	1.04	12.76	7	89.0	143.33	71.7	34.6
52	0.75	1.01	12.87	8	75.4	139.21	120.6	59.4
53	0.75	1.00	12.91	9	68.5	137.75	136.1	67.5
54	0.75	1.00	12.94	8	64.7	137.80	142.8	70.6
55	0.75	0.99	12.95	8	61.4	137.00	145.9	72.5
56	0.75	0.98	12.95	8	57.9	135.54	148.3	74.5
57	0.75	0.98	12.96	8	55.8	135.10	149.0	75.0
58	0.75	0.97	12.98	8	54 0	134.33	149.8	75.8
59	0.75	0.96	13 00	9	51.9	133.30	149.5	76.0
60	0.75	0.96	13.01	9	50.1	132.73	149.3	76.2
61	0.75	0.95	13.03	9	48.3	132.00	148.7	76 3

TABLE LXI.—Continued.

Test of a 96-inch Fourneyron Turbine Built in 1851 for the Tremont Mills, Lowell, Mass., after designs by James B. Francis.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
62........	0.75	0.95	13.04	9	46.3	130.99	147.9	76.4
63..........	0.75	0.95	13.03	8	42.5	130.89	145.1	75.0
64.........	0.75	1.03	12.72	11	103.0	149.55	0.0	0.0
65..........	0.49	0.88	13.17	11	92.9	121.97	0.0	0.0
66..........	0.49	0.86	13.08	6	81.1	118.55	52.8	30.0
67..........	0.49	0.84	13.13	6	73.1	116.1	78.3	45.3
68..........	0.49	0.83	13.18	8	65.0	114.26	96.6	56.6
69..........	0.49	0.82	13.21	7	60.2	113.24	103.3	60.9
70..........	0.49	0.81	13.25	6	55.4	111.52	106.7	63.7
71..........	0.49	0.79	13.28	7	50.6	109.71	107.8	65.2
72..........	0.49	0.78	13.31	6	46.5	108.05	106.8	65.5
73..........	0.49	0.78	13.31	6	46.5	107.95	107.0	65.6
74..........	0.49	0.76	13.33	9	41.2	105.53	102.3	64.1
75..........	0.49	0.75	13.36	8	36.9	103.85	97.3	61.9
76..........	0.49	0.73	13.41	7	27.4	100.54	83.7	54.8
77..........	0.87	0.99	12.88	7	51.3	137.36	156.8	78.1
78..........	0.87	0.99	12.90	7	49.3	136.97	157.0	78.4
79..........	0.87	0.99	12.91	7	47.4	136.55	156.6	78.3
80..........	0.25	0.58	13.35	5	74.9	80.45	0.0	0.0
81..........	0.25	0.57	13.37	6	68.8	78.84	168.2	14.1
82..........	0.25	0.56	13.40	6	57.6	76.62	33.6	33.2
83..........	0.25	0.54	13.43	6	46.3	74.06	49.7	44.0
84..........	0.25	0.52	13.48	8	40.3	71.87	50.9	46.3
85..........	0.25	0.51	13.51	8	33.6	70.01	48.8	45.5
86..........	0.25	0.49	13.56	7	27.7	67.82	44.4	42.7
87..........	0.25	0.47	13.56	11	15.0	64.51	32.7	33.0
88..........	0.25	0.44	13.52	....	0.0	60.36	0.0	0.0
89..........	0.25	0.44	13.53	....	0.0	60.42	0.0	0.0
90.........	0.067	0.28	13.98	7	37.2	38.22	9.09	15.0
91..........	0.067	0.28	14.00	7	41.3	38.57	6.23	10.2
92..........	0.067	0.27	14.02	17	23.3	37.17	14.21	24.0

TABLE LXII.

Test of a 57-inch Left Hand McCormick Turbine. Built by J. and W. Jolly, Holyoke, Mass. Testing Flume of the Holyoke Water Power Co. Tested on Conical Draft Tube. Test No. 1156. Oct. 31 and Nov. 1, 1898.

With the flume empty a strain of 17 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
40..........	1.000	1.011	13.77	4	79.87	244.50	305.80	80.10
39..........	1.000	1.007	13.86	4	83.25	244.34	309.12	80.49
38..........	1.000	1.004	13.92	4	86.75	244.16	311.49	80.82
37..........	1.000	1.001	13.98	4	90.25	243.88	311.78	81.10
36..........	1.000	0.994	14.08	4	94.62	242.58	312.08	80.86
35..........	1.000	0.984	14.10	4	99.87	240.66	305.68	79.43
34..........	0.770	0.890	14.69	4	79.87	222.19	299.08	80.81
33..........	0.770	0.886	14.72	4	82.87	221.41	302.63	81.89
32..........	0.770	0.888	14.73	4	85.75	220.81	303.24	82.22
31..........	0.770	0.881	14.75	4	88.75	220.88	303.58	82.36
30..........	0.770	0.876	14.76	4	92.00	219.17	302.19	82.35
29..........	0.770	0.868	14.82	4	95.50	217.65	298.75	81.68
28..........	0.770	0.857	14.86	5	98.90	215.23	292.57	80.67
27..........	0.770	0.847	14.94	5	101.50	213.41	283.00	78.28
26..........	0.615	0.762	15.36	4	77.75	194.56	261.73	77.28
25..........	0.615	0.761	15.36	4	81.87	194.25	266.69	78.62
24..........	0.615	0.757	15.40	5	86.20	193.52	269.07	79.62
23..........	0.615	0.753	15.39	4	89.62	192.50	267.55	79.64
22..........	0.615	0.745	15.45	4	92.87	190.92	262.57	78.80
21..........	0.615	0.739	15.47	4	95.62	189.80	257.51	77.56
20..........	0.615	0.729	15.52	4	98.75	187.15	251.16	76.26
19..........	0.615	0.719	15.63	4	102.25	185.14	243.37	74.17
18..........	0.483	0.632	15.90	4	78.12	164.25	215.69	72.83
17..........	0.483	0.630	15.87	4	82.37	163.42	218.46	74.28
16..........	0.483	0.626	15.85	4	86.37	162.32	217.32	74.40
15..........	0.483	0.621	15.81	4	89.50	160.80	213.08	73.90
14..........	0.483	0.615	15.81	4	93.00	159.31	208.71	73.07
13..........	0.483	0.609	15.74	4	96.00	157.40	202.88	72.04
12..........	0.483	0.603	15.75	4	99.25	156.03	195.74	70.24
11..........	0.483	0.598	15.72	4	102.37	154.55	187.97	68.23
10..........	0.483	0.593	15.76	4	105.50	153.45	179.36	65.41
9..........	0.360	0.500	16.27	4	74.75	131.52	161.14	66.41
8..........	0.360	0.498	16.37	4	79.62	131.26	163.52	67.11
7..........	0.360	0.495	16.42	4	83.75	130.63	163.46	67.20
6..........	0.360	0.492	16.41	4	87.12	129.75	161.15	66.74
5..........	0.360	0.488	16.43	4	90.37	128.87	157.94	65.76
4..........	0.360	0.485	16.42	6	93.75	127.96	153.65	64.49
3..........	0.360	0.481	16.45	4	96.62	127.21	149.15	62.86
2..........	0.360	0.479	16.54	4	100.25	126.83	143.17	60.19
1..........	0.360	0.474	16.59	4	104.37	125.70	134.86	57.05

TABLE LXIII.

Test of 56-inch Right Hand Samson Turbine, built by James Leffel Co., Springfield, O. Testing Flume of the Holyoke Water Power Co. Test No. 1257. June 20, 1900. Tested on Conical Cylinder.

With the flume empty a strain of 22 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency = 1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
19..........	1.000	0.995	13.27	3	99.67	245.41	303.23	82.29
18..........	1.000	0.996	13.27	4	104.00	245.75	307.27	83.26
17..........	1.000	1.001	13.27	4	108.62	246.86	309.85	83.58
16..........	1.000	0.999	13.30	4	111.75	246.69	307.40	82.79
15..........	1.000	0.992	13.33	4	113.50	245.26	302.19	81.68
14..........	1.000	0.980	13.50	4	117.00	243.81	294.03	78.94
25..........	0.919	0.945	13.52	4	99.37	235.43	301.69	83.75
24..........	0.919	0.947	13.50	4	104.25	235.56	304.47	84.60
23..........	0.919	0.943	13.52	4	107.75	234.80	301.52	83.93
22..........	0.919	0.936	13.56	4	110.12	233.37	295.43	82.50
21..........	0.919	0.928	13.63	4	112.37	231.98	288.50	80.63
20..........	0.919	0.916	13.71	4	115.12	229.82	281.48	78.91
32..........	0.846	0.885	13.80	4	96.62	222.77	293.34	84.32
31..........	0.846	0.888	13.79	4	101.00	223.37	296.35	85.01
30..........	0.846	0.882	13.80	4	103.12	221.99	292.06	84.21
29..........	0.846	0.876	13.82	4	105.00	220.47	286.69	83.14
28..........	0.846	0.868	13.91	5	107.80	219.26	283.35	82.10
27..........	0.846	0.855	14.09	4	110.87	217.31	278.62	80.41
26..........	0.846	0.843	14.11	4	113.50	214.58	269.81	78.75
54..........	0.771	0.824	14.15	4	97.50	210.02	282.77	84.08
53..........	0.771	0.819	14.18	4	100.37	208.95	280.18	83.56
52..........	0.771	0.812	14.21	4	102.12	207.29	274.66	82.40
51..........	0.771	0.802	14.24	4	104.00	205.01	269.13	81.46
50..........	0.771	0.794	14.27	4	106.25	203.08	264.12	80.56
49..........	0.771	0.786	14.33	3	109.67	201.51	260.71	79.78
48..........	0.696	0.736	14.63	4	97.50	190.73	254.95	80.74
47..........	0.696	0.727	14.69	4	100.37	188.70	254.28	81.06
46..........	0.696	0.725	14.70	4	103.75	188.28	253.68	80.99
45..........	0.696	0.717	14.76	4	106.87	186.67	250.42	80.81
44..........	0.696	0.715	14.81	4	110.50	186.38	247.67	79.29
43..........	0.626	0.663	15.11	4	98.25	174.50	238.60	80.06
42..........	0.626	0.660	15.12	4	102.00	173.83	239.01	80.36
41..........	0.626	0.655	15.16	4	105.00	172.83	236.77	79.85
40..........	0.626	0.651	15.13	4	107.37	171.55	231.17	78.70
39..........	0.626	0.647	15.17	4	110.75	170.70	225.66	77.01
38..........	0.564	0.603	15.45	4	95.50	160.63	219.89	78.29
37..........	0.564	0.600	15.44	4	98.62	159.54	218.36	78.33
36..........	0.564	0.598	15.46	4	102.25	159.11	217.37	78.09
35..........	0.564	0.592	15.49	4	105.75	157.75	214.01	77.40
34..........	0.564	0.587	15.51	4	109.87	156.65	208.95	75.99
33..........	0.564	0.581	15.56	4	114.00	155.30	201.31	73.62
6..........	0.497	0.537	15.91	4	101.25	145.14	195.99	75.00
5..........	0.497	0.532	15.95	4	105.12	143.93	192.77	74.20
4..........	0.497	0.527	16.00	5	109.20	142.87	189.13	73.11
3..........	0.497	0.522	16.04	4	113.37	141.70	184.80	71.85
2..........	0.497	0.520	16.04	4	117.50	140.91	179.56	70.20
1..........	0.497	0.517	16.07	4	125.75	140.27	170.82	66.96

TABLE LXIV.

Test of a 54-inch Right Hand Special Hercules Turbine. Built by the Holyoke Machine Co., Holyoke, Mass. Testing Flume of the Holyoke Water Power Co. No. 1051. Date Nov. 12, 1897.

Number of experiment.	Gate opening (proportional part).	Proportional (discharge discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
42	1.000	1.004	13.98	5	80.40	230.06	305 38	83.72
41..........	1.000	.997	14.07	4	84.12	228 98	306.94	84.00
40..........	1.000	.989	14.13	4	87.00	227.80	305.62	83.71
39..........	1.000	.981	14.22	4	90.50	226.71	305 62	83.58
38..........	1.000	.974	14.26	4	94 00	225.30	303.39	83.26
37..........	1.000	.964	14.29	4	98.00	223.16	299.65	82.83
36..........	1.000	.956	14.34	4	101.50	221.65	293.11	81.31
35..........	1.000	.944	14.38	4	104.87	219.38	285.03	79 66
34..........	.800	.881	14 69	4	80.00	206.93	287.01	83.25
33..........	.800	.875	14.73	5	83.20	205.72	287.19	83.56
32..........	.800	.868	14.80	4	86.60	204.66	287.16	83.59
31..........	.800	.859	14.87	4	90.25	202.98	286.38	83.65
30..........	.800	.852	14.94	4	93.75	201.77	284.75	83.28
29..........	.800	.844	15.01	4	97.12	200.41	281.78	82.59
28.	.800	.836	15.07	4	101.00	198.92	277.94	81.75
27..........	.800	.829	15.09	4	104.87	197.26	270.78	80.20
26..........	.800	.820	15.15	5	108.40	195.46	261.48	77.85
25..........	.650	.749	15.38	5	77.00	179.88	246.95	78.70
24..........	.650	.745	15.40	5	81.00	179.02	250.42	80.09
23..........	.650	.739	15.45	5	84.60	177.87	251.78	80.78
22..........	.650	.734	15.48	5	88.60	176.85	252.85	81.43
21..........	.650	.728	15.49	5	92.80	175.59	252.22	81.76
20..........	.650	.722	15.47	5	95.80	173.91	248.01	81.28
19..........	.650	.714	15.50	4	99.25	172.09	242.10	80.02
18..........	.650	.705	15.53	5	102.40	170.10	234.48	78.26
17..........	.650	.696	15.57	4	105.75	168.14	226.34	76.23
16..........	.527	.622	15.97	4	74.50	152.27	202.49	73.42
15..........	.527	.618	16.01	4	78.87	151.47	205.79	74.82
14..........	.527	.612	16.03	4	83.12	150.12	207.28	75.94
13..........	.527	.606	16.09	4	87.25	148.80	207.50	76.41
12..........	.527	.597	16.12	4	91.62	146.94	205.44	76.47
11..........	.527	.591	16.15	4	95.37	145.60	200.89	75.32
10..........	.527	.584	16.22	4	99.25	143.98	195.57	73.84
9..........	.527	.578	16.23	4	103.00	142.55	188.96	72.01
8..........	.410	.499	16.58	4	77.00	124.59	163.24	69.67
7..........	.410	.494	16.63	4	81.62	123.44	163.61	70.27
6..........	.410	.489	16.64	4	85.50	122.20	162.67	70.53
5..........	.410	.483	16.68	4	90.50	120.82	159.88	69.95
4..........	.410	.478	16.66	4	94.75	119.58	154.51	68.38
3..........	.410	.472	16.68	5	99.10	118.20	148.14	66.25
2..........	.410	.467	16.73	4	103.75	117.00	140.99	63.51
1..........	.410	.460	16.79	5	109.30	115.53	129.97	59.07

TABLE LXV.

Test of a 51-inch Left Hand McCormick Turbine. Built by J. and W. Jolly, Holyoke, Mass. Testing Flume of the Holyoke Water Power Co. Test No. 1444, Feb. 19 and 20, 1903. Tested on Conical Draft Tube.

With the flume empty a strain of 37 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
57..........	1.000	1.007	15.20	4	98.50	196.58	293.50	86.61
55..........	1.000	1.001	14.71	4	101.12	192.30	278.13	86.70
54..........	1.000	0.998	14.70	4	105.50	191.60	277.38	86.84
53..........	1.000	0.995	14.68	4	110.00	191.02	274.38	86.28
52..........	1.000	0.989	14.65	4	113.75	189.58	268.40	85.21
51..........	1.000	0.974	14.74	4	118.25	187.26	255.10	81.49
50..........	1.000	0.948	14.86	4	124.25	183.09	234.54	76.01
49..........	1.000	0.922	15.04	4	130.25	179.05	210.74	69.00
48..........	1.000	0.897	15.15	4	136.00	174.79	183.87	61.06
76..........	0.760	0.879	15.65	4	90.50	174.22	259.30	83.86
75..........	0.760	0.876	15.65	4	96.50	173.55	263.47	85.54
74..........	0.760	0.871	15.67	4	101.75	172.71	264.09	86.04
73..........	0.760	0.861	15.76	4	106.00	171.13	260.83	85.28
72..........	0.760	0.850	15.83	4	110.75	169.47	257.58	84.66
71..........	0.760	0.840	15.94	4	114.25	168.07	250.32	82.39
70..........	0.760	0.826	16.04	4	118.50	165.73	239.66	79.50
69..........	0.760	0.813	16.16	6	123.17	163.65	228.35	76.14
68..........	0.760	0.795	16.29	3	129.00	160.65	208.72	70.32
67..........	0.760	0.778	15.74	4	132.25	154.65	178.81	64.59
85..........	0.624	0.767	15.63	4	90.00	151.93	218.42	81.11
84..........	0.624	0.764	15.61	4	94.75	151.12	219.73	82.13
83..........	0.624	0.756	15.64	4	100.25	149.67	218.97	82.48
82..........	0.624	0.742	15.67	4	104.25	147.07	213.65	81.75
81..........	0.624	0.735	15.73	4	108.75	145.93	206.74	79.42
80..........	0.624	0.724	15.78	4	114.25	144.08	200.26	77.67
79..........	0.624	0.712	15.86	4	119.75	141.98	189.71	74.29
78..........	0.624	0.698	15.96	4	127.50	139.76	171.91	67.96
77..........	0.624	0.684	16.12	4	134.75	137.55	145.35	57.80
47..........	0.500	0.656	15.93	4	87.00	131.12	178.30	75.27
46..........	0.500	0.653	15.96	6	91.83	130.61	179.53	75.94
45..........	0.500	0.648	16.05	4	97.12	130.11	180.05	76.03
44..........	0.500	0.640	16.14	4	101.62	128.70	178.12	75.61
43..........	0.500	0.632	16.19	4	106.00	127.55	175.08	74.76
42..........	0.500	0.624	16.25	4	111.50	126.04	169.13	72.81
41..........	0.500	0.615	16.27	5	118.20	124.29	159.37	69.49
40..........	0.500	0.605	16.30	4	124.00	122.26	146.29	64.73
39..........	0.500	0.594	16.36	5	129.80	120.27	131.26	58.82
38..........	0.500	0.585	16.41	6	135.50	118.65	114.18	51.71

TABLE LXVI

Test of a 45-inch Right Hand Victor Turbine. Built by the Platt Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 1177, March 13 and 14, 1899. Tested on Conical Draft Tube.

With the flume empty a strain of 10 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
48..........	1.000	1.012	15.22	4	102.87	180.69	252.39	80.92
47..........	1.000	1.000	15.21	4	107.50	180.24	254.82	81.96
46..........	1.000	1.004	15.20	4	111.50	179.26	253.70	82.10
45..........	1.000	0.997	15.26	4	116.67	176.26	253.57	82.19
44..........	1.000	0.986	15.31	5	121.60	176.54	251.08	81.91
43..........	1.000	0.972	15.35	4	126.25	174.41	246.10	81.05
42..........	1.000	0.954	15.36	5	128.40	171.19	235.46	78.96
41..........	1.000	0.934	15.44	4	131.50	167.98	223.29	75.91
40..........	0.900	0.959	15.32	4	98.37	171.76	239.19	80.15
39..........	0.900	0.955	15.32	4	103.87	171.19	241.52	81.20
38..........	0.900	0.949	15.36	4	107.25	170.19	240.39	81.04
37..........	0.900	0.942	15.39	5	111.30	169.23	239.64	81.13
36..........	0.900	0.929	15.50	5	115.30	167.41	238.31	80.98
35..........	0.900	0.917	15.54	4	119.00	165.39	234.39	80.41
34..........	0.900	0.907	15.58	4	122.50	163.86	230.47	79.60
33..........	0.900	0.896	15.73	4	127.50	162.75	225.15	77.55
32..........	0.900	0.890	15.73	4	133.50	161.52	217.62	75.52
30..........	0.800	0.888	15.90	4	97.25	162.07	238.54	78.20
29..........	0.800	0.886	16.04	5	105.70	162.46	238.35	80.65
28..........	0.800	0.878	16.20	4	111.00	161.79	239.74	80.66
27..........	0.800	0.867	16.34	4	118.00	160.41	241.24	81.15
31..........	0.800	0.873	15.80	4	118.87	158.91	231.00	81.12
26..........	0.800	0.861	16.24	4	131.35	157.14	232.71	80.32
25..........	0.800	0.838	16.28	4	125.25	154.81	225.44	78.67
24..........	0.800	0.826	16.20	4	131.87	152.24	214.96	76.85
23..........	0.700	0.802	16.16	4	99.00	147.56	205.08	75.83
22..........	0.700	0.799	16.15	4	104.12	147.04	207.20	76.94
21..........	0.700	0.794	16.17	4	109.62	146.23	208.47	77.74
20..........	0.700	0.781	16.12	4	114.50	143.56	206.09	78.52
19..........	0.700	0.768	16.18	4	118.00	141.49	200.36	77.17
18..........	0.700	0.758	16.19	4	122.75	139.54	184.26	75.89
17..........	0.700	0.747	16.23	3	130.00	137.72	185.42	73.14
16..........	0.600	0.701	16.39	4	96.00	129.89	169.53	70.21
15..........	0.600	0.702	16.39	4	101.37	130.02	172.13	71.22
14..........	0.600	0.696	16.38	4	106.75	128.88	173.29	72.38
13..........	0.600	0.688	16.44	4	111.62	126.85	172.85	73.08
12..........	0.600	0.676	16.44	4	115.50	125.45	170.23	72.78
11..........	0.600	0.669	16.42	4	118.87	124.06	165.94	71.82
10..........	0.600	0.662	16.43	4	124.50	122.81	160.66	70.21
9..........	0.600	0.656	16.44	4	131.25	121.81	151.55	66.73
8..........	0.502	0.607	16.60	4	95.25	113.21	139.09	65.26
7..........	0.502	0.608	16.60	4	100.62	112.49	140.10	66.15
6..........	0.502	0.594	16.64	4	104.62	110.91	139.27	66.54
5..........	0.502	0.586	16.62	4	109.75	109.33	138.65	67.29
4..........	0.502	0.580	16.62	5	114.80	108.25	136.45	66.87
3..........	0.502	0.576	16.65	4	120.00	107.66	132.85	65.35
2..........	0.502	0.574	16.68	4	126.50	107.30	127.86	62.99
1..........	0.502	0.570	16.68	4	131.50	106.59	120,58	59.8

TABLE LXVII.

Test of a 45-inch Right Hand Samson Turbine. Built by The James Leffel Co., Springfield, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 979. Jan. 25 and 26, 1897. Tested with Conical Cylinder

With the flume empty a strain of 15 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
8	1.000	0.992	14.94	5	127.60	171.24	233.49	80.48
7	1.000	1.000	14.88	5	133.40	172.12	236.84	81.54
6	1.000	0.998	14.92	4	138.12	172.12	238.65	81.94
5	1.000	0.999	15.00	4	144.00	172.69	240.97	82.08
4	1.000	1.001	15.02	4	148.75	173.23	240.82	81.61
3	1.000	1.002	15.03	3	153.33	173.38	239.89	81.18
2	1.000	0.998	15.04	4	157.75	172.81	236.08	80.09
1	1.000	0.986	15.11	3	169.33	171.11	218.85	74.64
18	0.832	0.887	14.99	4	112.50	153.24	208.16	79.90
17	0.832	0.892	15.02	4	119.75	154.34	215 05	81.80
16	0.832	0.896	15.04	4	126.12	155.04	219.62	83.05
15	0.832	0.897	15.03	4	132.25	155.27	223.11	84.30
14	0.832	0.896	15.04	4	134.12	155.03	223.61	84.55
13	0.832	0.893	15.06	4	143.00	154.74	221.79	83.92
12	0.832	0.888	15.09	4	148.12	153.93	219.65	83.38
11	0.832	0.881	15.16	4	151.25	153.12	214 01	81.29
10	0.832	0.874	15.21	4	155.00	152.15	208.77	79.55
9	0.832	0.847	15.32	4	160.50	148.02	196.52	76.42
27	0 684	0.766	15.19	3	112.67	133.24	183.94	80.14
26	0.684	0.769	15.12	3	121 33	133.52	189.83	82.91
25	0.684	0.768	15.11	4	127.67	133.24	191.06	83.68
24	0.684	0.762	15.14	4	131.50	132.34	187 85	82.67
23	0.684	0.756	15.20	4	135.50	131.58	185.27	81.68
22	0.684	0.745	15.28	4	138 00	130.06	182.49	80.97
21	0.684	0.732	15.33	4	141.75	128.02	178.39	80.15
20	0.684	0.728	15.39	4	147.00	127.52	176.99	79.52
19	0.684	0.719	15.43	4	156.00	125.99	169.79	77.01
57	0.568	0.641	15.85	5	125.80	113.89	162.59	79.42
58	0.568	0.633	15.88	4	131.50	112.65	162.80	80 25
59	0.568	0.630	15 85	4	135.75	112.04	160.68	79 78
60	0.568	0.629	15.83	4	139 75	111.68	157.81	78.71
61	0.568	0.623	15.84	4	143.25	110.45	152.99	77.11
62	0.568	0.618	15.85	4	148.25	109.02	146.23	74 62
46	0.424	0.500	16.50	4	112.50	90.70	123.97	73.05
45	0.424	0.499	16.53	4	121.25	90.59	127.84	75.28
47	0.424	0.499	16.49	4	124.00	90.37	127.79	75.61
44	0.424	0.497	16.55	4	127 00	90.24	127.86	75.49
49	0.424	0 497	16.47	4	126.87	90.04	127.73	75.95
43	0.424	0.494	16.55	4	131.75	89.69	125.47	74.58
48	0.424	0.487	16.50	4	135.50	88.24	121.67	73.69
42	0.424	0.479	16.58	4	151.25	87.01	113.18	69.18

TABLE LXVIII.

Test of a 44-inch Left Hand Improved New American Turbine. Built by the Dayton Globe Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test. No. 1609. March 21, 1904. Tested on Long Conical Draft Tube. Balanced Gate.

With the flume empty a strain of 7 lbs. applied 3.5 feet from the center of the shaft, sufficed to start the wheel.

Number of experi-ment.	Gate opening (propor-tional part.)	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolu-tions per minute.	Dis-charge in sec-ond-feet.	Horse-power devel-oped.	Percent-age of effi-ciency.
1	2	3	4	5	6	7	8	9
8..........	1.000	0.986	15.31	4	132.25	172.28	228.61	76.43
7..........	1.000	0.988	15.24	4	137.75	172.13	231.64	77.86
9..........	1 000	0.992	15.23	4	141.75	172.89	233.60	78.22
6..........	1.000	1.001	15.22	4	146.50	174.40	236.50	78.56
10..........	1.000	1.014	15.19	4	150.50	176.85	237.89	78.31
5..........	1.000	1.011	15.18	4	155.00	175.81	234.58	77.50
4..........	1.000	0.999	15.23	4	159.00	174.09	224.59	74.69
3..........	1.000	0.988	15.32	4	161.75	172 57	217.59	72.57
2..........	1.000	0.976	15.37	5	165.40	170.73	205 82	69.16
1..........	1.000	0.772	16.15	4	210.25	138.47	....	....
49..........	0.907	0.944	15.42	4	131.50	165.47	231.74	80.08
48..........	0.907	0.948	15.42	4	136.00	166.15	235 10	80.91
47..........	0.907	0.949	15.42	4	139.00	166.27	235.61	81.03
46..........	0.907	0.952	15.39	4	143.50	166.70	236.48	81.28
45..........	0.907	0.945	15.47	4	147.75	165.87	233.54	80.25
44..........	0.907	0.935	15.52	4	151.00	164.50	228.52	78.92
21..........	0.823	0.878	15.61	4	115.75	154.80	218.00	79.55
20..........	0.823	0.882	13.59	5	121.00	155.32	225.20	82.06
19..........	0.823	0.884	13.56	4	131.00	155.60	229.10	83.44
18..........	0.823	0.880	15.57	4	135.25	155 05	227.43	83.07
17..........	0.823	0.871	15.61	4	138.50	153.55	223.58	82.25
16..........	0.823	0.863	15.67	4	142.50	152.60	220.45	81.29
15..........	0.823	0.855	15.79	4	147.50	151.67	216.28	79.63
14..........	0 823	0.848	15.75	4	151.50	150.20	208.90	77.86
13..........	0.823	0.838	15.74	4	156.75	146.45	200.32	75 60
12..........	0.823	0.821	15.82	4	161.00	145.80	189.51	72.45
11..........	0.823	0.796	15 94	4	164.00	141.85	176.50	68.83
29..........	0.684	0.729	16.31	3	98.00	131.50	174.68	71.82
30..........	0.684	0.739	16.24	4	109.00	132 91	190.62	77.87
28..........	0.684	0.746	16.19	4	117.75	134.05	201.96	82.05
27..........	0.684	0.741	16.19	4	122.75	133.15	202.28	82.74
26..........	0.684	0.736	16.16	4	126.50	132.00	199.95	82.65
25..........	0.684	0.725	16.17	4	132.00	130.22	195.33	81.80
24..........	0 684	0.703	16.24	4	139.25	126.92	187.33	80.14
23..........	0.684	0.680	16 35	4	145.00	122.77	175.56	77.12
22..........	0.684	0.662	16.41	4	151.50	119.79	163.04	73.14
36..........	0.581	0.628	16.54	4	108.50	114.06	160.12	74.84
37..........	0.581	0.633	16.55	4	112.00	115.00	169 50	78.53
35..........	0.581	0.633	16.56	4	114.50	114.90	169.43	78.52
34..........	0.581	0.614	16.63	4	124.25	111.84	167.15	79.24
33..........	0.581	0.586	16.75	4	131.50	107.13	158.32	77.80
32..........	0.581	0.566	16.87	4	145.00	103.84	146.30	73.64
31..........	0.581	0.551	16.97	4	156.25	101 26	131.37	67.41
43..........	0.459	0.496	17.02	4	99.75	91.31	127.48	72.33
42..........	0.459	0.493	17.03	4	107.25	90.75	129.85	74.09
41..........	0.459	0.488	17.04	4	112.50	89.87	128.64	74.07
40..........	0.459	0.468	17.10	4	128.75	86.39	125 57	74.95
39..........	0.459	0.450	17.11	4	143.00	82.01	115.43	71.66
38..........	0.459	0.441	17.08	4	152.00	81.30	102.24	64 92

TABLE LXIX.

Test of a 42-inch Right Hand Victor Turbine. Built by the Platt Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 1707. Tested on Conical Draft Tube. Swing Gate. Nov. 20, 1907.

With the flume empty a strain of 20 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
1..........	1.000	0.852	16.19	3	Still	154.79	....	....
15..........	1.000	0.946	15.98	2	100.00	170.72	203.27	65.69
14..........	1.000	0.969	15.93	3	122.00	174.66	231.46	73.35
13..........	1.000	0.985	15.79	3	143.00	176.80	247.08	78.03
11..........	1.000	0.989	15.75	3	151.33	177.23	251.22	79.35
10..........	1.000	0.998	15.74	3	159.67	178.78	254.24	79.66
12..........	1.000	1.002	15.70	3	165.00	179.22	254.91	79.87
9..........	1.000	1.006	15.82	4	172.25	180.64	256.77	79.22
8..........	1.000	1.005	15.86	4	178.75	180.79	254.35	78.21
7..........	1.000	0.979	16.01	4	178.75	176.80	236.18	73.57
6..........	1.000	0.956	16.10	4	183.25	173.26	217.29	68.63
5..........	1.000	0.931	16.23	3	189.67	169.33	192.77	61.85
4..........	1.000	0.878	16.39	3	202.67	160.52	137.33	46.02
3..........	1.000	0.808	16.60	4	214.25	148.59	72.59	25.95
2..........	1.000	0.762	16.67	3	224.00	140.43	....	....
28..........	0.900	0.874	15.98	4	89.75	157.76	182.44	63.80
27..........	0.900	0.901	15.93	4	111.75	162.30	215.80	73.59
26..........	0.900	0.908	15.90	4	127.00	163.54	232.34	78.78
25..........	0.900	0.911	15.84	4	138.50	163.67	239.30	81.38
24..........	0.900	0.918	15.81	4	150.25	164.76	244.34	82.70
23..........	0.900	0.915	15.81	4	158.50	164.22	241.64	82.06
22..........	0.900	0.894	15.88	4	160.25	160.79	228.02	78.74
21..........	0.900	0.877	15.94	4	164.00	158.15	216.69	75.79
20..........	0.900	0.862	15.94	4	168.50	155.31	205.51	73.19
19..........	0.900	0.839	16.02	4	174.00	151.68	188.64	68.45
18..........	0.900	0.798	16.21	4	184.25	145.14	156.05	58.48
17..........	0.900	0.743	16.38	3	199.00	135.86	101.13	40.07
16..........	0.900	0.690	16.59	2	218.50	126.81	....	....
38..........	0.800	0.783	16.32	3	66.33	142.88	130.31	49.28
37..........	0.800	0.837	16.15	4	119.25	151.91	218.16	78.40
36..........	0.800	0.847	16.14	4	135.50	153.56	234.12	83.29
35..........	0.800	0.847	16.13	4	146.00	153.56	237.42	84.51
34..........	0.800	0.824	16.23	4	148.00	149.80	225.63	81.82
33..........	0.800	0.807	16.30	4	151.75	147.12	215.93	79.39
32..........	0.800	0.773	16.42	4	161.50	141.45	196.97	74.77
31..........	0.800	0.726	16.56	4	173.75	133.30	164.82	65.83
30..........	0.800	0.690	16.67	4	187.00	127.18	126.71	52.69
29..........	0.800	0.627	16.86	3	215.00	116.21	....	....
47..........	0.700	0.717	16.57	3	89.33	131.75	157.37	63.56
48..........	0.700	0.738	16.51	3	110.67	135.45	189.72	74.80
46..........	0.700	0.754	16.42	3	126.00	137.95	209.17	81.42
45..........	0.700	0.752	16.38	3	133.00	137.41	211.78	82.96
44..........	0.700	0.732	16.44	3	136.00	133.93	203.73	81.18
43..........	0.700	0.708	16.53	4	142.25	129.98	192.77	79.10
42..........	0.700	0.678	16.63	4	150.50	124.90	178.46	75.75
41..........	0.700	0.650	16.70	4	160.00	120.02	162.62	71.53
40..........	0.700	0.618	16.82	4	179.50	114.42	121.63	55.72
39..........	0.700	0.568	17.02	3	209.00	105.73	....	....
58..........	0.600	0.623	16.94	3	94.67	115.74	147.54	66.35
57..........	0.600	0.634	16.89	3	105.00	117.55	169.08	71.03
56..........	0.600	0.643	16.83	3	116.00	119.02	172.92	76.11
55..........	0.600	0.645	16.80	4	122.50	119.28	178.46	78.52
54..........	0.600	0.625	16.81	4	126.50	115.74	171.43	77.66
53..........	0.600	0.601	16.90	4	134.50	111.47	164.04	76.77
52..........	0.600	0.582	16.97	4	144.25	108.20	156.38	75.09
51..........	0.600	0.560	17.04	4	160.25	104.44	135.73	67.23
50..........	0.600	0.543	17.14	4	182.50	101.49	92.74	47.01
49..........	0.600	0.501	17.28	3	204.00	94.03	....	....

TABLE LXIX.—Continued.

Test of a 42-inch Right Hand Victor Turbine. Built by the Platt Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 1707. Nov. 20, 1907. Tested on Conical Draft Tube. Swing Gate.

With the flume empty a strain of 20 lbs. applied 3.6 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
67	0.500	0.542	17.32	3	94.33	101.82	132.31	66.15
66..........	0.500	0.553	17.24	3	106.67	103.59	144.56	71.37
65..........	0 500	0.556	17.19	4	116.25	104.06	149.66	73.77
64..........	0.500	0.539	17.25	3	122.00	101.02	144.66	73.19
63..........	0.500	0.515	17.28	4	135.75	96.66	137.97	72.83
62..........	0.500	0.509	17.29	4	150.50	95.61	127.47	67.99
61..........	0.500	0.499	17.31	4	163.00	93.69	110.45	60.04
60..........	0.500	0.476	17.39	3	184.33	89.63	62.45	35.33
59..........	0.500	0.451	17.50	3	198.33	85.20	....	....

NOTE—For experiments 2, 16, 29, 39, 49, 59, Jacket Loose.

TABLE LXX.

Test of a 39-inch Left Hand McCormick Turbine. Built by the S. Morgan Smith Co., York, Penn. Testing Flume of the Holyoke Water Power Co. Tested on Conical Draft Tube. Test No. 1191. May 29, 1899.

With the flume empty a strain of 6 lbs. applied 3.2 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
8	1.000	1.009	15.79	1	126.00	117.83	177.36	84.15
7	1.000	1.006	15.79	4	131.75	117.46	178.98	85.19
6	1.000	1.001	15.82	4	138.00	116.98	179.84	85.78
5	1.000	0.998	15.85	4	143.00	116.73	179.32	85.56
4	1.000	0.990	15.87	4	150.00	115.89	177.03	84.27
3	1.000	0.977	15.91	5	154.60	114.53	171.06	82.87
2	1.000	0.961	15.96	4	160.25	112.80	162.53	79.70
1	1.000	0.945	15.98	4	185.75	110.99	152.83	76.07
43	0.796	0.903	15.95	4	118.25	105.98	159.91	83.51
42	0.796	0.899	15.97	4	123.75	105.51	161.27	84.49
41	0.796	0.896	15.97	4	129.25	105.02	162.08	85.31
40	0.796	0.889	15.99	4	133.00	104.44	160.24	84.70
39	0.796	0.882	15.98	5	136.60	103.62	157.86	84.11
38	0.796	0.874	16.03	4	140.50	102.78	155.46	83.29
37	0.796	0.864	16.08	4	145.00	101.72	151.52	81.78
36	0.796	0.853	16.06	4	149.00	100.45	146.54	80.19
35	0.796	0.843	16.11	4	153.25	99.41	141.30	77.89
34	0.621	0.760	16.26	4	123.75	89.97	136.92	82.62
33	0.621	0.754	16.29	4	127.25	89.41	135.32	82.02
32	0.621	0.748	16.29	5	130.80	88.73	133.47	81.51
31	0.621	0.742	16.30	4	134.00	87.94	130.97	80.66
30	0.621	0.734	16.34	4	138.25	87.18	129.17	80.05
29	0.621	0.728	16.35	4	142.25	86.51	126.79	79.13
28	0.621	0.716	16.37	4	147.75	85.09	122.61	77.70
27	0.621	0.703	16.40	4	156.25	83.65	115.26	74.17
26	0.498	0.645	16.55	3	115.00	77.10	109.57	75.80
25	0.498	0.640	16.55	4	120.25	76.47	110.14	76.82
24	0.498	0.636	16.55	5	123.60	75.95	108.65	76.30
23	0.498	0.630	16.56	4	127.75	75.31	107.58	76.15
22	0.498	0.625	16.59	4	131.75	74.77	106.09	75.50
21	0.498	0.619	16.60	4	136.25	74.12	104.69	75.11
20	0.498	0.611	16.69	4	143.00	73.30	101.09	72.94
19	0.498	0.600	16.71	4	150.25	72.03	95.13	69.77
18	0.498	0.588	16.78	4	157.75	70.71	87.27	64.93
17	0.390	0.527	16.72	4	116.75	63.25	83.97	70.09
16	0.390	0.522	16.73	4	121.50	62.66	82.90	69.81
15	0.390	0.516	16.75	4	125.50	62.06	81.77	69.44
14	0.390	0.512	16.78	4	129.50	61.66	80.40	68.60
13	0.390	0.509	16.77	4	133.00	61.28	78.48	67.42
12	0.390	0.506	16.76	4	135.50	60.86	76.63	66.32
11	0.390	0.502	16.75	4	139.75	60.38	73.88	64.48
10	0.390	0.495	16.77	4	145.00	59.60	70.41	62.19
9	0.390	0.489	16.77	5	150.60	58.81	64.80	58.00

TABLE LXXI.

Test of a 36-inch Right Hand Swain Turbine. Built by the Swain Turbine and Mfg. Co., Lowell, Mass. Testing Flume of the Holyoke Water Power Co. No. 977. Date Jan. 20-21, 1897.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
64..........	1.000	1.004	15.16	3	130.33	76.49	111.57	84 8.
63..........	1.000	.995	15.25	4	135.00	75.98	111.47	84.63
62..........	1.000	.984	15.40	3	140.33	75.52	111.61	84.69
61..........	1.000	.973	15.42	3	144.00	74.74	110.16	84.25
60..........	1.000	.966	15.43	4	146.50	74.19	108.51	83.58
59..........	1.000	.954	15.48	4	150.75	73.45	107.08	83.04
58..........	1.000	.945	15.47	4	154.00	72.73	104.72	82.07
57..........	1.000	.934	15.44	3	158.00	71.76	101.68	80.92
56..........	1.000	.922	15.33	4	161.37	70.58	97.97	79.84
55..........	.875	.932	15.16	3	132.00	70.95	102.58	84.10
54..........	.875	.925	15.15	4	135.75	70.42	102.20	84.47
53..........	.875	.916	15.28	4	140.75	70.01	102.54	84.52
52..........	.875	.907	15.41	4	147.00	69.61	102.63	84.37
51..........	.875	.896	15.50	4	153.50	68.99	101.58	83.75
50..........	.875	.877	15.60	3	162.33	67.75	98.55	82.22
49..........	.750	.866	15.66	4	130.00	67.03	100.24	84.20
48..........	.750	.857	15.74	4	136.00	66.52	100.73	84.53
47..........	.750	.849	15.76	4	141.00	65.96	100.16	84.96
46..........	.750	.844	15.70	2	146.00	65.41	99.28	85.24
45..........	.750	.838	15.54	4	149.75	64.64	97.28	85.39
44..........	.750	.829	15.62	3	157.33	64.09	96.47	84.97
43..........	.750	.826	15.16	4	156.75	62.88	91.36	84.51
42..........	.750	.814	15.20	4	162.75	62.06	88.93	83.13
41..........	.625	.783	15.30	4	127.50	59.90	85.92	82.67
40..........	.625	.775	15.38	4	134.75	59.45	86.72	83.63
39..........	.625	.767	15.43	4	142.12	58.95	87.15	84.48
38..........	.625	.758	15.47	4	149.50	58.30	86.23	84.30
37..........	.625	.749	15.51	4	154.50	57.70	84.42	83.18
36..........	.625	.733	15.58	4	162.75	56.62	81.02	80.99
35..........	.625	.715	15.65	4	169.50	55.32	76.15	77.56
34..........	.500	.683	15.74	5	123.20	52.98	74.80	79.09
33..........	.500	.676	15.78	4	131.25	52.54	75.70	80.51
32..........	.500	.668	15.83	4	137.50	51.98	75.13	80.51
31..........	.500	.660	15.85	4	144.00	51.42	74.31	80.40
30..........	.500	.652	15.91	4	150.25	50.87	72.98	79.51
29..........	.500	.644	15.95	4	157.50	50.31	71.72	78.81
28..........	.500	.635	15.96	3	163.33	49.62	69.41	77.29
26..........	.375	.557	15.21	4	120.37	42.50	54.08	73.77
25..........	.375	.552	15.19	4	127.50	42.06	54.19	74.78
24..........	.375	.545	15.21	4	123.50	41.58	53.49	74.58
23..........	.375	.538	15.21	4	139.00	41.06	52.82	73.87
22..........	.375	.531	15.22	3	145.67	40.53	51.30	73.32
21..........	.375	.524	15.21	3	152.33	40.01	49.94	72.36
20..........	.375	.514	15.30	4	160.25	39.36	48.65	71.23
27..........	.375	.504	15.30	5	167.20	38.57	45.68	68.25
19..........	.250	.420	15.53	4	113.50	32.40	36.52	64.00
18..........	.250	.416	15.54	4	120.25	32.11	36.50	64.50
17..........	.250	.412	15.57	4	126.50	31.83	36.10	64.22
16..........	.250	.407	15.61	4	133.25	31.46	35.60	63.91
15..........	.250	.401	15.60	4	139.50	31.00	34.72	63.81
14..........	.250	.396	15 61	4	146.25	30.59	33.74	62.31
13..........	.250	.386	15 66	4	155.75	29.91	32.15	60.52
12..........	.250	.378	15.70	4	163.50	29.26	29.78	57.16
11..........	.250	.367	15.74	4	171.75	28.45	26.07	51.33
10..........	.250	.356	15.79	4	179.50	27.64	21.80	44.04

TABLE LXXI.—Continued.

Test of a 30-inch Right Hand Swain Turbine. Built by the Swain Turbine and Mfg. Co., Lowell, Mass. Testing Flume of the Holyoke Water Power Co. No. 977. Date Jan. 20-21, 1897.

Number of experiment.	Gate opening (proportional part)	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
9..........	0.125	0.257	16.03	4	111.50	20.06	16.92	46.41
8..........	.125	.254	16.01	4	119.00	19.89	16.62	46.01
7..........	.125	.252	16.07	4	126.75	19.76	16.16	44.88
6..........	.125	.249	16.17	4	134.12	19.62	15.47	43.00
5..........	.125	.247	16.23	4	141.00	19.45	14.55	40.65
4..........	.125	.245	16.18	4	146.25	19.24	13.32	37.73
3..........	.125	.242	16.22	4	152.25	19.07	12.02	34.26
2..........	.125	.239	16.11	4	159.50	18.73	9.68	28.30
1..........	.067	.161	16.49	4	153.25	12.80	....	....

TABLE LXXII.

Test of a 36-inch Right Hand Victor Turbine. Built by the Platt Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 1061, December 14, 1897. Tested on Conical Draft Tube. Cylinder Gate.

With the flume empty a strain of 8 lbs. applied 3.2 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
12	1.000	1.010	16.75	3	133.33	116.88	175.20	79.24
11	1.000	1.009	16.76	4	139.25	116.88	176.99	80.01
10	1.000	1.009	16.74	4	144.87	116.27	177.91	80.59
9	1.000	1.002	16.79	4	150.75	115.68	177.73	80.71
8	1.000	0.997	16.80	4	155.00	115.04	176.07	80.33
6	1.000	0.992	16.73	4	156.50	114.30	172.97	79.75
7	1.000	0.982	16.82	4	161.75	113.46	173.81	80.30
5	1.000	0.975	16.69	4	162.75	112.21	169.89	79.98
4	1.000	0.965	16.58	4	167.00	110.63	164.07	78.87
3	1.000	0.953	16.59	4	172.25	109.31	158.65	77.14
2	1.000	0.941	16.65	4	177.25	108.09	152.37	74.65
1	1.000	0.923	16.70	4	184.00	106.21	141.23	70.20
53	1.000	0.748	17.33	4	240.50	87.70	....	....
52	0.900	0.967	16.92	4	133.75	112.07	172.47	80.19
51	0.900	0.965	16.99	4	139.50	112.07	174.74	80.92
50	0.900	0.959	17.01	4	144.00	111.37	174.19	81.07
49	0.900	0.953	17.04	4	148.50	110.76	173.25	80.93
48	0.900	0.947	17.03	4	152.00	110.03	171.73	80.81
47	0.900	0.939	17.04	4	157.00	108.19	170.63	80.86
46	0.900	0.925	17.05	4	164.00	107.62	168.17	80.81
45	0.900	0.914	17.06	2	170.00	106.32	161.80	78.65
44	0.801	0.900	17.10	3	132.33	104.79	160.07	78.76
43	0.801	0.900	17.07	4	137.75	104.67	162.40	80.14
42	0.801	0.899	17.02	4	142.25	104.32	162.46	80.64
41	0.801	0.892	17.02	4	147.00	103.61	161.57	80.78
40	0.801	0.884	17.02	4	151.25	102.68	159.74	80.59
39	0.801	0.870	17.02	4	155.25	101.04	157.29	80.64
38	0.801	0.863	16.97	4	159.25	100.09	154.50	80.20
37	0.801	0.856	16.92	4	163.50	99.18	150.59	79.12
36	0.801	0.845	16.89	4	168.25	97.79	144.64	77.21
35	0.701	0.814	16.89	3	125.67	94.25	134.27	74.37
34	0.701	0.814	16.90	4	133.25	94.25	139.09	76.99
33	0.701	0.812	16.89	4	138.75	94.03	140.58	78.04
32	0.701	0.807	16.92	4	144.25	93.47	140.83	78.51
31	0.701	0.794	17.00	4	148.75	92.20	139.75	78.61
30	0.701	0.787	17.07	5	153.60	91.54	137.70	77.70
29	0.701	0.776	17.15	4	158.75	90.52	134.52	76.40
28	0.701	0.768	17.15	4	163.25	89.52	130.31	74.84
27	0.601	0.717	17.24	4	129.00	83.87	117.23	71.49
26	0.601	0.714	17.29	4	137.00	83.65	120.29	73.33
25	0.601	0.705	17.36	4	143.25	82.77	120.51	73.94
24	0.601	0.696	17.44	4	148.00	81.91	119.05	73.48
23	0.601	0.685	17.47	4	153.75	80.62	116.12	72.69
22	0.601	0.676	17.53	4	159.00	79.77	112.28	70.79
21	0.601	0.636	17.56	4	166.00	78.57	107.02	68.39
20	0.502	0.622	17.60	3	123.33	78.47	96.93	66.09
19	0.502	0.617	17.55	3	131.67	72.85	98.64	68.02
18	0.502	0.609	17.55	3	138.00	71.81	98.29	68.77
17	0.502	0.598	17.56	3	145.00	70.57	96.16	68.41
16	0.502	0.592	17.55	4	150.25	69.87	93.18	67.00
15	0.502	0.588	17.56	4	155.37	69.34	90.63	65.63
14	0.502	0.580	17.56	3	163.50	68.44	85.34	62.61
13	0.502	0.573	17.55	4	170.25	67.61	78.40	58.26

For experiment 53, the jacket was removed from the dynamometer.

TABLE LXXIII.

Test of a 33-inch Special Left Hand Turbine. Built by the S. Morgan Smith Co., York, Penn. Testing Flume of the Holyoke Water Power Co. Test No. 1511. March 25 and 26, 1904. Tested on Conical Draft Tube. Balanced Gate.

With the flume empty a strain of 9 lbs. applied 3.3 feet from the center, sufficedto start the wheel.

Number of experiment.	Gate opening (proportional part.	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horsepower developed.	Percentage of efficiency.
	2	3	4	5	6	7		9
15..........	1.000	0.995	17.01	4	196.00	112.81	178.22	81.89
14..........	1.000	0.998	16.99	5	203.80	113.05	178.92	82.14
13..........	1.000	1.000	16 98	4	208.75	113.20	179.34	82.27
12..........	1.000	1.003	16.99	4	213.50	113.55	179.40	82.00
11..........	1.000	1.004	16.95	4	220.00	113 55	177.97	81.53
10..........	1.000	0.995	16.94	4	221.50	112.57	173.62	80.28
9..........	1.000	0.984	17.00	4	223.50	111.46	168.18	78.27
7..........	0.948	0.964	17.05	4	194.75	109.42	177.08	83.70
5..........	0.948	0.967	17.11	4	201.50	109 90	179.43	84.13
6..........	0.948	0.969	17.08	4	204.75	110.02	179.75	84.35
4..........	0.948	0.968	17.14	4	209 75	110.14	180.20	84.17
3..........	0.948	0.959	17.19	4	211.75	109.30	175.28	82.26
2..........	0.948	0.952	17.17	4	213.00	108.45	170.97	80.96
1..........	0.948	0.949	17.19	4	214.75	108.11	168.33	79.87
8..........	0.948	0 939	17.09	5	216.40	106.69	162.84	78.75
71..........	0.883	0.909	17.29	4	181.75	103.84	173.24	85.08
70..........	0.883	0.910	17.25	4	185.75	103.84	173 56	85.44
69..........	0.883	0.914	17.23	4	193.00	104.20	175.49	86.19
68..........	0.883	0.909	17.24	4	197.00	103.71	172.95	85.29
67..........	0.883	0.901	17.28	4	199.25	102.90	168.68	83.65
66..........	0.883	0.893	17.31	5	202.00	102.09	164.67	82.17
64..........	0.883	0.876	17.36	4	203.50	100.34	156.90	79.42
54..........	0.851	0.894	17.15	4	191.00	101.72	170.08	85.97
53..........	0.851	0.883	17.19	4	193.50	100.58	166.24	84.78
52..........	0.851	0.875	17.20	4	196.50	99.77	162.65	83.58
51..........	0.851	0.868	17.27	4	200.25	99.07	159.48	82.19
50..........	0.851	0.858	17.29	4	203.25	98.02	155.50	80.90
49	0.851	0.851	17.32	4	206.25	97.35	152.62	79.81
48..........	0.851	0.842	17.38	4	210.25	96.50	150.30	79 02
47..........	0.851	0.836	17.37	4	212 67	95.74	146.70	77.78
46..........	0.851	0.827	17.42	4	215 50	94.82	141.90	75.75
45..........	0.851	0.813	17.44	4	218.25	93.34	136.86	74.13
44..........	0.765	0.836	17.37	4	169.25	95.73	161.82	85.55
43..........	0.765	0.823	17.38	4	172 75	94.25	160.33	86.30
42..........	0.765	0.818	17.41	5	177.60	93.79	159.26	86.00
41..........	0.765	0.803	17.45	4	181.75	92.20	153.86	84.33
40..........	0.765	0.778	17.48	4	192.75	89.41	145.04	81.83
39..........	0.765	0.736	17.55	4	206.25	84.77	129.34	76.66
63..........	0.702	0.764	17.33	3	159.00	87.39	144.57	84.17
62..........	0.702	0.765	17.33	3	166.67	87.50	147.37	85.69
61..........	0.702	0.763	17.35	4	168.50	87.28	145.82	84.91
60..........	0.702	0.750	17.37	3	172.00	85.95	143.45	84.73
59..........	0.702	0.739	17.38	3	175.67	84.66	139.90	83.84
58..........	0.702	0.729	17.42	4	180.25	83.56	136.77	82.85
57..........	0.702	0.720	17.44	4	185.00	82.57	133.41	81.69
56..........	0.702	0.707	17.48	4	189.00	81.18	130.37	81.01
55..........	0.702	0.690	17.50	4	197.75	79.34	124.01	78.75
31..........	0.636	0.711	17.71	4	162.50	82.24	137.57	83.28
30..........	0.636	0.705	17.69	4	166.25	81.51	135.53	82.88
29..........	0.636	0.696	17.71	4	170.50	80.43	133.65	82.73
28..........	0.636	0.689	17.69	4	173.58	79.58	131.65	82.46
27..........	0.636	0.680	17.70	4	176.50	78.60	129.50	82.08
26..........	0.636	0.670	17.72	4	180.25	77.54	126.60	81.24
25..........	0.636	0.658	17.74	4	187.50	76.15	123.46	80.58
24..........	0.636	0.659	17.73	4	209.50	76.28	118.24	77.09

TABLE LXXIII.—Continued.

Test of a 33-inch Special Left Hand Turbine. Built by the S. Morgan Smith Co., York, Penn. Testing Flume of the Holyoke Water Power Co. Test No. 1511. March 25 and 26, 1904. Tested on Conical Draft Tube Balanced Gate.

With the flume empty a strain of 9 lbs. applied 3.3 feet from the center, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part.	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
22..........	0.556	0.616	17.77	4	126.25	71.40	102.92	71.53
23..........	0.556	0.638	17.75	5	153.60	73.82	120.89	81.35
21..........	0.556	0.632	17.74	4	156.00	73.20	119.35	81.04
20..........	0.556	0.619	17.78	4	161.00	71.73	116.10	80.27
19..........	0.556	0.605	17.79	4	184.50	70.10	109.91	77.71
18..........	0.556	0.589	17.80	4	205.75	68.88	103.22	74.83
17..........	0.556	0.564	17.83	4	235.00	65.42	78.68	65.70

TABLE LXXIV.

Test of a 33-inch Right Hand Victor Turbine. Built by the Platt Iron Works Co., Dayton, Ohio. Testing Flume of the Holyoke Water Power Co. Test No. 1250, May 29 and 31, 1900. Tested on Conical Cylinder, Wicket or Swing Gate.

With the flume empty a strain of 12 lbs. applied 3.2 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
6..........	1.000	0.998	16.95	4	156.50	114.36	146.64	66.76
7..........	1.000	0.999	16.99	4	161.25	114.61	148.13	67.13
5..........	1.000	1.001	16.96	4	163.00	114.74	147.74	67.00
4..........	1.000	1.002	16.95	4	169.37	114.86	147.29	66.77
3..........	1.000	1.004	16.93	4	174.25	114.99	146.20	66.27
2..........	1.000	1.005	16.98	4	181.50	115.11	144.50	65.44
1..........	1.000	1.006	16.98	5	191.20	115.34	140.51	63.32
19..........	0.878	0.977	16.96	4	155.75	112.02	150.71	70.01
18..........	0.878	.980	16.93	4	160.87	112.27	151.72	70.44
17..........	0.878	.981	16.95	5	167.00	112.40	152.39	70.59
16..........	0.878	.983	16.94	4	173.00	112.63	151.50	70.08
15..........	0.878	.982	16.98	5	101.20	112.63	149.81	69.10
24..........	0.785	.937	17.02	5	159.20	107.56	155.99	75.20
23..........	0.785	.938	17.03	4	163.25	107.70	155.96	75.04
22..........	0.785	.936	17.08	4	169.75	107.70	156.98	75.31
21..........	0.785	.935	17.07	4	177.75	107.56	156.75	75.35
20..........	0.785	.935	17.05	4	191.25	107.44	152.26	73.31
30..........	0.688	.872	17.15	3	162.00	100.51	154.77	79.24
29..........	0.688	.872	17.15	5	167.00	100.51	155.46	79.59
28..........	0.688	.870	17.16	4	171.50	100.27	155.44	79.73
27..........	0.688	.867	17.18	4	175.50	100.04	154.77	79.47
26..........	0.688	.863	17.20	4	181.00	99.57	152.97	78.83
25..........	0.688	.853	17.22	4	187.75	98.53	149.47	77.75
38..........	0.595	.796	17.53	4	152.75	92.81	147.80	80.17
37..........	0.595	.793	17.50	4	158.50	92.35	148.51	81.10
36..........	0.595	.791	17.48	4	163.50	92.02	148.19	81.31
35..........	0.595	.784	17.47	4	169.00	91.24	146.97	81.37
34..........	0.595	.779	17.43	4	173.50	90.57	145.57	81.38
33..........	0.595	.773	17.39	4	178.25	89.68	143.00	80.92
32..........	0.595	.765	17.38	4	181.50	88.75	138.94	79.50
31..........	0.595	.756	17.40	5	184.60	87.76	135.66	78.40
14..........	0.472	.677	17.63	4	148.00	79.15	126.89	80.25
13..........	0.472	.670	17.67	4	155.25	78.41	128.35	81.76
12..........	0.472	.662	17.69	5	160.80	77.45	128.02	82.46
11..........	0.472	.654	17.68	4	164.50	76.50	125.93	82.17
10..........	0.472	.646	17.69	4	170.00	75.64	123.89	81.71
9..........	0.472	.639	17.68	4	177.00	74.80	121.40	81.02
8..........	0.472	.633	17.70	4	184.00	74.17	118.32	79.54
51..........	0.386	.555	17.92	4	150.25	65.43	101.22	76.18
50..........	0.386	.548	17.94	5	156.40	64.63	100.57	76.55
49..........	0.386	.547	17.95	4	161.25	64.44	99.74	76.10
48..........	0.386	.542	17.97	4	167.75	63.93	98.62	75.76
47..........	0.386	.539	17.97	4	175.25	63.54	96.59	74.66
46..........	0.386	.531	18.00	4	188.00	62.74	92.11	71.98
45..........	0.304	.450	18.17	4	146.25	53.41	78.82	71.68
44..........	0.304	.447	18.17	4	151.25	53.01	77.81	71.29
43..........	0.304	.445	18.14	4	156.50	52.74	76.67	70.73
42..........	0.304	.443	18.14	4	162.50	52.54	75.63	70.03
41..........	0.304	.442	18.13	4	168.50	52.35	74.30	69.09
40..........	0.304	.439	18.15	4	176.75	52.10	71.44	66.67
39..........	0.304	.435	18.15	4	187.75	51.63	66.69	62.81

TABLE LXXV.

Test of a 30-inch Special Chase Jonval Turbine. Built by the Chase Turbine Mfg. Co., Orange, Mass. Testing Flume of the Holyoke Water Power Co. No. 256. June 7, 1884.

With the flume empty a strain of 4 lbs. applied 2.4 feet from the center of the shaft sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part.	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse-power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
1..........	1.000	0.960	14.72	3	Still.	39.74		
8..........	1.000	1.008	14.51	3	169.67	41.42	49.66	72.98
7..	1.000	1.004	14.43	3	181.67	41.16	50.68	75.36
6..........	1.000	1.004	14.61	3	196.33	41.42	52.08	76.00
5..........	1.000	0.998	14.57	4	204.75	41.10	51.50	75.96
4..........	1.000	0.999	14.41	3	215.00	40.91	51.13	76.60
3..........	1.000	1.001	14.41	3	225.00	41.02	50.42	75.34
2..........	1.000	0.998	14.49	5	244.80	40.99	50.38	74.91
22..........	0.930	0.922	14.68	4	185.75	38.12	49.27	77.76
21..........	0.930	0.920	14.72	6	194.17	38.07	49.73	78.37
23..........	0.930	0.919	14.78	4	203.75	38.12	50 32	78 88
24..........	0.930	0.916	14.87	4	213.00	38.13	50.66	78.90
25..........	0.930	0.913	14.95	4	226.00	38.07	50.65	78.59
17..........	0.837	0.831	15.28	3	180.00	35.07	46.10	75.98
16..........	0.837	0.827	15.42	4	191.00	35.03	47.17	77 12
15..........	0.837	0.826	15,42	3	199.67	35.00	47.49	77.71
18..........	0.837	0.823	15.33	3	207.00	34.75	47.34	78.47
19..........	0.837	0.822	15.28	4	215.00	34.65	47.20	78.73
20..........	0.837	0.818	15.26	4	228.25	34.49	46.98	78.82
14..........	0.674	0.666	16.13	4	163.50	28.85	35.15	66.70
13..........	0.674	0.663	16.14	4	174.25	28.75	35.86	68.25
12..........	0.674	0.661	16.07	3	185.33	28.60	36.45	70.04
11..........	6.674	0.669	16.11	4	195.50	28.52	36.66	70.46
10..........	0.674	0.655	16.14	4	206.50	28.40	36.83	70.97
9..........	0.674	0.650	16.20	4	217.00	28.25	36.72	70.86
30..........	0.488	0.462	17.10	4	142.25	20.62	16.26	40.74
29..........	0.488	0.460	17.11	4	158.50	20.55	16.67	41.88
28..........	0.488	0.458	17.08	3	174.33	20.43	16.74	42.38
27..........	0.488	0.459	17.02	4	182.50	20.43	16.69	42.40
26..........	0.488	0.458	17.07	3	190.67	20.43	16.57	41.96
31..........	0.488	0.457	17.09	5	206.20	20.37	16.03	40.67

TABLE LXXVI.

Test of a 30-inch Regular Chase Jonval Turbine. Built by the Chase Turbine Mfg. Co., Orange, Mass. Testing Flume of the Holyoke Water Power Co. June 10, 1884.

With the flume empty a strain of 33 lbs. applied 2.4 feet from the center of the shaft, sufficed to start the wheel.

Number of experiment.	Gate opening (proportional part).	Proportional discharge (discharge at full gate with highest efficiency=1).	Mean head in feet.	Duration of test in minutes.	Revolutions per minute.	Discharge in second-feet.	Horse power developed.	Percentage of efficiency.
1	2	3	4	5	6	7	8	9
1..........	1.000	0.938	15.62	3	Still.	33.33		
6..........	1.000	0.993	15.32	3	194.00	34.93	43.48	71.70
5..........	1.000	0.995	15.29	3	201.67	34.96	43.35	71.58
4..........	1.000	0.996	15.30	5	211.14	35.00	43.46	71.62
3..........	1.000	1.001	15.27	3	222.33	35.14	43.72	71.92
2..........	1.000	1.007	15.26	3	237.00	35.34	43.36	70.96
29..........	0.889	0.894	15.78	4	174.75	31.93	41.56	72.80
28..........	0.889	0.897	15.77	4	190.25	32.01	42.64	74.55
27..........	0.888	0.897	15.75.	4	200.25	32.01	43.05	75.36
26..........	0.889	0.898	15.80	4	211.25	32.10	43.48	75.66
23..........	0.889	0.901	15.77	3	220.67	32.17	43.40	75.50
24..........	0.889	0.903	15.74	4	232.00	32.20	43.50	75.76
25..........	0.889	0.907	15.72	3	242.67	32.33	43.29	75.17
22..........	0.733	0.756	16.30	4	184.50	27.43	36.28	71.63
21..........	0.733	0.757	16.24	4	195.25	27.41	36.61	72.59
20..........	9.733	0.758	16.27	4	207.00	27.47	36.92	72.91
18..........	0.733	0.756	16.28	3	218.67	27.43	37.00	73.14
19..........	0.733	0.757	16.32	4	230.50	27.47	36.90	72.64
14..........	0.611	0.644	16.65	5	175.80	23.63	27.34	61.33
13..........	0.611	0.644	16.65	4	188.75	23.60	27.62	62.05
12..........	0.611	0.641	16.68	3	202.00	23.54	27.72	62.16
15..........	0.611	0.644	16.77	3	209.33	23.63	27.76	62.21
16..........	0.611	0.644	16.67	3	221.33	23.64	27.33	61.21
17..........	0.611	0.647	16.61	3	236.33	23.71	27.02	60.56
7..........	0.411	0.469	17.14	3	141.33	17.47	12.93	38.10
8..........	0.411	0.469	17.20	4	157.00	17.47	12.92	37.96
9..........	0.411	0.468	17.17	4	166.00	17.43	12.91	38.06
10..........	0.411	0.469	17.13	4	184.00	17.43	12.62	36.46
11..........	0.411	0.471	17.16	3	200.00	17.53	11.89	34.89

APPENDIX E.

EFFECT OF AN "UMBRELLA" UPON THE FORMATION OF VORTICES.

Report of Test Made on 30-Inch Horizontal Wheel With "Umbrella" at the Holyoke Water Power Company's Flume, April 25th to 27th, 1907, by F. Moeller, Engineer Power and Mining Department of the Wellman, Seaver, Morgan Co. for The Southern Wisconsin Power Company.

The general arrangement of the wheel and testing apparatus is shown by Fig. 407.

Before beginning the test it was desired to note the action of the water without umbrella in place. The penstock was filled, the level of the water being 8' above the center of the shaft, making the total head of water 16.2'. Under this condition, with the head stationary and the wicket gates wide open, a large vortex was formed immediately above the wheel.

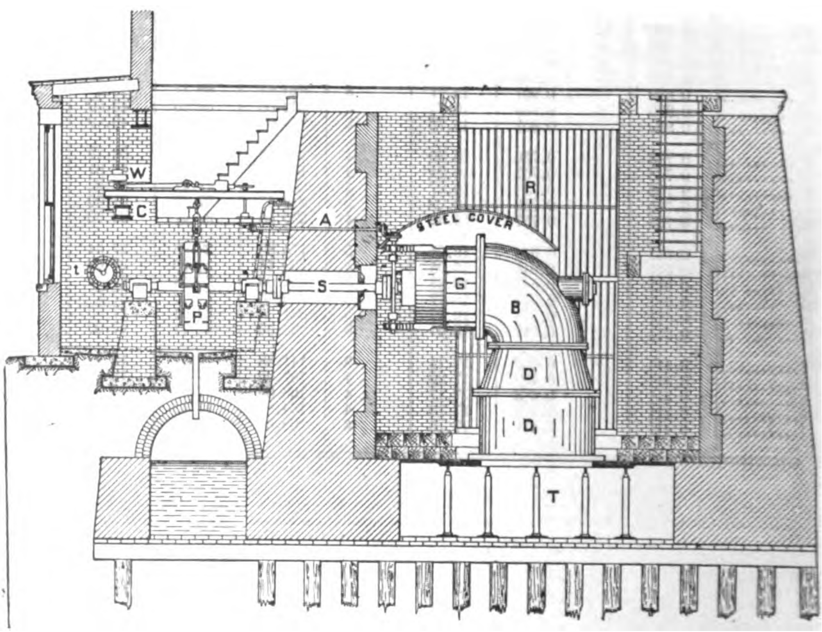

Fig. 407.

The umbrella which was first made 7′ in diameter and dished 11″, was lowered into the penstock until the edge was 3.1′ above the center of the shaft, with the level of the water the same as before. With this arrangement no vortex was formed immediately above the wheel, but there were vortices near the edge of the umbrella, (see Fig. 408). The umbrella was then removed and a raft 8′ square was built of matched pine about 1½″ thick, tongued and grooved and placed as nearly as possible over the center of the wheel on the surface of the water. This did not prevent the formation of vortices. The raft was then increased from 8′x8′ to 8′x12′, and placed in position as shown in Figure 409. This entirely prevented the formation of vortices under the same condition of head as before and under all the running conditions of the wheel.

Regarding these vortices it was observed that all of them were formed at the right hand side of the wheel (standing at the point marked "A," Fig. 408) and towards the upper face of the penstock. The water enters the penstock

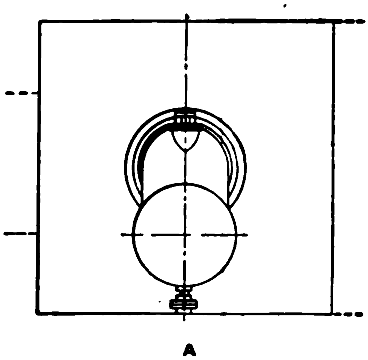

Fig. 408.

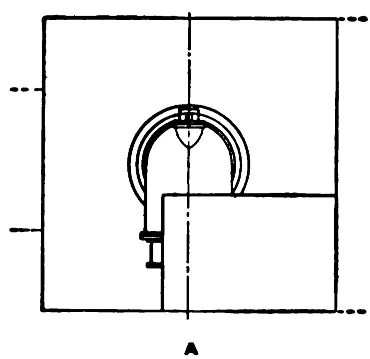

Fig. 409.

from the left hand side, flows through the wheel and draft tube and off at the right hand side. The most reasonable explanation of this tendency for the vortices to form at the place mentioned was that the wheel, being right hand, the gates at the right hand side of the wheel pointed upward (see Figure 410) and formed a comparatively direct path for the vortex into the wheel, while the gates on the right hand side pointing downward, formed an effectual barrier. An examination of Figure 409 shows that the left hand edge of the large raft does not project beyond the gates so that there was every chance for the vortices to form at this point, yet none formed on this side in any of the experiments.

As a result of these preliminary trials it was decided to increase the umbrella to 10½′ in diameter, and meanwhile a test was run off at full gate and three-quarter gate opening, with the large raft in place, to determine the efficiency of the wheel under this condition. These efficiencies are shown on the report of the Holyoke Water Power Company and are numbered 1 to 18.

It may be here noted that the Holyoke Water Power Company finds it necessary to use a raft on practically all of the horizontal tests made by them, the exceptions being only in the case of the smallest wheels, and it is the opinion of the Hydraulic Engineer of that Company, as a result of his observations on the various tests, that the employment of rafts to prevent the formation of vortices does not affect the efficiency of the wheels. This is verified in at least one instance, in the test made of two 33″ runners built for the "Soo," the maximum efficiency obtained was 84%, it being necessary in making this test to use a raft, and this efficiency has not been exceeded by the same wheels when tested in a vertical setting when no raft was used.

The next test was made on the wheel with the enlarged umbrella in place, the edge of the umbrella being 2′ 2″ above the center of the shaft, the center of the umbrella being in the vertical plane of the shaft. The head of the water was 16.2′. With the wheel standing still (with gate wide open), vortices formed occasionally, but only for an instant, immediately disappearing. With the wheel allowed to run under the brake, no vortices formed, but the

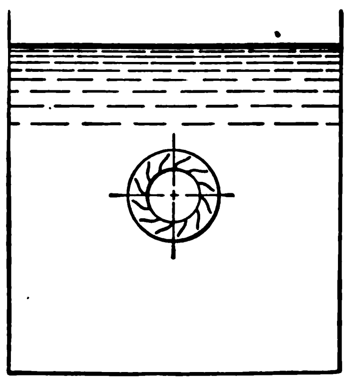

Fig. 410.

surface of the water was disturbed by the formation of whirls, which, however, disappeared without becoming vortices. This action took place at all speeds of full gate opening. The same peculiarities of the action of the water were noticed under three-quarter gate opening, but at no time were any actual vortices formed.

A test was then made of the wheel with the umbrella in the last named position, and the results of this test are noted under Nos. 19 to 33 in the report of the Holyoke Water Power Company.

It was then decided to suspend the umbrella towards the right side of the wheel. With the umbella in this position there were no whirls or vortices at any gate opening, and the level of the water was entirely smooth except such disturbances as were created by the current of the water flowing in. With the umbrella in this position it was decided to make a few tests to determine whether or not there was any difference in the efficiencies between the two positions of the umbrella.

A test was then made with the head lowered about 2′ and it was decided to confine the test to only full gate. The action of the water during this test showed the formation of irregular whirls, but no actual vortices resulted. The results of this test are numbered 38 to 43.

The head was then lowered about 1′. Under this condition the level of the water was about 15″ above the umbrella. No whirls or vortices were formed and there was less disturbance to the water than in previous tests, but owing to the method used for changing the level of the water in the penstock, it was necessary with the water at this head, to allow the incoming water to fall over the gate so that the water when flowing into the wheel was rather full of air bubbles. The results of this test are numbered 44 to 49.

The level of the water was then reduced 2′ more so that the top of the umbrella projected 11″ above the level of the water. Under this condition there were absolutely no disturbances of the water, except that it was full of air bubbles in the head race, and upon examining the water in the tail race it was found that the water there was also full of air bubbles. This condition of the water probably accounts for the lower efficiencies obtained under these conditions. The results of the tests are given in numbers 50 to 56.

A final test was made with the umbrella raised so that the top was about flush with the level of the water. Under these conditions there were small whirls forming around the edge of the umbrella, but no vortices occurred. The surface of the water on the whole was quieter than with the umbrella placed immediately above the wheel. The results of this test are numbered 57 to 65.

COMPARISON OF RESULTS.

It must be noted from an examination of all of the tests that the best efficiency obtained on this wheel was practically at about .8 gate, so that in making comparisons for similar speeds under different gate openings, this must be allowed for.

ONE—Comparing the results obtained with the raft, numbered 1 to 18, with the results obtained with the umbrella placed immediately above the wheels, numbered 18 to 33:—

Take	No. 5	Head 16.09	Revolutions 163	Efficiency 76.18
	No. 24	Head 16.16	Revolutions 161.25	Efficiency 76.51
Also	No. 25	Head 16.19	Revolutions 164.20	Efficiency 76.05

These show that the umbrella, if anything, is better.

Take	No. 16	Head 16.95	Revolutions 142.25	Efficiency 78.99
	No. 17	Head 16.93	Revolutions 137.5	Efficiency 79.11
			and compare with	
	No. 33	Head 16.84	Revolutions 140.5	Efficiency 79.07

This also indicates that the umbrella is a little better than the raft.

TWO—Comparing the umbrella at the surface with the umbrella immediately above the wheel:—

Take	No. 61	Head 16.48	Revolutions 158.25	Efficiency 78.7
	No. 26	Head 16.16	Revolutions 157.	Efficiency 76.19
	No. 29	Head 16.8	Revolutions 159.75	Efficiency 74.08

show that the umbrella should be placed near the surface of the water. Also—

Take No. 58 Head 16.53 Revolutions 168. Efficiency 75.73
No. 22 Head 16.24 Revolutions 168.75 Efficiency 74.81
which indicates the same.

THREE—Comparing the results obtained in tests numbered 38 to 43 with those obtained in tests 1 to 18:—

Take No. 43 Head 14.58 Revolutions 129.75 Efficiency 77.36 with
No. 17 Head 16.93 Revolutions 137.50 Efficiency 79.11
(Giving 137½ revolutions at 16.95 head.)

This shows a falling off in the efficiency, but as specified above, the point of gate opening for No. 17 is at the point of maximum efficiency of the wheel, whereas the point of gate opening under No. 43 is considerably larger and therefore of itself would be less efficient.

Compare No. 40 Head 14.62 Revolutions 148.5 Efficiency 77.14
(Giving 156 revolutions at 16.07 head) with
No. 6 Head 16.07 Revolutions 154.5 Efficiency 75.98
No. 8 Head 16.08 Revolutions 157.6 Efficiency 76.03
Also No. 40 (Giving 160 Revolutions at 16.99 Head) with
No. 12 Head 16.99 Revolutions 159.75 Efficiency 75.09

The result of these comparisons would show no loss in efficiency.

FOUR—Comparing numbers 44 to 49 with 1 to 18 tests:—

No. 47 Head 13.54 Revolutions 138. Efficiency 77.41
(Giving 151 Revolutions at 16.08 Head) with
No. 7 Head 16.08 Revolutions 150. Efficiency 75.63
Also No. 47 (Giving 155 Revolutions at 16.99 Head) with
No. 13 Head 15.99 Revolutions 155.5 Efficiency 75.09

This shows no loss.

FIVE—Comparing tests Nos. 50 to 56 with 1 to 18:—

Take No. 54 Head 11.5 Revolutions 136. Efficiency 74.92
(Giving 161 Revolutions at 16.09 Head) with
No. 5 Head 16.09 Revolutions 163. Efficiency 76.18

This shows a loss, as was to be expected from the condition of the water, as stated above.

As a result of our calculations from the tests we should say as follows:—

(A) That the use of an umbrella or hood does not reduce the efficiency of the wheel.

(B) That the hood should be kept as close to the surface of the water as possible.

(Signed) F. Moeller.

APPENDIX—F.

EVAPORATION TABLES.

Depth of evaporation, in inches, at signal service stations, in thermometer shelters, computed from the means of the tri-daily determination of dew-point and wet-bulb observations.*

Stations and Districts.	Jan. 1888	Feb. 1888	Mar. 1888	April 1888	May 1888	June 1888	July 1887	Aug. 1887	Sept. 1887	Oct. 1887	Nov. 1887	Dec. 1887	Year
New England:													
Eastport..........	0.9	1.4	1.5	2.4	2.5	2.7	2.2	2.9	2.5	2.6	2.2	1.4	25.2
Portland..........	1.0	1.2	1.8	2.6	1.8	3.3	3.8	3.9	3.4	3.0	2.5	1.4	29.7
Manchester	0.9	1.6	2.2	3.3	3.8	5.0	4.1	3.3	2.5	2.8	2.4	1.4	33.3
Northfield	0.8	1.0	1.5	2.3	2.5	3.4	3.5	2.7	2.3	1.8	1.1	1.0	23.9
Boston............	1.2	1.6	2.2	3.4	3.1	4.7	4.4	4.0	3.5	2.7	2.2	1.4	34.4
Nantucket	1.1	1.1	1.2	1.5	1.8	2.1	3.3	3.8	3.4	2.7	1.8	1.8	25.6
Wood's Holl	0.5	0.8	1.8	2.4	1.8	2.7	2.7	2.4	2.7	1.2	0.8	0.5	20.3
Block Island	1.1	1.1	1.2	2.0	1.8	2.6	2.5	3.1	2.8	2.6	1.8	1.4	24.0
New Haven.......	1.1	1.6	1.8	2.7	2.7	4.1	3.7	3.8	3.1	3.2	2.4	1.6	31.8
New London.....	1.5	1.3	1.5	2.6	2.8	4.0	3.4	3.9	3.2	3.1	2.4	2.1	31.8
Mid-Atlantic States:													
Albany	0.9	1.2	1.6	3.3	3.9	4.5	5.0	4.7	3.2	3.0	2.1	1.4	34.8
New York City...	1.8	1.4	2.0	3.4	3.3	4.6	5.0	5.2	4.3	4.1	3.3	2.2	40.6
Philadelphia.....	1.6	2.1	2.5	4.4	4.0	5.7	5.7	5.2	4.3	4.0	3.3	2.2	45.0
Atlantic City.....	1.2	1.6	1.5	2.4	1.8	3.6	2.9	3.3	2.4	1.8	1.2	1.5	25.2
Baltimore....... ..	2.0	2.2	2.8	5.1	4.7	5.6	6.0	5.0	4.4	4.3	3.6	2.4	48.1
Washington City.	1.8	1.7	2.5	4.2	3.8	6.0	5.4	4.9	4.1	4.2	4.5	2.5	45.6
Lynchburg.......	2.6	2.7	3.4	5.2	4.5	5.6	4.7	4.3	3.3	3.4	3.2	2.6	45.5
Norfolk..........	1.8	1.6	2.3	3.5	3.2	4.2	4.6	3.7	3.7	2.9	2.3	1.8	35.6
So. Atlantic States:													
Charlotte	2.6	2.6	4.3	6.4	4.5	5.8	4.0	4.0	4.6	4.0	3.6	2.6	49.0
Hatteras.........	1.8	1.6	1.6	2.5	2.2	3.0	3.3	4.1	3.8	3.2	2.6	1.6	31.3
Raleigh..........	2.0	1.8	2.6	3.8	4.1	5.4	4.2	3.2	3.0	2.7	2.4	1.8	37.0
Wilmington......	2.4	2.2	2.7	3.3	3.3	4.3	4.3	3.1	3.9	3.4	2.8	2.7	38.4
Charleston.......	2.5	2.5	3.5	3.7	3.9	4.4	4.5	4.8	4.2	4.0	3.2	2.5	43.7
Columbia	2.2	2.3	2.6	4.8	4.3	5.4	4.2	3.8	4.2	3.4	3.6	2.4	43.2
Augusta	3.0	2.6	3.4	5.3	4.8	5.0	4.8	4.5	5.1	4.1	3.6	3.1	49.3
Savannah	3.3	2.8	4.1	4.7	4.3	4.6	4.2	4.7	3.4	3.6	3.5	2.8	46.0
Jacksonville	2.9	2.6	3.8	4.3	4.6	5.3	5.0	4.7	3.8	3.6	3.0	2.1	45.7
Florida Peninsula:													
Titusville	3.5	2.6	3.3	3.8	3.8	4.3	3.8	4.3	4.0	4.1	3.6	3.1	44.2
Cedar Keys	3.3	2.8	4.0	4.6	4.5	5.1	5.0	5.5	4.5	4.1	3.5	2.6	49.5
Key West........	3.8	3.7	3.8	4.5	4.4	4.8	5.1	5.1	4.7	4.3	3.8	3.6	51.6

***From Monthly Weather Review, September, 1888.**

Depth of evaporation, in inches, at signal service stations—Continued.

Stations and Districts.	Jan. 1888	Feb. 1888	Mar. 1888	April 1888	May 1888	June 1888	July 1887	Aug. 1887	Sept. 1887	Oct. 1887	Nov. 1887	Dec. 1887	Year.
Eastern Gulf States:													
Atlantic	2.7	2.6	4.0	6.2	4.7	5.0	4.5	4.7	5.8	4.6	4.2	2.5	51.5
Pensacola	2.9	2.8	4.1	4.0	4.3	4.6	5.0	5.4	5.2	4.5	3.6	2.4	48.8
Mobile	2.6	2.5	2.8	3.5	3.7	4.0	4.1	4.6	4.6	4.1	3.4	2.2	42.1
Montgomery	3 5	3.3	5.1	6.5	5.9	5.8	4.3	4.5	5.7	4.6	4.3	3.1	56.6
Vicksburg	2.1	2.5	3.6	5.1	5.7	4.8	4.0	5.0	4.7	3.4	4.0	2.2	47.1
New Orleans	2.8	2.8	4.1	3 8	4.2	4.1	4.1	4.3	4.4	4.6	3.7	2.5	45.4
West. Gulf States:													
Shreveport	1.6	2.1	3.0	4.8	4.9	4.2	4.9	5.2	5.0	4.1	3.4	2.4	45.6
Fort Smith	2.2	2.7	3.5	5.3	4.4	4.6	5.6	4 6	4.7	5.9	3.9	2.2	49.6
Little Rock	2.1	2.8	3.5	5.5	4.8	4.1	5 4	5.9	5.8	5.2	4.3	2.3	51.7
Corpus Christie	1.4	1.6	3.3	3.0	3.2	3.9	4.4	4.3	4.3	4.1	3.0	2.3	38.8
Galveston	1.6	2.8	3.2	2.9	4.3	4.2	5.3	5.2	5.2	4.7	4.2	2.4	46.0
West. Gulf States—Continued.													
Palestine	2.1	3.0	3.3	4.2	4.3	4.5	5.8	4.6	4.8	4.4	4.0	2.1	47.1
San Antonio	2.4	3.3	4.1	3.8	4.0	4.5	6.6	5.8	5.2	5.4	4.2	3.1	52.4
Rio Grande Valley:													
Rio Grande City	2.7	3.5	3.5	3.6	4.5	4.6	6.9	7.0	5.2	4.9	3.6	3.1	53.1
Brownsville	1.8	2.6	2.9	3.0	3.5	3.9	4.0	4.1	3.3	3.0	2.6	2.3	37.0
Ohio Valley and Tennessee:													
Chattanooga	2.0	3.3	3.3	5.3	3.7	4.3	4.3	5.0	5.4	4.0	3.9	1.9	46.4
Knoxville	2.4	2.6	3.4	5.0	3.5	4.2	4.9	5.0	4.9	4.1	3.8	2.1	45.9
Memphis	2.1	2.3	3.1	5.9	5.3	4.8	4.9	5.4	5.5	4.2	4.1	2.4	50.0
Nashville	1.9	2.1	3.2	5.9	5.0	5.1	5.5	6.3	5.9	4.0	3.3	1.9	50.1
Louisville	1.7	2.1	2.8	5.6	5.4	5.8	6.8	7.4	6.4	4.9	3.8	2.1	54.8
Indianapolis	1.3	1.4	2.2	4.6	4.8	5.7	7.7	6.9	5.2	4.1	3.1	1.6	48.6
Cincinnati	1.8	1.8	2.6	4.9	5.2	6.4	6.5	6.6	6.1	4.7	3.3	2.1	52.0
Columbus	1.6	2.0	2.3	4.5	4.8	5.8	6.9	6.4	5.1	4.0	2.6	1.8	47.8
Pittsburg	1.4	1.9	2.2	3.8	4.2	5.4	6.6	5.6	4.9	3.4	2.8	2.3	44.5
Lower Lake Region:													
Buffalo	0.8	1.1	1.3	2.2	3.3	3.9	4.9	5.2	3.9	2.8	1.9	1.6	32.9
Oswego	0.6	1.0	1.1	2.2	2.8	3.8	3.9	4.0	3.6	2.7	2.2	1·0	28.9
Rochester	0.5	1.1	0.9	2.6	3.8	4.9	4.6	4.1	3.8	2.6	2.2	1.3	32.4
Erie	1.0	1.4	1.4	2.7	3.7	4.6	5.5	4.8	3.1	2.5	1.9	1.2	33.8
Cleveland	1.1	1.4	1.5	2.9	3.3	4.4	5.2	4.9	3.8	3.4	2.4	1.4	35.7
Sandusky	0.8	1.4	1.5	3.2	3.7	4.6	5.4	5.4	3.7	3.4	2.2	1.3	36.6
Toledo	0.9	1.1	1.5	3.5	3.8	4.6	6.0	6.4	3.7	3.4	2.4	1.3	38.6
Detroit	0.8	1.1	1.6	3.0	4.1	4.8	5.9	5.2	3.4	2.8	2.0	1.3	36.0
Upper Lake Region:													
Alpena	0.7	0.6	0.9	1.6	2.1	3.6	3.8	3.7	2.8	2.2	1.5	0.8	24.3
Grand Haven	0.5	0.7	1.3	2.6	3.1	3.8	4.7	3.8	2.7	2.6	1.7	1.1	28.6
Lansing	0.6	1.2	1.4	2.7	2.8	4.0	4.3	3.9	2.4	1.9	1.4	1.0	27.6
Marquette	0.8	0.8	0.9	1.7	2.4	3.3	3.4	3.3	3.1	2.2	1.3	1.3	24.5
Port Huron	0.6	1.0	1.1	2.6	3.0	3.8	4.6	4.2	3.2	2.5	1.7	1.0	29.3
Chicago	1.0	1.2	1.8	3.2	3.3	4.8	5.4	5.3	4.1	3.2	2.3	1.2	36.8
Milwaukee	0.5	1.0	1.1	2·4	2.6	3.8	4.8	3.7	3.4	2.9	1.9	0.9	29.0
Green Bay	0.5	0.6	0.8	1.7	2.5	4.1	5.6	4.2	3.0	2.4	1.9	0.9	28.2
Duluth	0.5	0.5	0.6	1.5	2.4	2.5	3.9	3.4	3.0	2.5	1.2	1.0	23.0

Depth of evaporation, in inches, at signal service stations—Continued.

Stations and Districts.	Jan. 1888	Feb. 1888	Mar. 1888	April 1888	May 1888	June 1888	July 1887	Aug. 1887	Sept. 1887	Oct. 1887	Nov. 1887	Dec. 1887	Year.
Extreme Northwest:													
Moorhead	0.2	1.4	0.5	2.1	3.6	3.8	3.7	3.3	3.5	2.4	1.3	0.5	26.3
Saint Vincent	0.3	0.3	0.5	1.8	3.8	3.9	3.1	2.6	2.6	2.0	0.9	0.3	22.1
Bismarck	0.4	0.6	0.6	3.0	4.3	4.1	5.6	4.2	4.0	2.6	1.2	0.4	31.0
Fort Buford	1.4	0.7	0.6	3.0	4.7	5.0	6.2	4.9	4.8	3.0	1.7	0.5	35.5
Fort Totten	0.2	0.3	0.4	2.2	4.6	3.8	4.2	3.7	3.7	2.3	1.4	0.4	27.2
Upper Mississippi Valley:													
St. Paul	0.7	0.7	2.2	2.0	2.3	4.1	5.0	3.7	2.8	2.4	1.5	0.7	28.1
LaCrosse	0.4	1.2	1.4	3.3	3.5	4.4	5.4	4.7	3.0	3.0	1.8	0.8	32.9
Davenport	0.5	1.0	1.8	3.8	3.4	4.6	6.9	6.2	4.4	3.0	2.3	1.1	39.0
DesMoines	0.6	1.0	1.5	3.7	3.1	4.2	6.6	4.7	4.1	3.3	2.3	0.9	36.0
Dubuque	0.7	1.0	1.4	2.2	2.9	4.2	6.2	4.8	3.3	2.8	1.8	0.9	33.2
Keokuk	0.8	1.1	2.1	4.2	3.7	4.3	7.0	6.8	5.0	3.8	2.9	1.2	42.9
Cairo	1.6	2.1	2.9	5.8	4.4	4.3	5.6	6.5	5.1	4.5	3.8	2.3	48.9
Springfield, Ill.	0.8	1.1	2.0	4.6	3.8	4.3	5.4	6.5	4.5	3.5	2.9	1.4	40.8
St. Louis	1.3	1.6	2.5	5.5	4.7	5.0	7.5	8.0	5.9	4.9	3.9	1.4	52.2
Missouri Valley:													
Lamar	1.1	1.6	2.4	4.4	3.8	4.0	6.0	4.6	3.7	3.6	2.9	1.5	39.6
Springfield, Mo.	1.1	1.7	2.4	5.0	4.8	4.0	5.0	3.4	3.4	3.5	3.1	1.4	38.8
Leavenworth	0.9	1.5	2.3	4.6	4.5	5.0	6.3	4.5	4.0	3.9	2.7	1.4	41.6
Topeka	1.1	1.2	2.0	4.0	4.1	4.1	6.3	3.5	3.2	3.0	2.2	1.4	36.1
Omaha	0.8	1.5	1.4	4.4	3.8	5.2	6.2	5.2	4.3	4.3	3.0	1.4	41.7
Crete	0.7	1.1	1.2	3.5	3.3	4.5	5.6	4.7	3.8	3.6	2.4	1.1	35.5
Valentine	1.2	1.6	1.8	5.0	3.2	5.3	6.9	5.0	5.2	3.8	3.3	1.5	43.8
Fort Sully	0.6	0.9	1.3	4.4	4.1	5.2	7.7	4.9	5.7	3.6	2.8	0.7	41.9
Huron	0.3	0.7	0.8	3.7	3.7	4.1	5.7	4.2	4.1	3.1	2.4	0.7	33.0
Yankton	0.4	1.4	1.2	3.3	3.1	4.4	4.6	3.7	2.9	3.0	2.2	0.8	31.0
Northern Slope:													
Fort Assiniboine	0.8	1.2	1.2	3.8	4.1	4.2	6.8	5.5	4.8	3.5	2.5	1.1	39.5
Fort Custer	0.6	1.5	1.3	5.4	6.8	4.9	9.6	8.0	6.1	3.4	2.9	1.5	52.0
Fort Maginnis	1.1	1.4	1.1	3.3	3.2	4.6	6.8	4.6	3.8	2.8	2.0	1.1	35.8
Helena	1.1	3.6	2.1	6.1	4.3	5.5	7.2	7.7	6.4	4.3	3.0	2.1	53.4
Poplar River	0.4	0.8	0.8	2.7	4.9	5.7	6.0	4.8	4.4	2.5	1.7	0.7	35.4
Cheyenne	3.3	5.7	4.0	8.2	5.2	10.4	8.0	7.7	8.6	5.8	6.1	3.5	76.5
North Platte	0.8	1.8	1.8	5.4	3.9	6.9	6.0	4.8	3.7	2.8	2.3	1.1	41.3
Middle Slope:													
Colorado Springs	3.0	3.3	4.1	6.7	5.6	4.3	6.7	7.2	6.8	4.6	4.2	2.9	59.4
Denver	2.8	3.7	3.5	7.6	5.8	10.5	8.3	8.5	6.1	4.9	4.2	3.1	69.0
Pike's Peak	2.1	1.3	1.5	2.1	1.8	1.9	3.0	4.0	3.0	2.3	2.8	1.0	26.8
Concordia	1.3	2.8	1.8	4.8	4.3	5.7	7.3	5.2	4.3	4.5	3.4	1.8	47.2
Dodge City	1.4	2.4	2.8	4.1	4.6	7.4	8.3	6.6	5.5	5.2	4.2	2.1	54.6
Fort Elliott	1.3	1.9	3.2	5.1	5.4	8.2	7.6	6.2	5.4	4.7	4.2	2.2	55.4
Southern Slope;													
Fort Sill	1.6	2.0	2.6	3.8	4.0	4.4	4.8	7.5	5.1	4.2	4.1	2.0	46.1
Abilene	1.8	1.7	3.1	4.2	5.0	5.8	9.5	7.5	6.2	4.5	3.4	1.7	54.4
Fort Davis	5.4	5.7	6.7	8.5	11.0	12.0	11.4	9.0	5.9	5.2	5.7	4.9	96.4
Fort Stanton	3.9	3.9	5.2	7.3	9.5	10.9	9.4	11.6	3.9	4.0	3.6	3.8	76.0

Depth of evaporation, in inches, at signal service stations—Continued.

Stations and Districts.	Jan. 1888	Feb. 1888	Mar. 1888	April 1888	May 1888	June 1888	July 1887	Aug. 1887	Sept. 1887	Oct. 1887	Nov. 1887	Dec. 1887	Year.
Southern Plateau:													
El Paso	4.0	3.9	6.0	8.4	10.7	13.6	9.4	7.7	5.6	5.2	4.6	2.9	82.0
Santa Fe	3.0	3.4	4.2	6.8	8.8	12.9	9.2	9.8	6.6	6.7	5.7	2.7	79.8
Fort Apache	2.6	3.0	3.6	6.8	9.4	9.1	7.1	6.7	5.3	5.2	4.1	2.6	65.5
Fort Grant	5.2	4.8	6.4	9.2	10.2	13.8	12.4	10.5	9.0	7.9	7.2	4.6	101.2
Prescott	1.4	2.8	3.6	5.4	6.2	8.1	6.6	6.5	4.7	4.9	3.6	2.2	56.0
Yuma	4.4	5.2	6.6	9.6	9.6	12.6	11.0	10.2	8.2	8.2	5.5	4.6	95.7
Keeler	3.0	4.6	6.3	8.7	9.3	11.9	12.8	13.9	10.6	8.8	5.9	4.8	100.6
Middle Plateau:													
Fort Bidwell	0.8	1.8	1.8	4.6	5.2	4.0	8.8	8.1	5.0	4.6	2.4	1.3	48.9
Winnemucca	0.9	2.8	6.2	9.1	9.3	10.1	11.5	12.0	9.9	6.6	3.7	1.8	83.9
Salt Lake City	1.8	2.7	3.6	7.2	6.9	8.9	9.2	10.7	9.6	6.5	5.0	2.3	74.4
Montrose	1.8	2.7	3.7	6.2	7.0	11.1	10.2	8.3	6.9	5.2	3.4	2.0	68.3
Fort Bridger	1.6	2.5	2.7	4.3	4.3	6.5	7.7	6.8	5.6	4·2	5.2	4.7	56.1
Northern Pleateau:													
Boise City	1.6	2.5	3.8	6.1	6.5	6.6	10.0	9.2	7.4	5.2	3.2	1.8	63.9
Spokane Falls	0.7	1.7	2.7	4.4	5.4	4.4	7.7	6.4	3.8	2.5	1.7	1.4	42.8
Walla Walla	1.1	2.9	3.6	6.2	7.7	5.7	9.9	7.9	5.1	3.4	1.8	2.4	57.7
No. Pacific Coast:													
Fort Canby	1.2	1.1	1.8	2.1	2.8	2.3	1.8	2.9	1.8	1.8	1.5	0.9	21.1
Olympia	1.3	1.2	1.8	2.5	4.1	3.3	3.2	3.1	2.4	1.5	1.3	1.1	26.8
Port Angeles	1.0	0.9	1.8	1.8	2.5	2.1	2.1	1.8	1.5	1.2	1.3	1.1	19.1
Tatoosh Island	1.2	1.1	1.8	1.4	1.8	1.8	1.4	1.4	1.4	1.6	1.8	1.4	18.1
Astoria	1.1	1.0	1.6	2.1	3.0	2.7	3.0	2.9	2.6	2.3	1.8	1.2	25.3
Portland	0.9	1.1	2.4	3.4	5.0	3.2	5.4	4.2	3.4	2.7	1.8	1.2	34.7
Roseburg	1.2	1.6	2.7	3.9	4.7	3.5	5.4	4.7	5.0	3.2	1.7	1.6	39.2
Middle Pacific Coast:													
Red Bluff	3.0	4.6	5.4	6.1	7.0	6.9	11.0	10.7	10.1	10.5	5.9	3.6	84.8
Sacramento	1.8	3.1	3.7	4.3	4.2	5.6	5.9	5.6	6.5	7.3	3.9	2.4	54.3
San Francisco	2.7	2.7	3.3	3.1	2.8	3.1	2.4	2.5	3.3	5.0	2.8	3.0	36.7
So. Pacific Coast:													
Fresno	1.8	2.8	3.0	5.6	6.0	7.0	9.1	10.2	7.6	6.7	3.8	2.2	65.8
Los Angeles	2.3	2.0	2.8	3.4	3.0	3.8	3.2	3.5	3.1	4.1	3.0	3.0	37.2
San Diego	2.9	2.7	2.5	2.7	3.3	2.8	3.2	3.3	2.9	4.3	3.2	3.7	37.5

APPENDIX.—G.

TWO NEW WATER WHEEL GOVERNORS.

The Glocker-White Turbine Governor.—The I. P. Morris Company has built a governor for the Electrical Development Company of Ontario, Canada, which has one novel feature.* A cross section of its distinctive feature is shown in Fig. 411.

The governor ball is hollow and contains two chambers, a and b, communicating with each other through a small opening, c.

The balls are partially filled with mercury which, when running at normal speed, the axis of the ball being vertical, is divided between the two chambers. When an increase of speed throws the balls outward, centrifugal force causes a flow of mercury from chamber, a, to chamber, b. This raises the center of gravity of the ball and increases its lever-arm about the knife edge, j, thus increasing its effectiveness by making its movement increase in a greater ratio than the speed increases. Similarly a reduction in speed causes the balls to incline inward and the mercury therefore to flow from chamber, b, to chamber, a, which tends to cause a still greater inward inclination.

The charge of mercury hence increases the sensitiveness of the governor balls to small changes in speed.

The centrifugal force of the balls is resisted through knife edges, K, K, by a spiral spring. This movement is transmitted by levers to a small pilot valve which controls a larger relay valve admitting oil under 250 pounds pressure to the cylinder. The gate to be moved is a cylinder gate opening upward, a force of 15,000 pounds being required for the purpose. The weight of the gate is sufficient to close it and the power-cylinder of the governor is therefore made single acting. The entire governor is not shown as there are no other unusual features.

The Allis-Chalmers Governor.—This Company has recently developed a water wheel governor, the following description of which is taken from their bulletin No. 1612:

* See "The Glocker-White Turbine Governor" by W. M. White and L. F. Moody in "Power," Aug. 4, 1908

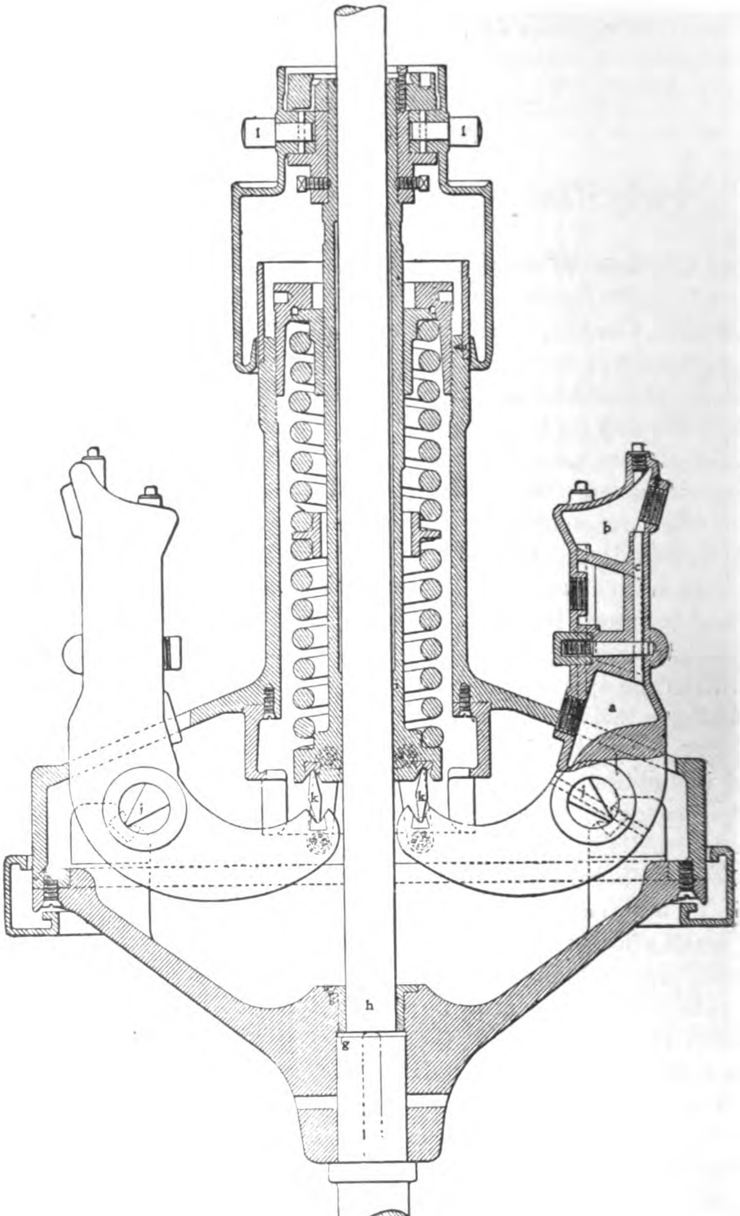

Fig. 411.—Cross-Section of the Glocker-White Governor Head.

"The Allis-Chalmers Governor is of the oil pressure type and consists of three distinct elements:

"First—Governor Stand (see Fig. 412) containing the apparatus

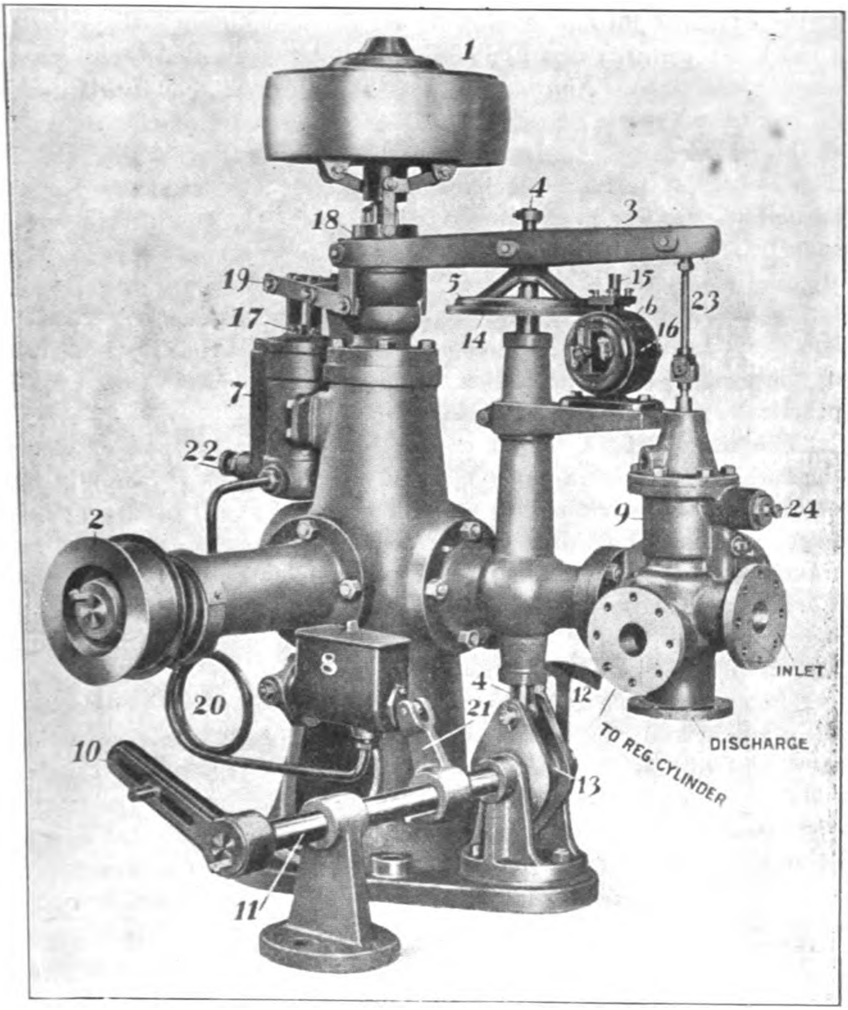

Fig. 412.—View of the Governor Stand of the Allis Chalmer Governor.

for controlling the time of application of energy for actuating the gates.

"Second—Regulating Cylinder for applying energy.

"Third—Pressure System for supplying energy.

"The Governor Head (1), designed to be a highly sensitive yet stable apparatus and driven from the Turbine Shaft by Pulley (2), forms the basic governing element. Any change in its position moves the Governor Collar (18), thereby shifting the Floating Lever (3), and through it and its connection with the Relay (4) (which momentarily acts as a stationary fulcrum) actuates the Regulating Valve (9). Any movement of this Regulating Valve admits oil from the Pressure System to either the opening or closing side of the Regulating Cylinder and thereby actuates the Turbine gates. The Relay (4) forms a mechanical connection between the Regulating Cylinder Piston and the Floating Lever (3), constituting what may be termed a moving fulcrum, so that every movement of the Regulating Piston shifts the fulcrum point and brings the Regulating Valve (9) back to mid position, thereby making the mechanism "dead beat." If this movement is adjusted so that the position of these parts have the proper relation, the Governor Collar will practically retain a fixed position.

"The Regulating Cylinder cannot however, fully open or close the turbine gates instantaneously and the above result can only be obtained within certain limits, a difference of speed occurring between no load and full load that requires a certain movement or travel of the Governor Collar (18). Consequently, the speed of the Turbine at different gate openings will vary slightly and depend upon the speed of the Governor at corresponding positions of the Regulating Piston Stroke.

"Under favorable conditions (open flume and short penstocks) the opening and closing time of the gates depends solely upon the inertia of the moving masses and "aperiodical regulation" can be obtained; i. e., the stroke of the Regulating Piston and the travel of the Governor Collar correspond in time. Under favorable conditions (long penstocks) the closing time is often so influenced by the "critical time," already mentioned, and by other considerations that "aperiodical regulation" is no longer practicable since a travel of Governor Collar would be required that would cause a greater difference in speed between no load and full load than is commercially allowable. To meet such conditions, the "Compensating Dash Pot" (7) is utilized.

"In the diagram, Fig. 413, the full travel of the Governor Collar is shown as corresponding to a speed change "x". The Relay Stroke, however, is designed so that only a portion of this travel corresponding to a speed change "y" is utilized; i. e., within this limit the Governor, without other mechanism than the Relay, is "dead beat" and the Regulating Valve by relay action is returned to mid-position after each movement. The Compensating Dash Pot, (7), consists of a cylinder having an adjustable bypass and containing a compound piston with auxiliary spring device, the rod of which is connected through a suitable lever to the Governor Collar. Arranged so that its piston takes motion from the Relay actuating shaft, is a positive displacement pump connected by a pipe to the "Compensating Dash Pot" cylinder. For slight changes of

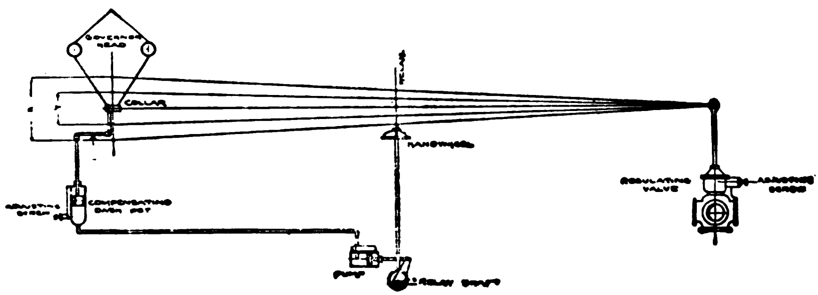

Fig. 413.—Diagram of Allis-Chalmers Governor.

load, a negligible displacement of oil takes place and the Dash Pot has a slight damping action only on the governor head, but when any load change occurs of sufficient magnitude to produce a speed variation greater than "y" as shown on the diagram, enough oil displacement takes place to bring the auxiliary spring effect of the Dash Pot piston strongly into action until the fluctuation is controlled and the Governor Collar is again brought within the limits corresponding to "y" speed variation when action ceases. By this means, a governing element of maximum sensitiveness can be used and the regulation of ordinary slight fluctuations made "aperiodical", even under the most unfavorable conditions. These elements in design, therefore, result in the Allis-Chalmers Governor operating with great quickness and holding the speed variation, due to ordinary fluctuations, within the narrowest limits, yet being absolutely safe from hunting or overtravel after heavy load fluctuation, even under the most difficult operating conditions."

APPENDIX—H.

MISCELLANEOUS TABLES.

TABLE LXXVIII.

EQUIVALENT MEASURES AND WEIGHTS OF WATER AT 4° CENTIGRADE—39.2° FAHRENHEIT.

U. S. Gallons	Imperial Gallons	Liters	Cubic Meters	Pounds	Cubic Feet	Cubic Inches	Circular Inch 1 Foot Long
1	.83321	3.7853	.0037853	8.34112	.13368	231	24.5096
1.20017	1	4.54303	.004543	10.0108	.160439	277.274	29.4116
.264179	.22012	1	.001	2.20355	.035316	61.0254	6.4754
264.179	220.117	1000	1	2203.55	35.31563	61025.4	6475.44
.119888	.099892	.453813	.0004538	1	.0160266	27.694	2.9411
7.48055	6.23287	28.3161	.0283161	62.3961	1	1728	183.346
.004329	.003607	.0163866	.0000164	.0361089	.0005787	1	.10613
.0408	.034	.1544306	.0001544	.340008	.005454	9.4224	1

TABLE LXXIX.

EQUIVALENT UNITS OF ENERGY

WORK				HEAT		ELECTRIC	HYDRAULICS			
Foot Pound	Foot Ton 2240 Lbs.	Kilogram Meter	Tonne Meter	B. T. U. Deg. Fah. Pound	Calorie Deg. Cent Kilogram	Volt Columb	Foot Gallon	Foot Cubic	Pound Gallon	Pound Cu. Ft.
1	.000446	1383	.000138	.001285	.000324	.000377	.12	.016	.0519	.0069
2240.	1	309.688	.3097	2.8785	.7262	.8439	268.817	35.906	116.414	15.456
7.233	.00323	1	.001	.0093	.00235	.00272	.8673	.1159	.3755	.0499
7233.18	3.2291	1000	1	9.302	2.3452	2.7241	867.303	115.928	375.516	49.90
778.	.3474	107.562	.1076	1	.2520	.2929	93.28	12.448	40.394	5.368
3085.34	1.3774	426.394	.4264	3.9683	1	1.1623	370.17	49.396	160.29	21.221
2655.4	1.1854	371.123	.3671	3.414	.8603	1	318.39	42.486	137.87	183.23
8.341	.00372	1.1532	.00115	.1072	.0027	.00314	1	.1334	.433	.05754
62.39	.02785	8.6257	.00863	.0803	.00202	.02353	7.48	1	3.245	.4312
19.259	.00859	2.6626	.00266	.0248	.00624	.00726	2.309	.3082	1	.1329
144.92	.0647	20.036	.02004	.1863	.04712	.05457	17.37	2.318	7.524	1

TABLE LXXX.

Velocities, in feet per second, due to Heads—from 0 to 50 feet.

Head in feet.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.....	0.000	2.536	3.587	4.393	5.072	5.671	6.212	6.710	7.173	7.609
1.....	8.020	8.412	8.786	9.144	9.490	9.823	10.145	10.457	10.760	11.055
2.....	11.342	11.622	11.896	12.163	12.425	12.681	12.932	13.179	13.420	13.658
3.....	13.891	14.121	14.347	14.569	14.789	15.004	15.217	15.427	15.634	15.839
4.....	16.040	16.240	16.437	16.631	16.823	17.013	17.201	17.387	17.571	17.753
5.....	17.934	18.112	18.289	18.464	18.637	18.809	18.979	19.148	19.315	19.481
6.....	19.645	19.808	19.970	20.131	20.290	20.448	20.604	20.760	20.914	21.067
7.....	21.219	21.370	21.520	21.669	21.817	21.964	22.110	22.255	22.399	22.542
8.....	22.685	22.826	22.966	23.106	23.245	23.383	23.520	23.656	23.793	23.927
9.....	24.061	24.194	24.326	24.458	24.589	24.720	24.850	24.979	25.107	25.235
10.....	25.362	25.489	25.614	25.740	25.864	25.988	26.112	26.235	26.357	26.479
11.....	26.600	26.721	26.841	26.960	27.079	27.198	27.316	27.433	27.550	27.667
12.....	27.783	27.898	28.013	28.128	28.242	28.356	28.469	28.582	28.694	28.806
13.....	28.917	29.028	29.139	29.249	29.359	29.468	29.577	29.686	29.794	29.901
14.....	30.009	30.116	30.222	30.329	30.435	30.540	30.645	30.750	30.854	30.958
15.....	31.062	31.165	31.268	31.371	31.474	31.576	31.677	31.779	31.880	31.980
16.....	32.081	32.181	32.281	32.380	32.480	32.579	32.677	32.775	32.873	32.971
17.....	33.068	33.165	33.262	33.359	33.455	33.551	33.647	33.742	33.837	33.932
18.....	34.027	34.121	34.215	34.309	34.403	34.496	34.589	34.682	34.775	34.867
19.....	34.959	35.051	35.143	35.234	35.325	35.416	35.507	35.597	35.688	35.778
20.....	35.867	35.957	36.046	36.135	36.224	36.313	36.401	36.490	36.578	36.666
21.....	36.753	36.841	36.928	37.015	37.102	37.188	37.275	37.361	37.447	37.532
22.....	37.618	37.703	37.789	37.874	37.959	38.043	38.128	38.212	38.296	38.380
23.....	38.464	38.547	38.630	38.714	38.797	38.879	38.962	39.014	39.127	39.209
24.....	39.291	39.373	39.454	39.536	39.617	39.698	39.779	39.860	39.940	40.021
25.....	40.101	40.181	40.261	40.341	40.421	40.500	40.579	40.659	40.738	40.816
26.....	40.895	40.974	41.052	41.130	41.209	41.287	41.364	41.442	41.520	41.597
27.....	41.674	41.751	41.828	41.905	41.982	42.058	42.135	42.211	42.287	42.363
28.....	42.439	42.515	42.590	42.666	42.741	42.816	42.891	42.966	43.041	43.116
29.....	43.190	43.264	43.339	43.413	43.487	43.561	43.635	43.708	43.782	43.855
30.....	43.928	44.002	44.075	44.148	44.220	44.293	44.366	44.438	44.510	44.582
31.....	44.655	44.727	44.798	44.870	44.942	45.013	45.085	45.156	45.227	45.298
32.....	45.369	45.440	45.511	45.581	45.652	45.722	45.792	45.863	45.933	46.003
33.....	46.073	46.142	46.212	46.281	46.351	46.420	46.489	46.559	46.628	46.697
34.....	46.765	46.834	46.903	46.971	47.040	47.108	47.176	47.244	47.312	47.380
35.....	47.448	47.516	47.584	47.651	47.719	47.786	47.853	47.920	47.987	48.054
36.....	48.121	48.188	48.255	48.321	48.388	48.454	48.521	48.487	48.653	48.719
37.....	48.785	48.851	48.917	48.982	49.048	49.113	49.179	49.244	49.310	49.375
38.....	49.440	49.505	49.570	49.635	49.699	49.764	49.829	49.893	49.958	50.022
39.....	50.086	50.150	50.214	50.278	50.342	50.406	50.470	50.534	50.597	50.661
40.....	50.724	50.788	50.851	50.914	50.977	51.040	51.103	51.166	51.229	51.292
41.....	51.354	51.417	51.479	51.542	51.604	51.667	51.729	51.791	51.853	51.915
42.....	51.977	52.039	52.100	52.162	52.224	52.285	52.347	52.408	52.470	52.531
43.....	52.592	52.653	52.714	52.775	52.836	52.897	52.958	53.018	53.079	53.139
44.....	53.200	53.260	53.321	53.381	53.441	53.501	53.561	53.621	53.681	53.741
45.....	53.801	53.861	53.921	53.980	54.040	54.099	54.159	54.218	54.277	54.336
46.....	54.396	54.455	54.514	54.573	54.632	54.690	54.749	54.808	54.867	54.925
47.....	54.984	55.042	55.101	55.159	55.217	55.275	55.334	55.392	55.450	55.508
48.....	55.566	55.623	55.681	55.739	55.797	55.854	55.912	55.969	56.027	56.084
49.....	56.141	56.199	56.256	56.313	56.370	56.427	56.484	56.541	56.598	56.655

TABLE LXXXI.

*Table of three-halves (3/2) power of number.**

Head in feet.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0....	0.0000	0.0316	0.0894	0.1643	0.2530	0.3536	0.4648	0.5857	0.7155	0.8538
1....	1.0000	1.1537	1.3145	1.4822	1.6565	1.8371	2.0238	2.2165	2.4150	2.6190
2....	2.8284	3.0432	3.2631	3.4881	3.7181	3.9529	4.1924	4.4366	4.6853	4.9385
3....	5.1962	5.4581	5.7243	5.9947	6.2693	6.5479	6.8305	7.1171	7.4076	7.7019
4....	8.0000	8.3019	8.6074	8.9167	9.2295	9.5459	9.8659	10.1894	10.5163	10.8466
5....	11.1803	11.5174	11.8578	12.2015	12.5485	12.8986	13.2520	13.6086	13.9682	14.3311
6....	14.6969	15.0659	15.4379	15.8129	16.1909	16.5718	16.9557	17.3425	17.7322	18.1248
7....	18.5203	18.9185	19.3196	19.7235	20.1302	20.5396	20.9518	21.3666	21.7842	22.2047
8....	22.6274	23.0530	23.4812	23.9121	24.3455	24.7815	25.2202	25.6613	26.1050	26.5512
9....	27.0000	27.4512	27.9050	28.3612	28.8199	29.2810	29.7445	30.2105	30.6789	31.1496
10....	31.6228	32.0983	32.5762	33.0564	33.5390	34.0239	34.5111	35.0006	35.4924	35.9865
11....	36.4829	36.9815	37.4824	37.9855	38.4908	38.9984	39.5082	40.0202	40.5343	41.0507
12....	41.5692	42.0910	42.6128	43.1388	43.6648	44.1952	44.7256	45.2600	45.7914	46.3332
13....	46.8720	47.4148	47.9576	48.5048	49.0520	49.6032	50.1544	50.7096	51.2648	51.8240
14....	52.3832	52.9464	53.5096	54.0768	54.6440	55.2152	55.7864	56.3616	56.9368	57.5154
15....	58.0944	58.6776	59.2608	59.8472	60.4336	61.0244	61.6152	62.2096	62.8040	63.4030
16....	64.0000	64.6020	65.2040	65.8096	66.4152	67.0244	67.6336	68.2464	68.8592	69.4760
17....	70.0928	70.7132	71.3336	71.9572	72.5808	73.2084	73.8360	74.4672	75.0984	75.7328
18....	76.3672	77.0056	77.6440	78.2856	78.9272	79.5724	80.2176	80.8664	81.5152	82.1672
19....	82.8192	83.4748	84.1304	84.7892	85.4480	86.1104	86.7728	87.4384	88.1040	88.7732
20....	89.4424	90.1152	90.7880	91.4636	92.1392	92.8184	93.4976	94.1800	94.8624	95.5484
21....	96.2344	96.9232	97.6120	98.3044	98.9968	99.6924	100.3880	101.0868	101.7856	102.4872
22....	103.1888	103.8940	104.6008	105.3076	106.0160	106.7276	107.4392	108.1540	108.8688	109.5864
23....	110.3040	111.0248	111.7456	112.4700	113.1944	113.9216	114.6488	115.3788	116.1088	116.8420
24....	117.5752	118.3128	119.0496	119.7876	120.5272	121.2696	122.0120	122.7576	123.5032	124.2516
25....	125.0000	125.7516	126.5032	127.2576	128.0120	128.7706	129.5292	130.2876	131.0480	131.8112
26....	132.5744	133.3406	134.1072	134.8764	135.6456	136.4180	137.1904	137.9652	138.7400	139.5180
27....	140.2960	141.0768	141.8576	142.6416	143.4256	144.2120	144.9984	145.7880	146.5776	147.3700
28....	148.1624	148.9572	149.7520	150.5500	151.3480	152.1488	152.9496	153.7532	154.5568	155.3632
29....	156.1696	156.9788	157.7880	158.6000	159.4120	160.2268	161.0416	161.8588	162.6760	163.4964
30....	164.3168	165.1396	165.9624	166.7884	167.6144	168.4428	169.2712	170.1020	170.9328	171.7666
31....	172.6008	173.4372	174.2736	175.1128	175.9520	176.7940	177.6360	178.4804	179.3248	180.1720
32....	181.0192	181.8692	182.7192	183.5716	184.4240	185.2792	186.1344	186.9920	187.8496	188.7100
33....	189.5704	190.4336	191.2968	192.1624	193.0280	193.8960	194.7640	195.6348	196.5056	197.3788
34....	198.2520	199.1460	200.0400	200.9008	201.7616	202.6424	203.5232	204.4068	205.2904	206.1764
35....	207.0624	207.9512	208.8400	209.7312	210.6224	211.5204	212.4184	213.3104	214.2024	215.1012
36....	216.0000	216.9012	217.8024	218.7060	219.6096	220.5760	221.4224	222.3812	223.2400	224.1512
37....	225.0624	225.9760	226.8896	227.8056	228.7216	229.6404	230.5592	231.4800	232.4008	233.3244
38....	234.2480	235.1736	236.0992	237.0276	237.9560	238.8868	239.8176	240.7508	241.6840	242.6196
39....	243.5552	244.4932	245.4312	246.3712	247.3112	248.2540	249.1968	250.1420	251.0872	252.0348
40....	252.9824	253.9320	254.8816	255.8340	256.7864	257.7412	258.6960	259.6528	260.6096	261.5688
41....	262.5280	263.4896	264.4512	265.4152	266.3792	267.3456	268.3120	269.2804	270.2488	271.2200
42....	272.1912	273.1644	274.1376	275.1132	276.0888	277.0672	278.0456	279.0252	280.0048	280.9872
43....	281.9696	282.9544	283.9392	284.9264	285.9136	286.9028	287.8920	288.8836	289.8752	290.8692
44....	291.8632	292.8597	293.8552	294.8536	295.8520	296.8528	297.8536	298.8564	299.8592	300.8640
45....	301.8688	302.8764	303.8840	304.8936	305.9032	306.9148	307.9264	308.9404	309.9544	310.9708
46....	311.9872	313.0056	314.0240	315.0448	316.0656	317.0877	318.1112	319.0556	320.0000	321.108
47....	322.2160	323.2452	324.2744	325.3060	326.3376	327.3716	328.4056	329.4416	330.4776	331.5136
48....	332.5536	333.5927	334.6333	335.4753	336.7188	337.7588	338.8051	339.8529	340.8972	341.9479
49....	343.0000	344.0486	345.0986	346.1500	347.2079	348.2622	349.3179	350.3750	351.4336	352.4936
50....	353.5500	354.6128	355.6720	356.7376	357.7996	358.8681	359.9329	360.9992	362.0719	363.1409

*From Water-Supply and Irrigation Paper No. 180.

TABLE LXXXI—Continued.

Table of three-halves ($\frac{3}{2}$) power of number.

Head in feet.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
51....	364.2114	365.2832	366.3564	367.4311	368.5020	369.5794	370.6582	371.7383	372.8149	373.8927
52....	374.9772	376.0578	377.1397	378.2231	379.3078	380.3940	381.4815	382.5703	383.6606	384.7522
53....	385.8453	386.9348	388.0301	389.1219	390.2205	391.3150	392.4163	393.5136	394.6122	395.7122
54....	396.8136	397.9163	399.0204	400.1258	401.2326	402.3408	403.4448	404.5557	405.6679	406.7759
55....	407.8855	409.0017	410.1139	411.2273	412.3477	413.4639	414.5814	415.7002	416.8204	417.9419
56....	419.0648	420.1833	421.3089	422.4257	423.5583	424.6879	425.8181	426.9458	428.0782	429.2080
57....	430.3386	431.4704	432.6036	433.7380	434.8738	436.0110	437.1494	438.2892	439.4302	440.5726
58....	441.7106	442.8556	443.9961	445.1438	446.2869	447.4372	448.5830	449.7300	450.8842	452.0359
59....	454.0849	455.3271	455.4907	456.6455	457.8017	458.9592	460.1179	461.2720	462.4384	463.5960
60....	464.7540	465.9192	467.0797	468.2475	469.4106	470.5750	471.7467	472.9137	474.0819	475.2514
61....	476.4222	477.3942	478.7676	479.9423	481.1181	482.2891	483.4676	484.6473	485.8222	487.0044
62....	488.1880	489.3666	490.5465	491.7329	492.9163	494.1000	495.2912	496.4774	497.6648	498.8536
63....	500.0136	501.2348	502.4278	503.6211	504.8161	506.0061	507.2036	508.4024	509.5961	510.7974
64....	512.0000	513.1974	514.3960	515.6024	516.8035	518.0059	519.2160	520.4209	521.6270	522.8344
65....	524.0430	525.2528	526.4639	527.6762	528.8898	530.1046	531.3120	532.5313	533.7498	534.9630
66....	536.184[illegible]	537.2996	538.6230	539.8411	541.0670	542.2875	543.5092	544.7389	545.9680	547.1894
67....	548.4151	549.6429	550.8720	552.1022	553.3337	554.5665	555.6179	557.0356	558.2652	559.5027
68....	560.7416	561.9748	563.2160	564.4516	565.6953	566.9334	568.1795	569.4199	570.6616	571.9118
69....	573.1554	574.4006	575.6478	576.8947	578.1436	579.3937	580.6449	581.8974	583.1510	584.4059
70....	585.6620	586.9122	588.1707	589.4303	590.6841	591.9462	593.2023	594.4668	595.7253	596.9921
71....	598.2531	599.5152	600.7836	602.0500	603.3157	604.5825	605.8505	607.1197	608.3901	609.6616
72....	610.9344	612.2083	613.4840	614.7596	616.0371	617.3085	618.5883	619.8692	621.0841	622.4274
73....	623.7120	624.9903	626.2699	627.5579	628.8398	630.1302	631.4144	632.6997	633.9862	635.2813
74....	636.5702	637.8602	639.1518	640.4437	641.7372	643.0318	644.3276	645.6246	646.9152	648.2145
75....	649.5150	650.8166	652.1118	653.4157	654.7208	656.0195	657.3268	658.6278	659.9375	661.2408
76....	662.5452	663.8593	665.1650	666.4728	667.7894	669.0996	670.4108	671.7131	673.0368	674.3514
77....	675.6673	676.9842	677.2048	679.6216	680.9419	682.2635	683.5784	684.9021	686.2271	687.5454
78....	688.8726	690.2009	691.5226	692.8533	694.1771	695.5100	696.8361	698.8361	699.1718	700.8292
79....	702.1599	703.4995	704.8324	706.1665	707.5016	708.8379	710.1752	711.5137	712.8534	714.1941
80....	715.5360	716.8789	718.2230	719.5683	720.9146	722.2540	723.6026	724.9523	726.2950	727.6496
81....	729.0000	730.3460	731.7613	733.0496	734.3989	735.7575	737.1091	738.4699	739.8237	741.1876
82....	742.5346	743.8998	745.2580	746.6173	747.9776	749.3392	750.7018	752.0653	753.4308	734.7962
83....	756.1632	757.5312	758.9004	760.2621	761.6338	763.0068	764.3798	765.7461	767.1219	768.4904
84....	769.8684	771.2474	772.6192	774.0004	775.3743	776.7493	778.1338	779.5110	780.8892	782.2770
85....	783.6575	785.0389	686.4215	787.8052	789.1984	790.5843	791.9712	793.3591	794.7482	796.1383
86....	797.5296	798.9219	800.3066	801.7011	803.0966	804.4932	805.8909	807.2810	808.6808	810.0833
87....	811.4731	812.8781	814.2736	815.6788	817.0763	818.4837	819.8834	821.2929	822.6947	824.1064
88....	825.5704	826.9154	828.3214	829.7374	831.1456	832.5549	833.9652	835.3766	836.7890	838.2095
89....	839.6171	841.0327	842.4444	843.8671	845.2859	846.7058	848.1267	849.5487	850.9627	852.3868
90....	853.8120	855.2382	856.6564	858.0847	859.5051	860.9355	862.3670	863.7905	865.2241	866.6496
91....	868.0763	869.5130	870.9417	872.3806	873.8114	875.2432	876.6761	878.1192	879.5541	880.9901
92....	882.4272	883.8652	885.3041	886.7445	888.1857	889.6280	891.0712	892.5156	893.9609	895.4073
93....	896.8548	898.3032	899.7528	901.1946	902.6456	904.0982	905.5519	906.9972	908.4530	909.9097
94....	911.3582	912.8170	914.2675	915.7284	917.1809	918.6439	920.0985	921.5541	923.0202	924.4778
95....	925.9365	927.4056	928.8664	930.3281	931.7908	933.2642	934.7290	936.1948	937.6616	939.1295
96....	940.5984	942.0683	943.5392	945.0111	946.4841	947.9581	949.4331	950.9091	952.3764	953.8545
97....	955.3336	956.8136	958.2948	959.7672	961.2503	962.7345	964.2099	965.6961	967.1785	968.6617
98....	970.1412	971.6314	973.1129	974.6051	976.0886	977.5829	979.0684	980.5548	982.0522	983.5407
99....	985.0302	986.5206	988.0220	989.5145	991.0080	992.5025	993.9980	995.4945	996.9920	998.4905
100...	1,000.0000	....	....	....	....	....	....	....	....	....

TABLE LXXXII.

Table of five-halves ($\frac{5}{2}$) powers of numbers.

Head in feet.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.....	0.000	.003	.018	.049	.101	.177	.279	.410	.572	.768
1.....	1.000	1.269	1.578	1.927	2.319	2.756	3.238	3.769	4.347	4.976
2.....	5.657	6.390	7.179	8.022	8.923	9.883	10.901	11.980	13.118	14.320
3.....	15.589	16.920	18.317	19.784	21.315	22.918	24.588	26.333	28.150	30.038
4.....	32.000	34.038	36.149	38.343	40.612	42.957	45.384	47.888	50.477	53.150
5.....	55.901	58.736	61.662	64.671	67.765	70.945	74.211	77.671	81.014	84.553
6.....	88.182	91.903	95.716	99.622	103.622	107.718	111.910	116.198	120.578	125.063
7.....	129.642	134.325	139.104	143.985	148.962	154.050	159.235	164.526	169.915	175.410
8.....	181.019	186.729	192.544	198.470	204.506	210.647	216.892	123.251	229.724	236.313
9.....	243.000	249.804	256.726	263.757	270.908	278.170	285.552	293.047	300.654	308.385
10.....	316.228	324.190	332.275	340.477	348.806	357.252	365.817	374.511	383.314	392.258
11.....	401.311	410.500	419.798	429.242	438.797	448.477	458.293	468.234	478.301	488.507
12.....	498.820	509.301	519.879	530.610	541.446	552.438	563.548	574.802	586.168	597.696
13.....	609.336	621.187	633.048	645.170	657.297	669.641	682.094	694.727	707.457	720.354
14.....	733.865	746.539	759.842	773.301	786.873	800.618	814.476	828.521	842.668	856.986
15.....	871.416	886.038	900.767	915.659	930.684	945.872	961.194	976.697	992.303	1008.062
16.....	1024.000	1040.092	1056.305	1072.703	1089.206	1105.896	1122.724	1139.708	1156.831	1174.144
17.....	1191.578	1209.192	1226.945	1244.856	1262.909	1281.140	1299.514	1318.066	1336.744	1355.621
18.....	1374.606	1393.809	1413.121	1432.634	1452.257	1472.082	1492.055	1512.194	1532.482	1552.956
19.....	1573.561	1594.373	1615.296	1636.428	1657.691	1679.145	1700.751	1722.529	1744.459	1766.583
20.....	1788.840	1811.312	1833.918	1856.719	1879.636	1902.769	1926.059	1949.536	1973.130	1996.953
21.....	2020.914	2045.075	2069.374	2093.875	2118.536	2143.378	2168.381	2193.588	2218.935	2244.465
22.....	2270.186	2296.057	2322.142	2348.368	2374.758	2401.380	2428.121	2455.096	2182.213	2509.519
23.....	2536.992	2564.678	2592.507	2620.551	2648.740	2677.167	2705.716	2734.482	2763.394	2792.524
24.....	2821.800	2851.343	2881.010	2910.848	2940.859	2971.115	3001.495	3032.123	3062.874	3093.875
25.....	3125.000	3156.875	3187.876	3219.627	3251.505	3283.661	3315.942	3348.402	3381.038	3413.905
26.....	3446.924	3480.200	3513.603	3547.239	3581.054	3615.077	3649.254	3683.666	3718.232	3753.034
27.....	3787.992	3823.187	3858.538	3894.127	3929.872	3965.830	4001.945	4038.328	4074.868	4111.623
28.....	4148.536	4185.692	4223.006	4260.565	4298.283	4336.247	4374.370	4412.711	4451.242	4489.991
29.....	4528.930	4568.089	4607.410	4646.980	4686.713	4726.697	4766.843	4807.212	4847.745	4888.530
30.....	4929.510	4970.714	5012.052	5053.676	5095.466	5137.512	5179.693	5222.131	5264.736	5307.600
31.....	5350.631	5393.891	5437.849	5481.087	5524.893	5569.011	5613.298	5657.816	5702.536	5747.487
32.....	5792.608	5837.995	5883.552	5929.376	5975.338	6021.568	6067.948	6114.638	6161.450	6208.550
33.....	6255.810	6303.365	6351.060	6398.995	6447.185	6495.516	6544.070	6592.899	6641.903	6691.148
34.....	6740.568	6790.879	6841.368	6890.904	6940.618	6991.149	7041.896	7092.923	7144.092	7195.542
35.....	7247.170	7299.080	7351.168	7403.504	7456.019	7508.960	7562.081	7615.167	7668.432	7722.126
36.....	7776.000	7830.133	7884.447	7939.028	7993.789	8051.024	8104.060	8159.555	8215.232	8271.179
37.....	8327.809	8383.709	8440.293	8497.149	8554.188	8611.515	8669.026	8726.796	8784.750	8842.995
38.....	8901.424	8960.114	9018.989	9078.157	9137.510	9197.142	9256.959	9317.056	9377.339	9437.902
39.....	9498.653	9559.684	9620.903	9682.388	9744.061	9806.033	9868.198	9930.637	9993.271	10056.189
40.....	10119.296	10182.673	10246.240	10310.110	10374.171	10438.519	10503.056	10567.869	10632.872	10698.164
41.....	10763.648	10829.423	10895.389	10961.648	11028.099	11094.842	11161.779	11228.993	11296.340	11364.118
42.....	11432.030	11502.221	11568.607	11637.288	11706.165	11775.356	11844.743	11939.996	11984.205	12054.351
43.....	12124.698	12195.335	12266.173	12337.818	12408.650	12480.272	12552.091	12624.213	12696.634	12769.197
44.....	12841.761	12915.113	12988.840	13062.014	13135.829	13209.930	13284.271	13358.881	13433.692	13508.794
45.....	13584.096	13659.726	13735.557	13811.680	13888.005	13964.623	14041.444	14118.576	14195.912	14273.560
46.....	14351.411	14429.558	14507.909	14586.574	14665.444	14744.578	14823.982	14899.897	14976.000	15059.963
47.....	15144.152	15224.849	15305.752	15386.971	15468.402	15550.151	15632.107	15714.364	15796.829	15879.597
48.....	15962.573	16045.809	16129.325	16203.457	16297.190	16381.302	16465.928	16552.036	16635.783	16722.212
49.....	16807.000	16892.786	16978.851	17065.195	17152.070	17238.979	17326.168	17413.647	17501.393	17589.181
50.....	17677.500	....	....	....	....	....	....	....	....	....

TABLE LXXXIII.

Showing relation of mean rainfall to the maximum and minimum discharge of various rivers.

DRAINAGE AREA, 500 TO 1,000 SQUARE MILES

STREAM AND LOCALITY.	Drainage Area, Sq. Miles	Mean Annual Rainfall, Inches	Discharge Cu. Ft. per Sec. per Sq. Mile Max.	Min.
I. AMERICAN STREAMS.				
Broad river at Carlton, Ga	762	47.73	22.21	.394
Coosawattee river at Carters, Ga	532	52.73	15.17	.588
Des Plaines river at Riverside, Ill	630	29.75	14.23	.000
Etowah river at Canton, Ga	604	52.73	31.50	.405
Flint river at Molina, Ga	892	52.73	7.37	.062
French Broad river at Asheville, N. C	987		7.98	.660
Greenbriar river, mouth Howard's cr., W. Va.	810	40.70		.120
Housatonic river, Massachusetts	790			.165
Little Tennessee river at Judson, N. C	675		56.40	.408
Mahoning river at Warren, O	596			.017
Mahoning river	967			.026
Monocacy river at Frederick, Md	665	38.77	16.98	.116
North river at Port Republic, Va	804	38.77	29.78	.220
North river at Glasgow, Va	831	38.77	44.80	.180
Olentangy river at Columbus, O	523			.014
Passaic river at Paterson, N. J	791	45.00		.190
Potomac river, no. branch at Cumberland, Md.	891	38.77	22.82	.045
Potomac river at Cumberland, Md	920	38.77	19.46	.022
Raritan river at Bound Brook, N. J	879	45.94	59.30	.140
Schoharie creek at Fort Hunter, N. Y.	948	39.25	44.00	
Shenandoah river at Fort Republic, Va	770	38.77		.167
Tuckasagee river at Bryson, N. C	662		45.30	.003
II. FRENCH STREAMS.				
Armancon river at Aisy	575		49.20	.011
Armancon river at Tonnerre	853			.034
Marne river at St. Dizier	915	30.70	7.73	.101
Meuse river at Pagny-la-Blanchecote	573			.039
Meuse river at Chalaines	607	31.51		.041
Meuse river at Pagny-sur-Meuse	734			.056
Meuse river at Vignot	817			.085
Meuse river at Mt. Mihiel	914			.078
III. GERMAN STREAMS.				
Ihna river at Stargard	672	26.60	15.50	.137
Jagst river at its mouth	708	29.50		.200
Kocher river at its mouth	768	29.50		.221
Lippe river at Hamm	965		9.75	.235
Malapane river at Czarnowanz	773	25.04	14 35	.274
Oppa river at Strebowitz	805	24.40	21.95	.256
Stober river at its mouth	620	22.70	3.65	

*From paper on Water Supply for New York State Canals, Report of State Engineer on Barge Canal, 1901.

TABLE LXXXIII—Continued.

DRAINAGE AREA, 1,000 TO 2,500 SQUARE MILES.

STREAM AND LOCALITY.	Drainage Area, Sq. Mile.	Mean Annual Rainfall, Inches.	Discharge Cu. Ft. per Sec. per Sq. Mile. Max.	Min.
I. AMERICAN STREAMS.				
Androscoggin river at Rumford Falls, Me...	2,220	40.39	25.00	.475
Broad river at Gaffney, S. C.	1,435	47.73		.49
Catawba river at Catawba, N. C.	1,535		49.1	.465
Chattahoochee river at Oakdale, Ga.	1,560	48.91	21.75	.432
Genesee river at Mt. Morris, N. Y.	1,070	38.09	39.20	.094
Greenbriar river at Aederson, W. Va.	1,344	44.86	41.55	.041
James river at Buchanan, Va.	2,058	40.83	15 56	.146
Neuse river at Raleigh, N. C.	1,000			.193
Neuse river at Selma, N. C.	1,175		6.70	.064
Ocmulgee river at Macon, Ga.	2,425	49.23	14.92	.157
Oconee river at Carey, Ga.	1,346	49.31	7.44	.283
Oostannala river at Resaca, Ga.	1,527	52.47	14.50	.389
Potomac river at Cumberland, Md.	1,364	35.28		.018
Saluda river at Waterloo, S. C.	1,056		12.08	.275
Schuylkill river at Philadelphia, Pa.	1,800			.170
Schuylkill river at Fairmount, Pa.	1,915		12.17	.013
Scioto river at Columbus, O.	1,070			.004
Scioto river at Shadeville, O.	1,670			.015
Tar river at Tarboro, N. C.	2,290		6.38	.074
Youghiogheny river at Ohio Pyle, Pa.	1,775			.060
II. FRENCH STREAMS.				
Aisne river at Biermes.	1,341			.085
Aisne river at Berry-au-Bac.	2,120			.092
Aisne river at Berry-au-Bac.	2,120		7.58	
Loing river at its junction with the Seine.	1,785	28.40		.046
Lys river.	1,420		1.74	.099
Marne river at La Chaussee.	2,297			.010
Marne river at Chalons.	2,497			.010
Meuse river at Verdun.	1,219	28.33		.110
Oise river at Chauny.	1,575			.104
Seine river at Troyes.	1,314			.051
III. GERMAN STREAMS.				
Bober river at Sagan.	1,638	39.20	17.40	.389
Drage river at its mouth.	1,234		2.11	.356
Ill river at Strasburg.	1,294		9.15	.327
Kuldow river at Usch.	1,830	18.90	19.30	.405
Lahn river at Diez.	2,008	25.60	12 80	.123
Lippe river at Wesel.	1,890		11.62	.198
Main river above mouth of the Regnitz river	1,725	27.44		.224
Netze river at Antonsdorf.	1,086			.063
Netze river above Eichhorst.	1,130			.046
Oder river at Hoschialkowitz.	1,440	21.60		.155
Oder river at Annaberg.	1,800	24.60	27 00	.219
Oder river at Olsau.	2,250	24.60	43.00	.274
Obra river at Moschin.	1,325			.101
Ruhu river at Mulheim.	1,728		33.80	.176
Saale river at its junction with the Main.	1,070	27.76		.081
Welna river at Kowanowko, near mouth.	1,013		3.14	.077

TABLE LXXXIII.—Continued.

DRAINAGE AREA, 2,500 TO 5,000 SQUARE MILES.

STREAM AND LOCALITY.	Drainage Area, Sq. Miles.	Mean Annual Rainfall, Inches.	Discharge Cu. Ft. per Sec. per Sq. Mile. Max.	Min.
I. American Streams.				
Black Warrior river at Tuscaloosa, Ala....	4,900		38.80	.018
Broad river at Alston, S. C............	4,609		23.2	.12
Cape Fear river at Fayetteville, N. C..	4,493		16.3	.09
Catawa river at Rock Hill, S. C.........	2,987		48.4	.355
Chattahoochee river at West Point, Ga....	3,300	52.92	17.37	.252
Connecticut river at Dartmouth, N. H.....	3,287			.306
Coosa river at Rome, Ga.................	4,001	52.73	11.42	.225
Crow Wing river, Minnesota	3,576	30.84	2.84	.250
Dan river at Clarksville, Va.............	3,749	38.28	8.80	.107
Hudson river at Mechanicsville, N. Y......	4,500	41.61	15.50	.189
Kennebec river at Waterville, Me..........	4,410		25.20	.006
Merrimac river at Lowell, Mass...........	4,085		19.83	.310
*Merrimac river at Lawrence, Mass.......	4,551		20.00	.27
Mohawk river at Rexford Flats, N. Y......	3,384		23.10	
Mohawk river at Cohoes, N. Y............	3,444	38.65		.232
Ocanee river at Dublin, Ga.	4,182	49.31	6.69	.021
Potomac river at Dam No. 5, Md.........	4,640	38.77	22.15	.078
Savannah river at Calhoun Falls, Ga	2,712	47.73	.96	.518
Shenandoah river at Millville, W. Va	2,995	30.56	11.44	.203
Staunton river at Clarksville, Va..........	3,546	38.28	10.30	.157
Susquehanna river, w. br., Williamsport, Pa.	4,500		11.60	.178
Tallapoosa river at Milstead, Ala..........	3,840		9.50	.091
Yadkin river at Salisbury, N. C.	3,399		15.0	.225
Yadkin river at Norwood, N. C............	4,614		13.70	.284
II. French Streams.				
Aisne river at Soissons..................	3,040		6.43	.081
Aisne river, above junction with the Oise river	3,285	23.50	5.95	.096
Eure river at its mouth.........	2,980	22.30	2.72	.076
Isere river at its mouth..................	4,360		21.00	.780
Marne river at Chateau Thierry...........	3,332			.127
Meuse river at Sedan.....................	2,560	28.33	8.05	.194
Meuse river at Fumay....	3,700	28.33	4.04	.191
Seine river at Bray......................	3,750		4.05	.003
Seine river at Nogent-sur-Seine.	3,594			.103
Yonne river at Sens......................	4,270		9.09	.106
Yonne river at Nogent-sur-Seine..........	4,300	30.80	6.37	.140
III. German Streams.				
Main river, below mouth of the Regnitz river	4,650	27.44		.186
Moselle river at Metz.............	3,550	29.48	14.92	.199
Mur river at Graz	2,959		12.98	.243
Neckar river at Heilbronn................	3,155			.146
Neckar river at Offenau.	4,770		33.35	.167
Oder river at Ratibor....................	2,580	24.60	21.20	.306
Oder river at Kosel......................	3,520	24.60	14.10	.128
Oder river at Krappitz...................	4,150	24.60	3.86	.187
Regnitz river at its junc. with the Main river	2,920	25.60		.164

*Figures supplied by Mr. Rich. A. Hale, Lawrence, Mass.

TABLE LXXXIII.—Continued.

DRAINAGE AREA, 5,000 AND OVER SQUARE MILES.

STREAM AND LOCALITY.	Drainage Area, Sq. Miles.	Mean Annual Rainfall, Inches.	Discharge Cu. Ft. per Sec. per Sq. Mile. Max.	Min.
I. AMERICAN STREAMS.				
Connecticut river at Holyoke, Mass........	8,660		13.26	.029
Connecticut river at Hartford, Conn.......	10,234	44.53		.310
Connecticut river at Hartford, Conn.......	10,234	44.53	20.27	.510
Coosa river at Riverside, Ala..............	6,850	48.08	10.53	.197
Delaware river, New Jersey................	6,750		50.00	.300
Delaware river at Stockton, N. J......	6,790	45.29	37.50	.170
Delaware river at Lambertsville, N. J......	6,855	45.29	9.71	.364
James river at Richmond, Va.	6,800	40.83		.191
Kanawha river at Charleston, W. Va.......	8,900	40.70	13.49	.123
Mississippi river.........................	7,283	32.64	1.49	.261
Mississippi river above St. Paul...........	36,085	25.75	10.73	.045
Mississippi river...........	164,534			.190
Mississippi river......	526,500			.050
Mississippi river	1,214,000			.210
Missouri river	17,615	15.70		.100
New river at Fayette, W. Va..............	6,200	40.70	13.49	.189
Ohio river at Pittsburg, Pa..........	19,990			.114
Ohio river	200,000	41.50		.270
Oswego river at Oswego, N. Y............	5,013	37.69		.230
Potomac river at Point of Rocks, Md......	9,654	30.35	19.40	.083
Potomac river...............	11,043	38.77	42.60	.170
Potomac river at Georgetown, D. C........	11,124	38.77	15.70	
Potomac river at Great Falls, Md..........	11,427	45.36	41.15	.215
Potomac river at Great Falls, Md.........	11,476	45.36	15.25	.093
Potomac river at Chain Bridge, D. C......	11,545	38.77	17.16	.165
Red river, Arkansas.......................	97,000	30.00	2.32	
Roanoke river at Neal, N. C...............	8,717	38.21	7.38	.229
St. Croix river, Minnesota............... .	5,950	32.58	6.00	.424
Savannah river at Augusta, Ga.............	7,294	47.73	42.50	.272
Susquehanna, w. branch, at Northumberland	6,800		17.53	.074
Susquehanna river at Harrisburg, Pa.......	24,030		18.88	.092
Tennessee river at Chattanooga, Tenn......	21,418		20.78	.199
II. FRENCH STREAMS.				
Loire river at Nevers	6,560		23.10	.070
Loire river, between Maine and Vienne rivers	9,950			.255
Marne river at Charenton..................	5,657			.016
Marne river at its junction with the Seine...	5,295	30.70	4.67	.080
Meuse river at Maestricht.................	8,240	42.50	5.61	.146
Meuse river at Maeseyck..................	8,480	42.50	7.36	.244
Meuse river above Ruremond..............	8,750		3.01	.317
Oise river at Creil........................	5,622		3.14	.194
Rhone river at Lyons.........	18,000	36.32	11.83	.333
Seine river at Port a l'Anglais.............	17,624			.046
Seine river at Paris.......................	20,000	21.27	5.80	.085
Seine river at Mantes........	25,135		3.09	.091
Seine river at mouth of the Eure river......	28,583		3.09	
III. GERMAN STREAMS.				
Elbe river at Torgau......................	22,000	27.09	2.89	.144
Main river above mouth of Saale river......	5,820			.182
Main river below mouth of Saale river	6,900			.166
Main river above mouth of Tauber river....	7,290			.167
Main river below mouth of Tauber river....	8,000			.167
Main river at Frankfort...........	9,610		12.50	.121
Memel river at Tilsit......................	38,600		4.02	.813
Moselle river at Kochem....	10,253		8.52	.174
Moselle river at Coblenz.................	10,840	24.76	13.01	.166

TABLE LXXXIII.—Continued.

DRAINAGE AREA, 5,000 AND OVER SQUARE MILES.

TREAM AND LOCALITY	Drainage Area Sq. Miles.	Mean Annual Rainfall Inches.	Discharge Cu. Ft Per Sec. Per Sq. Mile,	
			MAX.	MIN.
III. GERMAN STREAMS.				
Neckar river at Heidelberg................	5,321		32.17	.215
Neckar river at Mannheim................	5,395		31.02	
Oder river at Ohlau......................	7,750	24.60	4.17	.215
Oder river at Breslau, below the Ohle river.	8,330	24.60	10.40	.209
Oder river at Steinau.....................	11,412	24.02	.95	.229
Oder river below mouth of the Warthe-river	28,319	23.62	.61	.212
Saale river at Rothenburg.................	7,282	27.76	5.41	.120
Warthe river at Pogorzelice.	7,900			.104
Warthe river at Posen..............	9,620		6.37	.100
Warthe river at Landsberg..............	20,020	21.65	2.56	.192

TABLE LXXXIV.

*Mean average rainfall, run-off, and evaporation for storage, growing and replenishing periods for 12 streams of the United States.**

Period.	Muskingum River, from 1888 to 1895, eight years. Catchment area, 5,828 square miles.			Genesee River, from 1890 to 1898, nine years. Catchment area, 1,070 square miles.			Croton River, from 1877 to 1899, twenty-three years. Catchment area, 338.8 square miles.		
	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.
Storage	18.8	9.6	9.2	19.4	10.5	8.9	23.7	16.8	6.9
Growing	11.6	1.7	9.9	11.5	1.7	9.8	13.6	2.6	11.0
Replenishing	9.3	1.8	7.5	9.4	2.0	7.4	12.1	3.4	8.7
Year	39.7	13.1	26.6	40.3	14.2	26.1	49.4	22.8	26.6

Period.	Lake Cochituate, from 1863 to 1900, thirty-eight years. Catchment area, 18.9 square miles.			Sudbury River, from 1875 to 1900, twenty-six years. Catchment area, 78.2 square miles.			Mystic Lake, from 1878 to 1895, eighteen years. Catchment area, 26.9 square miles.		
	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.
Storage	23.1	14.9	8.2	23.5	17.9	5.6	22.4	15.1	7.3
Growing	11.6	2.1	9.5	10.7	1.7	9.0	10.9	2.3	8.6
Replenishing	12.4	3.3	9.1	11.9	3.0	8.9	10.8	2.6	8.2
Year	47.1	20.3	26.8	46.1	22.6	23.5	44.1	20.0	24.1

Period.	Neshaminy Creek, from 1884 to 1899, sixteen years. Catchment area, 139.3 square miles.			Perkiomen Creek, from 1884 to 1899, sixteen years. Catchment area, 152 square miles.			Tohickon Creek, from 1884 to 1898, fifteen years. Catchment area, 102.2 square miles.		
	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.
Storage	23.1	17.2	5.9	23.2	16.7	6.5	24.2	20.5	3.7
Growing	13.4	2.7	10.7	13.7	3.1	10.6	14.6	3.5	11.1
Replenishing	11.1	3.2	7.9	11.1	3.8	7.3	11.3	4.4	6.9
Year	47.6	23.1	24.5	48.0	23.6	24.4	50.1	28.4	21.7

Period.	Hudson River, from 1888 to 1901, fourteen years. Catchment area, 4,500 square miles.			Pequannock River, from 1891 to 1899, nine years. Catchment area, 63.7 square miles.			Connecticut River, from 1872 to 1885, eleven years. Catchment area, 10,234 square miles.		
	Rain.	Run off.	Evaporation.	Rain.	Run-off.	Evaporation.	Rain.	Run-off.	Evaporation.
Storage	20.6	16.1	4.5	23.0	19.7	3.3	18.9	15.1	3.8
Growing	12.7	3.5	9.2	12.7	3.1	9.6	13.8	3.3	10.5
Replenishing	10.9	3.7	7.2	11.1	4.0	7.1	10.3	3.6	6.7
Year	44.2	23.3	20.9	46.8	26.8	20.0	43.0	22.0	21.0

*From W. S. and I. Paper No. 80. Rafter.

TABLE LXXXV—*Croton River, 1868-1899, inclusive.*

[Catchment area=[illegible].8 square miles.]

Period.	1868.			1869.			1870.		
	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
Storage	23.24	17.25	5.99	21.89	15.75	6.14	28.42	19.01	9.41
Growing	13.64	5.75	7.89	7.77	2.01	5.76	10.59	1.56	9.03
Replenishing	14.85	11.06	3.79	15.09	4.39	10.70	10.09	.96	9.13
Year	51.78	34.06	17.67	44.75	22.15	22.60	49.10	21.53	27.57
	1871.			1872.			1873.		
Storage	19.83	9.72	10.11	14.57	10.31	4.26	22.19	18.52	3.67
Growing	16.04	2.61	13.43	14.33	3.01	11.32	8.65	1.54	7.11
Replenishing	11.95	5.65	6.30	10.75	4.38	6.37	12.58	3.20	9.38
Year	47.82	17.98	29.84	39.65	17.70	21.95	43.42	23.26	20.16
	1874.			1875.			1876.		
Storage	23.74	22.86	0.88	17.10	14.81	2.29	22.64	19.89	2.75
Growing	12.30	2.77	9.53	16.45	5.86	10.59	7.14	1.07	6.07
Replenishing	8.68	1.60	7.08	10.33	3.41	6.92	10.11	1.35	8.76
Year	44.72	27.23	17.49	43.88	24.08	19.80	39.89	22.31	17.58
	1877.			1878.			1879.		
Storage	17.49	12.36	5.13	20.99	14.19	6.80	25.17	20.81	4.36
Growing	13.17	.96	12.21	11.29	2.57	8.72	18.09	2.63	15.46
Replenishing	18.46	5.49	12.97	16.72	5.01	11.71	6.96	1.88	5.08
Year	49.12	18.81	30.31	49.00	21.77	27.23	50.22	25.32	24.90
	1880.			1881.			1882.		
Storage	19.78	12.19	7.59	24.53	14.79	9.74	27.91	16.85	11.06
Growing	11.42	.68	10.74	9.61	1.95	7.66	9.03	2.06	6.97
Replenishing	7.57	.84	6.73	8.96	.97	7.99	19.10	6.21	12.89
Year	38.77	13.71	25.06	43.10	17.71	25.39	56.04	25.12	30.92
	1883.			1884.			1885.		
Storage	19.03	11.37	7.66	24.81	16.85	7.96	21.86	15.36	6.50
Growing	12.10	1.09	11.01	15.72	2.34	13.38	12.89	.88	12.01
Replenishing	10.41	1.28	9.13	8.01	1.87	6.14	12.23	2.92	9.31
Year	41.54	13.74	27.80	48.54	21.06	27.48	46.98	19.16	27.82
	1886.			1887.			1888.		
Storage	25.45	18.16	7.29	23.05	16.44	6.61	30.33	21.74	8.59
Growing	11.68	1.53	10.15	24.75	6.71	18.04	11.25	2.63	8.62
Replenishing	9.82	1.23	8.59	7.78	2.60	5.18	18.76	8.23	10.53
Year	46.95	20.92	26.03	55.58	25.75	29.83	60.34	32.60	27.74

TABLE LXXXV—Continued.—*Croton River, 1868–1899, inclusive.*

Period.	1889.			1890.			1891.		
	Rainfall.	Run-off.	Evaporation.	Rainfall.	Run-off.	Evaporation.	Rainfall.	Run-off.	Evaporation.
Storage	22.40	16.86	5.54	25.81	19.10	6.21	26.66	21.22	5.44
Growing	17.37	6.49	10.88	18.31	2.51	10.80	11.26	1.14	10.12
Replenishing	18.83	8.70	10.13	14.60	7.02	7.58	7.78	1.11	6.67
Year	58.60	32.05	26.55	58.22	28.63	24.59	45.70	23.47	22.23
	1892.			1893.			1894.		
Storage	22.93	12.87	10.06	27.84	21.41	5.93	23.24	15.65	7.59
Growing	15.37	2.60	12.77	12.89	1.84	10.55	7.95	1.82	6.13
Replenishing	10.30	2.31	7.99	11.08	3.51	7.57	17.05	4.41	12.64
Year	48.60	17.78	30.82	50.81	26.76	24.05	48.24	21.88	26.36
	1895.			1896.			1897.		
Storage	19.55	14.78	4.77	24.84	18.01	6.83	20.55	14.64	5.91
Growing	11.19	1.05	10.14	12.25	2.03	10.22	20.79	6.93	13.86
Replenishing	9.54	1.27	8.27	11.27	3.13	8.14	8.76	2.73	6.03
Year	40.28	17.10	23.18	48.36	23.17	25.19	50.10	24.30	25.80
				1898.			1899.		
Storage				28.81	20.08	8.73	22.66	21.38	1.28
Growing				17.17	4.83	12.34	12.19	1.57	10.62
Replenishing				13.36	3.99	9.37	10.37	1.96	8.41
Year				59 34	28.90	30.44	45.22	24.91	20.31
				Mean 1868–1876, inclusive.			Mean 1877–1899, inclusive.		
Storage				21.51	16.46	5.05	23.68	16.83	6.85
Growing				11.88	2.91	8.97	13.58	2.57	11.01
Replenishing				11.61	4.00	7.61	12.08	3.42	8.66
Year				45.00	23.37	21.63	49.33	22.81	26.52

TABLE LXXXVI—*Lake Cochituate, 1863–1900, inclusive.*

[Catchment area = 18.9 square miles, not including catchment of Dudley Pond.]

Period.	Rain fall.	Run-off.	Evaporation.	Rain-fall.	Run-off.	Evaporation.	Rain-fall.	Run-off.	Evaporation.
	1863.			1864.			1865.		
Storage	29.49	16.31	13.18	24.70	14.44	10.26	29.63	17.28	12.35
Growing	21.71	5.15	16.56	5.20	1.58	3.62	7.37	1.27	6.10
Replenishing	16.49	5.25	11.24	13.47	3.17	10.30	13.43	2.15	11.28
Year	67.69	26.71	40.98	43.37	19.19	24.18	50.43	20.70	29.73
	1866.			1867.			1868.		
Storage	22.87	9.38	13.49	27.02	16.47	10.55	23.02	16.95	6.07
Growing	22.13	2.94	19.19	20.67	3.54	17.33	12.49	3.22	9.27
Replenishing	16.81	3.26	13.05	10.98	2.43	8.55	15.65	4.76	10.89
Year	61.81	15.58	45.73	58.67	22.44	36.43	51.16	24.93	26.23
	1869.			1870.			1871.		
Storage	28.91	12.83	16.08	36.50	23.72	12.78	19.77	10.19	9.58
Growing	8.65	2.39	6.26	9.18	1.91	7.27	11.72	2.15	9.57
Replenishing	21.25	4.77	16.48	13.00	2.85	10.15	13.85	2.38	11.47
Year	58.81	19.99	38.82	58.68	28.48	30.20	45.34	14.72	30.62
	1872.			1873.			1874.		
Storage	14.51	8.88	5.63	20.00	18.51	1.49	20.76	16.23	4.53
Growing	19.58	2.95	16.63	11.63	2.47	9.16	12.78	3.83	8.95
Replenishing	14.20	5.39	8.81	13.27	4.68	8.59	4.64	1.63	3.01
Year	48.29	17.22	31.07	44.90	25.66	19.24	38.18	21.69	16.49
	1875.			1876.			1877.		
Storage	17.80	10.76	7.04	20.45	14.91	5.54	21.61	15.65	5.96
Growing	15.34	2.35	12.99	13.28	1.64	11.64	8.76	2.24	6.52
Replenishing	13.11	3.75	9.36	12.57	3.22	9.35	15.54	4.31	11.23
Year	46.25	16.86	29.39	46.30	19.77	26.53	45.91	22.20	23.71
	1878.			1879.			1880.		
Storage	23.38	19.08	4.30	19.96	16.83	3.13	18.47	8.55	9.92
Growing	13.74	2.07	11.67	13.95	2.05	11.90	12.06	.62	11.44
Replenishing	12.36	3.09	9.27	5.62	1.93	3.69	6.34	1.56	4.78
Year	49.48	24.24	25.24	39.53	20.81	18.72	36.87	10.73	26.14
	1881.			1882.			1883.		
Storage	22.23	12.74	9.49	23.10	12.39	10.71	16.62	8.31	8.31
Growing	8.74	1.56	7.18	6.50	.75	5.75	5.08	.16	4.92
Replenishing	8.85	1.25	7.60	12.35	2.39	9.96	8.53	1.62	6.91
Year	39.82	15.55	24.27	41.95	15.53	26.42	30.23	10.09	20.14

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TABLE LXXXVI.—Continued.—*Lake Cochituate, 1863–1900, inclusive.*

Period.	1884. Rain-fall.	1884. Run-off.	1884. Evapo-ration.	1885. Rain-fall.	1885. Run-off.	1885. Evapo-ration.	1886. Rain-fall.	1886. Run-off.	1886. Evapo-ration.
Storage	24.79	15.70	9.09	22.80	11.90	10.90	24.14	18.97	5.17
Growing	12.79	1.54	11.25	11.70	.76	10.94	8.26	.57	7.69
Replenishing	5.82	1.09	4.73	12.15	3.09	9.06	11.12	1.92	9.20
Year	43.40	18.33	25.07	46.65	15.75	30.90	43.52	21.46	22.06

Period.	1887. Rain-fall.	1887. Run-off.	1887. Evapo-ration.	1888. Rain-fall.	1888. Run-off.	1888. Evapo-ration.	1889. Rain-fall.	1889. Run-off.	1889. Evapo-ration.
Storage	26.97	19.91	7.06	24.22	15.44	8.78	21.79	17.26	4.53
Growing	10.05	2.87	7.18	10.06	1.94	8.12	16.84	6.24	10.60
Replenishing	6.53	1.83	4.70	20.79	9.09	11.70	14.56	6.65	7.91
Year	43.55	24.61	18.94	55.07	26.47	28.60	53.19	30.15	23.04

Period.	1890. Rain-fall.	1890. Run-off.	1890. Evapo-ration.	1891. Rain-fall.	1891. Run-off.	1891. Evapo-ration.	1892. Rain-fall.	1892. Run-off.	1892. Evapo-ration.
Storage	23.42	17.17	6.25	27.73	28.21	−0.48	21.11	12.47	8.64
Growing	7.43	2.20	5.23	11.68	1.99	9.69	10.49	1.38	9.11
Replenishing	17.82	6.29	11.53	9.10	2.38	6.72	9.43	2.26	7.17
Year	48.67	25.66	23.01	48.51	32.58	15.93	41.03	16.11	24.92

Period.	1893. Rain-fall.	1893. Run-off.	1893. Evapo-ration.	1894. Rain-fall.	1894. Run-off.	1894. Evapo-ration.	1895. Rain-fall.	1895. Run-off.	1895. Evapo-ration.
Storage	22.84	12.40	10.44	21.00	10.25	10.75	20.18	11.29	8.89
Growing	11.01	1.90	9.11	7.79	1.24	6.55	11.79	1.45	10.34
Replenishing	7.58	2.51	5.07	10.94	2.04	8.90	18.66	6.17	12.49
Year	41.43	16.81	24.62	39.73	13.53	26.20	50.63	18.91	31.72

Period.	1896. Rain-fall.	1896. Run-off.	1896. Evapo-ration.	1897. Rain-fall.	1897. Run-off.	1897. Evapo-ration.	1898. Rain-fall.	1898. Run-off.	1898. Evapo-ration.
Storage	20.91	15.96	4.95	19.87	11.05	8.82	26.61	16.15	10.46
Growing	7.69	1.55	6.14	12.34	2.57	9.77	12.71	2.45	10.26
Replenishing	14.74	3.70	11.04	9.92	2.58	7.34	16.76	4.26	12.50
Year	43.34	21.21	22.13	42.13	16.20	25.93	56.08	22.86	33.22

Period.	1899. Rain-fall.	1899. Run-off.	1899. Evapo-ration.	1900. Rain-fall.	1900. Run-off.	1900. Evapo-ration.
Storage	22.31	13.33	8.98	28.30	14.09	14.21
Growing	8.16	.23	7.93	9.25	1.49	7.76
Replenishing	10.01	1.63	8.38	13.01	2.72	10.29
Year	40.48	20.24	20.24	50.56	18.30	32.26

Period.	Mean for 5 years, 1896–1900, inclusive. Rain-fall.	Run-off.	Evapo-ration.	Mean for 38 years, 1863–1900, inclusive. Rain-fall.	Run-off.	Evapo-ration.
Storage	23.60	15.13	8.47	23.15	14.92	8.23
Growing	10.03	1.66	8.37	11.59	2.08	9.51
Replenishing	12.89	2.98	9.91	12.38	3.32	9.06
Year	46.52	19.77	26.75	47.13	20.32	26.81

TABLE LXXXVII—***Neshaminy Creek, 1884–1899, inclusive.***

[Catchment area = 139.3 square miles.]

Period.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
	1884			1885.			1886.		
Storage	25.77	25.61	0.16	20.13	17.85	2.28	26.61	21.45	5.16
Growing	13.71	1.85	11.86	10.25	1.08	9.17	12.67	1.87	10.80
Replenishing	7.05	.45	6.60	11.22	1.73	9.49	7.60	.66	6.94
Year	46.53	27.91	18.62	41.60	20.66	20.94	46.88	23.98	22.90
	1887.			1888.			1889.		
Storage	21.88	15.92	5.96	26.48	21.17	5.31	22.32	13.44	8.88
Growing	19.26	4.44	14.82	11.83	1.01	10.82	22.42	10.00	12.42
Replenishing	7.59	1.03	6.56	14.18	6.02	8.16	22.18	12.37	9.81
Year	48.73	21.39	27.34	52.49	28.20	24.29	66.92	35.81	31.11
	1890.			1891.			1892.		
Storage	22.06	14.85	7.21	23.48	17.74	5.74	22.55	15.01	7.54
Growing	14.28	2.15	12.13	15.90	2.53	13.37	11.58	1.31	10.27
Replenishing	10.23	3.33	6.90	8.08	2.38	5.70	10.13	1.94	8.19
Year	46.57	20.33	26.24	47.46	22.65	24.81	44.26	18.26	26.00
	1893.			1894.			1895.		
Storage	22.16	18.52	3.64	26.68	18.16	8.52	20.97	15.84	5.13
Growing	12.21	1.70	10.51	8.95	1.82	7.13	11.41	2.07	9.34
Replenishing	11.07	3.74	7.33	16.45	6.12	10.33	6.21	.24	5.97
Year	45.44	23.96	21.48	52.08	26.10	25.98	38.59	18.15	20.44
				1896.			1897.		
Storage				20.52	11.54	8.98	19.28	10.60	8.68
Growing				10.80	1.65	9.15	17.70	6.50	11.20
Replenishing				12.65	3.41	9.24	9.06	2.11	6.95
Year				43.97	16.60	27.37	46.04	19.21	26.83
				1898.			1899.		
Storage				25.68	16.87	8.81	23.09	20.50	2.59
Growing				12.34	1.69	10.65	9.41	1.76	7.65
Replenishing				12.80	3.33	9.47	10.91	1.96	8.95
Year				50.82	21.89	28.93	43.41	24.22	19.19

TABLE LXXXVIII—*Perkiomen Creek, 1884–1899, inclusive.*

[Catchment area = 152 square miles.]

Period.	1884.			1885.			1886.		
	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
Storage	25.25	25.19	0.06	20.47	15.29	5.18	26.03	19.74	6.29
Growing	15.53	4.07	11.46	9.83	1.68	8.15	11.76	3.35	8.41
Replenishing	7.54	1.59	5.95	9.49	2.38	7.11	9.00	2.02	6.98
Year	48.32	30.85	17.47	39.79	19.35	20.44	46.79	25.11	21.68
	1887.			1888.			1889.		
Storage	21.63	14.66	6.97	27.48	19.67	7.81	22.99	14.28	8.71
Growing	17.26	4.26	13.00	12.42	2.17	10.25	23.38	10.02	13.36
Replenishing	6.70	1.45	5.25	14.18	7.40	6.78	20.45	11.81	8.64
Year	45.59	20.37	25.22	54.08	29.24	24.84	66.82	36.11	30.71
	1890.			1891.			1892.		
Storage	24.68	18.15	6.53	22.89	17.35	5.54	23.64	15.89	7.75
Growing	14.35	3.11	11.24	18.32	3.25	15.07	11.06	2.38	8.68
Replenishing	10.31	4.52	5.79	8.15	2.69	5.46	9.33	2.66	6.67
Year	49.34	25.78	23.56	49.36	23.29	26.07	44.03	20.93	23.10
	1893.			1894.			1895.		
Storage	22.16	17.21	4.95	24.37	15.77	8.60	23.22	15.51	7.71
Growing	12.20	1.82	10.38	8.77	2.05	6.72	10.88	1.32	9.56
Replenishing	10.18	3.33	6.85	15.40	5.18	10.22	6.25	.75	5.50
Year	44.54	22.36	22.18	48.54	23.00	25.54	40.35	17.58	22.77
				1896.			1897.		
Storage				19.99	10.26	9.73	20.00	12.37	7.63
Growing				15.05	2.83	12.22	13.69	3.08	10.61
Replenishing				14.62	4.19	10.43	10.07	2.26	7.81
Year				49.66	17.28	32.38	43.76	17.71	26.05
				1898.			1899.		
Storage				24.24	15.74	8.50	22.79	20.49	2.30
Growing				9.98	1.39	8.59	14.12	2.46	11.66
Replenishing				13.85	3.90	9.95	11.36	4.01	7.35
Year				48.07	21.03	27.04	48.27	26.96	21.31

TABLE LXXXIX—*Tohickon Creek, 1884–1898, inclusive.*

[Catchment area=102.2 square miles.]

Period.	1884.			1885.			1886.		
	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.	Rain-fall.	Run-off.	Evapo-ration.
Storage	26.06	27.27	−1.21	21.86	19.45	2.41	28.54	27.79	0.75
Growing	17.52	6.53	10.99	11.31	1.54	9.77	11.10	2.27	8.83
Replenishing	7.97	1.35	6.62	10	2.94	7.06	9.05	2.04	7.01
Year	51.55	35.15	16.40	43.17	23.93	19.24	48.69	32.10	16.59
	1887.			1888.			1889.		
Storage	21.60	18.44	3.16	28.52	27.37	1.15	25.13	17.82	7.31
Growing	19.19	4.80	14.39	12.96	1.99	10.97	23.90	12.45	11.45
Replenishing	6.71	.91	5.80	16.04	10.14	5.90	21.34	13.70	7.64
Year	47.50	24.15	23.35	57.52	39.50	18.02	70.37	43.97	26.40
	1890.			1891.			1892.		
Storage	25.09	19.01	6.08	23.07	20.23	2.84	23.43	19.76	3.67
Growing	15.49	2.54	12.95	19.77	4.99	14.78	11.22	1.52	9.70
Replenishing	10.20	5.45	4.75	7.16	2.03	5.13	10.65	3.47	7.18
Year	50.78	27	23.78	50	27.25	22.75	45.30	24.75	20.55
	1893.			1894.			1895.		
Storage	22.82	22.05	0.77	27.04	21.65	5.39	21.35	19.91	1.44
Growing	14.82	2.10	12.72	6.95	.84	6.11	12.45	1.46	10.99
Replenishing	11.31	4.06	7.25	17.63	8.11	9.52	6.63	.28	6.35
Year	48.95	28.21	20.74	51.62	30.60	21.02	40.43	21.65	18.78
	1896.			1897.			1898.		
Storage	21.69	12.30	9.39	20.82	13.93	6.89	26.40	21.20	5.20
Growing	13.76	2.91	10.85	17.32	5.12	12.20	10.87	1	9.87
Replenishing	12.58	4.52	8.06	8.78	1.98	6.80	13.80	5.19	8.61
Year	48.03	19.73	28.30	46.92	21.03	25.89	51.07	27.39	23.68

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